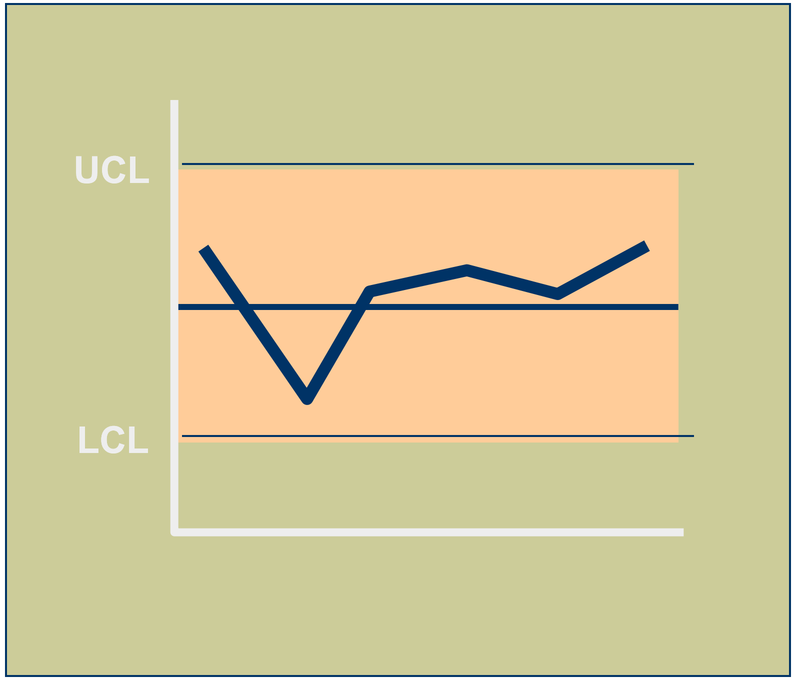
Basics of Statistical Process Control

- Statistical Process Control (SPC)
  - monitoring production process to detect and prevent poor quality
- Sample
  - subset of items produced to use for inspection
- Control Charts
  - process is within statistical control limits



Variability

- Random
  - common causes
  - inherent in a process
  - can be eliminated only through improvements in the system
- Non-Random
  - special causes
  - due to identifiable factors
  - can be modified through operator or management action

Quality Measures

- **Attribute**
  - a product characteristic that can be evaluated with a discrete response
  - good – bad; yes - no
- **Variable**
  - a product characteristic that is continuous and can be measured
  - weight - length

Applying SPC to Service

- Nature of defect is different in services
- Service defect is a failure to meet customer requirements
- Monitor times, customer satisfaction

Applying SPC to Service

Hospitals

- timeliness and quickness of care, staff responses to requests, accuracy of lab tests, cleanliness, courtesy, accuracy of paperwork, speed of admittance and checkouts

Grocery stores

- waiting time to check out, frequency of out-of-stock items, quality of food items, cleanliness, customer complaints, checkout register errors

Airlines

- flight delays, lost luggage and luggage handling, waiting time at ticket counters and check-in, agent and flight attendant courtesy, accurate flight information, passenger cabin cleanliness and maintenance

Fast-food restaurants

- waiting time for service, customer complaints, cleanliness, food quality, order accuracy, employee courtesy

Catalogue-order companies

- order accuracy, operator knowledge and courtesy, packaging, delivery time, phone order waiting time

Insurance companies

- billing accuracy, timeliness of claims processing, agent availability and response time

Where to Use Control Charts

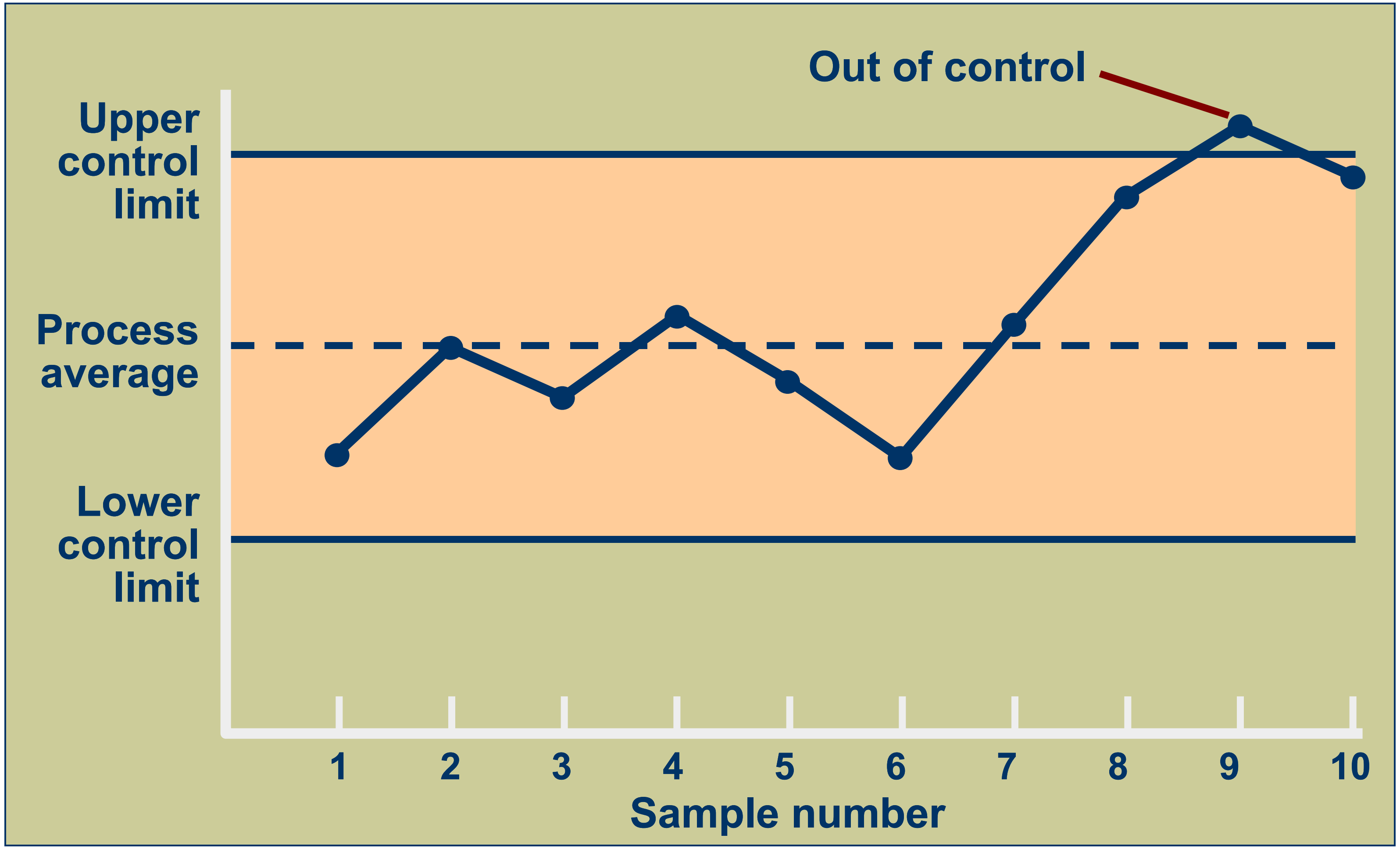
- Process has a tendency to go out of control
- Process is particularly harmful and costly if it goes out of control
- Examples
  - at the beginning of a process because it is a waste of time and money to begin production process with bad supplies
  - before a costly or irreversible point, after which product is difficult to rework or correct
  - before and after assembly or painting operations that might cover defects
  - before the outgoing final product or service is delivered

Control Charts

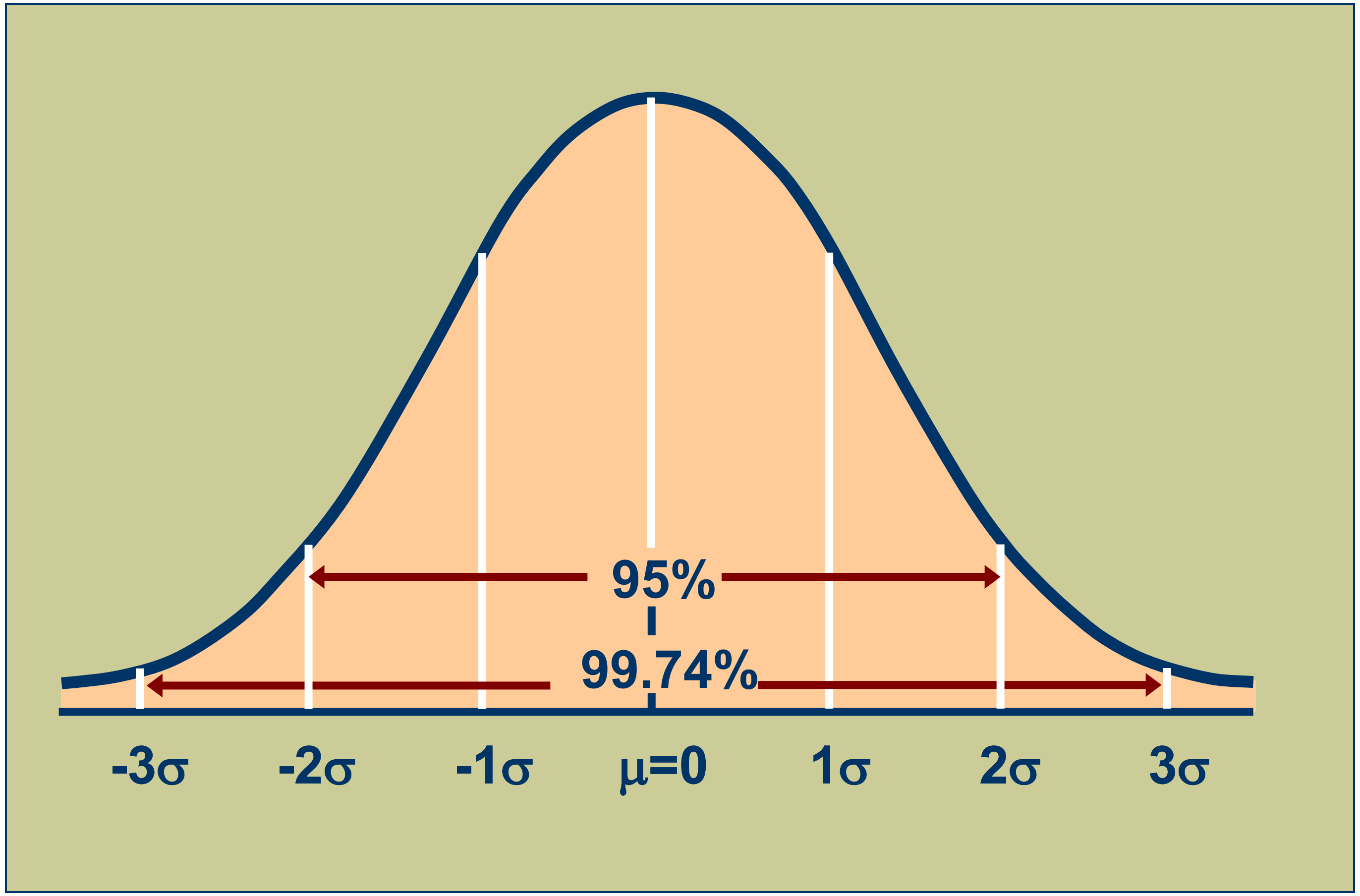
- A graph that establishes control limits of a process
- Control limits
  - upper and lower bands of a control chart

- Types of charts
  - Attributes
    - p-chart
    - c-chart
  - Variables
    - range (R-chart)
    - mean (x bar – chart)

Process Control Chart



Normal Distribution



A Process Is in Control If …

1. … no sample points outside limits
2. … most points near process average
3. … about equal number of points above and below centerline
4. … points appear randomly distributed

Control Charts for Attributes

- p-charts
  - uses portion defective in a sample
- c-charts
  - uses number of defects in an item

p-Chart

$$UCL = \bar{p} + z\sigma_p$$

$$LCL = \bar{p} - z\sigma_p$$

z = number of standard deviations from process average

$\bar{p}$ = sample proportion defective; an estimate of process average

σ_p = standard deviation of sample proportion

$$\sigma_p = \sqrt{\frac{\bar{p}(1 - \bar{p})}{n}}$$

p-Chart Example

SAMPLE	NUMBER OF DEFECTIVES	PROPORTION DEFECTIVE
1	6	.06
2	0	.00
3	4	.04
:	:	:
:	:	:
20	18	.18
	200	

20 samples of 100 pairs of jeans

p-Chart Example (cont.)

$$\bar{p} = \frac{\text{total defectives}}{\text{total sample observations}} = 200 / 20(100) = 0.10$$

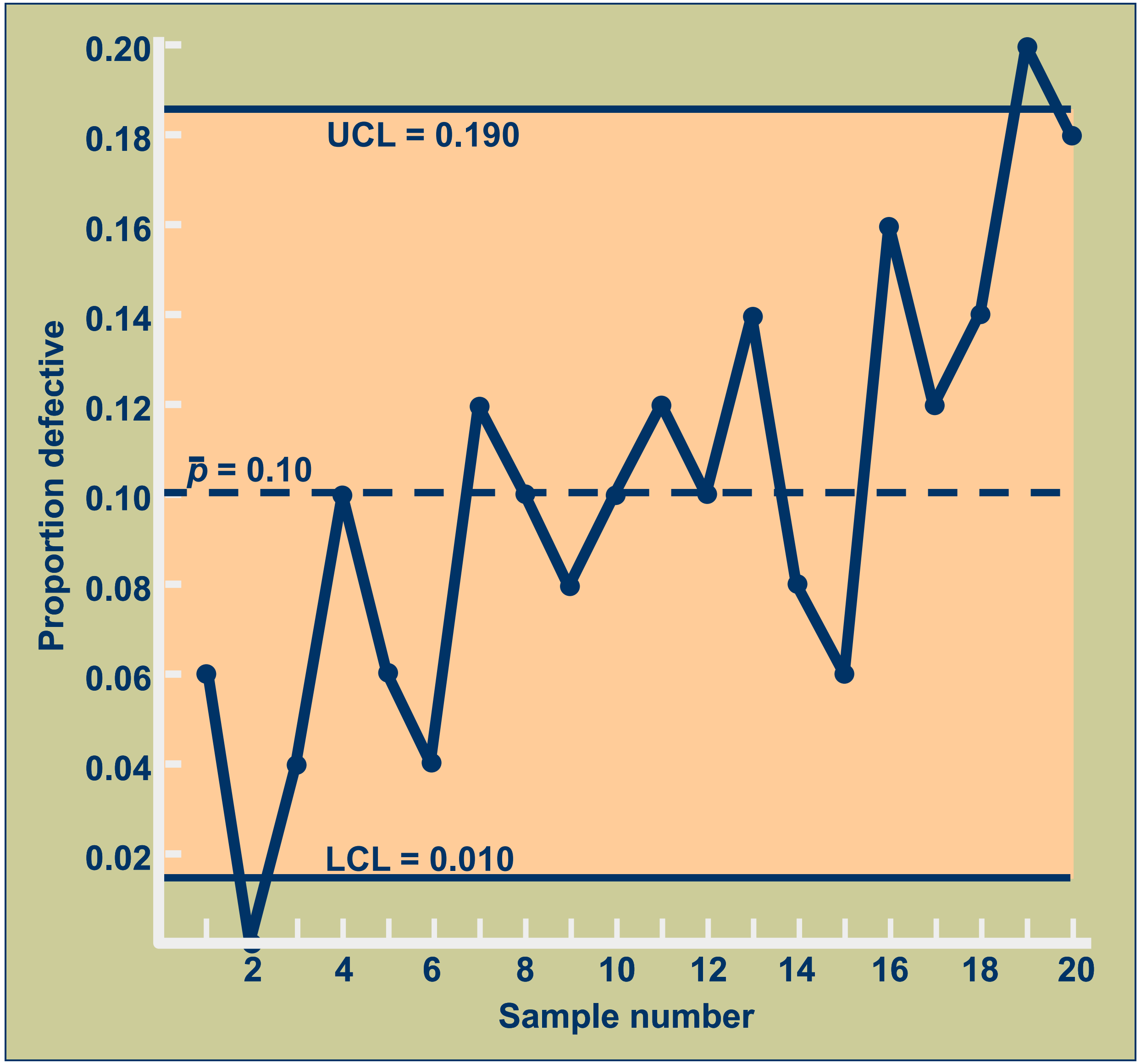
$$UCL = \bar{p} + z\sqrt{\frac{\bar{p}(1 - \bar{p})}{n}} = 0.10 + 3\sqrt{\frac{0.10(1 - 0.10)}{100}}$$

UCL = 0.190

$$LCL = \bar{p} - z\sqrt{\frac{\bar{p}(1 - \bar{p})}{n}} = 0.10 - 3\sqrt{\frac{0.10(1 - 0.10)}{100}}$$

LCL = 0.010

p-Chart Example (cont.)



c-Chart

$$UCL = \bar{c} + z\sigma_c$$

$$LCL = \bar{c} - z\sigma_c$$

$$\sigma_c = \sqrt{\bar{c}}$$

where

c = number of defects per sample

c-Chart (cont.)

Number of defects in 15 sample rooms

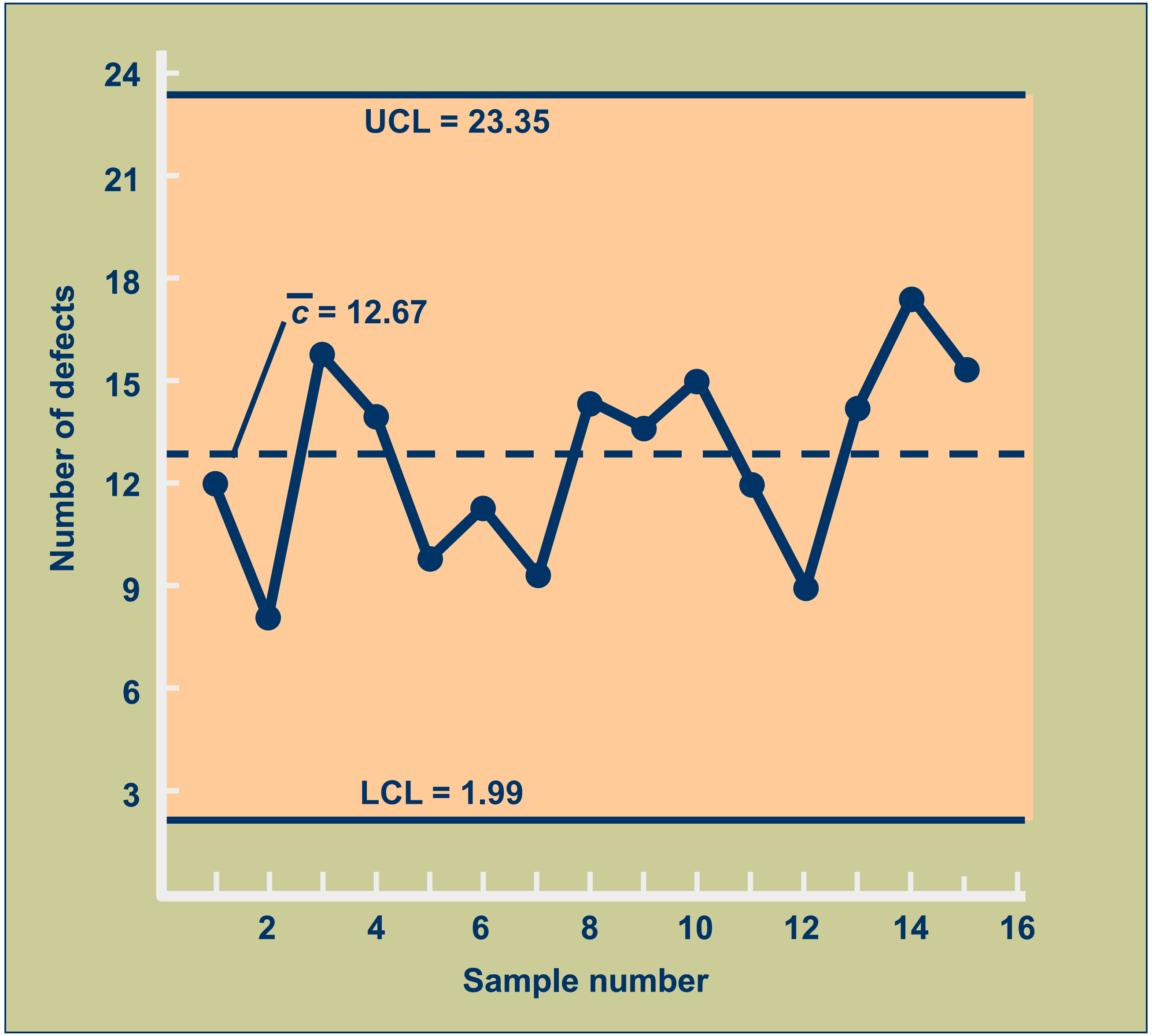
SAMPLE	NUMBER OF DEFECTS
1	12
2	8
3	16
:	:
:	:
15	15
	190

$$\bar{c} = \frac{190}{15} = 12.67$$

$$\begin{aligned} UCL &= \bar{c} + z\sigma_c \\ &= 12.67 + 3\sqrt{12.67} \\ &= 23.35 \end{aligned}$$

$$\begin{aligned} LCL &= \bar{c} + z\sigma_c \\ &= 12.67 - 3\sqrt{12.67} \\ &= 1.99 \end{aligned}$$

c-Chart (cont.)



24
21
18
15
12
9
6
3
UCL = 23.35
c̄ = 12.67
LCL = 1.99
Number of defects
2
4
6
8
10
12
14
16
Sample number

Control Charts for Variables

- Mean chart ($\bar{x}$ -Chart)
  - uses average of a sample
- Range chart (R-Chart)
  - uses amount of dispersion in a sample

x-bar Chart

$$\bar{\bar{x}} = \frac{\bar{x}_1 + \bar{x}_2 + \ldots \bar{x}_k}{k}$$

$$UCL = \bar{\bar{x}} + A_2\bar{R} \qquad LCL = \bar{\bar{x}} - A_2\bar{R}$$

where

$\bar{\bar{x}}$ = average of sample means

x-bar Chart Example

SAMPLE k	OBSERVATIONS (SLIP- RING DIAMETER, CM) 1	2	3	4	5	$\bar{x}$	R
1	5.02	5.01	4.94	4.99	4.96	4.98	0.08
2	5.01	5.03	5.07	4.95	4.96	5.00	0.12
3	4.99	5.00	4.93	4.92	4.99	4.97	0.08
4	5.03	4.91	5.01	4.98	4.89	4.96	0.14
5	4.95	4.92	5.03	5.05	5.01	4.99	0.13
6	4.97	5.06	5.06	4.96	5.03	5.01	0.10
7	5.05	5.01	5.10	4.96	4.99	5.02	0.14
8	5.09	5.10	5.00	4.99	5.08	5.05	0.11
9	5.14	5.10	4.99	5.08	5.09	5.08	0.15
10	5.01	4.98	5.08	5.07	4.99	5.03	0.10
						50.09	1.15

Example 15.4

x- bar Chart Example (cont.)

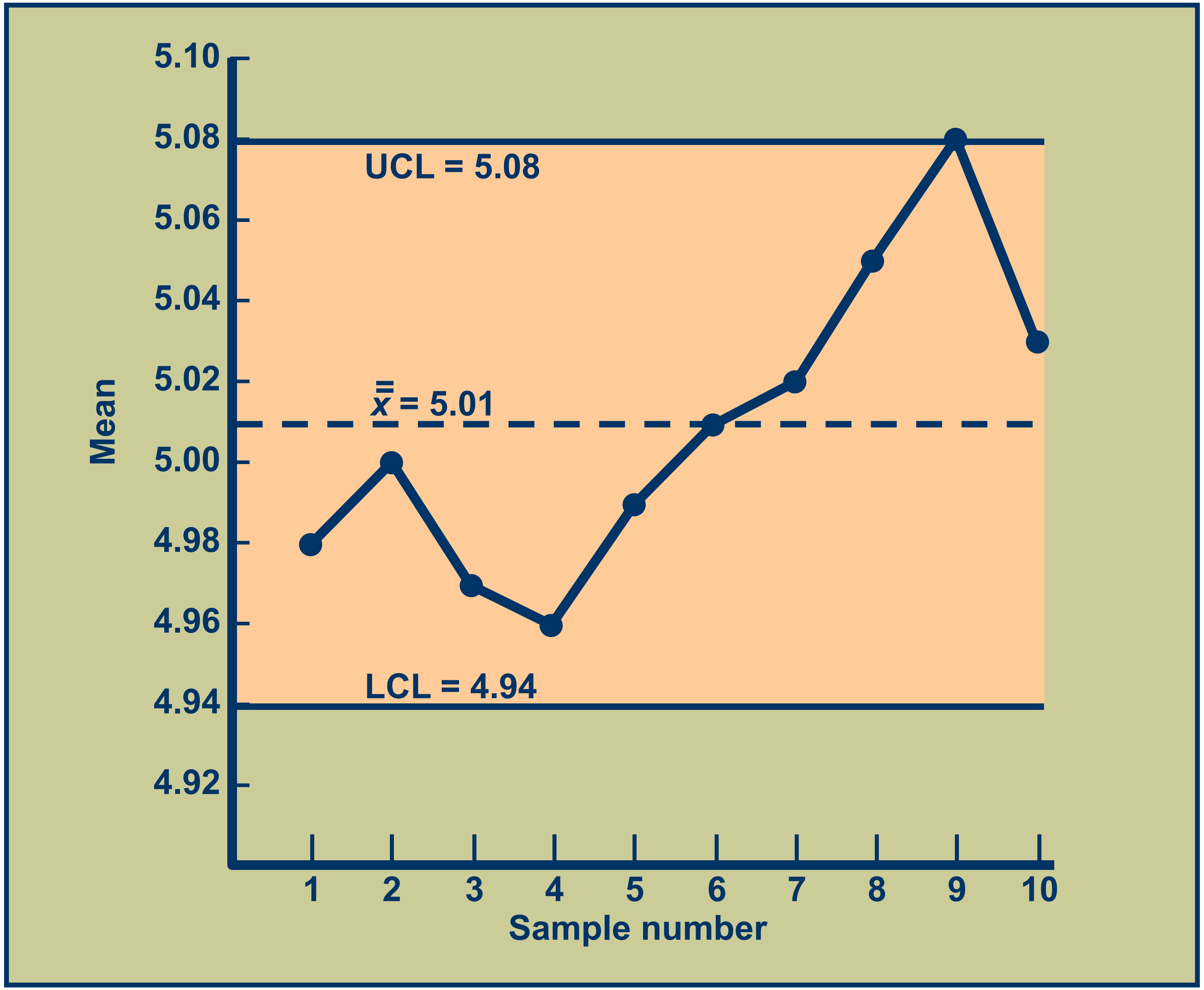
$$\bar{\bar{x}} = \frac{\sum \bar{x}}{k} = \frac{50.09}{10} = 5.01 \text{ cm}$$

$$UCL = \bar{\bar{x}} + A_2\bar{R} = 5.01 + (0.58)(0.115) = 5.08$$

$$LCL = \bar{\bar{x}} - A_2\bar{R} = 5.01 - (0.58)(0.115) = 4.94$$

Retrieve Factor Value A_2

x- bar Chart Example (cont.)



R- Chart

$$UCL = D_4\bar{R} \qquad LCL = D_3\bar{R}$$

$$\bar{R} = \frac{\sum R}{k}$$

where

$\bar{R}$ = range of each sample
k = number of samples

R-Chart Example

	OBSERVATIONS ()						
SAMPLE *k*	*1*	*2*	*3*	*4*	*5*	$\bar{x}$	*R*
1	5.02	5.01	4.94	4.99	4.96	4.98	0.08
2	5.01	5.03	5.07	4.95	4.96	5.00	0.12
3	4.99	5.00	4.93	4.92	4.99	4.97	0.08
4	5.03	4.91	5.01	4.98	4.89	4.96	0.14
5	4.95	4.92	5.03	5.05	5.01	4.99	0.13
6	4.97	5.06	5.06	4.96	5.03	5.01	0.10
7	5.05	5.01	5.10	4.96	4.99	5.02	0.14
8	5.09	5.10	5.00	4.99	5.08	5.05	0.11
9	5.14	5.10	4.99	5.08	5.09	5.08	0.15
10	5.01	4.98	5.08	5.07	4.99	5.03	0.10
						50.09	1.15

Example 15.3

R-Chart Example (cont.)

$$\bar{R} = \frac{\sum R}{k} = \frac{1.15}{10} = 0.115$$

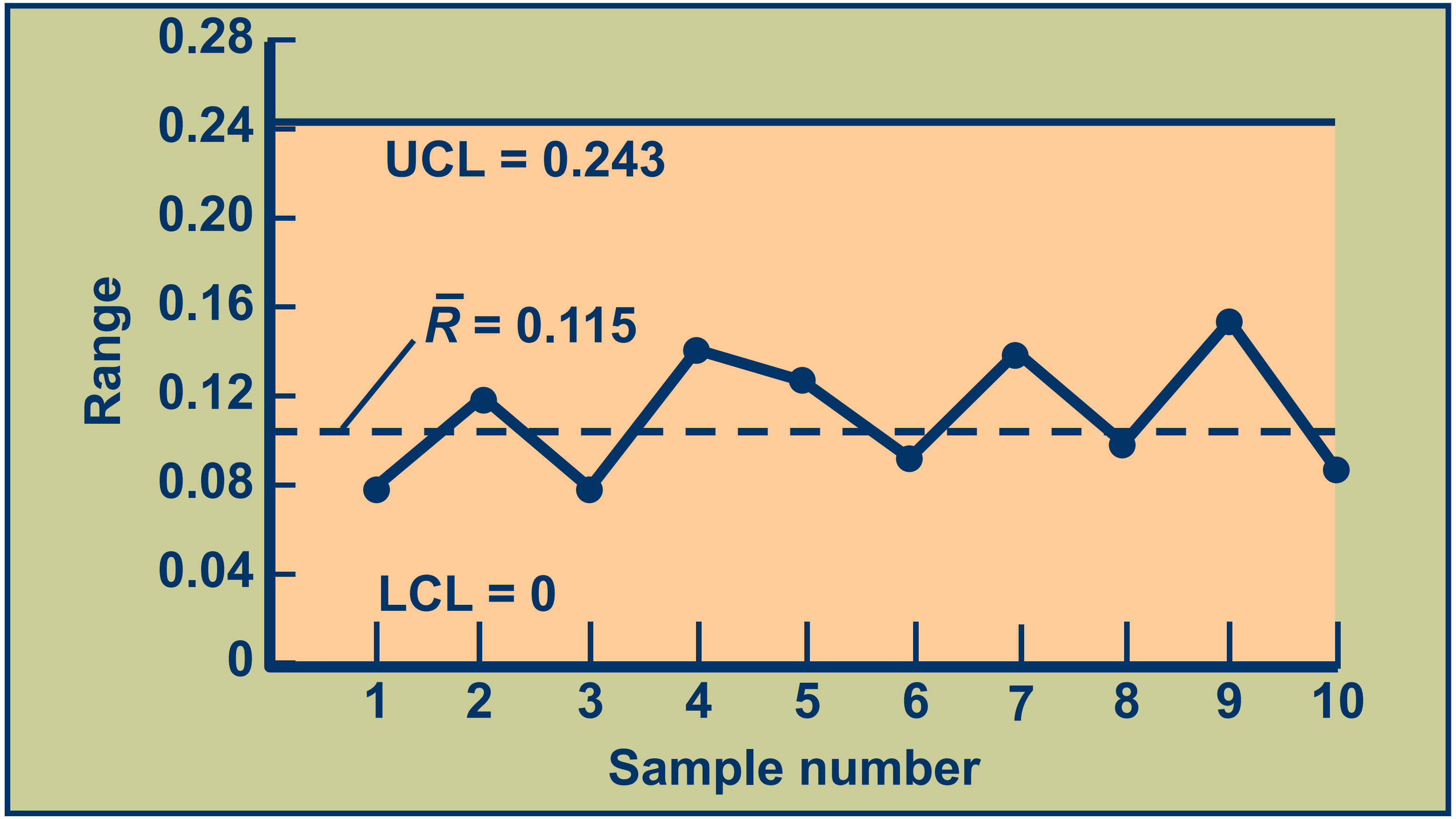
$$UCL = D_4\bar{R} = 2.11(0.115) = 0.243$$

$$LCL = D_3\bar{R} = 0(0.115) = 0$$

Retrieve Factor Values D_3 and D_4

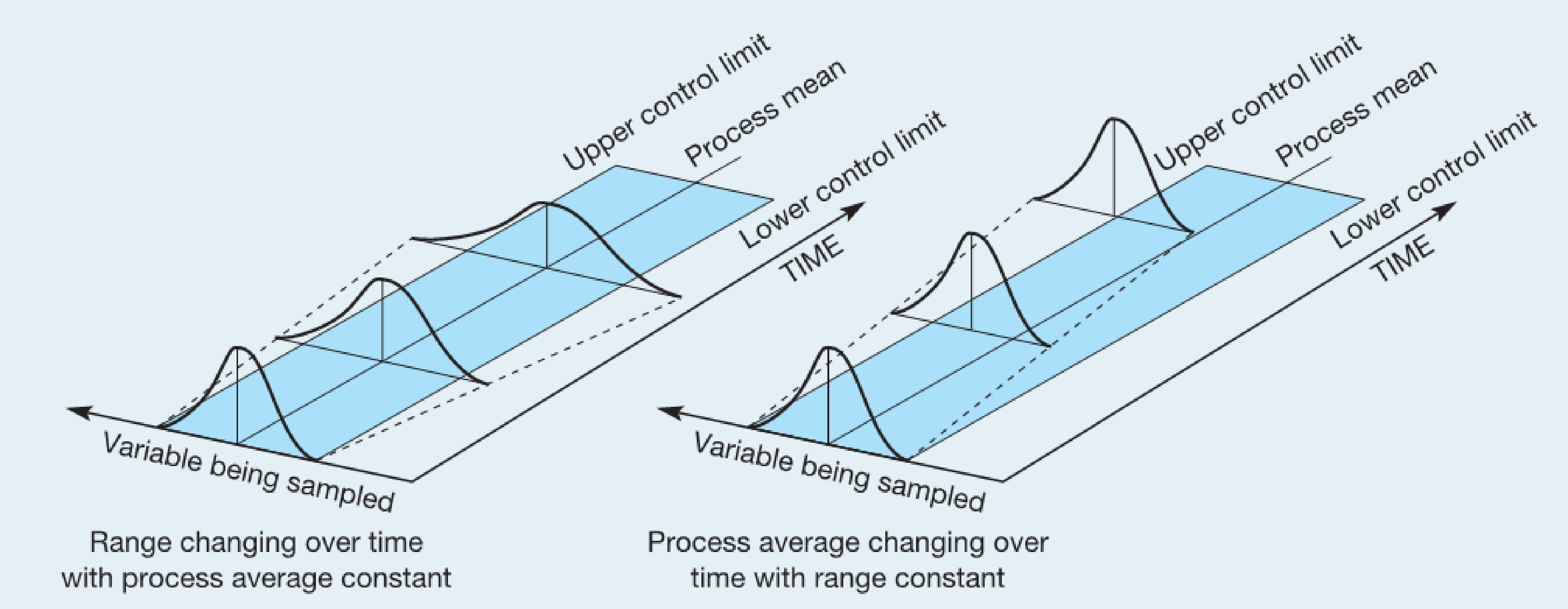
Example 15.3

R-Chart Example (cont.)

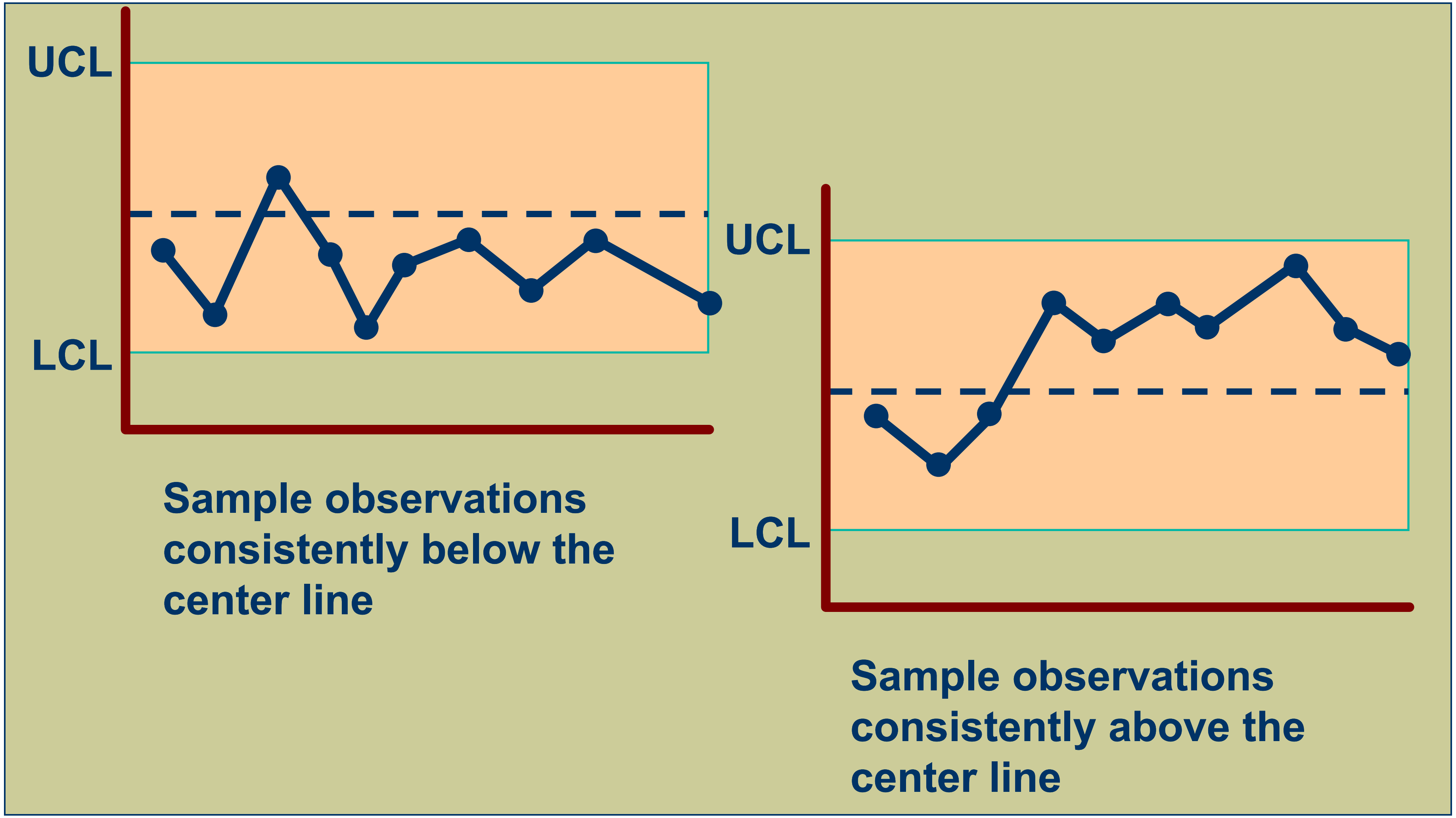


Using x- bar and R-Charts Together

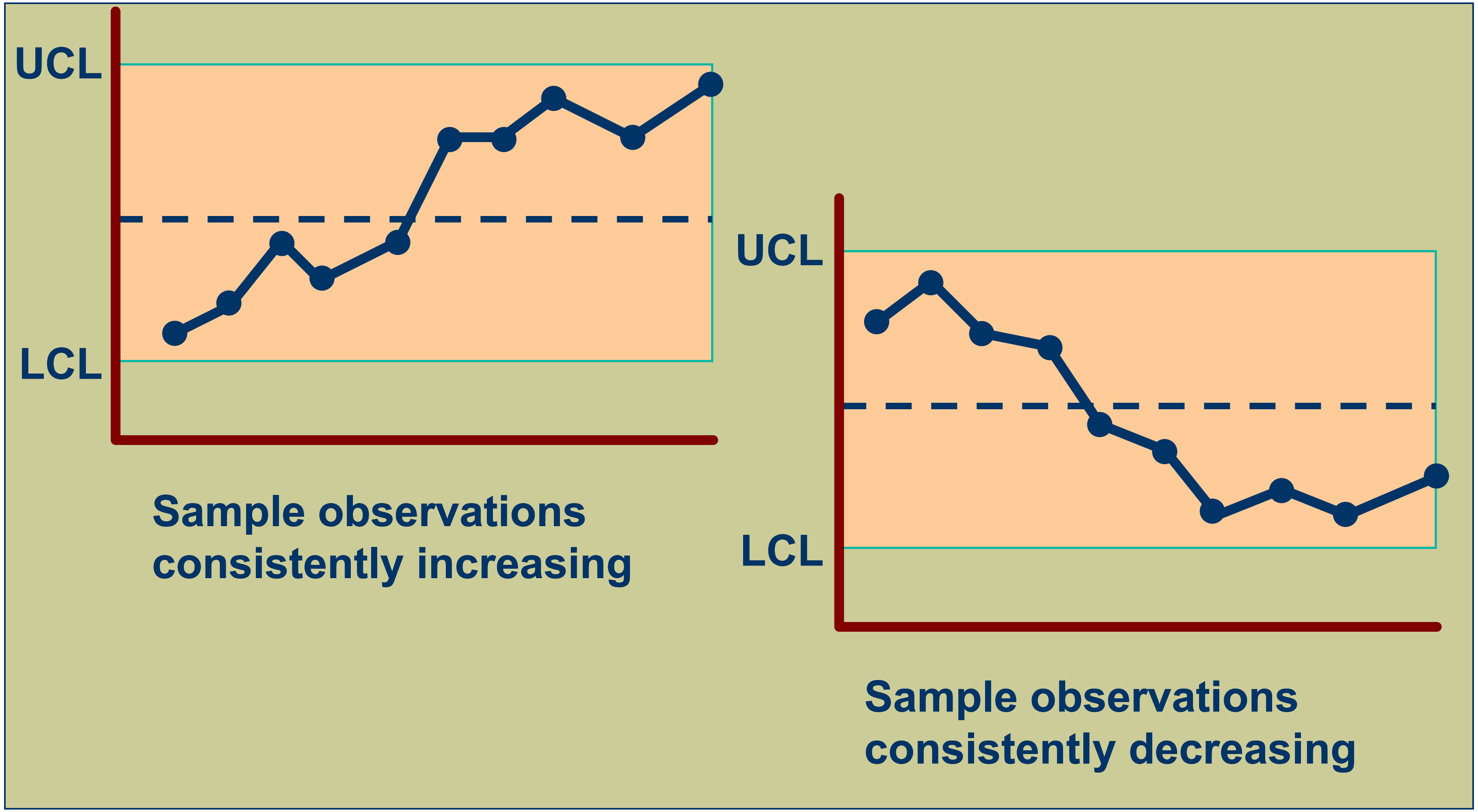
- Process average and process variability must be in control
- It is possible for samples to have very narrow ranges, but their averages is beyond control limits
- It is possible for sample averages to be in control, but ranges might be very large


Upper control limit
Process mean
Lower control limit
TIME
Variable being sampled
Range changing over time
with process average constant
Upper control limit
Process mean
Lower control limit
TIME
Variable being sampled
Process average changing over
time with range constant

Control Chart Patterns



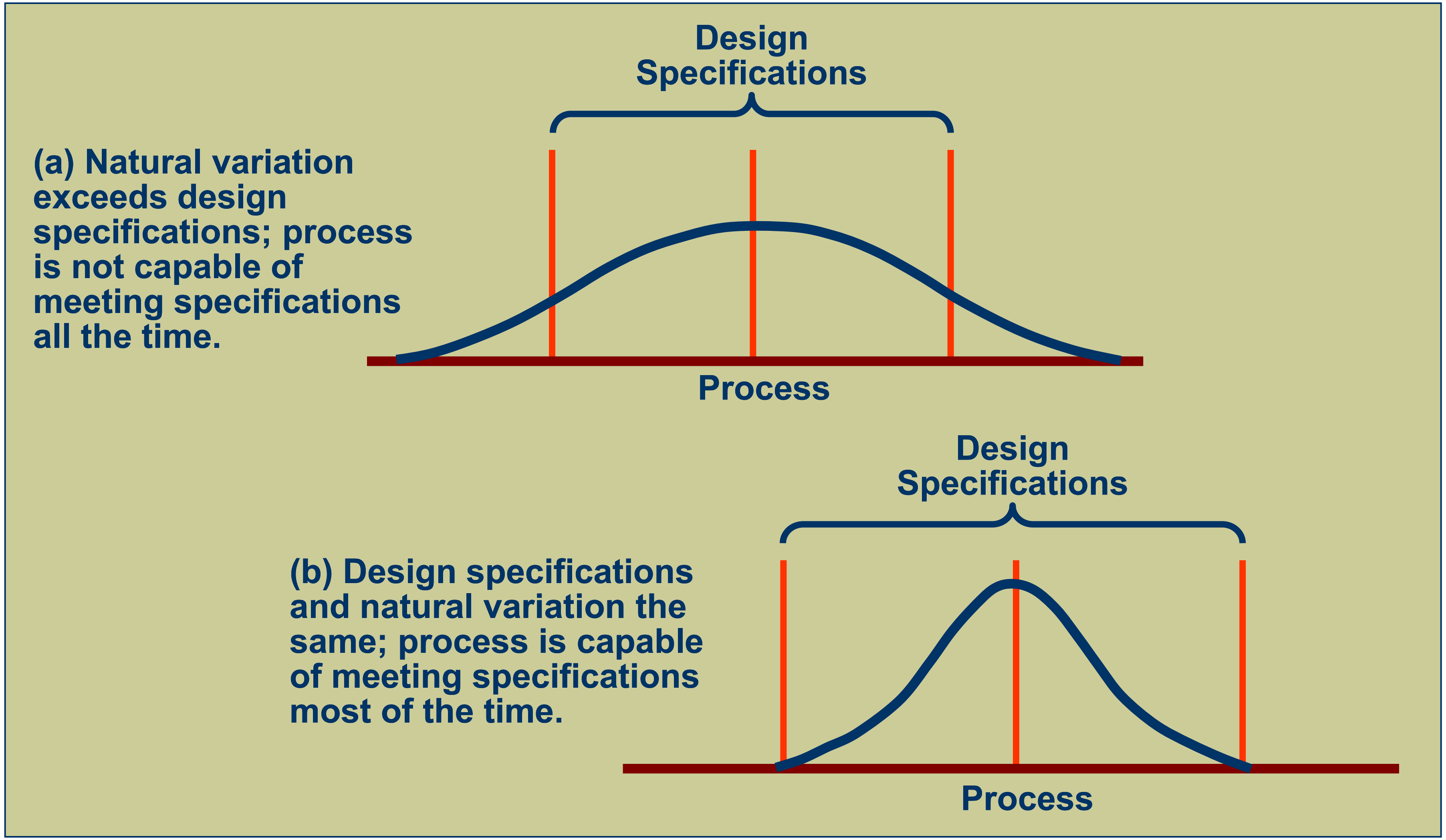
Control Chart Patterns (cont.)



Process Capability

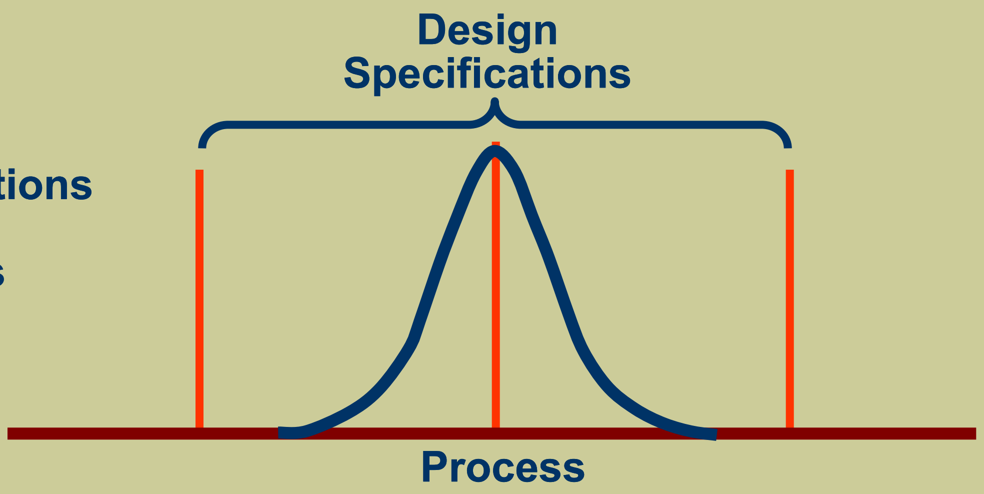
- Tolerances
  - design specifications reflecting product requirements
- Process capability
  - range of natural variability in a process what we measure with control charts

Process Capability



Process Capability (cont.)

(c) Design specifications greater than natural variation; process is capable of always conforming to specifications.



(d) Specifications greater than natural variation, but process off center; capable but some output will not meet upper specification.