PORTFOLIO THEORY – EXAM 20/6/2024

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EXERCISE 1

In a perfectly competitive market where the interest rate r remains constant over time, security A pays 1200 euro after three years and it costs 1000 euro.

Security B pays 100 euro after one year and 1100 euro after two years. How much does the investor who buys B earn after two years?

The interest rate is:

$$1000 = \frac{1200}{(1+r)^3}$$

$$(1+r)^3 = \frac{1200}{1000}$$

$$1 + r = 1.2^{1/3}$$

$$r = 1.2^{1/3} - 1 \approx 0.0627 = 6.27\%$$

Since the interest rate is 6.27% and we are in a perfect market, the price of B has to be:

$$P_B = \frac{100}{1+\mathrm{r}} + \frac{1100}{(1+r)^2} = \frac{100}{1+0.0627} + \frac{1100}{(1+0.0627)^2} = 1068.128$$

The investor earned $1200 - 1068.128 = 131.872$ euro.

EXERCISE 2

Suppose that the conditions for the CAPM are fully respected. The security A, whose returns have zero correlation with the returns of the market portfolio, has a 2% expected return. If the expected return of the market portfolio is 6%, what is the expected return of the security B, whose exposure to systemic risk is twice that of the market portfolio?

A security uncorrelated with the market portfolio has a beta equal to zero, and therefore its returns are equal to the risk-free rate. Formally:

$$E[R_A] = R_f + \beta_A * \left(E[R_M] - R_f\right) = R_f + 0 * \left(E[R_M] - R_f\right) = R_f$$

Therefore, $R_f = E[R_A] = 2\% = 0.02$

The beta measures the exposure to systemic (market) risk, and by definition the beta of the market portfolio is equal to 1. Hence, the beta of B is equal to 2.

Therefore, the expected return of B is:

$$E[R_B] = R_f + \beta_B * \left(E[R_M] - R_f\right) = 0.02 + 2 * (0.06 - 0.02) = 0.02 + 0.08 = 0.1$$

EXERCISE 3

Consider a portfolio of four assets whose weights are:

$$\boldsymbol{w} = \begin{bmatrix} 0.4 \\ 0.5 \\ 0.3 \\ -0.2 \end{bmatrix}$$

Compute the weights of the shrinkage portfolio from Tu and Zhou (2011) with shrinkage parameter $\delta = 0.2$

The weights are:

$$\boldsymbol{w}^* = \delta \boldsymbol{w_{NAIVE}} + (1-\delta)\boldsymbol{w} = 0.2 \begin{bmatrix} 0.25 \\ 0.25 \\ 0.25 \\ 0.25 \end{bmatrix} + 0.8 \begin{bmatrix} 0.4 \\ 0.5 \\ 0.3 \\ -0.2 \end{bmatrix} = \begin{bmatrix} 0.05 \\ 0.05 \\ 0.05 \\ 0.05 \end{bmatrix} + \begin{bmatrix} 0.32 \\ 0.40 \\ 0.24 \\ -0.16 \end{bmatrix} = \begin{bmatrix} 0.37 \\ 0.45 \\ 0.29 \\ -0.11 \end{bmatrix}$$

EXERCISE 4

The vector of weights and the covariance matrix of a portfolio with two assets are:

$$\boldsymbol{w} = \begin{bmatrix} 0.7 \\ 0.3 \end{bmatrix} \quad \boldsymbol{\Sigma} = \begin{bmatrix} 0.002 & 0.001 \\ 0.001 & 0.003 \end{bmatrix}$$

Compute, using matrix form, the standard deviation of the portfolio.

$$Var(R_P) = \begin{bmatrix} 0.7 & 0.3 \end{bmatrix} \begin{bmatrix} 0.002 & 0.001 \\ 0.001 & 0.003 \end{bmatrix} \begin{bmatrix} 0.7 \\ 0.3 \end{bmatrix} =$$

$$\begin{bmatrix} 0.7 * 0.002 + 0.3 * 0.001 & 0.7 * 0.001 + 0.3 * 0.003 \end{bmatrix} \begin{bmatrix} 0.7 \\ 0.3 \end{bmatrix} =$$

$$\begin{bmatrix} 0.0017 & 0.0016 \end{bmatrix} \begin{bmatrix} 0.7 \\ 0.3 \end{bmatrix} = 0.0017 * 0.7 + 0.0016 * 0.3 = 0.00167$$

The standard deviation is the square root of the variance, therefore it is $\sqrt{0.00252} \approx 0.04087$

EXERCISE 5

Given the following series of returns at different periods:

$$R_{t=1} = 0.05\,, \quad R_{t=2} = -0.01\,, \quad R_{t=3} = 0.002\,, \quad R_{t=4} = 0.06\,, \quad R_{t=5} = 0.005$$

and the series of risk-free rate:

$$R_{f,t=1} = 0.005\,, \quad R_{f,t=2} = 0.005\,, \quad R_{f,t=3} = 0.005\,, \quad R_{f,t=4} = 0.01\,, \quad R_{f,t=5} = 0.01$$

Compute the Sortino ratio for an investor that sets the benchmark equal to the risk-free rate.

As the benchmark is equal to the risk-free rate, the most appropriate way is to work with excess returns:

$$0.05 - 0.005 = 0.045$$

$$-0.01 - 0.005 = -0.015$$

$$0.002 - 0.005 = -0.003$$

$$0.06 - 0.01 = 0.05$$

$$0.005 - 0.01 = -0.005$$

We compute the semivariance:

$$\sigma_B^2 = \frac{1}{T}\sum_{t=1}^{T}[\mathrm{Min}(R_t - B, 0)]^2$$

$$= \frac{1}{5}[0 + (-0.015)^2 + (-0.003)^2 + 0 + (-0.005)^2] = \frac{1}{5}0.000259 = 0.0000518$$

From which we obtain the downside deviation:

$$\sigma_B = \sqrt{0.0000518} \approx 0.0072$$

We compute the mean:

$$\bar{R} - B = \frac{0.045 - 0.015 - 0.003 + 0.05 - 0.005}{5} = 0.0144$$

So finally we compute the Sortino ratio:

$$\mathrm{Sortino} = \frac{\overline{R} - B}{\sigma_B} = \frac{0.0144}{0.0072} = 2$$

EXERCISE 6

1. *What are the three assumptions of the Arbitrage Pricing Theory?*
2. *When testing the significance of the alpha, what are Newey-West standard errors used for?*
3. *When is it theoretically equivalent to minimize the variance or the semivariance of a portfolio?*
4. *What is an ETF?*
5. *What is estimated in the first stage of the Fama-MacBeth regression? And in the second?*

1. APT rests on three assumptions: (1) Capital markets are perfectly competitive. (2) Investors always prefer more wealth for a given risk. (3) The stochastic process generating asset returns can be expressed as a linear function of a set of K risk factors, and all unsystematic risk is diversified away.
2. They are used to account for heteroskedasticity and autocorrelation in the returns.
3. When the distribution is symmetric and (1) the benchmark is equal to the mean, or (2) we set a target return (mean-variance/semivariance).
4. It is a fund that is traded on the financial markets and which tries to replicate the index
5. In the first stage we estimate the factor loadings. In the second stage we estimate the risk premia.