PORTFOLIO THEORY – LECTURE NOTES 2

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FACTOR INVESTING WITH LONG-SHORT PORTFOLIOS

Computing optimal weights is not the only possibility. An alternative approach involves creating **long-short portfolios**. Suppose we want to invest into N assets. First, assets are ranked according to their predicted return. We then assemble a portfolio with two legs: a long leg which contains a given number of assets with the highest predicted returns, and a short leg with a given number of assets predicted to have the lowest returns. Within a certain leg the assets are often equally weighted, although other weighting systems that assign different weights based on the predicted return are of course possible.

One of the advantages of such an approach is that it allows the investor to consider predicted returns without suffering the full consequences of the estimation error in the mean. Optimization procedures like the mean-variance one are in fact error-maximizing: errors in the inputs lead to extreme weights, which can lead to abysmal performance. This is why standard mean-variance optimization rarely works well and is generally replaced either with more advanced techniques that limit extreme allocations, or with a minimum variance portfolio. A long-short portfolio does not have theoretically optimal weights, but it can still work better by avoiding this error-maximization trap.

The other advantage is that a long-short portfolio can be self-financing: the money obtained from shorting the assets predicted to perform poorly is used to go long on the assets with predicted high return. As short positions tend to be more risky than long positions, using a partially self-financing portfolio is also common. In this case, less than half (e.g., 30%) of the wealth is placed on the short leg, and the long leg is financed partly from the shorting and partly from the investor's initial wealth (in our example where 70% of the invested sum goes to the long leg, 30% of the money comes for the shorting and the other 40% from the investor's funds).

A practical disadvantage of such a portfolio is that in the real world it is generally difficult to short a large number of stocks. So in practice N has to be relatively small, and therefore it can be more risky, as it is not very well diversified. A long-short portfolio can also be particularly vulnerable in turbulent market conditions, when the price of virtually all stocks are either increasing or decreasing at the same time. The latter problem can be eased by making the portfolio construction more flexible (e.g., by varying the number of stocks and/or the amount of wealth in the long and short leg depending on the market conditions).

Obviously, a pre-condition for creating a long-short portfolio is having a ranking based on how we expect the assets to perform. How can we obtain it? Using the sample means is not appropriate, as we pointed out that such estimates are too unreliable. A much better alternative is to use a factor-based approach. In fact, long-short portfolios are a typical way **factor investing** is performed. This generally involves computing expected returns using multifactor models.

Remember that the formula of a multifactor model with k factors is:

$$R_i = \alpha_i + b_{i1}f_1 + b_{i2}f_2 + \cdots + b_{ik}f_k + \varepsilon_i$$

In practice, the expected return of a stock given a certain multifactor model is computed as:

$$E[R_i] = \alpha_i + b_{i1}\gamma_1 + b_{i2}\gamma_2 + \cdots + b_{ik}f\gamma_k$$

where γ is the factor risk premium.[1]

Therefore, we need to estimate the loadings b_{ik} and the risk premia. This is typically done using the **Fama-MacBeth regression**. It is a two-stage linear regression. Consider an estimation sample with N assets and T periods.

In the first stage, the loadings are estimated by regressing the returns of each asset i on the k factors, using the entire set of T periods:

$$\begin{gathered} R_{1t} = \alpha_1 + b_{11}f_{1t} + b_{12}f_{2t} + \cdots + b_{1k}f_k \\ R_{2t} = \alpha_2 + b_{21}f_{1t} + b_{22}f_{2t} + \cdots + b_{2k}f_k \\ \vdots \\ R_{it} = \alpha_i + b_{i1}f_{1t} + b_{i2}f_{2t} + \cdots + b_{ik}f_k \\ \vdots \\ R_{Nt} = \alpha_N + b_{N1}f_{1t} + b_{N2}f_{2t} + \cdots + b_{Nk}f_{kt} \end{gathered}$$

The estimated loadings are then used as explanatory variables in a second regression that, for each period t, regresses the asset returns of the entire set of N assets:

$$\begin{gathered} R_{i1} = \gamma_{10} + \gamma_{11}\widehat{b_{i1}} + \gamma_{12}\widehat{b_{i2}} + \cdots + \gamma_{1k}\widehat{b_{ik}} \\ R_{i2} = \gamma_{20} + \gamma_{21}\widehat{b_{i1}} + \gamma_{22}\widehat{b_{i2}} + \cdots + \gamma_{2k}\widehat{b_{ik}} \\ \vdots \\ R_{it} = \gamma_{t0} + \gamma_{t1}\widehat{b_{i1}} + \gamma_{t2}\widehat{b_{i2}} + \cdots + \gamma_{tk}\widehat{b_{ik}} \\ \vdots \\ R_{iT} = \gamma_{T0} + \gamma_{T1}\widehat{b_{i1}} + \gamma_{T2}\widehat{b_{i2}} + \cdots + \gamma_{Tk}\widehat{b_{ik}} \end{gathered}$$

Ideally we should use the true loadings, but their value is of course unknown in practice.

To compute the expected returns of each asset i we need the loadings, estimated in the first regression, and the risk premia, estimated in the second regression. Notice however that the risk premia are time-varying. A common approach is to compute their average value over the T periods (just like it is common to compute the average market excess return when using the CAPM). The expected return of asset i according to the chosen multifactor model is given by (we omit the ^ to keep the notation light):

$$E[R_i] = b_{i1}\gamma_1 + b_{i2}\gamma_2 + \cdots + b_{ik}\gamma_k$$

For greater clarity, let us consider how this works with the Fama-French three-factor model, which is probably the most important factor model. Recall that the model is:

$$R_i = R_f + b_{i1}(R_m - R_f) + b_{i2}SMB + b_{i3}HML$$

In practice the expected return of asset i will be computed as:

$$E[R_i] = R_f + b_{i1}\gamma_{(R_m - R_f)} + b_{i2}\gamma_{SMB} + b_{i3}\gamma_{HML}$$

We use the Fama-MacBeth regression to estimate the loadings and the risk premia. Usually, the excess return is used as dependent variable, to focus on the component of the return that is dependent on factor exposure. Therefore, the first stage regression for each asset i is:

$$R_{it} - R_{ft} = \alpha_i + b_{i1}(R_{mt} - R_{ft}) + b_{i2}SMB_t + b_{i3}HML_t$$

[1] In the CAPM, and in single factor models in general, we can directly use the factor value (the excess market return in the case of CAPM). In multifactor models we cannot do this, and we need to use the risk premia of the factors instead.

To simplify the notation, we indicate the first factor as MKT:

$$R_{it} - R_{ft} = \alpha_i + b_{i1}MKT_t + b_{i2}SMB_t + b_{i3}HML_t$$

As explained before, this regression needs to be carried out separately for each of the N assets.

Now that we have the estimates for the loadings, we can set up the second stage regression:

$$R_i - R_f = \gamma_{t0} + \gamma_{t1}\widehat{b_{i1}} + \gamma_{t2}\widehat{b_{i2}} + \gamma_{t3}\widehat{b_{i3}}$$

This regression needs to be carried out separately for each of the T periods in the estimation window, obtaining T values for γ_{t1}, γ_{t2} and γ_{t3} . We then compute their average in order to have a single value. We rename the average of γ_{t1}, γ_{t2} and γ_{t3} as γ_{MKT}, γ_{SMB} and γ_{HML} respectively, for better clarity. We also compute the average risk-free rate in order to have a single value for R_f.[2]

We can now compute the expected return of each asset i as:

$$E[R_i] = R_f + b_{i1}\gamma_{MKT} + b_{i2}\gamma_{SMB} + b_{i3}\gamma_{HML}$$

It is now straightforward to create the long-short portfolio. We simply rank the assets according to their expected return, and take a long position on those positioned in the upper part of the ranking, and a short position on those in lower part of the ranking.

Each time the portfolio has to be updated, we need to compute new estimates of the expected returns. So, for example, if we want to update the portfolio monthly, we ne need to repeat the procedure every month, using the up-to-date data.

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[2] Computing the average value of the risk premia and of the risk-free rate is a reasonable approach, and the one commonly used. However, it is not “the” right approach. Other approaches might also be appropriate depending on the specific situation.