Assignment 4

(Solution)

Financial Mathematics
Lecturer: Axel Araneda, PhD.
Masaryk University
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1. On September 1, 2021, Paul borrowed \$3000 at 12%(4). He paid \$900 on March 1, 2022, and \$1200 on December 1, 2022.

   (a) What equal payments on June 1, 2023, and December 1, 2023, will be needed to settle the debt?

   - Using December 1, 2023, as focal date:

   $$\$3000\left(1+\frac{0.12}{4}\right)^9 = \$900\left(1+\frac{0.12}{4}\right)^7 + \$1200\left(1+\frac{0.12}{4}\right)^4 + X\left(1+\frac{0.12}{4}\right)^2 + X$$

   $$\to \$3000\,(1.03)^9 = \$900\,(1.03)^7 + \$1200\,(1.03)^4 + X\,(1.03)^2 + X$$

   $$\to X\left[(1.03)^2+1\right] = \$3000\,(1.03)^9 - \$900\,(1.03)^7 - \$1200\,(1.03)^4$$

   $$\Rightarrow X = \frac{\$3000\,(1.03)^9 - \$900\,(1.03)^7 - \$1200\,(1.03)^4}{(1.03)^2+1} = \$706.89$$

   (b) If an additional payment of \$900 was done on March 1, 2023, what will be the Pauls' outstanding balance on September 1, 2023?

   - Using September 1, 2023, as focal date:

   $$\begin{aligned}\text{Balance}_{\text{1Sep23}} &= \$3000\left(1+\frac{0.12}{4}\right)^8 - \$900\left(1+\frac{0.12}{4}\right)^6 - \$1200\left(1+\frac{0.12}{4}\right)^3 \\ &\quad -\$900\left(1+\frac{0.12}{4}\right)^2 \\ &= \$3000\,(1.03)^8 - \$900\,(1.03)^6 - \$1200\,(1.03)^3 - \$900\,(1.03)^2 \\ &= \$459.58\end{aligned}$$

2. On January 1, 2020, \$100 was deposited in an account paying 12% compounded monthly, for the first year, and 6%(2) for the following terms (starting on January 1, 2021). On July 1, 2021 \$100 was deposited in another account which pays 10%(1). At what time will both accounts have the same amount?

- Amount in the account 1 by July 1, 2021:

$$A^1_{1\text{Jul21}} = \$100 \underbrace{\left(1+\frac{0.12}{12}\right)^{12}}_{1\text{Jan20}-31\text{Dec20}} \underbrace{\left(1+\frac{0.06}{2}\right)}_{1\text{Jan21}-30\text{Jun21}}$$

- Amount in the account 1 at any future time t (from July 1, 2021):

$$A^1_t = \$100\,(1.01)^{12}\,(1.03)\,(1.03)^{2t} = \$100\,(1.01)^{12}\,(1.03)^{2t+1}$$

- Amount in the account 2 at any future time t (from July 1, 2021):

$$A^2_t = \$100\,(1+0.1)^t = \$100\,(1.1)^t$$

- If the two accounts have the same amount:

$$\begin{aligned}
A^1_t = A^2_t \quad &\iff \quad \$100\,(1.01)^{12}\,(1.03)^{2t+1} = \$100\,(1.1)^t \\
&\iff \quad (1.01)^{12} = \frac{(1.1)^t}{(1.03)^{2t+1}} \\
&\iff \quad 12\log(1.01) = t\log(1.1) - (2t+1)\log(1.03) \\
&\iff \quad 12\log(1.01) + \log(1.03) = t\left[\log(1.1) - 2\log(1.03)\right] \\
&\iff \quad t = \frac{12\log(1.01)+\log(1.03)}{\log(1.1)-2\log(1.03)} = 4.11583....
\end{aligned}$$

  - It corresponds roughly to 1502 days after July 1, 2021. Then the two accounts will have approximately an equal amount on Agust 11, 2025.

3. A deposit of \$1000 made 3.5 years ago, generates a current amount of \$1581.72. Find the nominal interest rate interest compounded quarterly for this saving account.

$$\begin{aligned}
A_t = P\left(1+\frac{i}{m}\right)^{mt} \quad &\Longrightarrow \quad \$1581.72 = \$1000\left(1+\frac{i}{4}\right)^{4\cdot(3.5)} = \$1000\left(1+\frac{i}{4}\right)^{14} \\
&\Longrightarrow \quad \frac{1581.72}{1000} = \left(1+\frac{i}{4}\right)^{14} \\
&\Longrightarrow \quad i = 4\left[\left(\frac{1581.72}{1000}\right)^{1/14} - 1\right] = 13.75\%
\end{aligned}$$

- The saving account has a interest rate equivalent to 13.75% (4).