

M U N I
E C O N

Financial Mathematics

Class 2: Simple and discount interest

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Simple Interest

– It pays interest at the end of the term over the initial amount or principal.

$$\begin{aligned} A_t &= P + I_t \\ &= P + \underbrace{Prt}_{I_t} \end{aligned}$$

- A_t : amount.
- P : principal
- I_t : Interest
- r : interest rate (per unit of time, e.g., years)
- t : term (length) of the loan/investment.

Examples

1. A simple interest loan for \$800 at 7% per annum is paid off after 6 months. What are the interest charges?

$$I_t = Prt = (\$800) \cdot (0.07) (0.5) = \$28$$

2. How long will it take \$4000 invested at 7% per annum to earn \$350?

$$t = \frac{I_t}{Pr} = \frac{\$350}{(\$4000) \cdot (0.07)} = 1.25$$

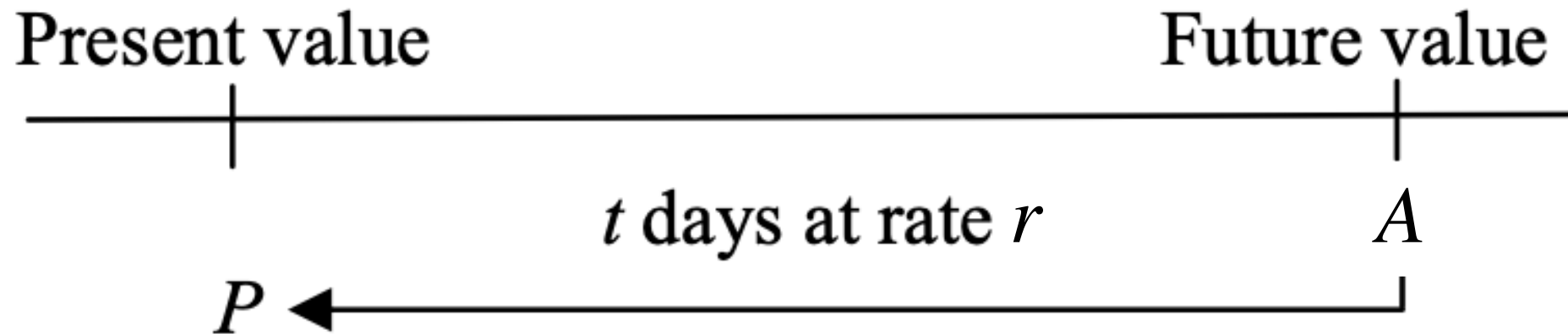
Exercises

1. If you hope to double your money investing at 6%, how long will it take you?
2. If money earns 5.5% simple interest, is it better to buy a piano for \$8200 cash or for \$8800 in 15 months?

More examples on simple interest

1. Anthony has \$5680 in a Money Market account paying 4.25%; interest is paid every quarter. Any money withdrawn between interest dates will earn no interest for the entire quarter. If Anthony has a pressing \$4550 financial need 45 days before interest accrues, should he withdraw the money or take a credit card loan that will cost him 18%?
 - a) If he withdraws \$4550 he will lose the following interest: $I_t = Prt = 4550 * 0.0425 * 0.25 = \48.34
 - b) The credit card loan will cost the following: $I_t = Prt = 4550 * 0.18 * 45 / 360 = \102.38

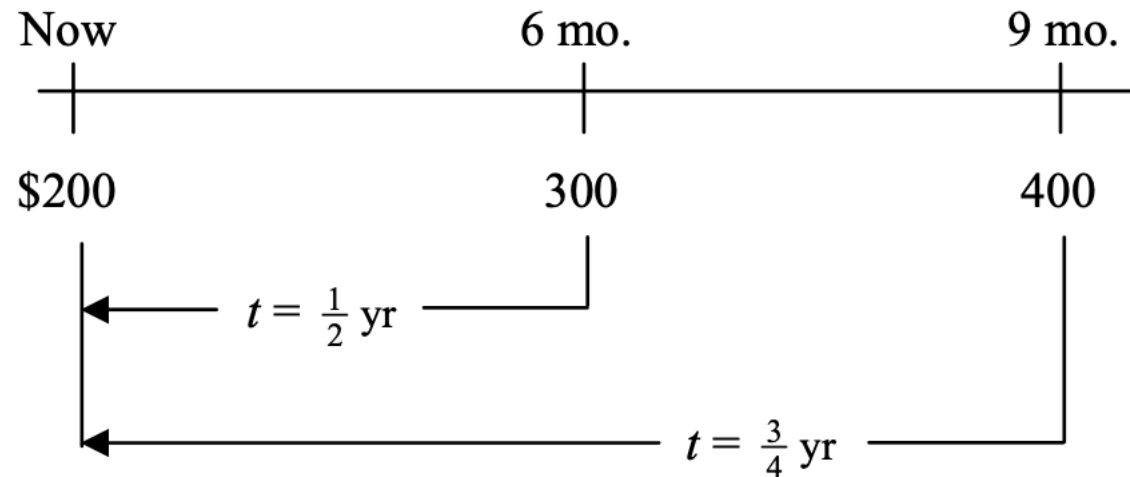
Present value



The Golden Rule of Finance: *Monies cannot be added or reconciled unless they are valued at the same point in time.*

Example 1

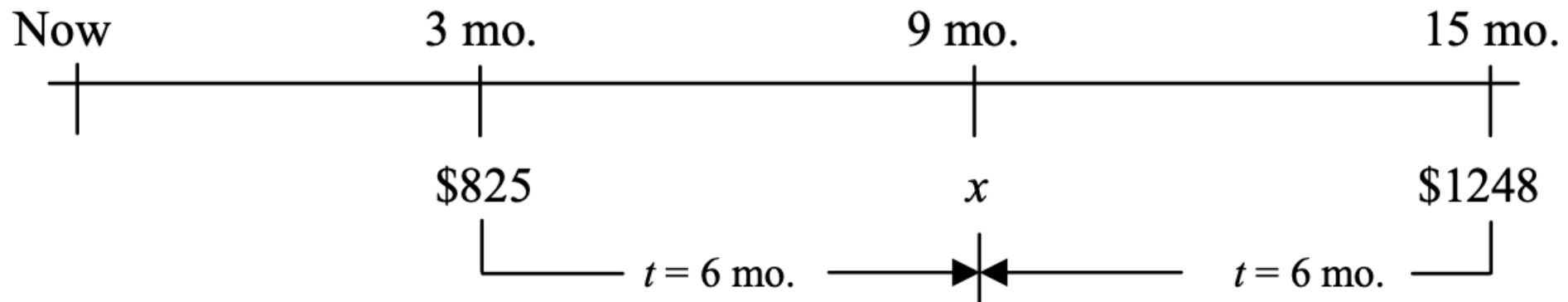
- A purchase agreement calls for \$200 down and two payments of \$300 in 6 months and \$400 in 9 months. If the interest rate is 12%, what is the cash price for the item purchased.



$$x = \$200 + \frac{300}{1 + (.12)(\frac{1}{2})} + \frac{400}{1 + (.12)(\frac{3}{4})} = \$200 + \$283.02 + \$366.97 = \$849.99$$

Example 2

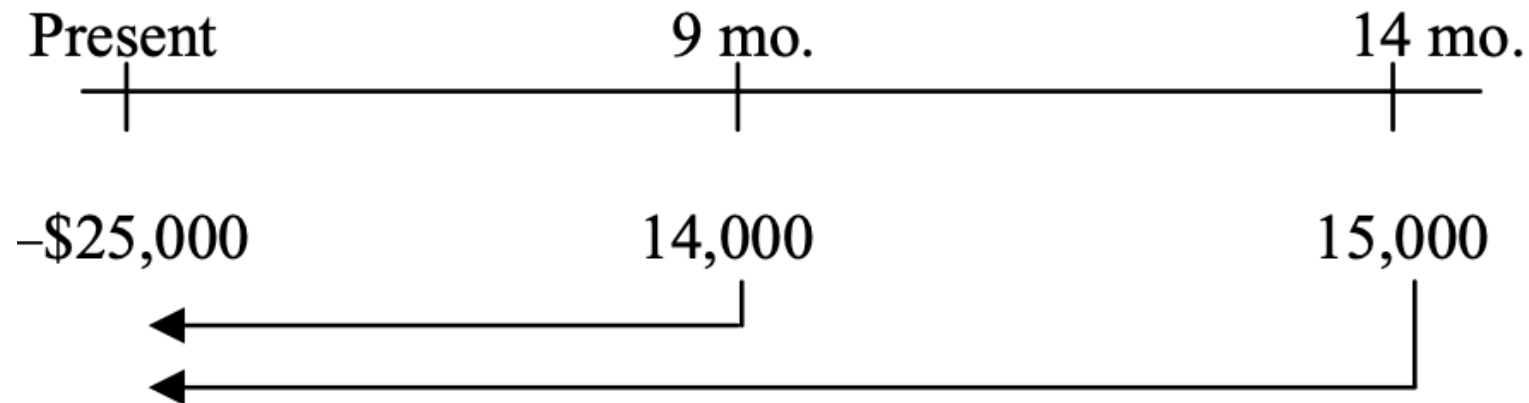
- A man owes two obligations, one for \$825 due in 3 months and one for \$1248 due in 15 months. If the interest rate is 5.5%, what payment in 9 months will settle the obligations?



$$x = 825(1 + (.055)(.5)) + \frac{1248}{1 + (.055)(.5)} = \$847.69 + \$1214.60 = \$2062.29$$

Example 3

- Find the net present value at 14% of an investment of \$25,000, which is expected to bring returns of \$14,000 in 9 months and \$15,000 in 14 months.



$$\text{NPV}_{@14\%} = -\$25,000 + \frac{14,000}{1 + (.14)\left(\frac{9}{12}\right)} + \frac{15,000}{1 + (.14)\left(\frac{14}{12}\right)} = \$563.66$$

Zero-coupon bonds

- A ZCB is an instrument that pays its facial value at a specified maturity. The owner can hold it until maturity or sold on secondary bond markets. Usually, is traded at discount.
- Exercise: A 182-day \$1,000,000 Treasury bill (T-bill) is bought with a bid of 96.2%. What's the rate of return (interest rate) that the investor earns.

Discount interest

- If the lending institution collects the interest “up front,” we are in presence of discount interest. The percent used to figure the interest charges is called the discount rate.
- Example: If you borrow \$500 for a year at a 10% discount rate, the banker would give you a check for \$450 and expect you to pay back \$500 at the end of a year.

Discount Interest

– It pays interest at the beginning of the term over the future amount.

$$\begin{aligned}P &= A_t - D_t \\ &= A_t - A_t dt \\ &= A_t (1 - dt)\end{aligned}$$

- A_t : amount.
- P : preceeds
- D_t : Discount interest
- d : Discount interest rate (per unit of time, e.g., years)
- t : term (length) of the loan/investment.

Discount Interest: Examples

- A bank charges 7% for short-term discount loans. What are the proceeds for an 8-month loan for \$6850?

$$P = A_t (1 - dt) = 6850 \cdot \left(1 - 0.07 \cdot \frac{8}{12}\right) = 6530.33$$

- A used car dealer charges his customers 12% discount for financing the balance on purchases. If a customer has a balance of \$5600 after trade allowances and taxes, what is the amount of the loan she will need for 15-month financing?

$$A_t = \frac{P}{1 - dt} = \frac{5600}{1 - .12 \cdot \left(\frac{15}{12}\right)} = 6588.24$$

Comparing Simple and Discount Interest

- Simple interest is based on present value.
- Discount interest is based on future value.
- A one-year loan with \$450 present value at a rate of 10% will cost \$45 with simple interest.
- The same loan at discount interest will cost \$50.
- The simple interest rate that is equivalent to a given discount rate.
- **Coupon equivalent.**

$$r = \frac{d}{1 - dt} \longleftrightarrow d = \frac{r}{1 + rt}$$

Comparing Simple and Discount Interest: Examples

1. What simple interest rate is equivalent to a discount rate of 12% for 15 months?

$$r = \frac{d}{1 - dt} = \frac{.12}{1 - .12 \cdot \frac{15}{12}} = 14.1\%$$

2. What discount rate is equivalent to a simple interest rate of 9.5% for 9 months?

$$d = \frac{r}{1 + rt} = \frac{0.095}{1 + 0.095 \cdot \left(\frac{9}{12}\right)} \approx 8.9\%$$