AMEM: Basic New Keynesian Model

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About New Keynesian Models

Basics

- One of the most common macroeconomic model frameworks. Go-to framework for business cycle modeling.
- Sits at the heart of most (all?) models used by central banks for monetary policy analysis and forecasting.
- We'll start with 3-equation New Keynesian model (3ENKM)

Why is 3ENKM popular?

- Simple core consisting of three equations that link basic macro variables: GDP, inflation, interest rates.
- Helps make sense of many stylized facts we observe in the economy, in general provides a good tool to understand business cycles.
- Introduces nominal rigidities and sticky prices, suitable for analysis of monetary policy and business cycle.
  - Overwhelming empirical support for sticky prices.
  - Other popular models (RBC, Solow, growth models) cannot replace 3ENKM.
    - RBC: Monetary neutrality, monetary policy in the model has no impact
    - Solow: Focus on modeling long-term trends, no focus on business cycle.
- Incorporates rational expectations.
- Can be derived from micro-foundations.
- Good description of demand shocks which matter for monetary policy.
- Large literature which extends the model in various directions.

Drawbacks:

- Model doesn't say anything about long-term trends. Trends are often exogenous, or ignored completely.

- Theoretical, micro-founded version of the model imposes unrealistic restrictions on parameters.
- Theoretical 3ENKM has problems fitting data, requires various modifications to be relevant in practice. We treat 3ENKM as a path to QPM (small open economy model).

Sources:

- The "Bible": https://books.google.cz/books/about/Monetary_Policy_Inflation_and_the_Busine.html?id=_1CUBgAAQBAJ&redir_esc=y
- Influential paper: https://www.aeaweb.org/articles?id=10.1257/jel.37.4.1661
- Or just use Google / AI

The Model

Some model-building principles

1. Clarify the question. What problem do you want the model to solve?
2. Consider the relevant mechanisms.
3. Collect and examine the data.
4. Include only what is relevant. Keep the model tractable.

Overview

The basic model has three types of agents:

1. Households - work, save, own firms, buy consumption goods. Fixed labor supply. There is an infinite amount of identical households.
2. Firms - monopolistically competitive firms with limited (but non-negligible) market power. Subject to menu costs / price adjustment costs. Maximize profits given their limited market power.
3. Monetary authority: sets interest rates in line with their objective (usually inflation).

Note what is **NOT** in the model: almost everything. When explaining model behaviors, we need to stick to what is in, not invoke e.g. investment.

Derivation, log-linearization

Derivation of the basic equations can be done by specifying optimization problems, budget constraints, taking first-order conditions, etc. This requires considerable effort. Good to do that once, but not necessary or even useful to work with the model.

When equations are derived, they are usually log-linearized (converted into log-deviations from steady-state). This also requires considerable effort and can be quite painful experience. For overview how log-linearization works, use Google. See e.g. here: https://www.macroeconomics.tu-berlin.de/fileadmin/fg124/advanced-macro/2014/Log-Linearization.pdf

We will skip both these steps for the sake of brewity, efficiency, and my sanity.

See Study materials (file "*Gali-odvozeni*") for detailed derivation.

Conventions

The following conventions will apply in the document:

- Hat denotes gap variables, e.g. output gap $\hat{y}_t$
- Bar denotes trend variables, e.g. output trend $\overline{y}_t$
- All variables will be expressed in $100 \cdot \log()$, for mathematical convenience

Output - IS Curve

Links household consumption (domestic demand, output gap) to interest rates: intertemporal decision, consumption now vs saving (consumption in future).

Simplest form of the IS Curve:

$$\hat{y}_t = E_t\hat{y}_{t+1} - \alpha \cdot \hat{r}_t + \epsilon_t^y$$

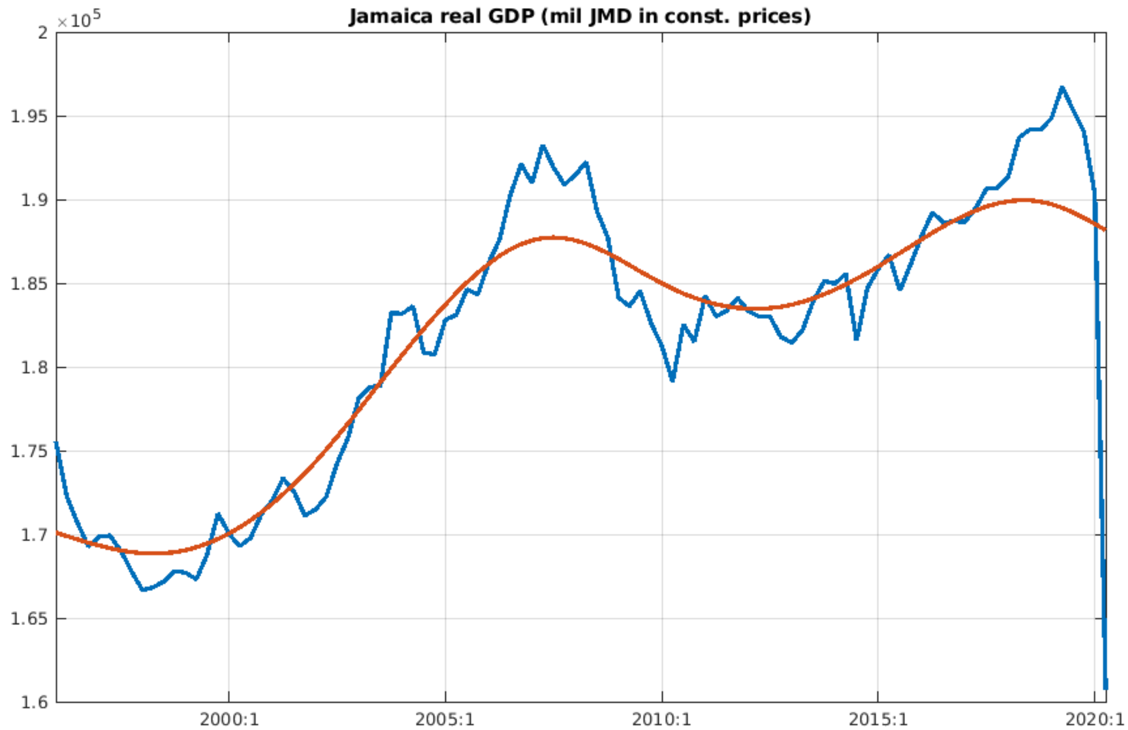
We usually assume shocks to follow normal distribution with zero mean:

$$\varepsilon_t^y \sim N(0, \sigma)$$

We add also equation linking output gap to the actual output y_t and potential (trend) output $\overline{y}_t$:

$$y_t = \overline{y}_t + \hat{y}_t$$

How do we model the trend though?



We need equations that allow potential output to grow over time:

$$\overline{y}_t = \overline{y}_{t-1} + \Delta\overline{y}_t/4 + \varepsilon_t^{\overline{y}}$$
$$\Delta\overline{y}_t = \rho \cdot \Delta\overline{y}_{t-1} + (1-\rho) \cdot \Delta\overline{y}^{ss} + \varepsilon_t^{\Delta\overline{y}}$$

Inflation - Phillips Curve

Links demand (= output gap) to inflation

We observe that in reality, some (most?) prices are sticky - firms are reluctant to change prices abruptly. Menu costs, costly decision making, uncertainty, ...

We've come up with various ~~fairy tales for modelers~~ ways of modeling sticky prices:

- Calvo pricing: Only some share of firms are allowed to change prices each period. https://www.karlwhelan.com/MAMacro/nkpc-details.pdf
- Rottenberg pricing: Each firm can change prices each period, but it's costly to do so. http://skchugh.com/images/Chapter22.pdf

Simplest form of the Phillips Curve:

$$\pi_t = \beta_1 E_t \pi_{t+1} + \beta_2 \hat{y}_t + \epsilon_t^{\pi}$$

We can choose our own specification. Taylor rule is most commonly used, but unlike Taylor we use *expected inflation*, rather than actual inflation.

For the moment, we'll cast the model in terms of real interest rate gap. We'll make the transition to more realistic structure later.

$$\hat{r}_t = \gamma_1 * (E_t \pi_{t+1} - \pi^{tar}) + \gamma_2 \hat{y}_t + \epsilon^r_t$$

What is missing?

Almost everything. But this is the core. When we understand the core, we can easily expand it.

Fixing the worst problem

The biggest problem of the model: it's entirely forward-looking, there is no persistence. That's against the data. We fix that by expanding the model backstory.

Habit in consumption:

In reality, people tend to smooth consumption over time.

In the model, we add *habit in consumption*: households do not care only about the current and future consumption, but also about the current consumption relative to past consumption - we get "used to" some level of consumption.

IS curve changes to:

$$\hat{y}_t = \alpha_1 E_t \hat{y}_{t+1} + \alpha_2 \hat{y}_{t-1} - \alpha_3 \cdot \hat{r}_t + \epsilon_t^y$$

Notice that we added coefficient α_1 in front of the output gap lead. While this coefficient should theoretically be equal to one, in practice that doesn't work well and we often set it to near-zero values.

Inflation persistence:

Inflation is empirically persistent. We add "modeling crutch" by assuming that default firm behavior is not to keep prices constant, but to increase prices by the same amount as the overall inflation in the previous period.

Phillips curve changes to:

$$\pi_t = \beta_1 E_t \pi_{t+1} + \beta_2 \pi_{t-1} + \beta_3 \hat{y}_t + \epsilon_t^\pi$$

Typical parameter values

IS curve

$$\hat{y}_t = \alpha_1 E_t \hat{y}_{t+1} + \alpha_2 \hat{y}_{t-1} - \alpha_3 \cdot \hat{r}_t + \epsilon_t^y$$

- α_1 - close to zero; usually in `[0; 0.2]`
- α_2 - larger for less flexible economies with ineffective / non-credible central banks; usually in `[0.4; 0.8]`
- α_3 - larger for economies where the monetary policy is more effective; usually in `[0.05; 0.2]`

Phillips curve

$$\pi_t = \beta_1 E_t \pi_{t+1} + \beta_2 \pi_{t-1} + \beta_3 \hat{y}_t + \epsilon_t^\pi$$

- β_1 - coefficients on the lag and lead should sum up to one (**why?**). We ensure this by rewriting the equation in the following form

  $$\pi_t = \beta_1 E_t \pi_{t+1} + (1 - \beta_1) \pi_{t-1} + \beta_2 \hat{y}_t + \epsilon_t^\pi$$

  β_1 should be larger for more developed economies with credible central banks; usually in `[0.25 0.8]`
- β_2 depends on the economic structure; usually in `[0.1 0.4]`

Monetary policy rule

$$\hat{r}_t = \gamma_1 * (E_t \pi_{t+1} - \pi^{tar}) + \gamma_2 \hat{y}_t + \epsilon_t^r$$

- parameters represent policy preferences - there is no "ideal" value, and values can change abruptly
- note that these parameters will change when we introduce nominal interest rates. These values are provisional
  - γ_1 - usually in `[0.5; 1]`
  - γ_2 - usually in `[0; 0.5]`, lower than γ_1

Simultaneity

Simultaneity: all three variables are determined by all three equations simultaneously.

We cannot determine the value of variables sequentially. Example of the problem:

- Output gap depends on interest rate.
- Interest rate depends on output gap and expected inflation.
- Expected inflation depends on today's inflation, which depends on output gap.

Values of all three variables have to be determined simultaneously. This is a difference compared to other models, e.g. Excel-based models.

The model therefore needs to be analyzed as a whole, not by looking at individual equations, but by looking at the interactions of all the model equations. The suitable tool for such analysis are impulse-response functions.

Expectations, model solution

There are forward-looking variables in the model: $E_t\hat{y}_{t+1}, E_t\pi_{t+1}$. Before we can use the model (simulations, filtration, etc), we need to solve the model to get rid of these.

Model solution

Model solution has the following recursive solution form:

$$X_t = AX_{t-1} + B\epsilon_t$$

where

- X_t is a vector of endogenous variables, in our case $X_t = [\hat{y}_t, \pi_t, r_t]'$
- ϵ_t is a vector of exogenous shocks $\epsilon_t = [\epsilon_t^y, \epsilon_t^\pi, \epsilon_t^r]'$
- **A** and **B** and matrices of fixed numbers

Note that the model solution implies that all forward-looking variables are expressed in terms of lagged variables and shocks.

To find model solution, we can employ several methods. See the following papers for more:
https://ideas.repec.org/p/nbr/nberwo/21862.html
http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.143.1003&rep=rep1&type=pdf

We leave this job to software (Matlab + IRIS).

Steady-state

A dynamic system is in steady-state if the variables do not change in time unless there are shocks

$$X_t = AX_{t-1} = X^{ss}$$

In the absence of shocks, the model does nothng more than converge to steady-state

In our case:

$$\begin{aligned}\hat{y}^{ss} &= ?\\ \pi^{ss} &= ?\\ \hat{r}^{ss} &= ?\\ \Delta\overline{y}^{ss} &= ?\\ \overline{y}^{ss} &= ?\end{aligned}$$

Can be quite complicated for non-linear models, we need to use numerical solvers.

In case of non-stationary variables, we introduce Balanced Growth Path (BGP). Steady-state is defined by a single fixed point and (constant) growth rate.

https://en.wikipedia.org/wiki/Balanced-growth_equilibrium

Expectations in the model

Expected future value of a variable is based on the information agents have at the time. Variable $E_t\hat{y}_{t+1}$ represents expectations about output gap in period $t+1$, based on information available in time t.

Information in time t consists of values of variables in the previous period (X_{t-1}) and shocks in this period (ϵ_t). In some cases, the information can also include shocks which are expected to come in future periods (anticipated shocks), but we leave this out now for simplicity.

Because we cannot forecast shocks, the expectations are formed as follows:

$$E_t X_{t+1} = A \cdot X_t$$

In time $t+1$, we receive new information about shocks. Therefore $E_t X_{t+1} \neq X_{t+1}$ if $\epsilon_{t+1} \neq 0$.

Recap from last time

These are the things you should remember:

- We have basic 3 equation model with:
    - Households, firms, central bank
    - Output gap, inflation, real interest rate (gap)

$$\hat{y}_t = \alpha_1 E_t \hat{y}_{t+1} + \alpha_2 \hat{y}_{t-1} - \alpha_3 \cdot \hat{r}_t + \epsilon_t^y$$
$$\pi_t = \beta_1 E_t \pi_{t+1} + (1 - \beta_1)\pi_{t-1} + \beta_2 \hat{y}_t + \epsilon_t^\pi$$
$$\hat{r}_t = \gamma_1 * (E_t \pi_{t+1} - \pi^{tar}) + \gamma_2 \hat{y}_t + \epsilon_t^r$$

- The equations have some "backstory" that we should know, makes life easier
    - Backstory: which agent does what, and why
    - Example: households try to maximize lifetime consumption, so they decide whether to consume today or tomorrow. Delaying consumption today (= saving) allows for higher consumption tomorrow, depending on the real interest rate.
- Model is forward-looking, but we can deal with that and find (backward-looking) model solution

$$X_t = AX_{t-1} + B\epsilon_t$$

- Model has steady-state = equilibrium, to which it will always return. The only thing that can move it away from equilibrium is a shock.
- To understand the model requires us to understand each equation, but also how these equations behave. That is shown by impulse response functions.

Understanding the Model

Understanding individual equations is not sufficient to understand the model. Interactions can be complex. We need to look at the model as a whole.

Model Code Language

We use the following notations:

- l_(variable) - 100*log of variable (necessary transformation)
- dl_(variable) - first difference, QoQ growth rate
- d4l_(variable) - fourth difference, YoY growth rate
- (variable)_gap, (variable)__tnd
- ss_(variable) - steady-state parameter
- c1_(variable) - parameter in equation for this variable
- shock_(variable) - shock to the equation for this variable

List of variables:

Variable	Model code
log output	l_y
log CPI	l_cpi
inflation	dl_cpi
real interest rate	r

Impulse Response Functions

We put one shock in one period, observe reaction of variables.

If you cannot explain IRFs, you don't understand the model.

Technically:

- We assume you have Matlab and IRIS working
- Go to study materials, zipfile "closed_model", download, unzip
- Open Matlab, start IRIS
- Open files "closed_model.model", "setparam.m", "run_toy_model_irf.m"