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ECON

Financial Mathematics

Seminar 2: Simple and discount interest

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Solved problems (Simple interest)

Find (a) the ordinary and (b) the exact simple interest, on a 60-day loan of \$1500 at $14\frac{1}{2}\%$.

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We have $P = 1500$ and $r = .0145$.

(a) Using a 360-day year,

$$t = \frac{60}{360} \quad \text{and} \quad I = Prt = 1500(0.145)\left(\frac{60}{360}\right) = \$36.25$$

(b) Using a 365-day year,

$$t = \frac{60}{365} \quad \text{and} \quad I = 1500(0.145)\left(\frac{60}{365}\right) = \$35.75$$

Solved problems (Simple interest)

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Solved problems (Simple interest)

At what rate of simple interest will \$1200 accumulate interest of \$72 in 6 months?

We have $P = 1200$, $I = 72$, and $t = 6/12 = 1/2$.

$$r = \frac{I}{Pt} = \frac{72}{1200(1/2)} = 0.12 = 12\%$$

Solved problems (Simple interest)

How long will it take for \$500 to accumulate to at least \$560 at $13\frac{1}{4}\%$ ordinary simple interest?

We have $P = 500$, $I = 60$, and $r = 0.1325$. From (3.1) we calculate

$$t = \frac{I}{Pr} = \frac{60}{500(0.1325)} = 0.90566038 \text{ years} \approx 326.03774 \text{ days}$$

It will take 327 days to accumulate to at least \$560.

Exercises (Simple interest)

1) A loan shark made a loan of \$100 to be repaid with \$120 at the end of one month. What was the annual interest rate? *Ans.* 240%

2) At what rate of simple interest will (*a*) \$1000 accumulate to \$1420 in $2\frac{1}{2}$ years? (*b*) Money double itself in 8 years? (*c*) \$500 accumulate \$10 interest in 2 months?
Ans. (*a*) 16.8%; (*b*) 12.5%; (*c*) 12%

3) How long will it take \$1000 (*a*) to earn \$100 at 15% simple interest? (*b*) To accumulate to at least \$1200 at $13\frac{1}{2}$% simple interest? *Ans.* (*a*) 8 months; (*b*) 534 days

4) A bank pays 10% per annum on savings accounts. Interest is credited quarterly on March 31, June 30, September 30, and December 31, based on the minimum quarterly balance. If a person opens an account with a deposit of \$200 on January 1 and withdraws \$100 on August 8, how much interest is earned in the first year? *Ans.* \$15.70

Solved problems (time value of money)

Ms. Hill owes \$500 due in 4 months and \$700 due in 9 months. What single payment (a) now, (b) in 6 months, (c) in 1 year, will liquidate these obligations if money is worth 11%?

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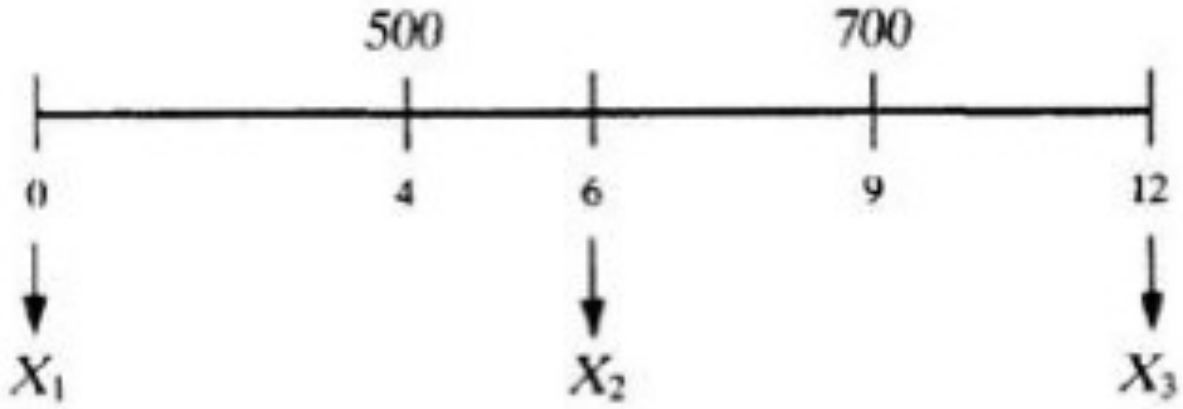

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Solved problems (time value of money)

Ms. Hill owes \$500 due in 4 months and \$700 due in 9 months. What single payment (*a*) now, (*b*) in 6 months, (*c*) in 1 year, will liquidate these obligations if money is worth 11%?

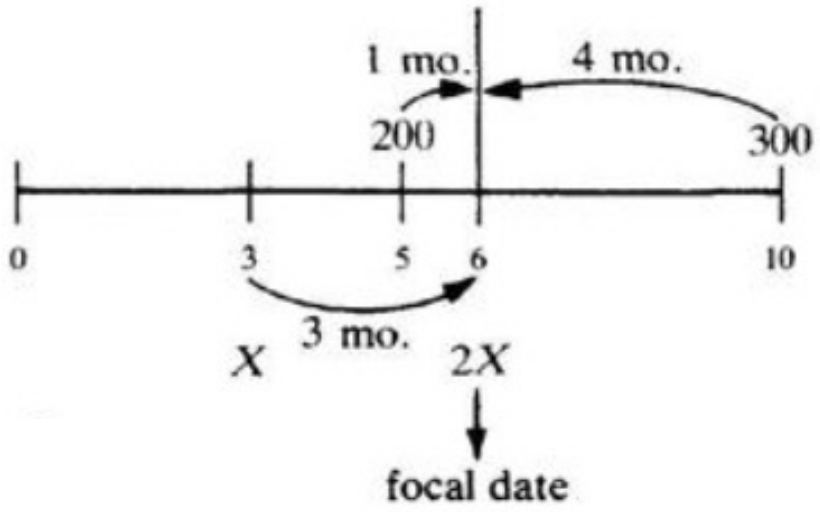
$$(a)\ X_1 = 500\left[1+(0.11)\left(\frac{4}{12}\right)\right]^{-1} + 700\left[1+(0.11)\left(\frac{9}{12}\right)\right]^{-1} = 482.32 + 646.65 = \$1128.97$$

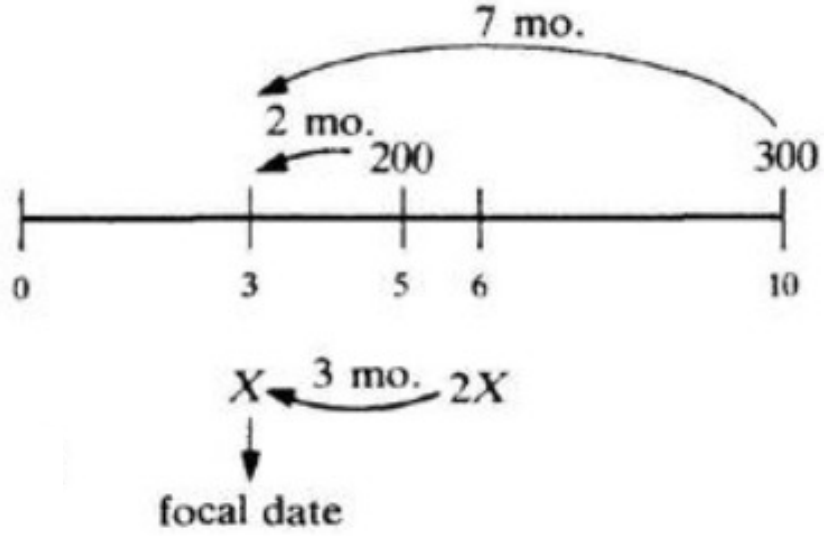
$$(b)\ X_2 = 500\left[1+(0.11)\left(\frac{2}{12}\right)\right] + 700\left[1+(0.11)\left(\frac{3}{12}\right)\right]^{-1} = 509.17 + 681.27 = \$1190.44$$

$$(c)\ X_3 = 500\left[1+(0.11)\left(\frac{8}{12}\right)\right] + 700\left[1+(0.11)\left(\frac{3}{12}\right)\right] = 536.67 + 719.25 = \$1255.92$$

Solved problems (time value of money)

Mrs. Adams has two options available in repaying a loan: she can pay \$200 at the end of 5 months and \$300 at the end of 10 months, or she can pay \$$X$ at the end of 3 months and \$$2X$ at the end of 6 months. If the options are equivalent and money is worth 12%, find X, using as the focal date (a) the end of 6 months; (b) the end of 3 months.





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Solved problems (time value of money)

(*a*) The equation of value at the end of 6 months is:

$$\begin{aligned}
\text{dated value of option 2} &= \text{dated value of option 1} \\
X\left[1+(0.12)\left(\frac{3}{12}\right)\right]+2X &= 200\left[1+(0.12)\left(\frac{1}{12}\right)\right]+300\left[1+(0.12)\left(\frac{4}{12}\right)\right]^{-1} \\
1.03X+2X &= 202+288.46 \\
3.03X &= 490.46 \\
X &= \$161.87
\end{aligned}$$

Solved problems (time value of money)

(*b*) The equation of value at the end of 3 months is:

$$X + 2X\left[1 + (0.12)\left(\frac{3}{12}\right)\right]^{-1} = 200\left[1 + (0.12)\left(\frac{2}{12}\right)\right]^{-1} + 300\left[1 + (0.12)\left(\frac{7}{12}\right)\right]^{-1}$$

$$X + 1.9417476X = 196.08 + 280.37$$

$$2.9417476X = 476.45$$

$$X = \$161.96$$

Solved problems (time value of money)

Blake borrowed \$5000 on January 1, 1995. He paid \$2000 on April 30, 1995, and \$2000 on August 31, 1995. The final payment was made on December 15, 1995. Find the size of the final payment if the rate of interest was 7% and the focal date was (*a*) December 15, 1995

(*a*) Equation of value on December 15, 1995:

$$\begin{aligned} \text{dated value of the payments} &= \text{dated value of the debt} \\ 2000\left[1+(0.07)\left(\frac{229}{360}\right)\right]+2000\left[1+(0.07)\left(\frac{106}{360}\right)\right]+X &= 5000\left[1+(0.07)\left(\frac{348}{360}\right)\right] \\ 2089.06+2041.22+X &= 5338.33 \\ X &= \$1208.05 \end{aligned}$$

Exercises (time value of money)

1) If money is worth 13% simple interest, find the values of a debt of \$1500 due in 8 months with interest at $14\frac{1}{2}\%$ (*a*) today, (*b*) 4 months from now, (*c*) 1 year from now.
Ans. (*a*) \$1513.80; (*b*) \$1576.68; (*c*) \$1716.28

2) Debts of \$500 due 20 days ago and \$400 due in 50 days are to be settled by a payment of \$600 now and a final payment 90 days from now. Find the value of the final payment at a simple interest rate of 11% with a focal date at the present. *Ans.* \$305.21

3) Paula owes \$100 due in 6 months and \$150 due in 1 year. She and the lender agree that she can pay off both debts today using a simple interest rate of 16% and putting the focal date now. How much will be paid in cash today? *Ans.* \$221.90

4) Carl owes \$300 due in 3 months and \$500 due in 8 months. What single payment (*a*) now, (*b*) in 6 months, (*c*) in 1 year will liquidate these obligations, if money is worth 8% and the focal date is the time of the single payment? *Ans.* (*a*) \$768.80; (*b*) \$799.42; (*c*) \$831.33

Solved Problems (Simple discount)

Find the present value at 12% simple discount of \$1000 due in 5 months. What is the simple discount?

We have $S = 1000$, $d = 0.12$, and $t = 5/12$. From (3.5),

$$P = S(1 - dt) = 1000\left[1 - (0.12)\left(\frac{5}{12}\right)\right] = \$950$$

The simple discount is $D = S - P = 1000 - 950 = \$50$.

Solved Problems (Simple discount)

A bank charges 11% simple interest in advance (that is, 11% bank discount) on short-term loans. Find the sum received by the borrower who requests (*a*) \$900 for 90 days, (*b*) \$1500 from May 3 to October 15.

(*a*) $S = 900$, $d = 0.11$, and $t = \frac{90}{360}$.

$$P = 900\left[1 - (0.11)\left(\frac{90}{360}\right)\right] = \$875.25$$

(*b*) $S = 1500$, $d = 0.11$, and $t = \frac{165}{360}$.

$$P = 1500\left[1 - (0.11)\left(\frac{165}{360}\right)\right] = \$1424.38$$

Solved Problems (Simple discount)

A bank charges 12% bank discount on short-term loans. A borrower needs \$2000 cash, to be repaid with interest in 9 months. What size loan should he ask for, and how much interest will he pay?

We have $P = 2000$, $d = 0.12$, and $t = 9/12$. From (3.6),

$$S = \frac{2000}{1 - (0.12)(9/12)} = \$2197.80$$

The borrower should ask for \$2197.80; the interest on the loan is \$197.80.

Solved Problems (Simple discount)

A bank bids 96.823 for a 91-day, \$1 million Treasury bill. (This means that the bank is willing to pay \$968 230 for the bill, which will be worth \$1 million 91 days after its issuance.) If the bid is accepted, what yield will the bank get, on (*a*) a bank discount basis? (*b*) A simple interest basis?

(*a*) We have $D = 1\,000\,000 - 968\,230 = \$31\,770$, $S = 1\,000\,000$, and $t = 91/360$. From (3.4),

$$d = \frac{D}{St} = \frac{31\,770}{1\,000\,000(91/360)} = 0.125683516 \approx 12.57\%$$

(*b*) We have $I = 31\,770$, $P = 968\,230$, and $t = 91/360$. From (3.1),

$$r = \frac{I}{Pt} = \frac{31\,770}{968\,230(91/360)} = 0.129807501 \approx 12.98\%$$

Exercises (Simple discount)

1) Find the bank discount on (*a*) \$2000 for 120 days at 15%, (*b*) \$10 000 for 91 days at 9.83%, (*c*) \$5000 from April 21 to May 17 at $12\frac{1}{2}\%$. *Ans.* (*a*) \$100; (*b*) \$248.48; (*c*) \$45.14

2) At $12\frac{3}{4}\%$ bank discount, find the value today of (*a*) \$1500 due in 9 months, (*b*) \$10 000 due in 182 days. *Ans.* (*a*) \$1356.56; (*b*) \$9355.42

3) A bank charges $13\frac{1}{2}\%$ interest in advance on short-term loans. Find the sums received by borrowers who request (*a*) \$1500 for 6 months, (*b*) \$2800 from June 1 to September 18, (*c*) \$5000 from March 15 to November 7. *Ans.* (*a*) \$1398.75; (*b*) \$2685.55; (*c*) \$4555.62

4) Robert borrows \$1000 for 8 months from a lender who charges a 16% discount rate. (*a*) How much money does Robert receive? (*b*) What size of loan should Robert ask for in order to receive \$1000 cash? (*c*) What is the equivalent simple interest rate he pays on the loan?
Ans. (*a*) \$893.33; (*b*) \$1119.40; (*c*) 17.91%