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ECON

Financial Mathematics

Seminar 5: Continuously compounded interest

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Solved problems

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$$S_t = P\mathrm{e}^{rt} = \$1500\mathrm{e}^{(0.13)\cdot(1.5)} = \$1822.97$$

Solved problems

2. The PC Corporation purchased a $200,000 piece of commercial paper paying 6.5% compounded continuously and maturing in 182 days. What is the value of this investment if held to maturity?

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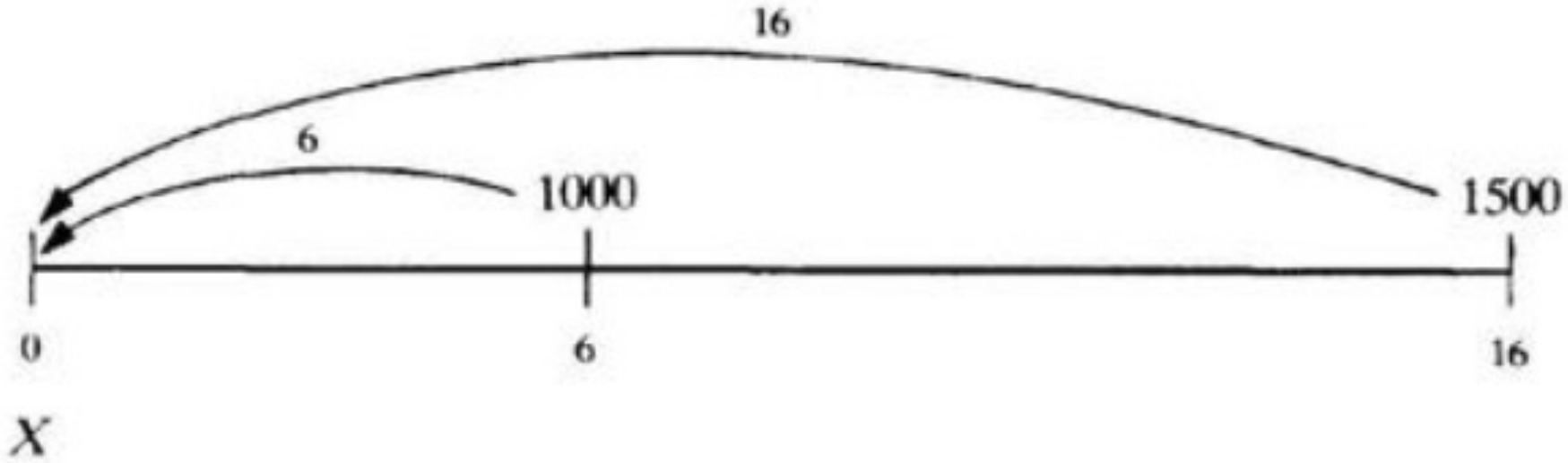
$$S_t = Pe^{rt} = \$200000e^{(0.065)\cdot(0.5)} = \$206606.78$$

Present value

3. A person owes $1000 due at the end of 18 months and $1500 due at the end of 4 years. If money is worth 6%(∞), what single payment liquidates these obligations now?



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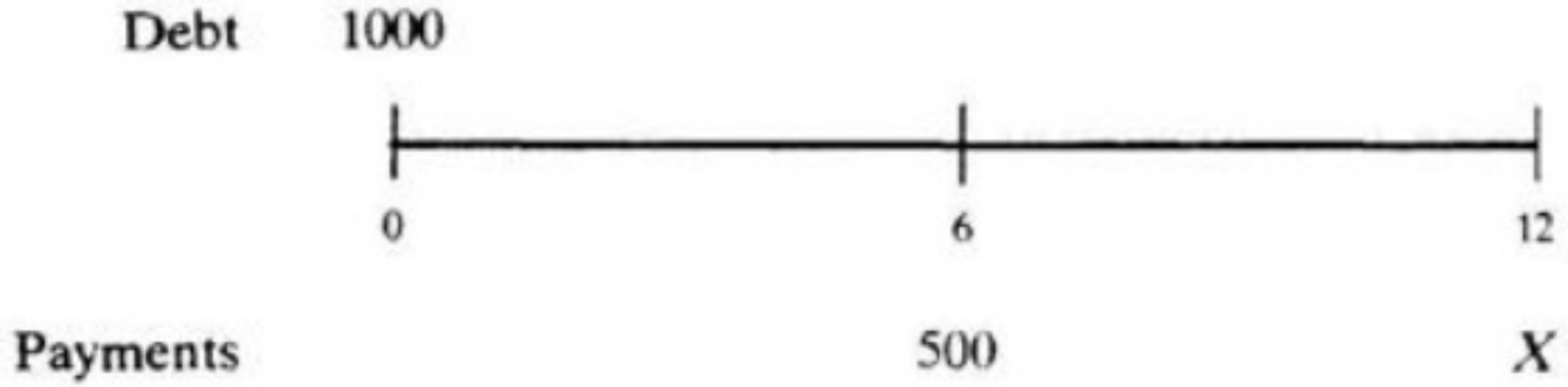
Time value of money

3. A consumer buys goods worth $1500 pating $500 down and $500 at the end of 6 months. If the stores charges interest at 8%(∞) on the unpaid balance, what final payment is necessary at the end of one year?



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Exercises

1) An obligation of $500 falls due at the end of 3 years. Given an interest rate of 12%(∞), find the equivalent debt at the end of (a) 3 months, (b) 3 years 9 months.

2) On June 1, a savings account paying 7%(∞) has a balance of $3568.25. What is the balance on October 8?

3) How much must be invested now at 9%(∞) in order to have $8500 in 5 years and 9 months?

More exercises

1. The Bank of Traveler's Rest advertises certificates of deposit with 15 months maturity and paying 6.75% compounded continuously. What is the maturity value of a $5000 CD?

2. The Greer Bank and Trust advertises certificates of deposit with 30-year maturity and paying 7.5% compounded continuously. What is the maturity value of a $5000 CD?

Implied rate

1. What is the continuous compounded rate of return on an investment that triples in 5 years?

$$S = Pe^{\delta t} \qquad 300 = 100e^{\delta(5)} \qquad 3 = e^{\delta(5)} \qquad \ln(3) = \ln[e^{\delta(5)}]$$

We use two properties of logs: $\ln(M^N) = N\ln(M)$ and $\ln e = 1$

The last equality becomes $\ln(3) = (\delta)(5)\ln e, \rightarrow \ln(3) = (\delta)(5).$

Thus, $\delta = \dfrac{\ln(3)}{5} = 21.97\%(\infty).$

Implied rate

2. In 5 years, the earnings per share of a company common stock increased from \$4.71 to \$9.38. What was the continuosly compounded nominal rate of increase?

Equivalent rates

1. Find
   a) The continuously compounded rate equivalent to 10.08% yearly.
   b) Continuously compounded rate equivalent to 12% compounded quarterly.

Exercises (implied rate)

1. An investor put $26,500 into a project that returned $67,850 in 75 months. What was her rate of return:
   a) Quarterlycompounding?
   b) Continuous compounding?

2. An investor put $2000 into a project that returned $3600 in 66 months. What was his ROR if it is figured via:
   a) Quarterly compounding?
   b) Continuous compounding?

Exercises (equivalent rates)

1. What is the equivalent annual interest rate of 9%(∞)?

2. What is the equivalent annual interest rate of 6%(∞)?

3. What is the equivalent semiannual interest rate of 9%(∞)? [Hint: Find the amount of $1 for 6 months at 9%(∞).]

4. What is the equivalent monthly interest rate of 6%(∞)?

5. Which rule of composition offers a higuer yield for a nominal rate of 9.2%: a) Monthly, or b) Continuosuly?

Finding the time

1. By what date will \$800 deposited on February 4, 1994, be worth at least \$1200 (*a*) at 12% compounded continuously? (*b*) at 12% compounded daily?

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(*a*) Given $P = 800$, $S = 1200$, $j_\infty = 0.12$

$$\begin{aligned} 1200 &= 800e^{0.12t} \\ e^{0.12t} &= 1.5 \\ 0.12t &= \ln 1.5 \\ t &= \frac{\ln 1.5}{0.12} = 3.378875901 \text{ years} \end{aligned}$$

Using exact time (1 year = 365 days), $t \approx 1234$ days. On June 22, 1997, the deposit will be worth at least \$1200.

Finding the time

1. By what date will \$800 deposited on February 4, 1994, be worth at least \$1200 (*a*) at 12% compounded continuously? (*b*) at 12% compounded daily?

(*b*) Given $P = 800$, $S = 1200$, $i = 0.12/365$,

$$\begin{aligned} 1200 &= 800\left(1+\frac{0.12}{365}\right)^n \\ \left(1+\frac{0.12}{365}\right)^n &= 1.5 \\ n\log\left(1+\frac{0.12}{365}\right) &= \log 1.5 \\ n &= \frac{\log 1.5}{\log\left(1+\frac{0.12}{365}\right)} = 1233.492437 \approx 1234 \text{ days} \end{aligned}$$

Exercises

1. Using logarithms, find the time it will take for (a) a deposit to double in value at $j_1 = 19.56\%$, (b) an investment to double in value at 15% compounded daily, (c) \$800 to grow to \$1500 at $j_2 = 9.8\%$, (d) an investment to triple in value at 15% compounded continuously.
Ans. (a) 3 years 10 months 17 days; (b) 1687 days; (c) 6 years 6 months 25 days; (d) 7 years 118 days