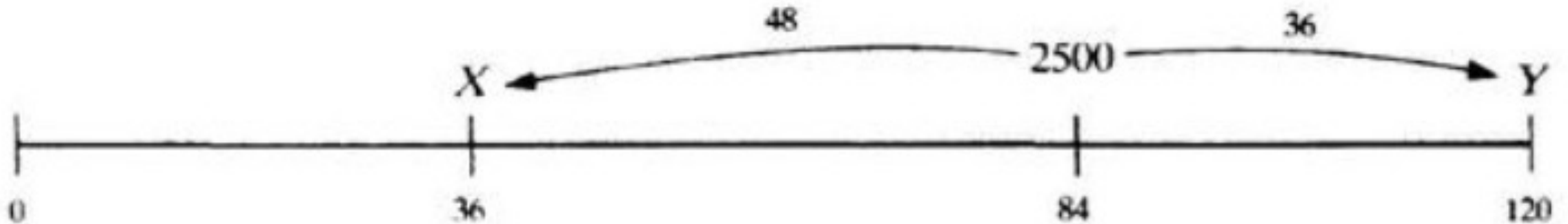# Financial Mathematics

Seminar 4: Compound interest (II)

Lecturer: Axel A. Araneda, Ph.D.

Solved problems (compound interest and time value of money)

1. An obligation of $2500 falls due at the end of of 7 years. If the money worths 10%(12), find the equivalent debt at the end of:
   a) 3 years
   b) 10 years

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Solved problems

1. An obligation of $2500 falls due at the end of of 7 years. If the money worths 10%(12), find the equivalent debt at the end of:

   a) 3 years

   $$X \left(1 + \frac{0.10}{12}\right)^{48} = \$2500 \iff X = \$2500 \left(1 + \frac{0.10}{12}\right)^{-48} = \$1678.58$$

   b) 10 years

   $$Y = \$2500 \left(1 + \frac{0.10}{12}\right)^{36} = \$3370.45$$

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Solved problems

1. An obligation of $2500 falls due at the end of of 7 years. If the money worths 10%(12), find the equivalent debt at the end of:
   a) 3 years
   b) 10 years
   Note that X and Y are equivalent!

$$Y = X \left(1 + \frac{0.10}{12}\right)^{84} = \$1678.58 \left(1 + \frac{0.10}{12}\right)^{84} = \$3370.45$$
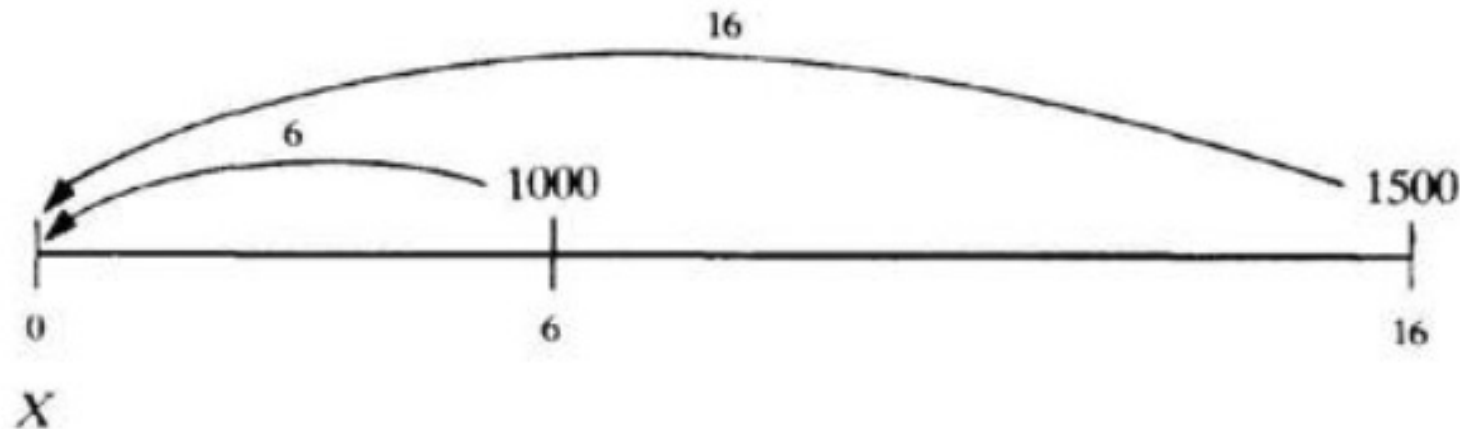
Solved problems

2. A person owes $1000 due at the end of 18 months and $1500 due at the end of 4 years. If money is worth 6%(4), what single payment liquidates these obligations

    a) Now?
    b) In two years?

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Solved problems

2. A person owes $1000 due at the end of 18 months and $1500 due at the end of 4 years. If money is worth 6%(4), what single payment liquidates these obligations
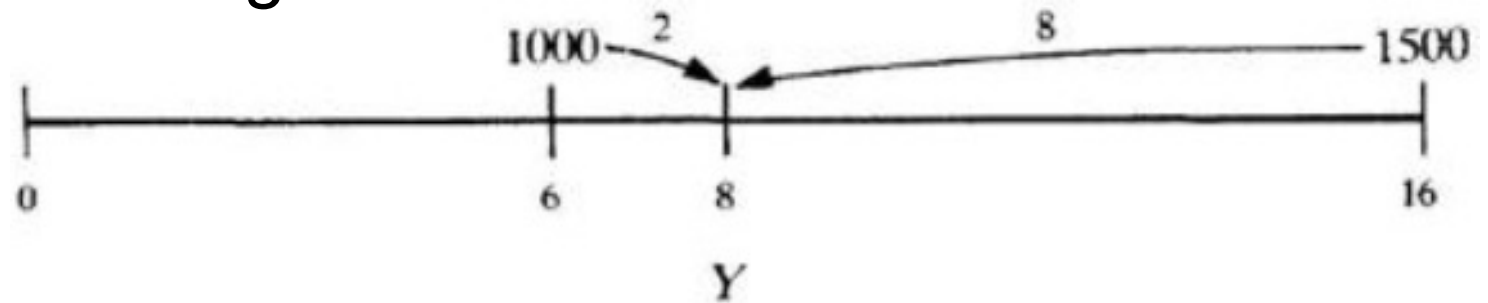
   a) Now?

$$X = 1000(1.015)^{-6} + 1500(1.015)^{-16} = 914.54 + 1182.05 = \$2096.59$$

Solved problems

2. A person owes $1000 due at the end of 18 months and $1500 due at the end of 4 years. If money is worth 6%(4), what single payment liquidates these obligations

   b) In two years?
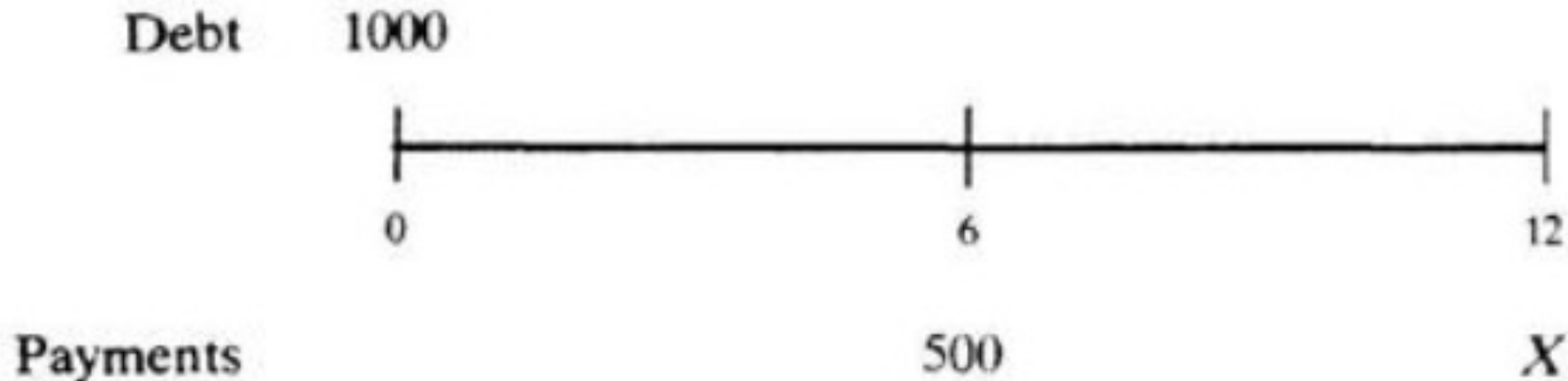

$$Y = 1000(1.015)^2 + 1500(1.015)^{-8} = 1030.22 + 1331.57 = \$2361.79$$

- Alternatively:

$$Y = X(1.015)^8 = 2096.59(1.015)^8 = \$2361.79$$

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Solved problems

3. A consumer buys goods worth $1500 pating $500 down and $500 at the end of 6 months. If the stores charges interest at 8%(12) on the unpaid balance, what final payment is necessary at the end of one year?

Debt   1000

```
        |------------------------|------------------------|
        0                        6                       12
```

Payments   500   X

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Solved problems

3. A consumer buys goods worth $1500 pating $500 down and $500 at the end of 6 months. If the stores charges interest at 8%(12) on the unpaid balance, what final payment is necessary at the end of one year?

- Equation of value at the end of 12 months:

$$\text{dated value of the payments} = \text{dated value of the debt}$$
$$500(1.015)^6 + X = 1000(1.015)^{12}$$
$$546.72 + X = 1195.62$$
$$X = \$648.90$$

Solved problems

3. A consumer buys goods worth $1500 pating $500 down and $500 at the end of 6 months. If the stores charges interest at 18%(12) on the unpaid balance, what final payment is necessary at the end of one year?

- Equation of value at the present:

$$\text{dated value of the payments} = \text{dated value of the debt}$$
$$500(1.015)^{-6} + X(1.015)^{-12} = 1000$$
$$457.27 + 0.83638743X = 1000$$
$$0.83638743X = 542.73$$
$$X = \$648.90$$

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Exercises

1) An obligation of $500 falls due at the end of 3 years. Given an interest rate of 12%(4), find the equivalent debt at the end of (a) 3 months, (b) 3 years 9 months.
   Ans. (a) $361.21; (b) $546.37

2) David borrows $4000 at $j_4 = 12\%$. He promises to pay $1000 at the end of one year, $2000 at the end of 2 years, and the balance at the end of 3 years. What will the final payment be?
   Ans. $2185.25

3) A debt of $1000 with interest at $j_4 = 10\%$ will be repaid by a payment of $200 at the end of 3 months and three equal payments at the ends of 6, 9, and 12 months. What will these payments be? Ans. $288.86

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Proposed problems

1) On September 1, 1992, Paul borrowed $3000, agreeing to pay interest at 12% compounded quarterly. He paid $900 on March 1, 1993, and $1200 on December 1, 1993.

    (a) What equal payments on June 1, 1994, and December 1, 1994, will be needed to settle the debt?

    (b) If Paul paid $900 on March 1, 1993, $1200 on December 1, 1993 and $900 on March 1, 1994, what would be his outstanding balance on September 1, 1994?
    Ans. (a) $706.89 (b) $459.58

2) Find the equated time for paying two debts of $1200 each, one due in 6 months and the other in 1 year, if money is worth 6% compounded monthly. Ans. 0.75 year

Solved problems (finding time)

1. How long will it take $2000 to accumulate $800 interest at 10% compounded quarterly, if compound interest is allowed for the fractional part of a conversion period?

Solved problems (finding time)

1. How long will it take \$2000 to accumulate \$800 interest at 10% compounded quarterly, if compound interest is allowed for the fractional part of a conversion period?

$$2800 = 2000(1.025)^n$$
$$(1.025)^n = 1.4$$
$$n \log 1.025 = \log 1.4$$
$$n = \frac{\log 1.4}{\log 1.025} = 13.62643323 \text{ quarters}$$

Using approximate time (1 month = 30 days), $n \approx 3$ years 4 months 26 days.

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Solved problems

2. By what date will $800 deposited on February 4, 1994, be worth at least $1200 at 12% compounded daily?

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Solved problems

2. By what date will $800 deposited on February 4, 1994, be worth at least $1200 at 12% compounded daily?

$$1200 = 800\left(1 + \frac{0.12}{365}\right)^n$$

$$\left(1 + \frac{0.12}{365}\right)^n = 1.5$$

$$n\log\left(1 + \frac{0.12}{365}\right) = \log 1.5$$

$$n = \frac{\log 1.5}{\log\left(1 + \frac{0.12}{365}\right)} = 1233.492437 \approx 1234 \text{ days}$$

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Solved problems

3. If an investment doubles it value in 6 years by a certain rate of interest compounded monthly, how long it takes for the same investment to triple in value?

Solved problems

3. If an investment doubles it value in 6 years by a certain rate of interest compounded monthly, how long it takes for the same investment to triple in value?

$$
\begin{aligned}
2x &= x(1+i)^{72} \\
(1+i)^{72} &= 2 \\
1+i &= 2^{1/72}
\end{aligned}
$$

$$\longrightarrow$$

$$
\begin{aligned}
3x &= x2^{n/72} \\
2^{n/72} &= 3 \\
\frac{n}{72}\log 2 &= \log 3 \\
n &= \frac{72\log 3}{\log 2} = 114.1173001 \text{ months}
\end{aligned}
$$

Using approximate time (1 month = 30 days), $n \approx 9$ years 6 months 4 days.

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Proposed problems (finding time)

1) Assuming that compound interest is allowed for the fractional part of a conversion period, how long will it take for: (*a*) \$1800 to accumulate to \$2200 at $j_4 = 8\%$? (*b*) \$100 to accumulate to \$130 at $j_2 = 9\%$? (*c*) \$500 to accumulate to \$800 at $j_{12} = 12\%$?
   Ans. (*a*) 2 years 6 months 12 days; (*b*) 2 years 11 months 23 days; (*c*) 3 years 11 months 7 days

2) \$500 was deposited on January 1, 1980, in an account paying 12% compounded semiannually. On January 1, 1983, \$400 was deposited in another account paying $15\frac{1}{2}\%$ compounded annually. Find the time when the two accounts will be of equal value, if the exact method is used for fractions of an interest period. *Ans.* 20.780279 years from January 1, 1983

Solved problems (implied rate)

1. An investment fund states that it will triplicate your money in 10 years. What rate of interest compounded monthly is implied?

Solved problems (implied rate)

1. And investment fund states that it will triplicate your money in 10 years. What rate of interest compounded monthly is implied?

- Alternative 1:

$$3X = X\left(1 + \frac{i}{12}\right)^{120} \iff 1 + \frac{i}{12} = 3^{\frac{1}{120}}$$

$$\iff i = 12\left(3^{\frac{1}{120}} - 1\right)$$

$$\iff i \approx 11.04\%$$

Solved problems (implied rate)

1. And investment fund states that it will triplicate your money in 10 years. What rate of interest compounded monthly is implied?

- Alternative 2:

$$3 = \left(1 + \frac{i}{12}\right)^{120} \quad \Longleftrightarrow \quad \log 3 = 120 \log \left(1 + \frac{i}{12}\right)$$

$$\Longleftrightarrow \quad \log \left(1 + \frac{i}{12}\right) = \frac{\log 3}{120}$$

$$\Longleftrightarrow \quad \left(1 + \frac{i}{12}\right) = 10^{\frac{\log 3}{120}}$$

$$\Longleftrightarrow \quad i = 12 \left(10^{\frac{\log 3}{120}} - 1\right) \approx 11.04\%$$

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Solved problems (implied rate)

2. In 5 years, the earnings per share of a company common stock increased from $4.71 to $9.38. What was the compounded annual rate of increase?

Solved problems (implied rate)

2. In 5 years, the earnings per share of a company common stock increased from \$4.71 to \$9.38. What was the compounded annual rate of increase?

$$9.38 = 4.71(1+i)^5 \qquad \text{or} \qquad i = \left(\frac{9.38}{4.71}\right)^{1/5} - 1 = 0.147721154 \approx 14.77\%$$

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Proposed problems (implied rate)

1) John deposited $1000 three and a half years ago. He has $1581.72 in his account now. What rate of interest compounded semiannually did he earn on his deposit? *Ans.* 13.54%

2) Find the nominal rate compounded quarterly at which $2000 will accumulate to $3000 in 3 years 9 months. *Ans.* 10.96%

3) On January 1 $500 000 is deposited in Fund X and $50 000 is deposited in Fund Y. No previous deposits exist. Fund X earns compound interest at rate of i per year. Fund Y earns simple interest at $(i + .01)$ per year. On April 1 one additional deposit of $50 000 is made to Fund Y. No additional deposits are made to Fund X. On December 31 the sum of the values of Funds X and Y is $642 000. Find i. *Ans.* 7%

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