Optimization technique in Portfolio creation

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Content

1. Short Selling
2. Portfolio optimization

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1 Short Selling

2 Portfolio optimization

…going Short

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An illustrative example with a long and a short position . . .

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Graphical representation of the portfolio's effective frontier when applying short selling . . .

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$$\sigma_p^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} w_i * w_j * \sigma_{i,j}$$

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$$\frac{\partial L(\vec{y})}{\partial y_i} \qquad \text{for } i = 1, 2, \ldots, n$$

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- Apply matrix calculation . . .

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$$L(\vec{y}) = \sigma_p^2(\vec{w}) + \lambda_1 (\sum_{i=1}^{n} w_i - 1) + \lambda_2 (\sum_{i=1}^{n} w_i * \bar{r}_i - E(R_p))$$