Portfolio Theory

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Lecture 2

Content:

- Interest rates
- Valuing bonds

Interest rates

Effective annual rate (EAR): also known as effective annual yield (EAY) or annual percentage yield (APY), it is the actual amount of interest that will be earned at the end of one year.

Discount Rate Period Conversion. We can convert a discount rate of *R* for one period to an equivalent discount rate for *n* periods using the following formula:

$$\text{Equivalent } n\text{-Period Discount Rate} = (1 + R)^n - 1$$

- $n>1$ to compute a rate over more than one period
- $n<1$ to compute a rate over a fraction of a period

Interest rates

Example:

The EAR is 5%. What is the monthly rate?

$$(1+0.05)^{1/12}-1 = 0.004074 = 0.4047\%$$

The monthly rate is 0.5% What is the yearly rate?

$$(1+0.005)^{12}-1 = 0.061678 = 6.1678\%$$

Taxes reduce the amount of interests the investor can keep.

Given a tax τ, the **After-Tax Interest Rate** is:

$$R-(\tau R) = R(1-\tau)$$

Interest rates

Nominal interest rate (*R*): the rate at which the money will grow if invested for a certain period

Real interest rate (R_r): the rate of growth of the purchasing power, after adjusting for the inflation rate *i*.

$$Growth\ in\ Purchasing\ Power\ =\ 1 + R_r = \frac{1+R}{1+i}$$

Nominal interest rate tends to move with inflation.

Interest rates affect firms' incentive to raise capital and invest

Interest rates

Term structure of interest rates: the relationship between the investment term and the interest rate.

We can plot this relationship on a graph called **yield curve**.

Present Value of a Cash Flow Stream Using a Term Structure of Discount Rates:

$$PV = \sum_{n=1}^{N} \frac{C_n}{(1 + R_n)^n}$$

Interest rates

Interest Rate Determination: the Federal Reserve determines very short-term interest rates through its influence on the **federal funds rate**(the rate at which banks can borrow cash reserves on an overnight basis).

The other interest rates are set on the market.

Usually, the yield curve is increasing (longer term rates higher than short term rates).

A sharply increasing curve indicates expectations that interest rates will increase in the future.

A decreasing (inverted) curve indicates expectations of decline interest rates. Usually this precedes a recession.

Valuing bonds

Bond certificate: a legal document describing the amounts and dates of all the payments to be made.

Maturity date of a bond: the date on which the final payment is made to the bondholder.

Term of a bond: the amount of time between the bond's issuance and its maturity.

Coupons: the promised interest payments of a bond.

Valuing bonds

Principal (or face value) of a bond: the amount paid to the bondholder at maturity (i.e. at the expiration date).

Coupon rate: the amount of each coupon payment, i.e. the rate of interest paid by the bond.

By convention, the coupon rate is expressed as an Annual Percentage Rate (APR), i.e., as the amount of interests earned in one year without compounding.
So the amount of each **coupon payment, CPN**, is:

$$CPN = \frac{Coupon\ Rate\ *\ Face\ Value}{Number\ of\ Coupon\ Payments\ per\ Year}$$

Valuing bonds

Zero-coupon bond: a coupon that does not make coupon payments. The only cash payment the investor receives is the face value of the bond on the maturity date.

Zero-coupon bonds trade at a **discount** (a price lower than the face value), also called sub-par price. So they are also called **pure discount bonds**.

Yield to maturity (YTM): the discount rate that sets the present value of the promised bond payments equal to the current market price of the bond.

Valuing bonds

Yield to maturity for a zero-coupon bond: the return you will earn as an investor from holding the bond to maturity and receiving the promised face value payment.

The YTM for a zero-coupon bond with *n* periods to maturity, current price *P*, and face value *FV* solves

$$P = \frac{FV}{(1 + YTM_n)^n}$$

Valuing bonds

Yield to Maturity of an *n*-Year Zero-Coupon Bond:

$$YTM_n = \left(\frac{FV}{P}\right)^{1/n} - 1$$

YTM_n is the <u>per-period</u> rate of return for holding the bond from today until maturity on date *n*.

For the law of one price, the risk-free interest rate over a certain period has to be equal to the YTM_n of a risk-free zero coupon bond with maturity equal to that period.

Valuing bonds

Coupon bonds: bonds that pay investors their face value at maturity, and also make regular coupon interest payments.

Yield to maturity of a coupon bond: the interest rate y that solves the following equation (assuming the first coupon will be paid one period from now), where CPN is the coupon:

$$P = CPN * \frac{1}{y}\left(1 - \frac{1}{(1 + y)^N}\right) + \frac{FV}{(1 + y)^N}$$

y is a rate per coupon interval. We can convert it to an APR by multiply it by the number of coupons per year.

Valuing bonds

Coupon bonds may trade:

- at a discount (or below par);
- at a **premium** (a price greater than their face value, also said above par);
- at **par** (a price equal to their face value).

The market price of a bond changes over time because:

- it gets closer to its maturity date
- changes in market interest rates affect its yield to maturity

Valuing bonds

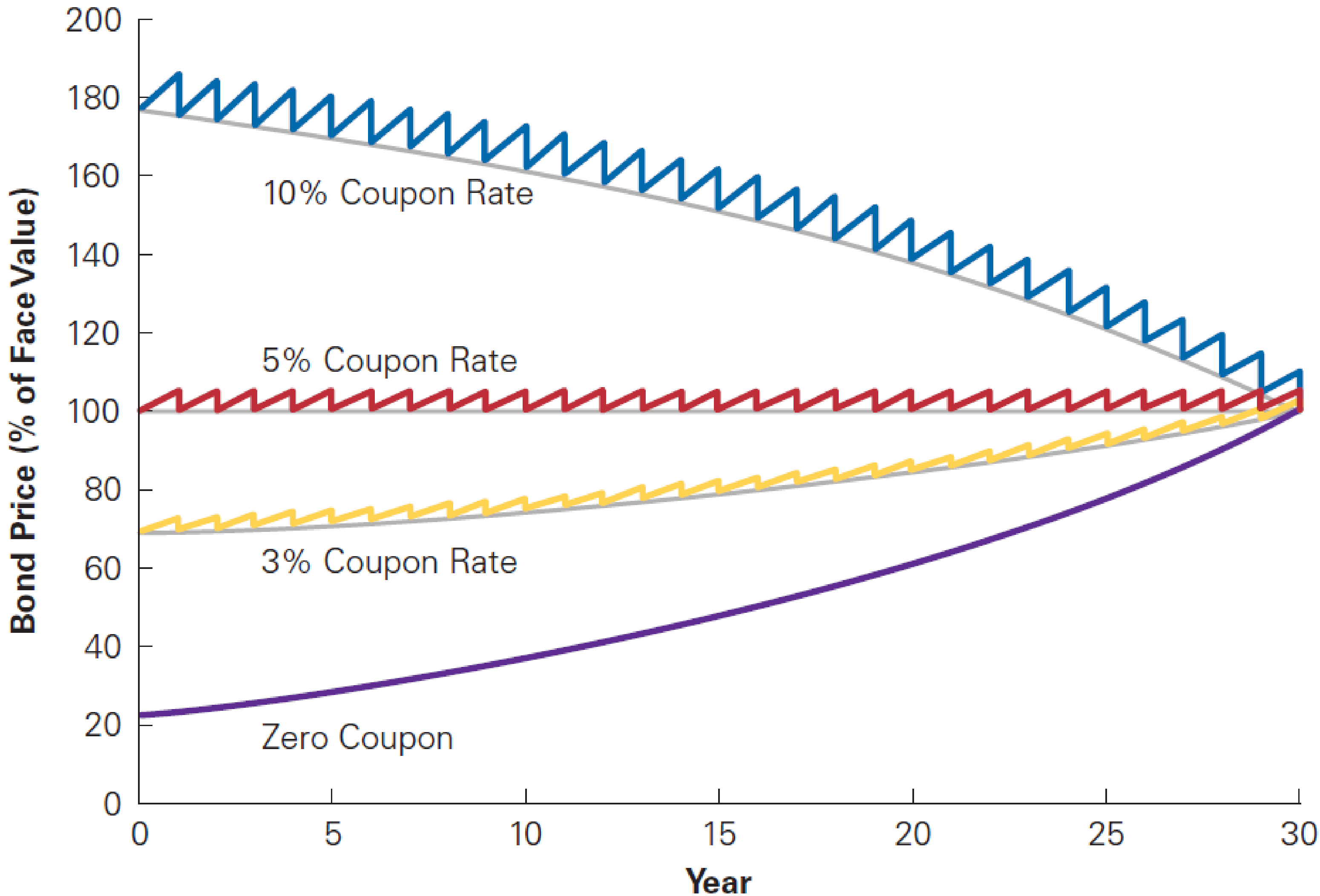
The price of a zero coupon bond will simply rise as it gets closer to maturity.

The price of a coupon bond will gradually drop if it trades at a premium, or rise if it trades at discount. Moreover, as each coupon is paid, its price drops by the amount of the coupon.

Ultimately, the prices of all bonds approach the bonds' face value when the bonds mature and their last coupon is paid.

Valuing bonds

Bond price over time if the yield to maturity remains at 5% (i.e., the investor earns a 5% return):



Valuing bonds

In reality, the yield does not stay the same, but reacts to changes in interest rates (and in the bond's risk level).

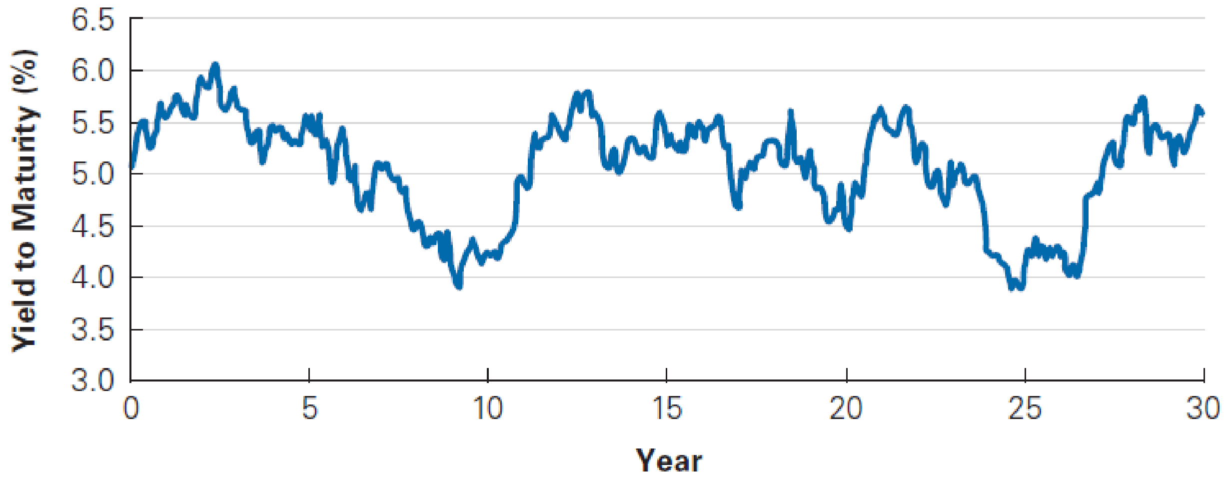
A higher yield to maturity implies a higher discount rate for a bond's remaining cash flows, reducing their present value and hence the bond's price.

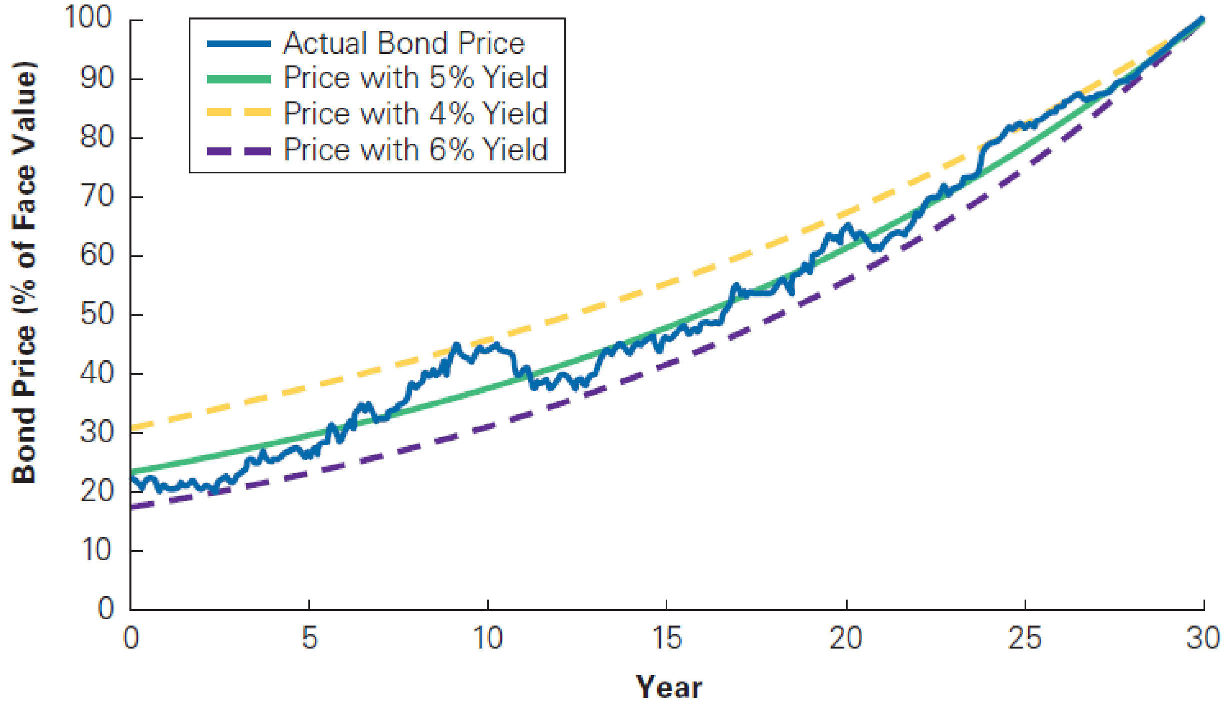
As interest rates and bond yields rise, bond prices will fall, and vice versa.

Bond prices fluctuate due to unpredictable changes in their yield while converging to their face value over time.

In the next slide, an example of a 30-year zero-coupon bond.

Valuing bonds





Valuing bonds

Duration: the sensitivity of a bond's price to changes in interest rates. Can be expressed in two ways:

- **Macaulay duration**:

$$MacaulayD = \sum_{t=1}^{n} \frac{PV(C_t)}{P} t$$

  where t is the time in years till the cash flow is received

- **Modified duration**:

$$ModifiedD = \frac{MacaulayD}{1 + \frac{YTM}{n}}$$

  where n is the number of coupon periods per year

Valuing bonds

The Macaulay duration is an estimate of the number of years required to repay the bond’s price.

Macaulay duration is a value-weighted maturity. Do not confuse it with the maturity (they are both expressed in years but their value is different)!

Modified duration measures the change in the price of a bond given a 1% change in the interest rates.

Valuing bonds

For the law of one price, we can replicate the cash flows of a coupon bond using zero-coupon bonds, obtaining the price of the coupon bond from the zero coupon bonds portfolio price.

Alternatively, we can use the zero-coupon bond yields. The price of a coupon bond must equal the present value of its coupon payments and face value discounted at the competitive market interest rates:

$$P = \frac{CPN}{1 + YTM_1} + \frac{CPN}{(1 + YTM_2)^2} + \cdots + \frac{CPN + FV}{(1 + YTM_n)^n}$$

where CPN is the coupon, YTM_n is the yield to maturity of a zero-coupon bond that matures at the same time as the nth coupon payment, and FV is the face value of the bond.

Valuing bonds

Corporate bonds: bonds issued by corporations. The issuer may default, i.e. it might not pay back the full amount promised in the bond prospectus.

Credit risk of the bond: the risk of default. Investors pay less for bonds with credit risk than they would for an otherwise identical default-free bond, because the cash flows are not known with certainty.

The yield to maturity for a bond is computed using the promised cash flows → the yield of a risky bond is higher than that of an otherwise identical default-free bond, and it exceeds the expected return of investing in the bond.

Valuing bonds

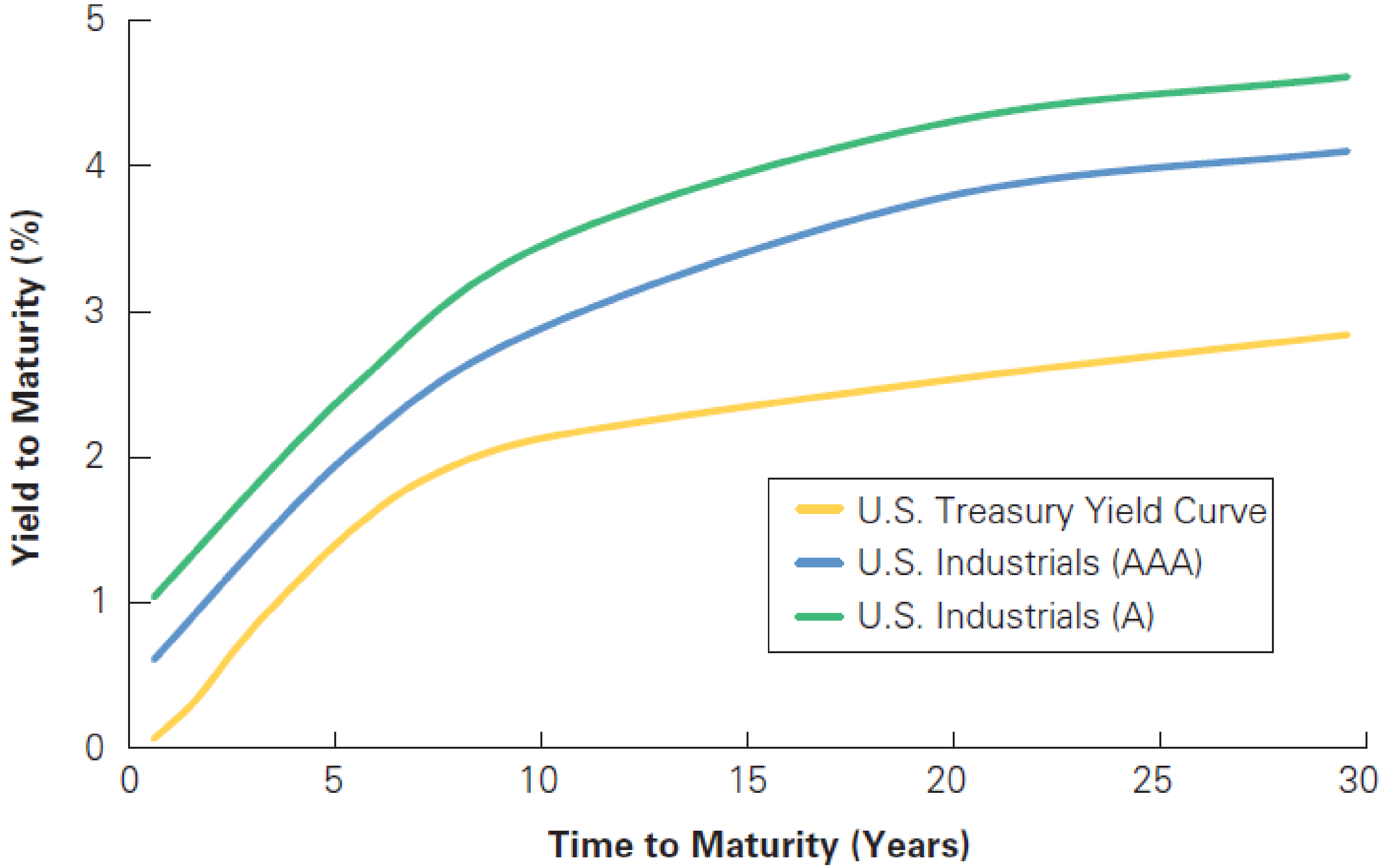
For a risky bond, a high yield to maturity does not necessarily imply a high expected return.

The creditworthiness of risky bonds is estimated by rating agencies. Bonds with low default risk are called **investment-grade bonds.** The others are called **speculative bonds**, **junk bonds**, or **high-yield bonds**.

Default spread (or credit spread): the difference between the yields of two bonds with different credit ratings.

Valuing bonds

Bond yield curves in 2015:



Valuing bonds

Sovereign bonds: bonds issued by national governments, many of which are not considered default free, and can therefore have high yields and low prices.

Countries might print money to repay debt in order to avoid default, therefore investors consider inflation expectations when determining the yield they are willing to accept.

Sometimes the required inflation rate would be too extreme, or there might be political reasons to not do it, so defaults still sometimes occur.