Cut-off portfolio

Luděk Benada

Department of Finance, office - 402

e-mail: *benada@econ.muni.cz*

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- It can very easy solve the problem with short sell ban

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- **Ultimately, it can be concluded that mutual covariances of assets are determined by their sensitivity to macroeconomic factors**

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- The subsequent ranking of the ratio results determines the asset's suitability for portfolio creation

Application of the decision criterion

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- If a certain asset with $\frac{r_i - r_f}{\beta_i}$ **is not** included in the portfolio, **all** other assets with **lower** ratio values will **not** be part of the portfolio

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Procedure for choosing the optimal portfolio

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3. All assets that satisfy the following condition will be included to the portfolio:

$$\frac{r_i - r_f}{\beta_i} > C^*$$

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Determining C*

The cut-off, or the last included asset according to the previous condition will with its C_i serve as the C^*

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$$w_i = \frac{Z_i}{\sum_{i=1}^{n} Z_i}$$

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