Assignment 3

Financial Mathematics
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(Solution)

1. \$2000 are invested for 10 years at: 8%(2) for the first 3 years, 6%(6) for the next 4 years, and 6%(12) for the last 3 years. Find the accumulated value after 10 years.

$$\begin{aligned} A_{t=10} &= \$2000\left(1+\frac{0.08}{2}\right)^{3\cdot 2}\left(1+\frac{0.06}{6}\right)^{4\cdot 6}\left(1+\frac{0.06}{12}\right)^{3\cdot 12} \\ &= \$2000\,(1.04)^{6}\,(1.01)^{24}\,(1.005)^{36} \\ &= \$3845.22 \end{aligned}$$

2. Bank A has an annual effective interest rate of 10%. Bank B has a nominal rate of 9.6%. What is the minimum frequency of compounding per year (integer) for Bank B in order that rate at bank B be at least as attractive as that at bank A.

   - Annual effective rate:

$$1+r=\left(1+\frac{i}{m}\right)^{m} \Leftrightarrow r=\left(1+\frac{i}{m}\right)^{m}-1$$

   - $m=1$

$$r=9.6\%$$

   - $m=2$

$$r=\left(1+\frac{0.096}{2}\right)^{2}=9.83\%$$

   - $m=3$

$$r=\left(1+\frac{0.096}{3}\right)^{3}=9.91\%$$

   - $m=4$

$$r=\left(1+\frac{0.096}{4}\right)^{4}=9.95\%$$

- $m = 5$

$$r = \left(1 + \frac{0.096}{5}\right)^5 = 9.98\%$$

- $m = 6$

$$r = \left(1 + \frac{0.096}{6}\right)^6 = 9.99\%$$

- $m = 7$

$$r = \left(1 + \frac{0.096}{7}\right)^7 = 10\%$$

We need at least 7 periods of composition per year in order to obtain an annual effective rate equal or higher to 10%