AMEM: Open Economy Model

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Introduction

- We follow up on the New Keynesian Three Equations model.
- We add external sector and impact of foreign economy on the domestic economy.
  - For simplification, foreign economy = US economy.
- Small open economy (SOE) concept: foreign economy impacts the domestic economy, but domestic economy doesn't impact the foreign economy.
- We arrive at the basic version of the QPM.
- Note: QPM is a forecasting model, not a story-telling model; you can build scenarios, but the model itself doesn't tell you much how the scenarios should look like

Impact of the external sector - overview of channels

The impact can be simplified into the following channels:

1. **Exchange rate movements + foreign inflation:**
   1. Part of the domestic CPI comprises of imported goods. Nominal exchange rate and foreign inflation therefore affect domestic inflation.
   2. Exports are not explicitly modeled, but changes in exchange rate impact competitiveness and thus also domestic demand (via exports).
2. **Foreign demand:** Domestic demand conditions (output gap) are impacted by foreign demand.
3. **Interest rates:** We assume open capital account and arbitrage between investment in LCY-denominated and FCY-denominated assets. The domestic interest rate therefore reacts to foreign interest rate.
4. **Other not featured here yet**: commodity prices (export, import),

Sources

- Canonical papers: http://www.douglaslaxton.org/fpas.html

Conventions

The following conventions will apply in the document:

- Hat denotes gap variables, e.g. output gap $\hat{y}_t$
- Bar denotes trend variables, e.g. output trend $\overline{y}_t$

Changes to the model

Foreign variables

We introduce several variables from the foreign economy:

name	Code
Foreign demand (output gap)	$\hat{y}_t^{US}$
Foreign nominal interest rate	i_t^{US}
Foreign real interest rate trend	$\bar{r}_t^{US}$
Foreign inflation	π_t^{US}

The equations for these variables are basically irrelevant. These variables will ***not*** be forecasted by our model. Values for foreign variables will imposed from external source - Consensus Forecast, GPMN, Bloomberg, etc.

Monetary policy rule

Modification of already existing equation.

Central bank sets *nominal* interest rate, following the same objectives as before

$$i_t = \gamma_1 i_{t-1} + (1 - \gamma_1)(i_t^{neutral} + \gamma_2(E_t\pi_{t+1} - \pi^{tar}) + \gamma_3\hat{y}_t) + \epsilon_t^i$$

We introduce the concept of *neutral interest rate:* $i_t^{neutral}$. The neutral rate represents the rate that would prevail with inflation on target and closed output gap. We can think of the neutral rate as "trend" interest rate. Note that without this term in the monetary policy rule, the steady-state value of interest rate would be zero.

$$i_t^{neutral} = \bar{r}_t + \pi^{tar}$$

Uncovered Interest Parity (UIP)

New equation.

Assumes open capital account - households can invest both in LCY and FCY assets with the aim of obtaining the highest return. There is *arbitrage* between LCY and FCY assets.

$$i_t = i_t^{US} + E_t\Delta s_{t+1} + prem_t$$

- i_t^{US} - foreign nominal interest rate

- $E_t \Delta s_{t+1}$ - expected depreciation of the LCY against the FCY (USD)
  - $ s_t$ denotes log of nominal exchange rate (LCY per FCY, JMD per USD)
- $prem_t$ - country risk premium

Explanation: Imagine a hypothetical US investor, looking to invest 1 USD for one period:

- Investing in USD, he gets $1 \cdot \frac{1+i_t^{US}}{100}$, guaranteed without any risk.
- Investing in LCY, he invests $1 \cdot \exp(s_t)$ units of LCY for interest rate i_t. In the next period, he will convert his investment back to USD for exchange rate s_{t+1}. He will get (in USD terms):

$$1 \cdot \exp s_t \cdot \frac{1+i_t}{100} \cdot \frac{1}{\exp E_t s_{t+1}}$$

- Note that because future exchange rate S_{t+1} is uncertain, we replace it with the expected value $E_t S_{t+1}$
- Because of the arbitrage, the two yields should be equal

$$1 \cdot \frac{1+i_t^{US}}{100} = 1 \cdot \exp s_t \cdot \frac{1+i_t}{100} \cdot \frac{1}{\exp E_t s_{t+1}}$$

- In log terms:

$$i_t^{US} = i_t - \Delta s_{t+1}$$

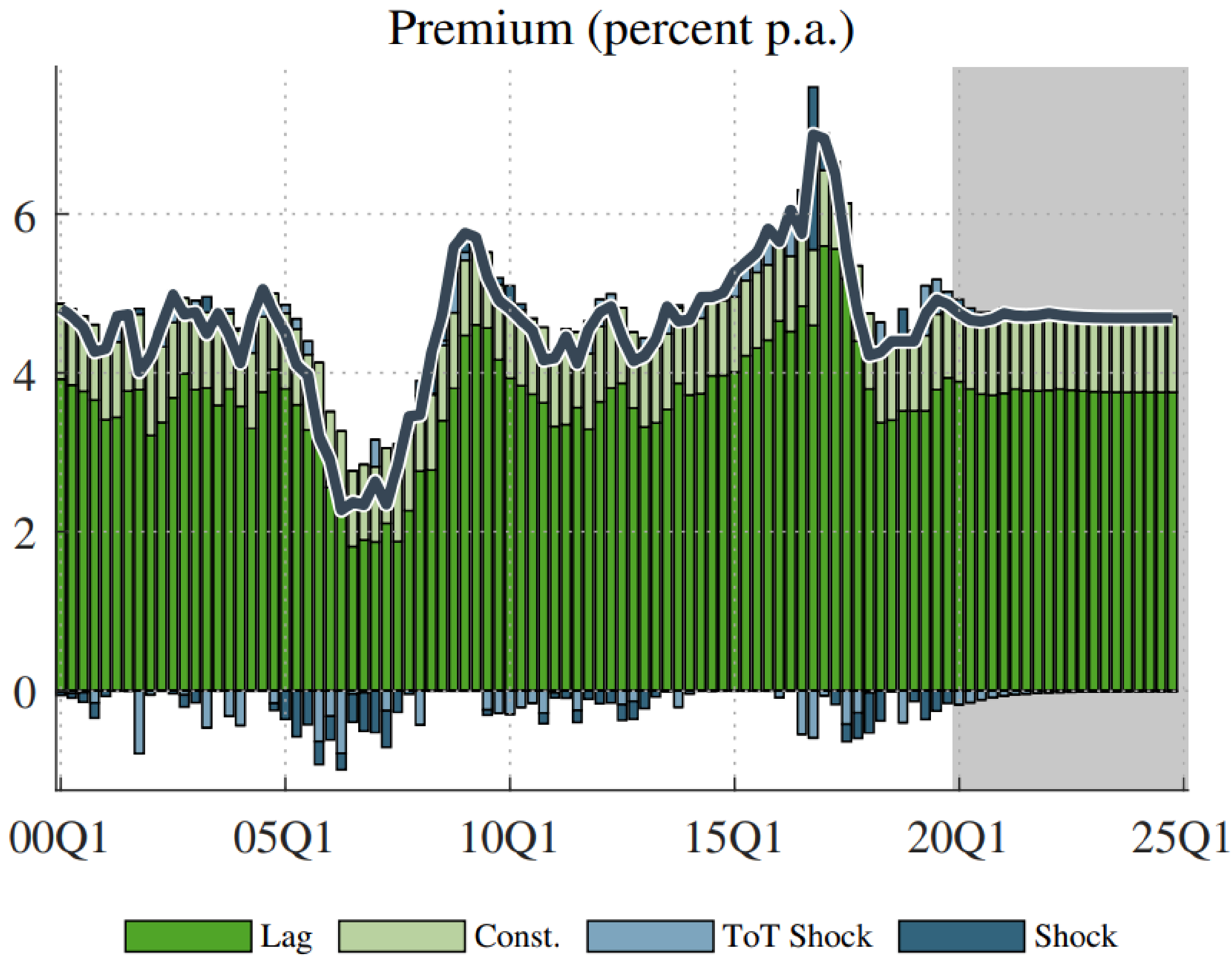
  where Δ is the difference operator.
- However, note that while the return on USD investment is certain (without risk), investment in LCY assets carries various risks:

  - Uncertainty about future FX rate path.
  - Higher costs of investment in some countries (underdeveloped financial markets).
  - Other risks (credit risk, nationalization, repatriation risk, ...).

  The investors in real world require additional compensation for these risks, which we represents as country risk premium $prem_t$. We model the country risk premium as simple AR(1) process:

$$prem_t = \rho \cdot prem_{t-1} + (1-\rho) \cdot prem_{ss} + \epsilon_t^{prem}$$

Example: Country risk premium for Mongolia estimate:



The final form of the UIP equation therefore becomes

$$i_t = i_t^{US} + E_t \Delta s_{t+1} + prem_t + \epsilon_t^s$$

Note that interest rate is on the LHS, but the equation still determines exchange rate. It is still the central bank that sets the nominal interest rate. When interest rate increases, there is a positive interest rate differential $i_t - i_t^{US} - E_t \Delta s_{t+1} - prem_t$, which means that local currency assets offer higher return than foreign currency assets. Investors react to this and capital flows into the country. The inflow of capital appreciates the current exchange rate s_t, which in turn increases the expected depreciation $E_t \Delta s_{t+1}$ and reduces the positive interest rate differential. The capital inflow continues until the expected depreciation is large enough to offset higher local interest rate.

Exchange rate expectations

UIP is purely forward-looking equation. This results in predictions that are usually at odds with data.

We introduce a rigidity into the equation via exchange rate expectations:

$$E_t s_{t+1} = \kappa \cdot s_{t+1}^{RE} + (1-\kappa)(s_{t-1} + \frac{2}{4}(\pi^{tar} + \Delta\overline{z}_t - \pi_t^{us}))$$

- Share κ of agents are rational and form their expectations about future exchange rate using rational (model-consistent) expectations.
- Share $(1-\kappa)$ of agents are not rational and have to use rule of thumb. Their rule of thumb is to take previous level of exchange rate s_{t-1} and update it using trends in the economy. Note that the term $(\pi^{tar} + \Delta\overline{z}_t - \pi_t^{us})$ can loosely be interpreted as trend for nominal exchange rate depreciation.

Parameter κ usually takes value of `[0.6; 1]`.

Real Exchange Rate

We introduce the real exchange rate (RER) as a measure of the domestic price level against the foreign price level. In practice, RER is more stable than nominal exchange rate and therefore significantly more useful for macroeconomic analysis and modeling.

$$z_t = s_t + p_t^{us} - p_t$$

where:

- z_t is the RER
- s_t is nominal exchange rate
- p_t^{US} is the US price level
- p_t is the domestic price level

Appreciation of the RER increases cost of domestically produced goods for foreign consumers (exports). Note that appreciation can come due to nominal FX rate appreciation or increase in domestic price level (usually via domestic nominal wages). When the RER appreciates, we can broadly define two scenarios:

1. The appreciation is supported by increased domestic productivity. Our exports are more expensive but the volume of exports does not decline because the increase in price corresponds to our increased productivity. An example for Mongolia would be to develop some mining sites, improve infratructure to lower costs, etc.. This kind of RER appreciation is in line with improving country fundamentals and is generally regarded as a positive development. There is no direct impact on output gap, inflation, etc.
2. The appreciation is not supported by increased domestic productivity. In this case, stronger RER causes problems with competitiveness - our exports become too expensive, we are unable to maintain external balance, and our trade balance deteriorates. Because of faltering exports, domestic demand also suffers, output gap becomes negative, and inflation declines.

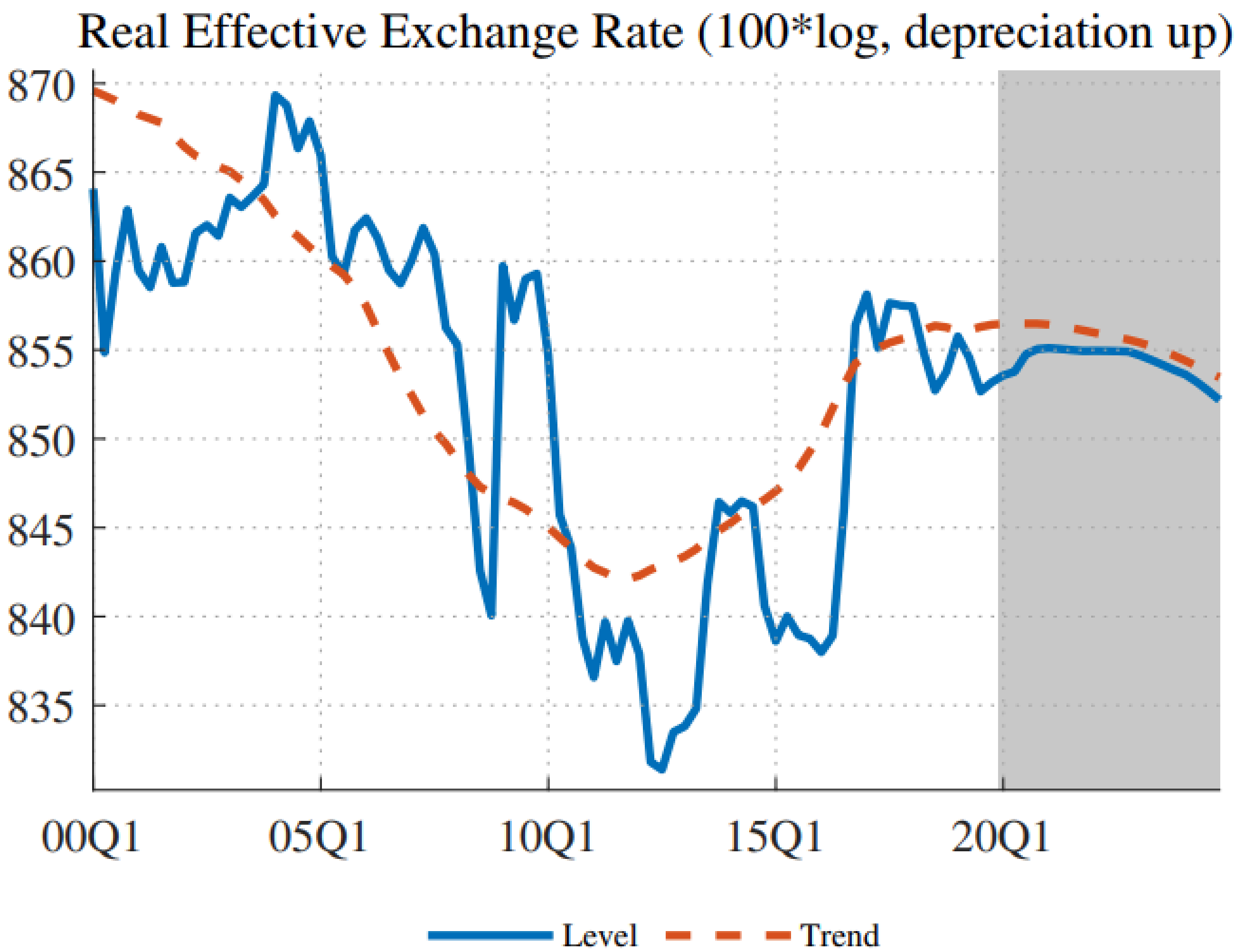
We need a way to distinguish the two scenarios above. We split the RER into gap and trend component:

$$z_t = \overline{z}_t + \hat{z}_t$$

Movements in the RER trend are considered to reflect country's productivity and therefore have no impact on output gap or inflation. Movements in the RER gap represent *misalignment of RER (overvaluation or undervaluation)* and have direct impact on output gap and inflation. RER gap features in key QPM equations (see below).

We assume that country productivity and RER trend do not change abruptly and we attribute most RER volatility to the gap.

Example: RER for Mongolia estimate:



Because the RER does not need to be stationary, we model the RER trend (similarly to the output trend) by modeling the RER trend depreciation as a simple AR(1) process.

$$\Delta\overline{z}_t = \rho \cdot \Delta\overline{z}_{t-1} + (1-\rho) \cdot \Delta\overline{z}_{ss} + \epsilon_t^{\overline{z}}$$

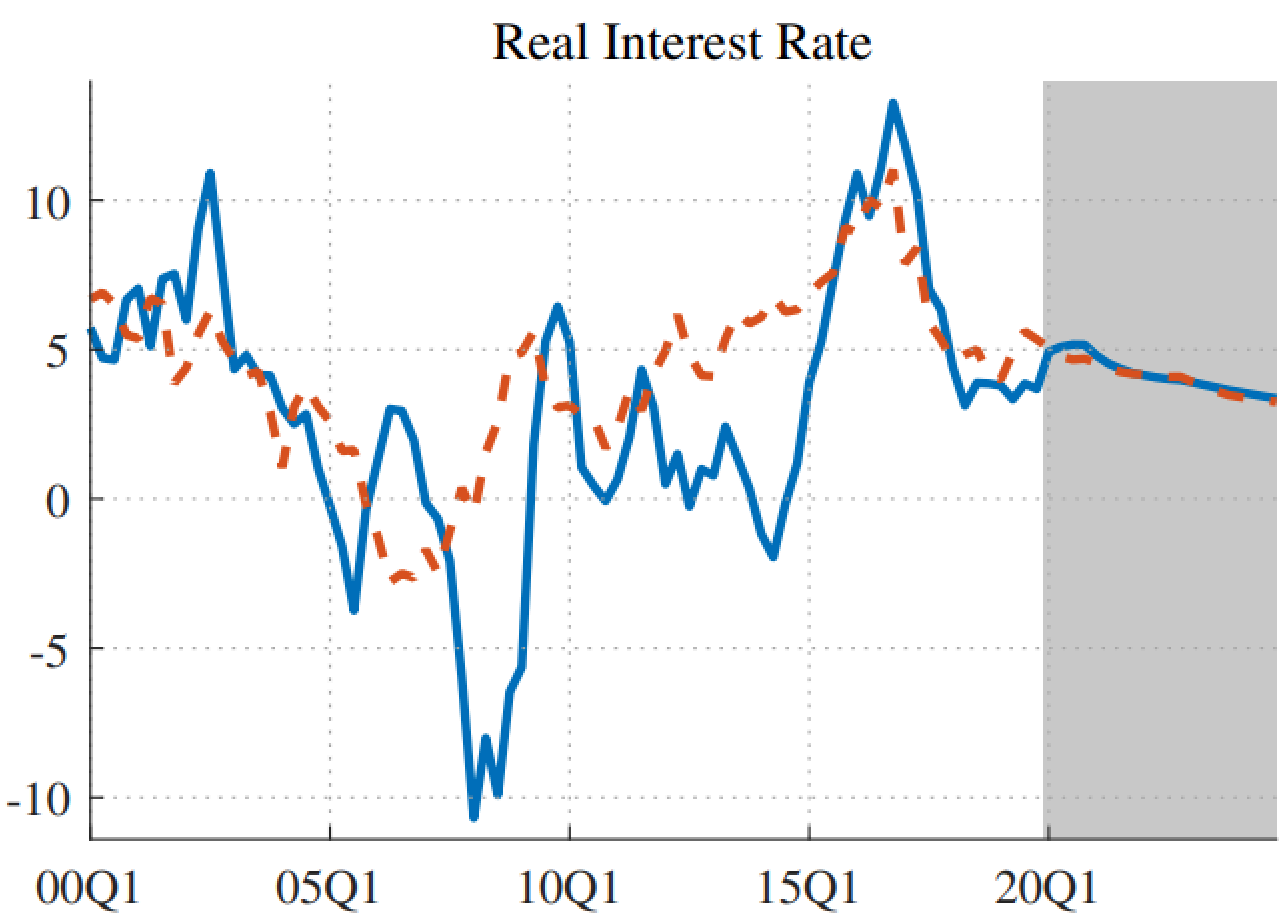
Real interest rate

We maintain the assumption that economic activity depends on the *real* interest rate (RIR). However, the central bank sets *nominal* interest rate. To link the two, we employ **Fisher equation**:

$$r_t = i_t - E_t \pi_{t+1}$$

However, equilibrium RIR can change over time and need not equal zero.

Example: RIR for Mongolia estimate:



We therefore again employ the gap-trend decomposition. For the trend, we employ UIP-like equation specified for trends and in real terms:

$$r_t = \bar{r}_t + \hat{r}_t \bar{r}_t = \bar{r}_t^{US} + \Delta \bar{z}_t + prem_t$$

IS Curve

We add two additional terms:

- α_4: Exchange rate effect - when exchange rate depreciates (movement up), Mongolia becomes more competitive and is able to sell more exports abroad. Foreign consumers (Russians, Chinese, Americans) want to consume the same amount as before, but now

prefer to buy Mongolian goods (cashmere clothes) over other producers, because Mongolian goods are now cheaper.
Moreover, when exchange rate depreciates (movement up), imports become more expensive and domestic consumers substite from imported goods to domestically produced goods.

- α_5: Higher foreign demand - Foreign consumers want to consume more of everything, so they also consume more Mongolian exports. We represent this by foreign output gap, estimated by an external model and taken over to our model.

$$\begin{aligned} \hat{y}_t =& \alpha_1 E_t \hat{y}_{t+1} \\ +& \ \alpha_2 \hat{y}_{t-1} \\ -& \ \alpha_3 \cdot \hat{r}_t \\ +& \ \alpha_4 \cdot \hat{z}_t \\ +& \ \alpha_5 \cdot \hat{y}_t^{US} \\ +& \ \epsilon_t^y \end{aligned}$$

Phillips Curve

Two additional terms:

- Part of the CPI basket comprises of imported goods, so all changes in foreign inflation or nominal exchange rate translate directly into changes in the domestic prices. This represents quick, short, direct pass-through of exchange rate into inflation. Real-world examples include fuel prices at fuel stations, fruits, toothpaste, chewing gum, and other goods where the import costs represent large share of the retail price and the retail price changes quickly.
  We can specify foreign imported inflation as

$$\pi_t^{US} + \Delta s_t$$

  which we can rewrite this conveniently as

$$\hat{z}_t - \hat{z}_{t-1}$$

- Domestic firms use imported goods as inputs into domestic production. When exchange rate depreciates, the inputs get more expensive in LCY-terms, but firms do not immediately pass this into final prices. Examples include fuel prices as input into transport services, imported food used by restaurants, consumer electronics, etc. It takes time before the domestic price level increases to match the weaker exchange rate. We represent this effect by RER gap $\hat{z}_t$.

$$
\begin{aligned}
\pi_t = {} & \beta_1 E_t \pi_{t+1} \\
& + (1-\beta_1)\pi_{t-1} \\
& + \beta_2 \hat{y}_t \\
& + \beta_3(\hat{z}_t - \hat{z}_{t-1}) \\
& + \beta_4 \hat{z}_t \\
& + \epsilon_t^{\pi}
\end{aligned}
$$

What's Still Missing?

We now have the basic open economy model.

Lots of things can be added to make the model more useful in real life. The choice depends on the particular economy and application:

- Fiscal policy - impact of deficits on demand, impact of debt on interest rates, ...
- Commodity prices (oil, food, ...)
- Subcomponents of CPI (core, food, energy, ...)
- Market interest rates with link to policy rate (imperfect policy transmission)
- CB interventions in the forex market
- Subcomponents of GDP (agriculture, mining, ...)

Next lecture

Exercises to be done in groups.

Exercise 1:

Analyze the four basic IRFs: demand shock, supply shock, policy (interest rate) shock, UIP (exchange rate) shock.

Exercise 2:

Make the monetary policy pass-through stronger = the central bank has to work *less* to accommodate shocks.

Additional

Model code language

We use the following notations:

- l_(variable) - 100*log of variable (necessary transformation)
- dl_(variable) - first difference, QoQ growth rate
- d4l_(variable) - fourth difference, YoY growth rate
- (variable)_gap, (variable)__tnd
- ss_(variable) - steady-state parameter
- c1_(variable) - parameter in equation for this variable
- shock_(variable) - shock to the equation for this variable

Variable	Model code	Notation
log output	l_y	y
log CPI	l_cpi	p
inflation	dl_cpi	π
nominal interest rate	i	i
real interest rate	r	r
log nominal exchange rate	l_s	s
log real exchange rate	l_z	z
country risk premium	prem	$prem$

Variable	Model code	Notation