Portfolio Theory

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Lecture 9

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- Benchmark portfolios
- Ledoit & Wolf shrinkage covariance estimator
- Shrinkage portfolios
- Grouping strategies

Benchmark portfolios

We need to compare portfolios with appropriate **benchmarks** to know if the results are satisfying.

A benchmark portfolio surprisingly difficult to beat is the one created with a **naïve 1/N rule** that assigns equal weights to all the assets in all periods. Three strengths:

- immune to estimation errors (as it needs no inputs)
- does not require to perform any optimization procedure
- has a very low turnover, which translates into very low transaction costs

Benchmark portfolios

Another possible benchmark is a large **stock market index**, like the S&P 500. While it is not possible to buy an index, it is possible to buy an **ETF (Exchange-Traded Fund)**, i.e., a fund that is traded on the financial markets and which tries to replicate the index. Advantages:

- only one instrument (the ETF) needs to be bought; no need to trade all the stocks contained in the index
- over the long run stock markets provide good returns (in countries with a solid economy)
- no estimation and optimization procedures required

Ledoit & Wolf shrinkage covariance estimator

Basic mean-variance optimization with sample estimates often struggles to beat common benchmarks, especially the naïve 1/N rule.

To improve optimization performance we need to reduce the severity of the estimation errors in the inputs, and/or reduce the impact of such errors on portfolio formation.

On the estimation side we focus on the covariance matrix:

- while $\boldsymbol{\mu}$ is generally more affected by estimation errors, Σ easier and more effective to improve
- the problem gets worse in Σ as N grows, due to the higher number of parameters (N^2)

Ledoit & Wolf shrinkage covariance estimator

Shrinkage estimators are arguably the most successful in dealing with the estimation of the covariance matrix.

Ledoit & Wolf (2004) define the shrinkage estimator

$$\sum_{LW} = \delta \boldsymbol{I} \mu_{\hat{s}} + (1 - \delta)\sum$$

where $\sum$ is the usual sample covariance matrix, $\boldsymbol{I}$ is the identity matrix, $\mu_{\hat{s}}$ is the average sample variance of all the variables, and $0 \leq \delta \leq 1$ is the shrinkage parameter.

The product $\boldsymbol{I}\mu_{\hat{s}}$ gives a diagonal matrix whose elements on the diagonal are equal to the average sample variance and all the other elements are equal to zero, and which is the target matrix toward which the sample covariance matrix is shrunk.

Ledoit & Wolf shrinkage covariance estimator

An optimal δ is selected, and this shrunk estimator improves over the sample covariance, which translate in better portfolio performance.

Moreover, it gives a nonsingular covariance matrix even when the number of periods T used for estimation is smaller than the number of assets N.

The sample covariance matrix, on the contrary, in such case is singular and therefore not invertible, which makes it impossible to perform the optimization procedures.

It is now standard to use this estimator instead of the sample covariance in portfolio optimization.

Shrinkage portfolios

Tu and Zhou (2011) propose a **shrinkage portfolio** that shrinks the mean-variance portfolio (optimized, but affected by estimation errors) toward the naïve 1/N portfolio (not optimized, but immune from estimation errors, improving over both.

The weights of such portfolio are given by

$$\boldsymbol{w}^* = \delta \boldsymbol{w}_{NAIVE} + (1-\delta)\boldsymbol{w}$$

where $\boldsymbol{w}_{NAIVE}$ is the vector of weights of the naïve 1/N portfolio, $\boldsymbol{w}$ is the vector of weights of the optimized portfolio, and δ controls the shrinkage intensity.

The value of δ can be chosen using optimization rules, heuristics, or cross-validation.

Grouping strategies

Branger et al. (2019) propose a **grouping strategy**.

The idea is that since the performance of mean-variance (or minimum variance) optimization suffers more and more as N increases, due to the growing number of parameters to estimate, one could achieve better performance by reducing the problem dimension.

Assets are grouped together in a certain number of groups. The optimization procedure is than performed between the groups, while within each group the assets are equally weighted.

Grouping strategies

Possible grouping criteria: how similar they are in terms of estimated mean, variance or beta.

The number of groups can be chosen using optimization rules, heuristics, or cross-validation.

The higher the number of groups, the closer we get to the usual optimization; the smaller the number of groups, the closer we get to the naïve 1/N rule.

In the extreme case where we only have one group we obtain the naïve 1/N portfolio; in the extreme case where the number of groups is equal to N, we get the usual optimized portfolio.