PORTFOLIO THEORY – EXAM 6/6/2024

Dr. Andrea Rigamonti

EXERCISE 1

In a perfectly competitive market where the interest rate r remains constant over time, there are two securities: A and B.

A pays 990 euro after one year and it costs 900 euro.

If B pays 1500 euro after two years, at what price can you buy it on the market?

The interest rate is:

$$900 = \frac{990}{1+r}$$

$$r = \frac{90}{900} = 0.1$$

Since the interest rate is 10% and we are in a perfect market, the price of B has to be:

$$P_B = \frac{1500}{(1+r)^2} = \frac{1500}{(1+0.1)^2} = 1239.669$$

EXERCISE 2

The market portfolio has an expected return of 5%, and the risk-free rate is 1%. Suppose that the conditions for the CAPM are fully respected. The expected return of a portfolio in which 40% of the wealth if placed in Acme stocks and 60% in an ETF that replicates the market portfolio with zero tracking error is 7%. What is the beta of the Acme stocks?

First, we determine the beta of the portfolio:

$$E[R_P] = R_f + \beta_P * \left(E[R_M] - R_f\right)$$

$$0.07 = 0.01 + \beta_P * (0.05 - 0.01)$$

$$0.06 = 0.04\beta_P$$

$$\beta_P = \frac{0.06}{0.04} = 1.5$$

The beta of a portfolio is equal to the weighted average of the beta of its components. The beta of the ETF is equal to 1 (because it replicates the market portfolio). Therefore, the beta of the Acme stocks is:

$$1.5 = 0.4 * \beta_{Acme} + 0.6 * 1$$

$$0.9 = 0.4 * \beta_{Acme}$$

$$\beta_{Acme} = \frac{0.9}{0.4} = 2.25$$

EXERCISE 3

There is a mean-variance utility investor with risk-aversion coefficient $\gamma = 5$, and the asset menu has two risky asset and one risk-free asset. The mean and covariance matrix of the excess returns of the two risky assets are:

$$\mu = \begin{pmatrix} 0.01 \\ 0.008 \end{pmatrix} \quad \Sigma = \begin{bmatrix} 0.005 & 0.002 \\ 0.002 & 0.003 \end{bmatrix}$$

Compute the weights for 1/N portfolio that optimally allocates the wealth between the risky assets and the risk-free asset.

We compute the weights for the risky assets:

$$w_{1/N} = \frac{1}{\gamma} \frac{\mathbf{1}'\boldsymbol{\mu}}{\mathbf{1}'\Sigma\mathbf{1}} \mathbf{1}$$

$$= \frac{1}{5} \frac{\begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} 0.01 \\ 0.008 \end{bmatrix}}{\begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} 0.005 & 0.002 \\ 0.002 & 0.003 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$= \frac{1}{5} \frac{1 * 0.01 + 1 * 0.008}{\begin{bmatrix} 1 * 0.005 + 1 * 0.002 & 1 * 0.002 + 1 * 0.003 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$= \frac{1}{5} \frac{0.018}{\begin{bmatrix} 0.007 & 0.005 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \frac{1}{5} \frac{0.018}{0.007 * 1 + 0.005 * 1} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$= \frac{1}{5} \frac{0.018}{0.007 * 1 + 0.005 * 1} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \frac{1}{5} \frac{0.018}{0.012} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \approx \frac{1}{5} * 1.5 \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$= \frac{1}{5} * \begin{bmatrix} 1.5 \\ 1.5 \end{bmatrix} \approx \begin{bmatrix} 0.3 \\ 0.3 \end{bmatrix}$$

The two risky assets get a weight of 0.3 each, so the weight of the risk-free asset is:

$1 - (0.3 * 2) = 0.4$

EXERCISE 4

Given the following series of returns

0.05 , -0.02 , -0.01 , 0.1

and the following series of risk-free rates

0.005 , 0.005 , 0 , 0

Compute:

1. *The Sharpe ratio*
2. *The Sortino ratio for an investor that sets the benchmark equal to zero*

1. The Sharpe ratio is given by:

$$SR = \frac{\bar{R} - R_f}{\sigma}$$

First we compute the excess return:

$$0.05 - 0.005 = 0.045$$

$$-0.02 - 0.005 = -0.025$$

$$-0.01 - 0 = -0.01$$

$$0.1 - 0 = 0.1$$

Then we compute the mean excess return and the standard deviation of excess returns:

$$\overline{R_{exc}} = \frac{0.045 - 0.025 - 0.01 + 0.1}{4} = 0.0275$$

$$\sigma^2 = \frac{1}{T-1}\sum_{t=1}^{T}\left(R_{exc,t} - \overline{R_{exc}}\right)^2$$

$$= \frac{(0.045 - 0.0275)^2 + (-0.025 - 0.0275)^2 + (-0.01 - 0.0275)^2 + (0.1 - 0.0275)^2}{3}$$

$$\approx \frac{0.0003 + 0.0028 + 0.0014 + 0.0053}{3} \approx 0.0032$$

$$\sigma = \sqrt{0.00425} \approx 0.0566$$

So the Sharpe ratio is:

$$SR = \frac{0.0275}{0.0566} \approx 0.48$$

2. As the investor sets the benchmark equal to zero, we can simply use the given returns to compute the semivariance:

$$\sigma_B^2 = \frac{1}{T}\sum_{t=1}^{T}[\mathrm{Min}(R_t - B, 0)]^2 = \frac{1}{4}[0 + (-0.02)^2 + (-0.01)^2 + 0] =$$

$$= \frac{1}{4}0.0005 = 0.00025$$

Then we compute the downside deviation:

$$\sigma_B = \sqrt{0.00025} \approx 0.0158$$

Finally we need the mean return:

$$\bar{R} = \frac{0.05 - 0.02 - 0.01 + 0.1}{4} = \frac{0.12}{4} = 0.03$$

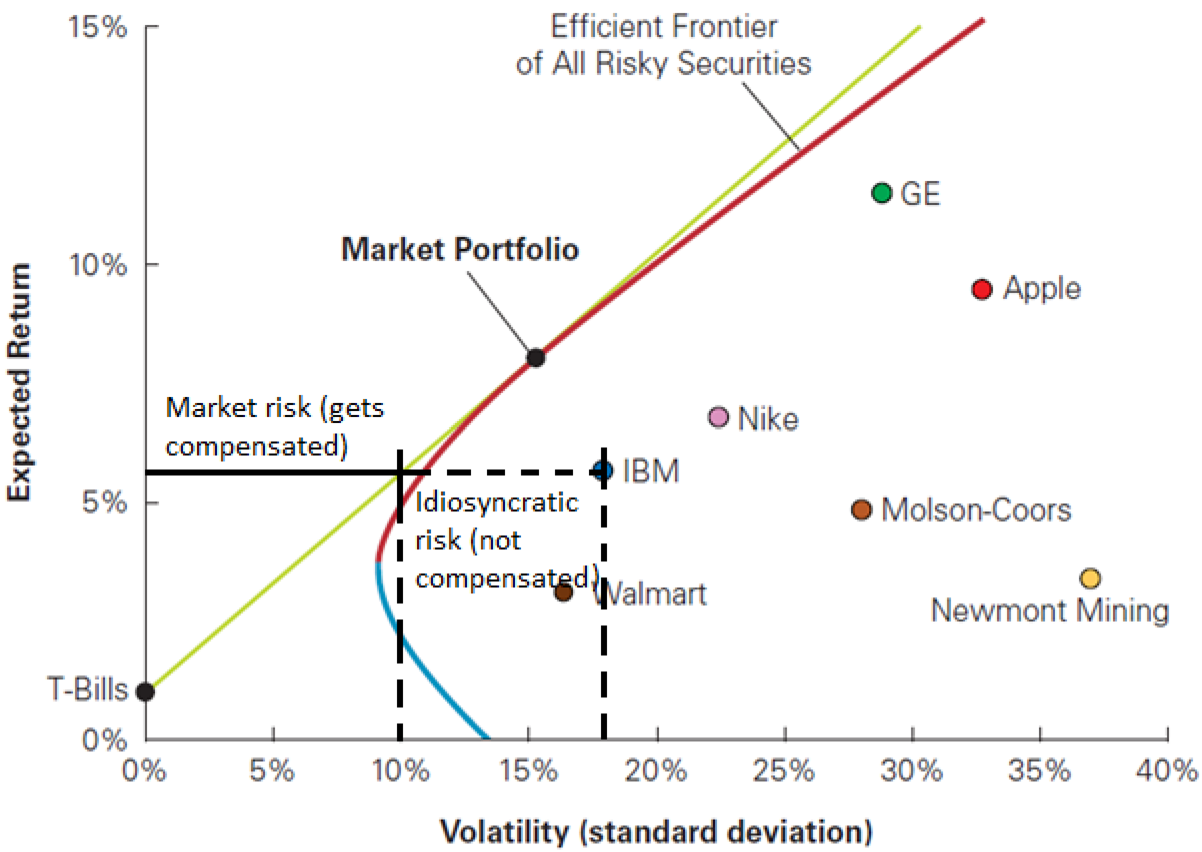
Now we can compute the Sortino ratio:

$$\text{Sortino} = \frac{\bar{R} - B}{\sigma_B} = \frac{0.03 - 0}{0.0158} \approx 1.899$$

EXERCISE 5

Suppose the assumptions of the CAPM are fully respected.

- *Can you show on the graph the fraction of volatility of IBM stocks for which the investors get compensated?*
- *How is the line passing through the "T-Bills" and the "Market Portfolio" called?*



The line is called Capital Market Line.

EXERCISE 6

1. *How does the price of a bond change right after a coupon is paid?*
2. *Assume the law of one price holds. A zero-coupon bond that pays 100 euro after one year costs 90 euro. What is the cost of a zero-coupon bond that pays 150 euro after one year?*
3. *If an portfolio manages to achieve returns that are not explained by its beta, the Jensen alpha of this portfolio is lower, equal, or greater than zero?*
4. *In an efficient market where the annual interest rate is always equal to 5%, what is the price of a security that pays the owner 100 euro once a year forever?*
5. *We want to estimate the Ledoit & Wolf shrinkage covariance matrix of three assets whose variances are 0.02, 0.03 and 0.01. Write the target matrix toward which the sample covariance matrix gets shrunk.*

1. The price of the bond drops by the amount of the coupon.
2. It costs $90 \times 1.5 = 135$ euro.
3. It is higher than 0.
4. It is a perpetuity. Its price is $P = \frac{100}{0.05} = 2000$ euro
5. The target matrix is a diagonal matrix whose elements on the diagonal are the average of the sample variances:

$$\begin{bmatrix} 0.02 & 0 & 0 \\ 0 & 0.02 & 0 \\ 0 & 0 & 0.02 \end{bmatrix}$$