PORTFOLIO THEORY – EXERCISES 19/03/2024

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EXERCISE 1

In a perfectly competitive market where the interest rate r remains constant over time, security A pays 1200 euro after two years and it costs 1000 euro.

Security B pays 1100 euro after one year. If there is a 20% tax on financial profits, how much does the investor who buys B earn after one year?

The interest rate is:

$$1000 = \frac{1200}{(1+r)^2}$$

$$(1+r)^2 = \frac{1200}{1000}$$

$$1 + r = 1.2^{1/2}$$

$$r = 1.2^{1/2} - 1 = 0.0954 = 9.54\%$$

Since the interest rate is 9.54% and we are in a perfect market, the price of B has to be:

$$P_B = \frac{1100}{1+\mathrm{r}} = \frac{1100}{1+0.0954} = 1004.199$$

The investor earned $1100 - 1004.199 = 95.801$ euro, but has to pay a 20% tax on it, so after taxes he actually earns:

$$95.801 - 95.801 * 0.2 = 76.6408$$

EXERCISE 2

A three-year 1000-euro coupon bond pays a 200-euro coupon each year.

Knowing that a bond that pays 100 euro after one-year costs 97 euro, one that pays 100 euro after two years costs 95 euro, and one that pays 100 euro after three years costs 90 euro, what is the price at which the coupon bond should trade in a market in which the law of one price holds?

Due to the law of one price, equal cash flows should have the same price. The coupon bond pays 200 euro after one year, 200 euro after two years and 1200 (the 1000 euro plus the 200 coupon) after three years. Therefore, we can replicate its cash flows with two one-year zero-coupon bonds, two two-year zero-coupon bonds and twelve three-year zero-coupon bonds. The price of this portfolio of zero-coupon bonds, which must be equal to the price of the coupon bond, is:

$$97 * 2 + 95 * 2 + 90 * 12 = 1464$$

EXERCISE 3

The log-return of the four assets included in an equally weighted portfolio is:

$r_1 = 0.1,\ r_2 = -0.06,\ r_3 = 0.07,\ r_4 = 0.05$

What is the return of the portfolio?

Log-returns are not asset additive. We first need to convert them to simple returns:

$$R_1 = \exp(r_1) - 1 = \exp(0.1) - 1 = 0.1052$$

$$R_2 = \exp(r_2) - 1 = \exp(-0.06) - 1 = -0.0582$$

$$R_3 = \exp(r_3) - 1 = \exp(0.07) - 1 = 0.0725$$

$$R_4 = \exp(r_4) - 1 = \exp(0.05) - 1 = 0.0513$$

We can now compute the return of the portfolio:

$$R_p = w_1 R_1 + w_2 R_2 + w_3 R_3 + w_4 R_4 =$$

$$= 0.25 * 0.1052 + 0.25 * (-0.0582) + 0.25 * 0.0725 + 0.25 * 0.0513 = 0.0427$$

EXERCISE 4

Suppose that the conditions for the CAPM are fully respected. The market portfolio expected return is 5% and the risk-free rate is 1%. We create a portfolio in which 30% of the wealth is placed in the security A whose beta is equal to that of the market, 50% in the security B whose beta is twice that of the market, and the remaining 20% is invested in the risk-free asset. What is the expected return of the portfolio?

The beta of the market portfolio is equal to 1 by definition. Therefore the beta of A is also equal to 1, and the beta of B is equal to 2. Since the CAPM holds, the expected return of A is equal to that of the market portfolio (because it has the same beta), while that of B is:

$$E[R_B] = R_f + \beta_B * \left(E[R_M] - R_f\right) = 0.01 + 2 * (0.05 - 0.01)$$
$$= 0.01 + 0.08 = 0.09$$

Hence, the expected return of the portfolio is:

$$E[R_P] = 0.3 * 0.05 + 0.5 * 0.09 + 0.2 * 0.01 = 0.062 = 6.2\%$$

EXERCISE 5

Consider the following series of unadjusted monthly closing prices (in euro) of a stock that undergoes the corporate events indicated next to the price.

January: 7

February: 9

March: 5 -- 2 for 1 stock split

April: 4.8

What is the monthly mean return for an investor that buys these shares at the end of January (i.e., for 7 euro per share) and keeps them until the end of April (i.e., until they are worth 4.8 euro per share)?

First we need to adjust for the stock split:

January: 7/2 = 3.5

February: 9/2 = 4.5

March: 5

April: 4.8

We can now compute the returns:

$$R_1 = \frac{4.5 - 3.5}{3.5} \approx 0.2857$$

$$R_2 = \frac{5 - 4.5}{4.5} \approx 0.1111$$

$$R_3 = \frac{4.8 - 5}{5} \approx -0.04$$

The mean return is:

$$\bar{R} = \frac{0.2857 + 0.1111 - 0.04}{3} \approx 0.1189$$