Tangency portfolio

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Portfolio creation considering a risk-free asset

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Four possible scenarios for creating a portfolio:

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The shape of efficient frontier by considering a r_f

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- Risk-free asset is changing the shape of affordable and efficient portfolios
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- *Lending* vs. *borrowing* and the **efficient frontier** . . .

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Reward to variability ratio:

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The relation between **SR** and **efficient frontier** . . .

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Tangency portfolio or max. SR portfolio

Objective function:

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Optimization problem:

max SR($\mathbf{w}$) with respect to $\mathbf{w}$

Calculation procedure

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Structure of the portfolio:

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$$\mathbf{w} = \frac{\sum^{-1}(\boldsymbol{\mu} - r_f\mathbf{1})}{\mathbf{1}^T\sum^{-1}(\boldsymbol{\mu} - r_f\mathbf{1})}$$

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Structure of the portfolio:

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Portfolio return:

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Portfolio return:

$$R_p = r_f + \frac{(\boldsymbol{\mu} - r_f \mathbf{1})^T \sum^{-1}(\boldsymbol{\mu} - r_f \mathbf{1})}{\mathbf{1}^T \sum^{-1}(\boldsymbol{\mu} - r_f \mathbf{1})}$$

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Portfolio variance:

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Portfolio variance:

$$\sigma_p^2 = \frac{(\mu - r_f\mathbf{1})^T\sum^{-1}(\mu - r_f\mathbf{1})}{(\mathbf{1}^T\sum^{-1}(\mu - r_f\mathbf{1}))^2}$$

More intuitive calculation approach

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$$R_p - r_f = \sum_{i=1}^{n} w_i(r_i - r_f)$$

$$\sigma_p = \sqrt{\sum_{i=1}^{n} w_i^2 * \sigma_i^2 + \sum_{i=1}^{n} \sum_{j=1|i \neq j}^{n} w_i * w_j * \sigma_{i,j}}$$

Getting the weights

Let´s assume:

$$\lambda = \frac{R_p - r_f}{\sigma_p^2}$$

and further, substitute:

$$Z_k = \lambda * w_k$$

then we can write:

$$r_i - r_f = Z_1\sigma_{i,1} + Z_2\sigma_{i,2} + Z_3\sigma_{i,3} + \cdots + Z_i\sigma_i^2 + \cdots + Z_{n-1}\sigma_{i,n-1} + Z_n\sigma_{i,n}$$

and finally:

$$w_k = \frac{Z_k}{\sum_{k=1}^{n} Z_k}$$