Portfolio Theory

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Lecture 10

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Downside risk

The variance measures volatility as a whole. This only reflects the preferences of the investors if:

1. Returns are symmetrically distributed
2. Or if investors treat downside and downside volatility in the same way

These assumptions are both unrealistic, but they greatly simplify optimization procedures. Moreover, targeting **downside risk** does not guarantee better results.

We do not address optimization techniques that target downside risk measures, but we will use these measures for portfolio evaluation.

Semivariance

The downside risk measure most closely related to the variance is the **semivariance**, which is defined as:

$$\sigma_B^2 = \frac{1}{T}\sum_{t=1}^{T}[\mathrm{Min}(R_t - B, 0)]^2$$

where T is the number of periods in the estimation window, and B is the benchmark below which the investor considers volatility to account as risk.

To apply this formula, one has to replace all the returns above the benchmark with 0.

Semivariance

B depends on the preferences of the investor. It is convenient to set it equal to the risk-free rate, so that if one works with excess returns, B can be treated as equal to zero. However, it can be set to any value.

The square root of σ_B^2 is called **downside deviation**, σ_B. The downside deviation is to the semivariance, what the standard deviation is to the variance.

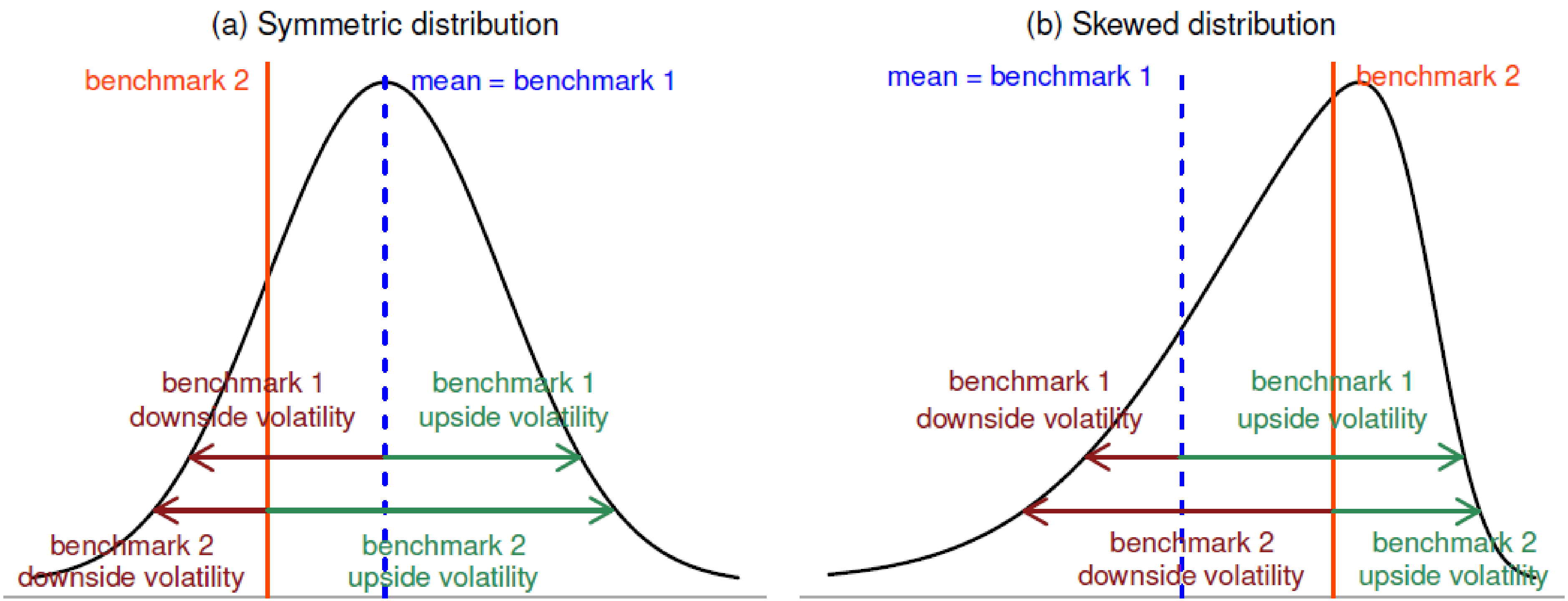
Theoretically, we can target semivariance by simply replacing the covariance matrix with the semicovariance matrix (the analogous to the covariance matrix in a downside risk setting) in the optimization procedures, but this matrix is difficult to estimate.

Semivariance

We do not address semivariance minimization, but we provide some intuition on it:

- If the distribution is symmetric and B is equal to the mean, targeting the variance or the semivariance is always equivalent, and one should target the former.
- If the distribution is symmetric but B is not equal to the mean, targeting the variance or the semivariance is only equivalent if we set a target return (i.e., mean-semivariance optimization).
- If the distribution is not symmetric, targeting the variance is never equivalent to targeting the semivariance.

Semivariance



Semivariance

To compute the risk-adjusted return in this context, the Sharpe ratio should be replaced by the **Sortino ratio**.

It is similar to the Sharpe ratio but replaces the risk-free rate with the benchmark B, and the standard deviation with the downside deviation σ_B:

$$\text{Sortino} = \frac{\overline{R} - B}{\sigma_B}$$

Value at Risk and Expected Shortfall

Value at Risk (VaR) measures the maximum potential loss that an investor can suffer over a certain period, with a $1-\alpha$ confidence level set by the investor. For example, an $\alpha = 0.05$ corresponds to a 95% confidence level.

Given a profit and loss distribution Y we can define VaR as:

$$VaR_{\alpha}(Y) = -\inf\{y \in \mathrm{R}: (Y \leq y) > \alpha\}$$

For example, if we set $\alpha = 0.05$ and when evaluating a set of returns we get $VaR = 0.04$, it means that we have a 5% chance of losing 4% or more in one period over the time horizon considered.

Value at Risk and Expected Shortfall

To compute the VaR we can use:

- The historical method: we rank the historical returns in increasing order and then check the (typically negative) return that we have at the α percentile.
- Parametric method: we assume that returns follow a certain distribution and we compute the loss at the chosen percentile.
- Simulation (“Monte Carlo”) approaches.

Value at Risk and Expected Shortfall

The main problem with VaR is that it is not a **coherent risk measure**. Consider the outcomes V_1 and V_2 of two investments. A risk measure is said to be coherent if it possesses the following desirable properties:

- Monotonicity: if V_1 is larger or equal to V_2 in every possible scenario, then the risk of V_1 must be lower than V_2. Formally: if $V_1 \geq V_2$, then $Risk(V_1) < Risk(V_2)$.
- Translation invariance: for any outcome V, adding an additional riskless outcome C reduces the risk by that amount. Formally: $Risk(V + C) = Risk(V) - C$.

Value at Risk and Expected Shortfall

- Positive homogeneity: multiplying all outcomes by a constant should result in a scaling of the risk measure by the same constant. Formally: $Risk(\lambda V) = \lambda Risk(V)$.
- Subadditivity: the risk of a combination of two risky positions should be lower or equal to that of the individual positions. In other words, diversification cannot increase risk. Formally: $Risk(V_1 + V_2) \leq Risk(V_1) + Risk(V_2)$.

The Value at Risk violates subadditivity: risk quantified using VaR can sometimes increase with greater diversification, which is not very meaningful.

Value at Risk and Expected Shortfall

To overcome this problem, the **Conditional Value at Risk (CVaR)**, or **Expected Shortfall (ES)**, has been proposed:

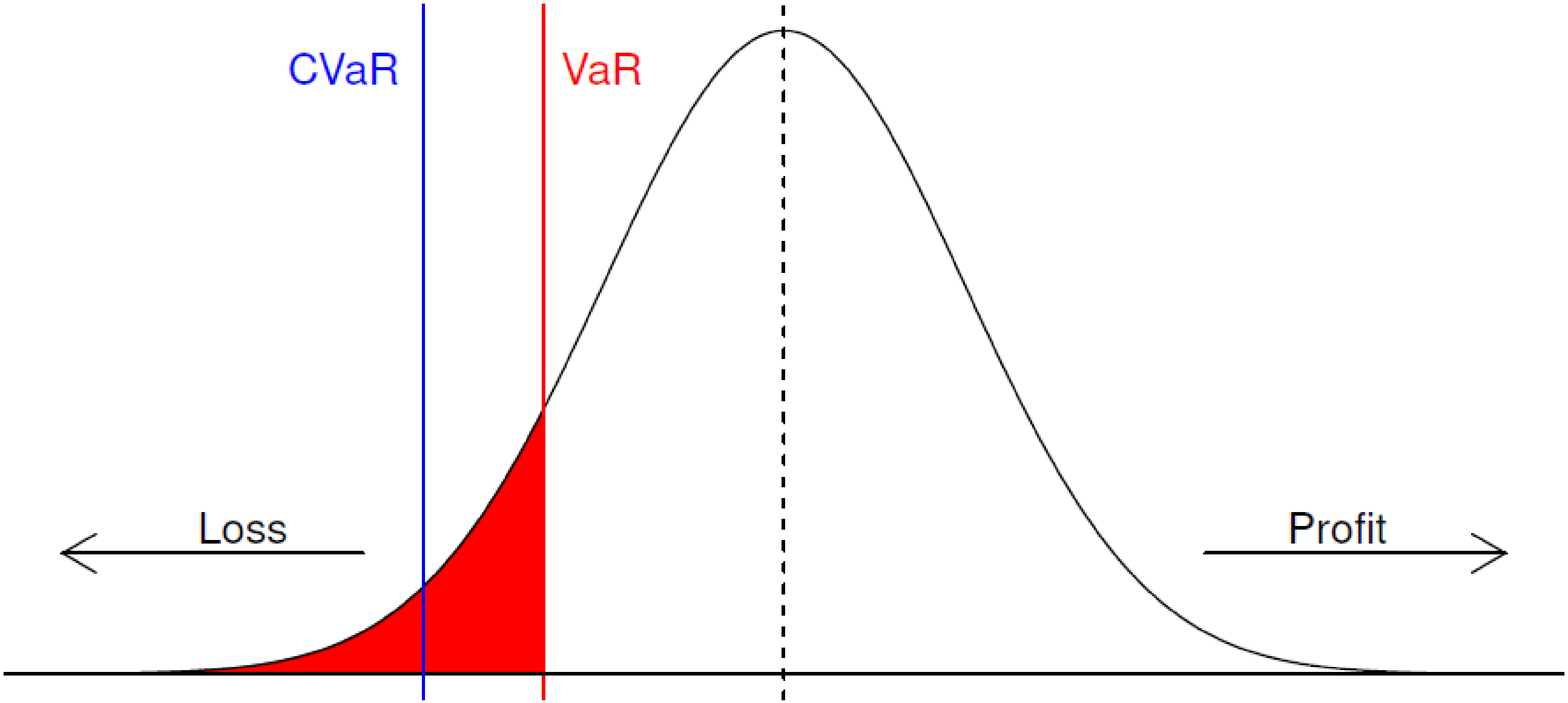
$$CVaR_\alpha(Y) = -\frac{1}{\alpha}\int_0^\alpha VaR_u du$$

where u is the variable of integration and du is the differential of this variable (i.e., we integrate from 0 to α using infinitesimal increments in u from 0 until we reach α).

The CVaR measures the average loss that we get, given that the loss exceeds the VaR. As it is a coherent measure, it is preferred and more commonly used than the VaR.

Value at Risk and Expected Shortfall

CVaR is always lower than the VaR, because it is the value that we get by computing the average loss that we have when we find ourselves in the red area left of the VaR.



Drawdown

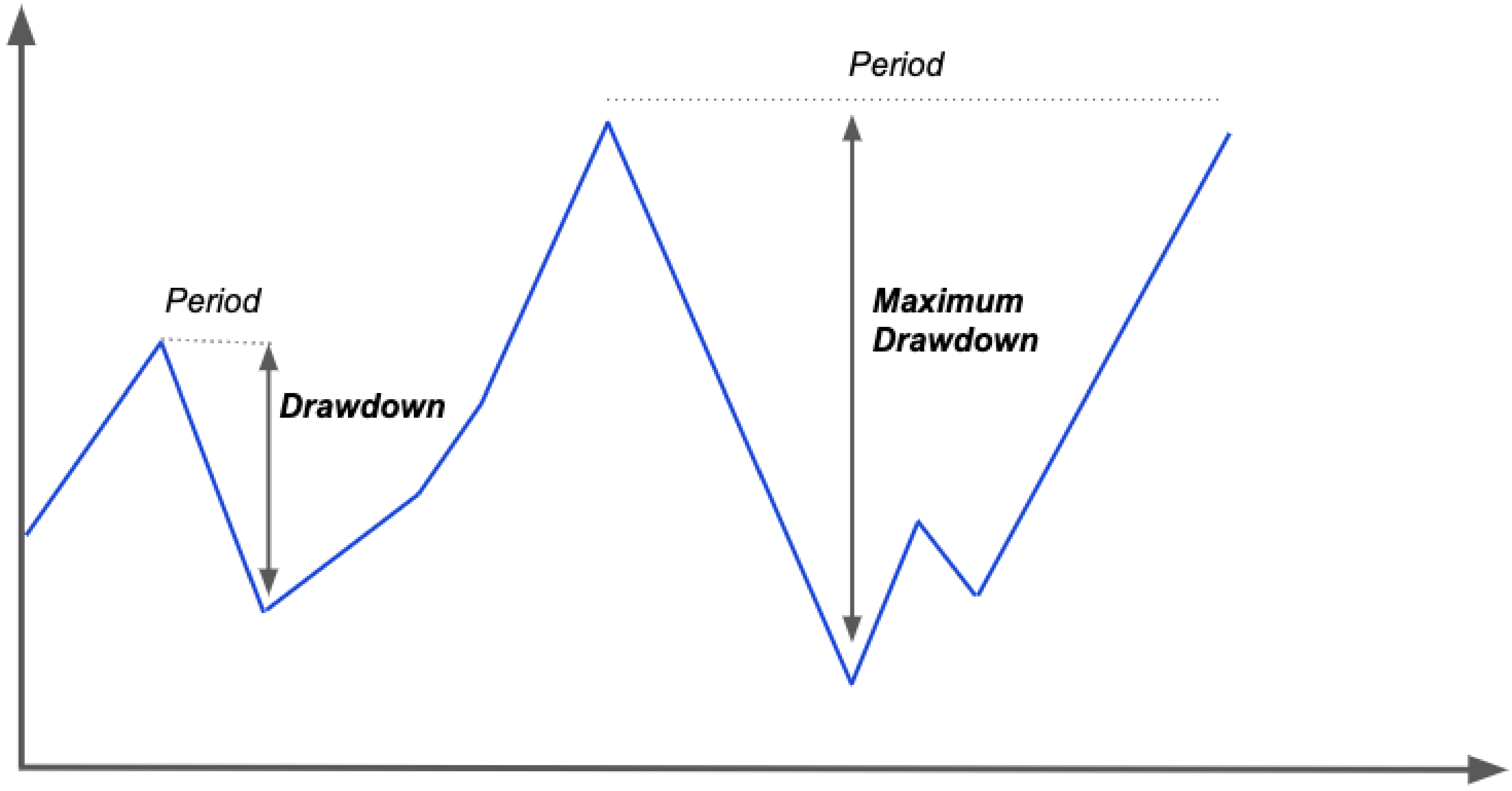
The **drawdown** is the decline in the value of an investment from a peak to a low point. Different drawdown measures can be computed. A popular and easy to compute one is the **maximum drawdown (MDD)**:

$$MDD = \frac{Trough\,Value - Peak\,Value}{Peak\,Vaue}$$

where the "Trough Value" is the lowest point in the series that is reached after the highest peak.

Obviously, a lower MDD is preferable to a higher MDD. In the worst possible case, MDD is equal to 100%, i.e., the value of the investment drops to zero.

Drawdown



Drawdown

MDD fails to consider the frequency and duration of losses, and does not account for the size of any gains.

To account for the gains, we can use the **Calmar Ratio**:

$$Calmar = \frac{\overline{R} - r_f}{MDD}$$

This is similar to the Sharpe ratio, but the MDD is used instead of the standard deviation.