and 10% blue M&Ms. The M&Ms are then transported along a special conveyor belt to a machine that stamps the "*m*" on the shell. A packaging machine weighs the candies, fills each package with the correct number and colors of candies, and then heat seals the package. SPC is used to monitor each step in this production process to make sure each individual M&M meets Mars' very high quality standards for taste, weight, shape, and color, and each package of M&Ms is as near perfect as possible.

In this chapter we will learn about the fundamentals of statistical process control (SPC) that companies like Mars and Hershey's that are committed to quality use as an important part of their quality management programs.

Source: Mars, Incorporated, www.mars.com and Hershey's Web site at www.thehersheycompany.com

fter World War II, W. E. Deming, the quality expert and consultant, was invited to Japan by the Japanese government to give a series of lectures on improving product reliability. This was probably the single most important event that started the Japanese toward a global quality revolution. These lectures were based on statistical quality control, and they became a cornerstone of the Japanese commitment to quality management.

A major topic in statistical quality control is *statistical process control*. **Statistical process control** (SPC) is a statistical procedure using control charts to see if any part of a production process is not functioning properly and could cause poor quality. SPC is used to inspect and measure the production process to see if it is varying from what it is supposed to be doing. If there is unusual or undesirable variability, the process is corrected so that defects will not occur. In this way, statistical process control is used to prevent poor quality before it occurs. It is such an important part of quality management that nearly all workers at all levels in companies committed to quality management are given extensive and continual training in SPC. Conversely, in many companies the reason cited for failure to achieve high quality is the lack of comprehensive training for employees in SPC methods. U.S. companies successful in quality management train employees in SPC methods and make extensive use of SPC for continuous process improvement.

THE BASICS OF STATISTICAL PROCESS CONTROL

Process control is achieved by taking periodic **samples** from the process and plotting these sample points on a chart, to see if the process is within statistical control limits. A sample can be a single item or a group of items. If a sample point is outside the limits, the process may be out of control, and the cause is sought so that the problem can be corrected. If the sample is within the control limits, the process continues without interference but with continued monitoring. In this way, SPC prevents quality problems by correcting the process before it starts producing defects.

No production process produces exactly identical items, one after the other. All processes contain a certain amount of variability that makes some variation between units inevitable. There are two reasons why a process might vary. The first is the inherent random variability of the process, which depends on the equipment and machinery, engineering, the operator, and the system used for measurement. This kind of variability is a result of natural occurrences. The other reason for variability is unique or special causes that are identifiable and can be corrected. These causes tend to be nonrandom and, if left unattended, will cause poor quality. These might include equipment that is out of adjustment, defective materials, changes in parts or materials, broken machinery or equipment, operator fatigue or poor work methods, or errors due to lack of training.

Statistical process control (SPC):

involves monitoring the production process to detect and prevent poor quality.

Employee training in SPC is a fundamental principle of TQM.

Sample:

a subset of the items produced to use for inspection.

All processes have variability—random and nonrandom (identifiable, correctable).



Richard Pasley Photography

A statistical control chart like this one is a graph to monitor a production process. Samples are taken from the process periodically, and the observations are plotted on the graph. If an observation is outside the upper or lower limits on the graph, it may indicate that something is wrong in the process; that is, it is not in control, which may cause defective or poor-quality items. By monitoring a process with a control chart, the employee and management can detect problems quickly and prevent poor-quality items from passing on through the remainder of the process and ending up as defective products that must be thrown away or reworked, thus wasting time and resources.

SPC IN QUALITY MANAGEMENT

Companies use SPC to see if their processes are in control—working properly. This requires that companies provide SPC training on a continuing basis that stresses that SPC is a tool individuals can use to monitor production or service process *for the purpose of making improvements*. Through the use of statistical process control, employees can be made responsible for quality in their area: to identify problems and either correct them or seek help in correcting them. By continually monitoring the production process and making improvements, the employee contributes to the goal of continuous improvement and few or no defects.

The first step in correcting the problem is identifying the causes. In Chapter 2 we described several quality-control tools used for identifying causes of problems, including brainstorming, Pareto charts, histograms, checksheets, quality circles, and fishbone (cause-and-effect) diagrams.

When an employee is unable to correct a problem, management typically initiates problem solving. This problem-solving activity may be within a group like a quality circle, or it may be less formal, including other employees, engineers, quality experts, and management. This group will brainstorm the problem to seek out possible causes. Alternatively, quality problems can be corrected through Six Sigma projects.

QUALITY MEASURES: ATTRIBUTES AND VARIABLES

The quality of a product or service can be evaluated using either an *attribute* of the product or service or a *variable measure*. An **attribute** is a product characteristic such as color, surface texture, cleanliness, or perhaps smell or taste. Attributes can be evaluated quickly with a discrete response such as good or bad, acceptable or not, or yes or no. Even if quality specifications are complex and extensive, a simple attribute test might be used to determine whether or not a product or service is defective. For example, an operator might test a light bulb by simply turning it on and seeing if it lights. If it does not, it can be examined to find out the exact technical cause for failure, but for SPC purposes, the fact that it is defective has been determined.

SPC is a tool for identifying problems in order to make improvements.

Attribute:

a product characteristic that can be evaluated with a discrete response (good/bad, yes/no).



Larry Lilac/Alamy

A variable measure is a product characteristic that is measured on a continuous scale such as length, weight, temperature, or time. For example, the amount of liquid detergent in a plastic container can be measured to see if it conforms to the company's product specifications. Or the time it takes to serve a customer at McDonald's can be measured to see if it is quick enough. Since a variable evaluation is the result of some form of measurement, it is sometimes referred to as a *quantitative* classification method. An attribute evaluation is sometimes referred to as a *qualitative* classification, since the response is not measured. Because it is a measurement, a variable classification typically provides more information about the product—the weight of a product is more informative than simply saying the product is good or bad.

SPC APPLIED TO SERVICES

Control charts have historically been used to monitor the quality of manufacturing processes. However, SPC is just as useful for monitoring quality in services. The difference is the nature of the "defect" being measured and monitored. Using Motorola's definition—*a failure to meet customer requirements in any product or service*—a defect can be an empty soap dispenser in a restroom or an error with a phone catalogue order, as well as a blemish on a piece of cloth or a faulty tray on a DVD player. Control charts for service processes tend to use quality characteristics and measurements such as time and customer satisfaction (determined by surveys, questionnaires, or inspections). Following is a list of several different services and the quality characteristics for each that can be measured and monitored with control charts.

Hospitals: Timeliness and quickness of care, staff responses to requests, accuracy of

- lab tests, cleanliness, courtesy, accuracy of paperwork, speed of admittance and checkouts. *Grocery stores:* Waiting time to check out, frequency of out-of-stock items, quality of food
- items, cleanliness, customer complaints, checkout register errors.
- *Airlines:* Flight delays, lost luggage and luggage handling, waiting time at ticket counters and check-in, agent and flight attendant courtesy, accurate flight information, passenger cabin cleanliness and maintenance.
- *Fast-food restaurants:* Waiting time for service, customer complaints, cleanliness, food quality, order accuracy, employee courtesy.
- *Catalogue-order companies:* Order accuracy, operator knowledge and courtesy, packaging, delivery time, phone order waiting time.
- Insurance companies: Billing accuracy, timeliness of claims processing, agent availability and response time.

Housekeepers at luxury hotels like the Ritz-Carlton strive to achieve the hotel's goal of a totally defect-free guest experience. Housekeeping processes are closely monitored for defects using statistical process control techniques.

Variable measure:

a product characteristic that is continuous and can be measured (weight, length).

A service defect is a failure to meet customer requirements.

WHERE TO USE CONTROL CHARTS

Most companies do not use control charts for every step in a process. Although that might be the most effective way to ensure the highest quality, it is costly and time consuming. In most manufacturing and service processes, there are clearly identifiable points where control charts should be used. In general, control charts are used at critical points in the process where historically the process has shown a tendency to go out of control, and at points where if the process goes out of control it is particularly harmful and costly. For example, control charts are frequently used at the beginning of a process to check the quality of raw materials and parts, or supplies and deliveries for a service operation. If material and parts are bad to begin with, it is a waste of time and money to begin the production process with them. Control charts are also used before a costly or irreversible point in the process, after which the product is difficult to rework or correct; before and after assembly or painting operations that might cover defects; and before the outgoing final product or service is shipped or delivered.

CONTROL CHARTS

Control chart:

a graph that establishes the control limits of a process.

Control limits:

the upper and lower bands of a control chart.

Types of charts: attributes, p, and c; variables, \overline{x} and R.

Sigma limits are the number of standard deviations.

Control charts are graphs that visually show if a sample is within statistical **control limits**. They have two basic purposes: to establish the control limits for a process and then to monitor the process to indicate when it is out of control. Control charts exist for attributes and variables; within each category there are several different types of control charts. We will present four commonly used control charts, two in each category: *p*-charts and *c*-charts for attributes and *mean* (\bar{x}) and *range* (R) control charts for variables. Even though these control charts differ in how they measure process control, they all have certain similar characteristics. They all look alike, with a line through the center of a graph that indicates the process average and lines above and below the center line that represent the upper and lower limits of the process, as shown in Figure 3.1.

The formulas for conducting upper and lower limits in control charts are based on a number of standard deviations, "z," from the process average (e.g., center line) according to a normal distribution. Occasionally, z is equal to 2.00 but most frequently is 3.00. A z value of 2.00 corresponds to an overall normal probability of 95%, and z = 3.00 corresponds to a normal probability of 99.73%.

The normal distribution in Figure 3.2 on page 114 shows the probabilities corresponding to z values equal to 2.00 and 3.00 standard deviations (σ).

The smaller the value of z, the more narrow the control limits are and the more sensitive the chart is to changes in the production process. Control charts using z = 2.00 are often referred to as having 2-sigma (2σ) limits (referring to two standard deviations), whereas z = 3.00 means 3-sigma (3σ) limits.





Figure 3.2 The Normal Distribution

Management usually selects z = 3.00 because if the process is in control it wants a high probability that the sample values will fall within the control limits. In other words, with wider limits management is less likely to (erroneously) conclude that the process is out of control when points outside the control limits are due to normal, random variations. Alternatively, wider limits make it harder to detect changes in the process that are not random and have an assignable cause. A process might change because of a nonrandom, assignable cause and be detectable with the narrower limits but not with the wider limits. However, companies traditionally use the wider control limits.

Each time a sample is taken, the mathematical average of the sample is plotted as a point on the control chart as shown in Figure 3.1. A process is generally considered to be in control if, for example,

- 1. There are no sample points outside the control limits.
- **2.** Most points are near the process average (i.e., the center line), without too many close to the control limits.
- **3.** Approximately equal numbers of sample points occur above and below the center line.
- 4. The points appear to be randomly distributed around the center line (i.e., no discernible pattern).

If any of these conditions are violated, the process may be *out of control*. The reason must be determined, and if the cause is not random, the problem must be corrected.

Sample 9 in Figure 3.1 is above the upper control limit, suggesting the process is out of control (i.e., something unusual has happened). The cause is not likely to be random since the sample points have been moving toward the upper limit, so management should attempt to find out what is wrong with the process and bring it back in control. Perhaps the employee was simply interrupted. Although the other samples display some degree of variation from the process average, they are usually considered to be caused by normal, random variability in the process and are thus in control. However, it is possible for sample observations to be within the control limits and the process to be out of control anyway, if the observations display a discernible, abnormal pattern of movement. We discuss such patterns in a later section.

After a control chart is established, it is used to determine when a process goes out of control and corrections need to be made. As such, a process control chart should be based only on sample observations from when the process is in control so that the control chart reflects a true benchmark for an in-control process. However, it is not known whether or not the process is in control until the control chart is first constructed. Therefore, when a control chart is first developed if the process is found to be out of control, the process should be examined and corrections made. A new center line and control limits should then be determined from a new set of sample observations. This "corrected" control chart is then used to monitor the process. It may not be possible to discover the cause(s) for the out-of-control sample observations. In this case a new set of samples is taken, and a completely new control chart constructed. Or it may be decided to simply use the initial control chart, assuming that it accurately reflects the process variation.

CONTROL CHARTS FOR ATTRIBUTES

The quality measures used in *attribute control charts* are discrete values reflecting a simple decision criterion such as good or bad. A *p*-chart uses the proportion of defective items in a sample as the sample statistic; a *c*-chart uses the actual number of defects per item in a sample. A *p*-chart can be used when it is possible to distinguish between defective and nondefective items and to state the number of defectives as a percentage of the whole. In some processes, the proportion defective cannot be

A sample point can be within the control limits and the process still be out of control.

The development of a control chart.

 p-chart: uses the proportion defective in a sample.

CONTROL CHARTS FOR VARIABLES

Variable control charts are used for continuous variables that can be measured, such as weight or volume. Two commonly used variable control charts are the range chart, or *R*-chart, and the mean chart, or \bar{x} -chart. A range (*R*-) chart reflects the amount of dispersion present in each sample; a mean (\bar{x} -) chart indicates how sample results relate to the process average or mean. These charts are normally used together to determine whether a process is in control.

MEAN (\overline{x} -) CHART

In a mean (or \bar{x}) control chart, each time a sample of a group of items is taken from the process, the mean of the sample is computed and plotted on the chart. Each sample mean (\bar{x}) is a point on the control chart. The samples taken tend to be small, usually around 4 or 5. The center line of the control chart is the overall process average, that is, the mean of the sample means.

The \bar{x} -chart is based on the normal distribution. It can be constructed in two ways depending on the information that is available about the distribution. If the standard deviation of the distribution is known from past experience or historical data, then formulas using the standard deviation can be used to compute the upper and lower control limits. If the standard deviation is not known, then a table of values based on sample ranges is available to develop the upper and lower control limits. We will first look at how to construct an \bar{x} -chart when the standard deviation is known.

The formulas for computing the upper control limit (UCL) and lower control limit (LCL) are

UCL =
$$\overline{x} + z\sigma_{\overline{x}}$$

LCL = $\overline{\overline{x}} - z\sigma_{\overline{x}}$

where

 $\overline{\overline{x}}$ = process average = $\frac{\overline{x}_1 + \overline{x}_2 + \dots + \overline{x}_n}{k}$

 σ = process standard deviation

 $\sigma_{\bar{x}}$ = standard deviation of sample means = σ/\sqrt{k}

k =sample (number of subgroups)

n = sample size (number of observations in each subgroup)

Example 3.3 illustrates how to develop an \bar{x} -chart using these formulas.

The Goliath Tool Company produces slip-ring bearings, which look like flat doughnuts or washers. They fit around shafts or rods, such as drive shafts in machinery or motors. At an early stage in the production process for a particular slip-ring bearing, the outside diameter of the bearing is measured. Employees have taken 10 samples (during a 10-day period) of 5 slip-ring bearings and measured the diameter of the bearings. The individual observations from each sample (or subgroup) are shown as follows:

	Observations (Slip-Ring Diameter, cm), n						
Subgroup k	1	2	3	4	5	\overline{x}	
1	5.02	5.01	4.94	4.99	4.96	4.98	
2	5.01	5.03	5.07	4.95	4.96	5.00	
3	4.99	5.00	4.93	4.92	4.99	4.97	
4	5.03	4.91	5.01	4.98	4.89	4.96	
5	4.95	4.92	5.03	5.05	5.01	4.99	
6	4.97	5.06	5.06	4.96	5.03	5.01	
7	5.05	5.01	5.10	4.96	4.99	5.02	
8	5.09	5.10	5.00	4.99	5.08	5.05	
9	5.14	5.10	4.99	5.08	5.09	5.08	
10	5.01	4.98	5.08	5.07	4.99	5.03	
						50.09	

• Range (*R*-) chart: uses the amount of dispersion in a sample.

 Mean (x̄-) chart: uses the process average of a sample.

Example 3.3 Constructing an x-Chart

(Continued)

From past historical data it is known that the process standard deviation is .08. The company wants to develop a control chart with 3-sigma limits to monitor this process in the future. The process average is computed as

$$\bar{\bar{x}} = \frac{50.09}{10} = 5.02$$

The control limits are

UCL =
$$\bar{x} + z\sigma_{\bar{x}}$$

= 5.01 + 3(.08/ $\sqrt{10}$)
= 5.09
LCL = $\bar{\bar{x}} - z\sigma_{\bar{x}}$
= 5.01 - 3(.08/ $\sqrt{10}$)
= 4.93

None of the sample means (\bar{x}) falls outside these control limits, which indicates that the process is *in control* and this is an accurate control chart.

In the second approach to developing an \bar{x} -chart, the following formulas are used to compute the control limits:

$$UCL = \overline{\overline{x}} + A_2 \overline{R}$$
$$LCL = \overline{\overline{x}} - A_2 \overline{R}$$

where \overline{x} is the average of the sample means and \overline{R} is the average range value. A_2 is a tabular value that is used to establish the control limits. Values of A_2 are included in Table 3.1. They were developed specifically for determining the control limits for \overline{x} -charts and are comparable to three-standard deviation (3σ) limits. These table values are frequently used to develop control charts.



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A Nestle's quality control team tests samples of candies. The sample results can be plotted on a control chart to see if the production process is in control. If not, it will be corrected before a large number of defective candies are produced, thereby preventing costly waste.

Sample Size	Factor for \overline{x} -Chart	Factors for <i>R</i> -Chart		
n	A_2	D_3	D ₄	
2	1.88	0	3.27	
3	1.02	0	2.57	
4	0.73	0	2.28	
5	0.58	0	2.11	
6	0.48	0	2.00	
7	0.42	0.08	1.92	
8	0.37	0.14	1.86	
9	0.34	0.18	1.82	
10	0.31	0.22	1.78	
11	0.29	0.26	1.74	
12	0.27	0.28	1.72	
13	0.25	0.31	1.69	
14	0.24	0.33	1.67	
15	0.22	0.35	1.65	
16	0.21	0.36	1.64	
17	0.20	0.38	1.62	
18	0.19	0.39	1.61	
19	0.19	0.40	1.60	
20	0.18	0.41	1.59	
21	0.17	0.43	1.58	
22	0.17	0.43	1.57	
23	0.16	0.44	1.56	
24	0.16	0.45	1.55	
25	0.15	0.46	1.54	

Table 3.1
Factors for
Determining
Control Limits for
x- and R-Charts

The Goliath Tool Company desires to develop an \bar{x} -chart using table values. The sample data collected for this process with ranges is shown in the following table.

		Observations (Slip-Ring Diameter, cm), n						
Sample k	1	2	3	4	5	\overline{x}	R	
1	5.02	5.01	4.94	4.99	4.96	4.98	0.08	
2	5.01	5.03	5.07	4.95	4.96	5.00	0.12	
3	4.99	5.00	4.93	4.92	4.99	4.97	0.08	
4	5.03	4.91	5.01	4.98	4.89	4.96	0.14	
5	4.95	4.92	5.03	5.05	5.01	4.99	0.13	
6	4.97	5.06	5.06	4.96	5.03	5.01	0.10	
7	5.05	5.01	5.10	4.96	4.99	5.02	0.14	
8	5.09	5.10	5.00	4.99	5.08	5.05	0.11	
9	5.14	5.10	4.99	5.08	5.09	5.08	0.15	
10	5.01	4.98	5.08	5.07	4.99	5.03	0.10	
						50.09	1.15	

The company wants to develop an \bar{x} -chart to monitor the process.

 $\frac{\text{Example 3.4}}{\text{An } \overline{x}\text{-Chart}}$

(Continued)

Solution

 \overline{R} is computed by first determining the range for each sample by computing the difference between the highest and lowest values as shown in the last column in our table of sample observations. These ranges are summed and then divided by the number of samples, k, as follows:

$$\overline{R} = \frac{\sum R}{k} = \frac{1.15}{10} = 0.115$$

 $\overline{\overline{x}}$ is computed as follows:

RANGE (R-) CHART

$$\overline{\overline{x}} = \frac{\sum \overline{x}}{10} = \frac{50.09}{10} = 5.01 \text{ cm}$$

Using the value of $A_2 = 0.58$ for n = 5 from Table 3.1 and $\overline{R} = 0.115$, we compute the control limits as

UCL =
$$\bar{x} + A_2 R$$

= 5.01 + (0.58)(0.115) = 5.08
LCL = $\bar{x} - A_2 \bar{R}$
= 5.01 - (0.58)(0.115) = 4.94

The \bar{x} -chart defined by these control limits is shown in the following figure. Notice that the process is on the UCL for sample 9; in fact, samples 4 to 9 show an upward trend. This would suggest that the process variability is subject to nonrandom causes and should be investigated.



Range:

the difference between the smallest and largest values in a sample. In an *R*-chart, the **range** is the difference between the smallest and largest values in a sample. This range reflects the process variability instead of the tendency toward a mean value. The formulas for determining control limits are

$$UCL = D_4 \overline{R}$$
$$LCL = D_3 \overline{R}$$

 \overline{R} is the average range (and center line) for the samples,

$$\overline{R} = \frac{\sum R}{k}$$

where

R = range of each sample k = number of samples (sub groups)

 D_3 and D_4 are table values like A_2 for determining control limits that have been developed based on range values rather than standard deviations. Table 3.1 also includes values for D_3 and D_4 for sample sizes up to 25.

The Goliath Tool Company from Examples 3.3 and 3.4 wants to develop an *R*-chart to control process variability.

From Example 3.4, $\overline{R} = 0.115$; from Table 3.1 for n = 5, $D_3 = 0$ and $D_4 = 2.11$. Thus, the control limits are,

UCL =
$$D_4 \overline{R}$$
 = 2.11(0.115) = 0.243
LCL = $D_3 \overline{R}$ = 0(0.115) = 0

These limits define the *R*-chart shown in the following figure. It indicates that the process appears to be in control; any variability observed is a result of natural random occurrences.



This example illustrates the need to employ the *R*-chart and the \bar{x} -chart together. The *R*-chart in this example suggests that the process is in control, since none of the ranges for the samples are close to the control limits. However, the \bar{x} -chart in Example 3.4 suggests that the process is not in control. In fact, the ranges for samples 8 and 10 were relatively narrow, whereas the means for these samples were relatively high. The use of both charts together provided a more complete picture of the overall process variability.

USING x- AND R-CHARTS TOGETHER

The \bar{x} -chart is used with the *R*-chart under the premise that both the process average and variability must be in control for the process to be in control. This is logical. The two charts measure the process differently. It is possible for samples to have very narrow ranges, suggesting little process variability, but the sample averages might be beyond the control limits.

For example, consider two samples, the first having low and high values of 4.95 and 5.05 centimeters, and the second having low and high values of 5.10 and 5.20 centimeters. The range of both is 0.10 centimeters, but \bar{x} for the first is 5.00 centimeters and \bar{x} for the second is 5.15 centimeters. The two sample ranges might indicate the process is in control and $\bar{x} = 5.00$ might be okay, but $\bar{x} = 5.15$ could be outside the control limit. Both the process average and variability must be in control.

Example 3.5 Constructing an *R*-Chart Conversely, it is possible for the sample averages to be in control, but the ranges might be very large. For example, two samples could both have $\bar{x} = 5.00$ centimeters, but sample 1 could have a range between 4.95 and 5.05 (R = 0.10 centimeter) and sample 2 could have a range between 4.80 and 5.20 (R = 0.40 centimeter). Sample 2 suggests the process is out of control.

It is also possible for an *R*-chart to exhibit a distinct downward trend in the range values, indicating that the ranges are getting narrower and there is less variation. This would be reflected on the \bar{x} -chart by mean values closer to the center line. Although this occurrence does not indicate that the process is out of control, it does suggest that some nonrandom cause is reducing process variation. This cause needs to be investigated to see if it is sustainable. If so, new control limits would need to be developed. Sometimes an \bar{x} -chart is used alone to see if a process is improving, perhaps toward a specific performance goal.

In other situations, a company may have studied and collected data for a process for a long time and already know what the mean and standard deviation of the process are; all they want to do is monitor the process average by taking periodic samples. In this case it would be appropriate to use the mean chart where the process standard deviation is already known as shown in Example 3.3.

CONTROL CHART PATTERNS

A pattern can indicate an out-of-control process even if sample values are within control limits. Even though a control chart may indicate that a process is in control, it is possible the sample variations within the control limits are not random. If the sample values display a consistent pattern, even within the control limits, it suggests that this pattern has a nonrandom cause that might warrant investigation. We expect the sample values to "bounce around" above and below the center line, reflecting the natural, random variation in the process that will be present. However, if the sample values are consistently above (or below) the center line for an extended number of samples or if they move consistently up or down, there is probably a reason for this behavior; that is, it is not random. Examples of nonrandom patterns are shown in Figure 3.3.

