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The Economic Theory of Clubs: An Evaluative Survey

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OH TO THE CLUB, THE SCENE OF SAVAGE JOYS,
THE SCHOOL OF COARSE GOOD FELLOWSHIP AND NOISE.
William Cowper

I. *Introduction*

ALTHOUGH THE MAJORITY of economic articles examining clubs have appeared since James Buchanan wrote his seminal piece, "An Economic Theory of Clubs" (1965), the origins of "club theory" can be traced to the works of A. C. Pigou (1920) and Frank H. Knight (1924) in their work on tolls on congested roads. These two authors assumed the existence of two alternative commuting routes: a narrow congested road of good quality and a broad uncongested road of poor quality. By determining the tolls on the congested road, Pigou and Knight were essentially solving a club problem, since the toll would restrict users and, thereby, determine "membership size" for the congested highway.[1]

Another pioneering club model is that of Charles Tiebout (1956), whose "voting with the feet" hypothesis attempted to show how jurisdictional size of local governments could be determined by voluntary mobility (or membership) decisions. In a private-good context, Jack Wiseman analyzed a club principle for sharing costs among users of a public utility (1957).

[1] E. J. Mishan (1971, pp. 4–5) showed that the solutions provided by Pigou and Knight were identical. That is, Pigou considered marginal costs (of congestion) excluding a rent concept, whereas Knight used average costs including rent; but under competitive conditions these two methods converge. For a modern treatment of this problem, see Noel Edelson (1971) and Martin Weitzman (1974).

Tiebout and Wiseman were the first researchers to focus on a cost-sharing rationale for clubs in which cost per user fell over some range of membership size.

The two most influential early club investigations are those of Mancur Olson, Jr. (1965) and Buchanan (1965). Olson recognized that clubs would form to exploit economies of scale and to share public goods. He also distinguished between inclusive and exclusive clubs (1965, pp. 34–43). Inclusive clubs share pure public goods and require no membership size restrictions, while exclusive clubs share impure public goods and require size restrictions owing to crowding and congestion. Impure public goods are those goods characterized by either partial rivalry *or* some excludability of benefits.[2] In Buchanan's attempt to bridge the Samuelsonian gap between private and public goods (see Paul Samuelson [1954; 1955]), the first analytical statement of the provision and membership conditions was derived for clubs sharing impure public goods. Moreover, Buchanan demonstrated how these two decisions interact (1965, pp. 6–12). Testimony to the importance of Buchanan's work can be found in the well over one hundred subsequent studies referencing his model.

Justifications for clubs have been based: on a pure taste for association (e.g., Thomas Schelling [1969], Martin McGuire [1974a]), cost reductions from scale economies, cost reductions from team production (e.g., McGuire [1972]), the sharing of public goods, and the sharing of public factors (e.g., Arye Hillman [1978b]). Consequently, diverse definitions for clubs have been stated, depending upon what was being shared.[3] In an attempt to provide a unifying definition, we define a club as *a voluntary group deriving mutual benefit from sharing one or more of the following: production costs, the members' characteristics, or a good characterized by excludable benefits.*

Club theory provides the theoretical foundation for the study of allocative efficiency of impure public goods. In fact, club theory can analyze some types of fully rival and fully nonrival goods (e.g., see McGuire [1972], Roland Artle and Christian Averous [1973], and John Chamberlin [1974]). Club theory can also be used in determining the need for exclusionary zoning; the efficacy of busing; and the optimal sizes for alliances, communities, and cities. In addition, an important linkage between welfare economics, public finance, and game theory is provided by club theory. Since the entry of a firm into an industry causes a market thinning (congestion) effect in the form of reduced sales to other competitors, Olson has suggested that club analysis can be used as a new paradigm for the determination of industry size (1965, p. 37). Finally, aspects of two-part tariffs, peak-load pricing, and cost allocation problems can be better understood with club theory.

The primary purpose of this survey is to evaluate critically the theoretical contribution of the literature on clubs and, in so doing, to indicate its far-ranging applications. In pursuing this goal, the paper examines some existing controversies: e.g., are mixed clubs (i.e., those with heter-

[2] A good is partially rival in consumption when one person's consumption *of a unit* of the good detracts, to some extent, from the consumption opportunities of another person. Since all club goods must be excludable for voluntary participation to occur, the *degree* of exclusion is initially ignored. Exclusion will be considered in *Section* IV-D where extensions to club theory are discussed.

[3] For example, Mark Pauly defined a club as a group of people who consume a public good as a single decision unit (1970b, pp. 53–54). Similar definitions were given by Elhanan Helpman and Hillman (1977, p. 293), and Todd Sandler (1979). Eitan Berglas recognized that clubs take advantage of scale economies and gains from public good sharing (1976a). Moreover, Allan DeSerpa equated clubs with any sharing group (1977, p. 33).

ogeneous members) optimal? what are the true Pareto-optimal membership conditions? how do mixed clubs differ from those with homogeneous members? what is the best institutional form for clubs? what is the optimal number of clubs? and can optimal tolls self-finance optimal provision of the shared good? Additionally, this survey presents two basic models of clubs: a "within club" model and a "total economy" model. Some future directions for club research are also suggested.

Resolution of the first controversy leads to an important result where mixed clubs are shown to be optimal unless a second-best constraint is imposed, whereby all members must share costs equally regardless of their utilization of the club. All articles that have argued against mixed clubs have explicitly imposed this constraint and, consequently, have not derived first-best results. This is an important finding, since the argument against mixed clubs can support segregation in schools, neighborhoods, and communities. Other solutions show that the optimal number of clubs, the club's membership composition, and the maximization objective are crucial in determining the true Pareto-optimal membership size. Additionally, self-financing depends on the form of the crowding function and the degree of returns in production.

The body of the survey contains seven sections. *Section* II presents the Buchanan model. *Section* III depicts a general club model encompassing many of the non-game-theory approaches that have a total economy point of view. The extensions of Buchanan's model and the controversies in the non-game-theory club literature are examined in *Section* IV. *Section* V analyzes the game theory approaches to clubs. The three remaining sections present the applications of club theory, future directions for research, and conclusions.

II. *The Buchanan Model of Clubs: A "Within-Club" Point of View*

The majority of club models are either identical to or else a slight variant of the Buchanan model (1965).[4] To understand how the more complex models of clubs operate, the Buchanan model must be examined first; both a mathematical and graphical treatment are presented here.

A. *Buchanan's Model: A Mathematical Representation*

Although Buchanan clearly stated that less restrictive assumptions could be invoked (1965, pp. 3–4, 7–8, 13), for simplicity he assumed that homogeneous members *equally share* the public good (e.g., a swimming pool, a golf course) and its associated costs. In addition, the club does not discriminate against any of its members and can costlessly exclude all nonmembers. The "Buchanan club" is a decentralized, voluntary organization operating without transaction costs.[5] Since all individuals are identical, no centralized control is needed; all members have the same goals. Only a representative *member's* standpoint is considered when opti-

[4] Models that are identical or closely related to that of Buchanan (1965) include: Roy Adams and Jeffrey Royer (1977); Larry Allen, Ryan Amacher, and Robert Tollison (1974); Richard Anderson, Philip Porter, and S. Charles Maurice (1979); Berglas (1976a); Rolf Färe and Shawna Grosskopf (1979b); Jerome Heavey and Miles Gunzenhauser (1978); James Litvack and Wallace Oates (1970); McGuire (1972; 1974a); Dennis Mueller (1976); Richard Musgrave and Peggy Musgrave (1976); A. Mitchell Polinsky (1974); and Richard Porter (1978). This list is not exhaustive.

[5] Few club researchers have examined transaction costs (e.g., decision-making and participation costs) for clubs and other nonmarket structures. Exceptions include Kenneth Arrow (1970), Artle and Averous (1973), Christoph Badelt (1978; 1979), Richard Cebula and Paul Gatons (1972), John Head and Carl Shoup (1969), Oates (1972, pp. 48–49), Olson (1965; 1973), Sandler and Jon Cauley (1976), Jan Smith (1976), and Clem Tisdell (1977). Nevertheless, no rigorous treatment of transaction costs in clubs can be found in the literature.

mal conditions are derived for the club; hence, a *within-club point of view* characterizes Buchanan's approach.

In order to display the essential features of Buchanan's model in the simplest possible terms, one private *numéraire* good (y) and one impure public good (X) are assumed.[6] Each of the s identical members attempts to maximize utility (U) subject to a cost (or production) constraint (F). The i^{th} (representative) member solves the following problem:

$$\text{Max } U^i(y^i, X, s) \text{ subject to } F^i(y^i, X, s) = 0, \quad (1)$$

where y^i is the i^{th} member's consumption of the private good.[7] Equation (1) implies that each member utilizes the entire available quantity of the shared good (i.e., $x^i = X$ for all i, where x^i is the utilization rate of the i^{th} member). In the utility function, the marginal utility of each good is assumed positive. The marginal utility derived from additional members may be positive for small memberships due to camaraderie, but eventually crowding occurs and marginal utility becomes negative. The cost function is associated with positive marginal costs of consuming the two goods; however, costs per person for the public good fall as more members share the costs of a given quantity (i.e., $\partial F^i/\partial s < 0$). If the utility and cost functions satisfy the required differentiability and convexity requirements (see Färe and Grosskopf [1979]), then maximizing (1) produces the following first-order conditions:

$$MRS^i_{xy} = MRT^i_{xy} \quad i = 1, \ldots, s \quad (2)$$
$$MRS^i_{sy} = MRT^i_{sy} \quad i = 1, \ldots, s. \quad (3)$$

Equation (2) is the provision condition for the shared good and indicates that for each member the marginal rate of substitution (MRS) between the public and private good (the numéraire) must be equated to the marginal rate of transformation (MRT) between these two goods. Thus, for the public good, members equate their marginal benefits with their marginal costs. If, at the margin, the club is breaking even in providing the public good, the sum of the members' marginal costs must equal the club's marginal cost of provision (i.e., $\Sigma_{i=1}^{s} MRT^i_{xy} = MRT_{xy}$). Equation (2) then indicates that the usual Samuelson provision condition for public goods holds (i.e., $\Sigma_{i=1}^{s} MRS^i_{xy} = MRT_{xy}$). In the Buchanan model, the provision condition of an impure public good does not differ significantly from that of a pure public good, except in terms of the number of individuals aggregated by the summation index *and* the interaction of the provision and membership conditions.[8]

The novel aspect of club analysis shows up in the membership condition, expressed in (3). For optimality, a representative member equates the MRS between group size and the private good (left-hand side of (3)) with the associated MRT (right-hand side of (3)), thereby achieving an equality between the marginal benefits and marginal costs from having another club member. These marginal benefits are normally negative due to crowding, and the corresponding marginal costs are negative due to cost reductions derived from cost sharing. Since by assumption a whole member must be added (or removed) from the club, the membership condition may not be satisfied as an equality. That is, going from s to $s + 1$ members may reverse an inequality between the two sides of (3). When this discreteness problem occurs, members should be added, provided marginal benefits exceed marginal costs. The membership size *prior* to the reversal of the inequality is optimal. A pure public

[6] Buchanan assumed n private goods and m public goods (1965, p. 4).

[7] Throughout this survey, a consistent set of notation is employed.

[8] Other differences will show up in more complex models when individuals can vary utilization rates (see *Section* III).

good club could accommodate the entire population because marginal benefits from new members are zero and, therefore, are always greater than the corresponding negative marginal costs. Consequently, pure public goods do not require a membership restriction in the Buchanan model.[9]

There are two crucial aspects of the Buchanan model to remember. First, the provision and membership conditions must be *simultaneously* determined, since the *MRS* and *MRT* expressions in (2) and (3) depend upon the same variables. Second, utility is maximized for the representative member; i.e., average net benefits are maximized.

B. *Buchanan's Model: A Graphical Representation*

To gain further insight into the operation of the Buchanan model, a graphical representation is depicted in the four quadrants of *Figure* 1. The graphical model invokes the same assumptions as the mathematical model. Optimal provision amounts are shown for three different membership sizes in quadrant I. The quantity of the public good is measured on the abscissa, while the total cost and benefit *per* member are measured on the ordinate. The shape of the benefit curve indicates diminishing returns to consumption, while that of the cost curve reflects constant returns to scale. For a given membership size (say s_1), optimal provision corresponds to the X value (i.e., X_2) that equates the slopes of the total benefit, $B(s_1)$, and the total cost, $C(s_1)$, curves confronting a member. Hence, quadrant I equilibriums satisfy (2). As membership size increases to s_2, the total benefit per member curve shifts down owing to crowding. Additionally, the total cost per member curve rotates down proportionally as more members share the costs for each facility size. When membership is s_2, optimal provision corresponds to X_3 units of public good. Similarly, X^* is optimal for s^* members. These optima can be transferred to quadrant IV in the form of the X_{opt} curve depicting the optimal provision amount for each membership size.

For each amount of the public good, an analogous exercise in quadrant II determines the membership size that maximizes per person net benefits. The benefit curves in quadrant II show the benefit per person associated with a changing membership size when the facility size is fixed at X_1, X_2, and X^* units. The shape of these curves indicates that camaraderie is eventually overpowered by crowding, and at that point the benefit per person begins to decline. In the same quadrant, the cost curves depict the cost per member when a given sized facility is shared by a varying number of members. Owing to the equal cost-sharing assumption, these cost curves are rectangular hyperbolas. As the facility size increases, both the benefit and the cost curves shift up. For each facility size, the optimal membership results when the slope of the corresponding benefit and cost curves are equal (i.e., equation (3) is satisfied): s_1 is optimal for facility size X_1; s_2 is optimal for X_2; and s^* is optimal for X^*. These optima can be transferred to quadrant IV in terms of the s_{opt} curve showing the optimal membership for each amount of provision. This transfer is accomplished via quadrant III, which transposes membership information from quadrant II to IV.

Both the provision and the membership conditions are satisfied in quadrant IV at point E where the s_{opt} and the X_{opt} curves

[9] Some nonrival goods with excludable benefits (e.g., the telephone) may require a finite club size owing to resource or "hook-up" costs needed to extend consumption services to additional people. For these goods, the marginal costs of new members account for hook-up costs *and* reductions due to cost sharing. Since these marginal costs are expected to be positive over some range of membership, finite memberships are optimal. On hook-up costs, see Artle and Averous (1973), Bryan Ellickson (1973), and Burckhard von Rabenau and Konrad Stahl (1974).

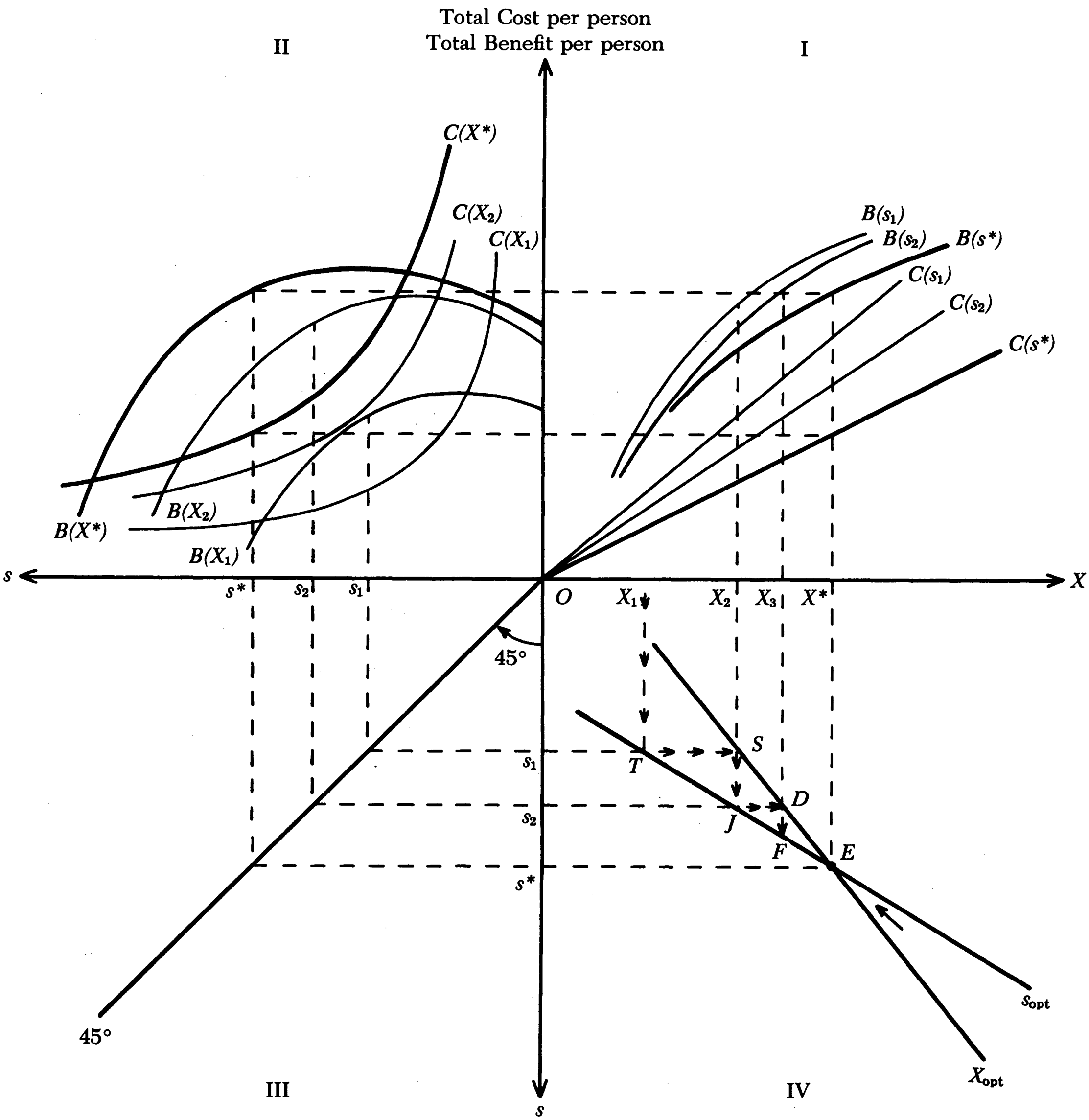

Figure 1

intersect. If public good provision is X_1, a series of iterations shown by path X_1*TSJDF* . . . *E* eventually forces the club to achieve optimal membership (s^*) and the associated optimal provision (X^*) at *E*. Three of these iterations are shown in quadrants I and II. For example, the club desires a membership of s_1, when output is X_1; however, a larger facility of size X_2 is required to maximize average net benefits (in quadrant I) when membership is s_1. And so it goes until equilibrium is reached (see the darker curves in quadrants I and II for the equilibrium).

The *interdependency* between the membership and the provision conditions is clearly displayed in *Figure* 1.[10] The figure shows that these conditions must be mutually consistent for equilibrium to result.

III. *A General Model from the Total Economy Point of View of Clubs: A Non-Game-Theory Approach*

Based upon its assumptions and within-club point of view, the Buchanan model is a consistent and correct treatment of clubs. Subsequent models have extended club theory to include heterogeneous members,[11] discrimination, variations in the utilization rate of the public good, and a *total economy point of view* in which members *and* nonmembers were considered.[12] We shall now present a "general model" of clubs from the total economy point of view. This model encompasses the major non-game-theory extensions to club analysis and serves as a backdrop to *Section* IV, where the value and meaning of these extensions are more fully evaluated. The general model is a modified version of William Oakland's analysis (1972). A major virtue of his model is its partial avoidance of the discreteness problem by allowing the utilization rate to vary between members.[13]

The general model here assumes the existence of two goods—a private *numéraire* good *(y)* and an impure public good *(x)*. Unlike the Buchanan model, both members and non-members are considered when the Pareto-optimal conditions are derived for a single club. A fixed population of $\tilde{s}$ individuals is assigned membership or nonmembership status; i.e., a centralized membership decision is made. The population (and therefore the members) consists of heterogeneous individuals whose preferences are represented by well-behaved utility functions. Club members are able to determine their utilization rate (x^i) of the impure public good by varying the number of visits or the time spent at the club. Moreover, non-members are costlessly excluded from the club.

The utility function of a member is depicted in (4) and that of a non-member in (5). All individuals are ordered by the i index according to their preference for the club good; i.e., individual $s-1$ has a greater preference for the club than indi-

[10] If the s_{opt} and X_{opt} curves switch positions, an unstable equilibrium results; hence, myriad cases are possible. A different four-graph treatment was given by Allen, Amacher, and Tollison (1974) based on Musgrave and Musgrave (1976, pp. 615–19). This latter presentation derived marginal curves corresponding to the benefit and cost curves drawn in *Figure* 1. Unfortunately, Allen *et al.* and Musgrave and Musgrave kept their demand curve for provision unchanged *even though* membership changed. This construction is in error, since it ignores the negative effects of congestion. Furthermore, Allen *et al.* diagrams were not correctly drawn to scale.

[11] Some of the models considering heterogeneous members are Berglas (1976a; 1976b); DeSerpa (1977; 1978a; 1978b); A. Myrick Freeman and Robert Haveman (1977); Morton Kamien, Nancy Schwartz, and Donald Roberts (1973); William Loehr and Sandler (1978); Yew-Kwang Ng (1973; 1974; 1978; 1980); Ng and Tollison (1974); Jerome Rothenberg (1970a; 1970b); Sandler (1975b; 1979); Sandler and Cauley (1976); and Tollison (1972).

[12] The following papers consider members and non-members when optimizing: Artle and Averous (1973); Berglas (1976a); Berglas and David Pines (1978); Helpman and Hillman (1976; 1977); Hillman and Peter Swan (1979a; 1979b); Ng (1973; 1974; 1978); Pauly (1967; 1970b); Sandler (1979); and John Sorenson, John Tschirhart, and Andrew Whinston (1978b).

[13] Many models in the literature (e.g., Berglas [1976a], McGuire [1974], Ng [1973; 1974; 1978]) have a discreteness problem, insofar as they differentiate with respect to s, a discrete variable. Mathematically, this problem can be avoided by using utilization rates and a continuum of traders. If a continuum of traders is used, aggregate utility can be summed with a Lebesque integral (see Helpman and Hillman [1976; 1977], Hillman and Swan [1979a; 1979b], and Sandler [1979]). However, the continuum of traders representation should *only* be employed when the number of members approaches infinity (e.g., some kinds of intergenerational clubs).

vidual s. Technically, the individuals' marginal evaluations of the club determine this ordering, and $\tilde{s}!$ (factorial) orderings must be examined before the proper one can be ascertained (see equation (8)). The importance of this ordering was emphasized by Freeman and Haveman, since different orderings may yield different optimum conditions (1977).

$$U^i = U^i(y^i, x^i, c(k))$$
$$\text{where } c_k = \partial c/\partial k > 0$$
$$\text{and } \partial U^i/\partial c \leqq 0, \; i = 1, \ldots, s \quad (4)$$
$$\tilde{U}^i = \tilde{U}^i(\tilde{y}^i, 0, 0)$$
$$i = s+1, \ldots, \tilde{s} \quad (5)$$

The utility of a member depends positively on the consumption rates of the two goods and negatively on congestion or crowding *(c)*. Crowding involves a reduction in the good's quality and assumes a variety of forms including, among others, poorer views, lost time, and less comfortable conditions. Although the crowding function can be represented in numerous ways, it must include two opposing influences; increases in the shared good's provision reduce crowding (i.e., $c_X = \partial c/\partial X < 0$), whereas increases in the utilization of the good increase crowding (i.e., $\partial c/\partial x^i > 0$). Because crowding depends upon the average utilization rate *(k)* of the shared good, both opposing influences are present.[14] The average utilization rate is equal to the total utilization rate divided by the quantity of the good (i.e., $k = \Sigma_{i=1}^{s} x^i/X$). A final important feature of the member's utility function is an "impurity" constraint requiring each member's consumption of the shared good to be less than or equal to its available quantity (i.e., $x^i \leqq X$ for $i = 1, \ldots, s$). This latter requirement and the form of the crowding function partially circumvent the discreteness problem because the member's club participation can be continuously varied. Moreover, each member can reveal a different intensity of public good utilization.

Since non-members are excluded from the club, their utility functions are independent of the club good and its associated crowding. The overhead tilde ($\tilde{}$) indicates that the variable applies to a non-member (however, the parameter $\tilde{s}$ refers to the entire population). The Pareto-optimal conditions are derived by maximizing the marginal entrant's utility function subject to the following: the constancy of all other members' and nonmembers' utility levels; the private good production–distribution constraint; the impurity constraints; and a transformation function.[15] When the associated Lagrangian expression is differentiated with respect to X, x^i, and s, the resulting first-order conditions can be written as the provision, toll, and membership conditions, respectively.[16] These three conditions follow:

[14] $\partial c/\partial X = -(kc_k)/X < 0$, since $c_k = \partial c/\partial k > 0$, and $\partial c/\partial x^i = c_k/X > 0$, where $k = \Sigma_{i=1}^{s} x^i/X$. Crowding is treated as a technical relationship dependent upon average utilization and independent of the member experiencing the crowding. The addition of a superscript to the crowding function allows crowding to differ between individuals; however, little is gained by way of insight by this complication.

[15] The Lagrangian has the following form:

$$L = \sum_{i=1}^{s} \lambda^i[U^i(\cdot) - K^i] + \sum_{i=s+1}^{\tilde{s}} \lambda^i[\tilde{U}^i(\cdot) - \tilde{K}^i] + \sigma F(X, Y) + \gamma(Y - \sum_{i=1}^{s} y^i - \sum_{i=s+1}^{\tilde{s}} \tilde{y}^i) + \sum_{i=1}^{s} \beta^i(X - x^i),$$

where the Greek letters are Lagrangian multipliers, K^i is a constant level of utility *except* for $i = s$ in which K^s is nonexistent, and Y is the amount produced of the private good. The production transformation function is $F(\cdot)$. An alternative approach yielding the same essential results is to maximize an additively separable Benthamite social welfare function of the members and nonmembers (i.e., $\Sigma_{i=1}^{s} \lambda^i U^i(\cdot) + \Sigma_{i=s+1}^{\tilde{s}} \lambda^i \tilde{U}^i(\cdot)$) subject to the same technical constraints. On this method, see Helpman and Hillman (1976; 1977) and Hillman and Swan (1979a; 1979b).

[16] These three conditions assume that no member's utilization of the public good equals the available amount (i.e., $x^i < X$). If this is *not* the case, then β terms must be included (see Sandler [1979]).

$$-(kc_k/X)\sum_{i=1}^{s} MRS^i_{cy} = MRT_{xy}$$

(Provision Condition) (6)

$$-(c_k/X)\sum_{i=1}^{s} MRS^i_{cy} = MRS^j_{xy}$$

$$j = 1, \ldots, s$$

(Toll Condition) (7)

$$\left[\frac{U^s(\cdot)}{(\partial U^s/\partial y^s)} - \frac{\tilde{U}^s(\cdot)}{(\partial \tilde{U}^s/\partial \tilde{y}^s)}\right] - (y^s - \tilde{y}^s)$$

$$\geqq -x^s(c_k/X)\sum_{i=1}^{s} MRS^i_{cy}$$

$$\text{for } 0 < s < \tilde{s}$$

(Membership Condition) (8)

Each of the three conditions permits a straightforward interpretation. Optimal provision requires that the marginal benefits from crowding reduction, resulting from increased provision, equal the marginal costs of provision as represented by the *MRT* expression. These marginal benefits are represented by the sum of the members' *weighted MRS* between crowding and the *numéraire*. The weighting factors indicate the effect on crowding relief attributable to a larger public good provision, and the *MRS* terms evaluate crowding. As in the Buchanan model, the provision condition is analogous to that of a pure public good because increased provision produces decreased crowding, which is a public good to the members.

The toll (or utilization) condition equates the member's marginal benefits from utilization (i.e., right-hand side of (7)) with the resulting marginal crowding costs imposed upon the membership (i.e., left-hand side of (7)). These marginal crowding costs are the sum of the members' weighted *MRS* between crowding and the *numéraire*, but here the weights indicate the effect on crowding resulting from a unit change in utilization. By making the per-unit toll *(T)* equal the marginal crowding costs imposed on the members, the crowding externality is internalized; hence, T equals the left-hand side of (7). The toll condition requires *an equal rate of toll for all members,* since the members' crowding costs resulting from an additional unit of utilization are the same at the margin, irrespective of the user. A member utilizes the club until the marginal crowding costs he creates equal his marginal benefits. Although each member pays an identical rate of toll, total toll payments vary between heterogeneous members based upon their revealed intensity of utilization (i.e., (7) is fulfilled at different utilization rates for diverse members).

In the toll condition, the sum of the marginal crowding costs includes the crowding costs imposed by the j^{th} member on himself. Some researchers have formulated models where this self-imposed crowding is eliminated from the toll (e.g., see Freeman and Haveman [1977], M. Bruce Johnson [1964], and Herbert Mohring and Mitchell Harwitz [1962]). These researchers have felt that self-imposed crowding is automatically taken into account by the club member. To derive a toll excluding self-inflicted crowding, k must be redefined in *each* individual's crowding function to exclude the individual's own consumption.

The membership size condition determines the optimal s, which appears in the provision and toll conditions. This membership condition requires the net benefits from membership for the entrant ($i = s$) to equal or exceed the crowding costs resulting from that person's entire utilization of the shared good. These crowding costs correspond to the right-hand side of (8). Since $\Sigma_{i=1}^{s} (-c_k/X)\ MRS^i_{cy}$ indicates only the crowding costs associated with a unit of utilization, this term must be multiplied by the *total* utilization of the entrant (i.e., by x^s) if full crowding costs of the entrant's membership are to be represented. On the left-hand side of (8), the two components of membership's net benefits are given. The first is the gain in utility from membership and consists of the difference in utility, evaluated in terms of the private good, between membership and non-membership. The second component relates to the required change in private good consumption that membership entails and can be considered as one part of the membership cost or fee to the entrant. This term appears because the private good must have the same benefit, at the margin, both before and after membership for the marginal entrant. If being a member alters the marginal benefit of the private good (i.e., $\partial^2 U^s/(\partial x^s \partial y^s) \neq 0$), then a reallocation of the private good is required for optimality (see Helpman and Hillman's discussion of this term when a social welfare function representation is used [1976, pp. 8–11; 1977]). A marginal entrant who views the private and shared goods as complements in the Edgeworth-Pareto sense (i.e., $\partial^2 U^s/(\partial x^s \partial y^s) > 0$) should receive more of the private good when becoming a member (i.e., $y^s > \tilde{y}^s$), whereas an entrant who views these goods as substitutes should receive less of the private good when becoming a member.

There appears to be some confusion about this term in the literature. In one place, Helpman and Hillman called it a resource cost (1976, p. 9), but *no* resources are needed owing to the unchanged level of private good production. Ng stated that this term appears because Helpman and Hillman use a social welfare function (SWF) presentation rather than a Pareto efficiency setup without an SWF (Ng, 1978, p. 408); however, a Pareto efficiency specification will always yield this term when the differentiation is performed and the goods are related in consumption (Sandler, 1979, Appendix; Hillman and Swan, 1979a).

Even though utilization is continuously variable, the discreteness aspect of membership has not altogether disappeared. This aspect shows up in the discrete change in utility and private good consumption (see (8)) that membership requires. In some instances, equation (8) cannot be satisfied at any membership size and an inequality is necessary.

Since the general model requires an ordering of the population based upon club preferences, this model is implicitly assuming cardinality of the utility functions. To circumvent this limitation, Hillman and Swan have proposed an ordinal Pareto-optimal representation that does not require an ordering of the population (1979a). In their model, an *arbitrary* individual's utility is maximized subject to the constancy of the other individual's utility levels. When this arbitrary individual is *not* the marginal entrant, the membership condition can be represented by (8), *provided* that the square-bracket term on the left-hand side is removed. This term drops out because the marginal entrant's utility level has been exogenously fixed and, hence, cannot change. In this case, Hillman and Swan interpreted the $(y^s - \tilde{y}^s)$ term as the compensated variation (1979a, p. 4). If, by chance, the arbitrary individual (whose utility is maximized) is the marginal entrant, then (8) is the membership condition when the square-bracket term is included. Only the form of the membership condition may be affected by the ordinal and cardinal alternative representations of Pareto optimality; the provision and toll conditions are unchanged in form. Of the two representations, the ordinal one has greater *positive* usefulness because in practice populations cannot be ordered.

The general model can be shown to be related to the Buchanan model when some further restrictions are imposed. First, all individuals must be identical and each must consume the available quantity of the shared good, $x^i = X$ for all i, and $k = s$. Second, the crowding function must be the identity mapping so that $c(s) = s$.

With these changes, the utility function is identical to that of the Buchanan model. Furthermore, only a representative member must be considered in the maximization process and, consequently, the transformation function must correspond to a member rather than to the economy.

By allowing the marginal utility of crowding to be positive over some range of membership, the model can account for a pure taste for association. Moreover, the exact specification of the production constraint can reflect such things as cost reductions from sharing and resource costs required to bring new members into the club.[17] The generality of the model is thus established.

IV. *An Evaluation of the Issues and Controversies in Club Theory: Non-Game-Theory Approaches*

A. *The Impact of Membership Heterogeneity*

An examination of the general model and its results can reveal the major impact of a membership heterogeneity assumption. In the first place, *both* a toll (or utilization) and a membership condition are required, since each member can utilize a different amount of the shared good.[18] The toll condition determines the price per unit of utilization, whereas the membership condition analyzes the net total effect on the club from having another member join. In other words, the toll condition depends upon a continuous change in utilization, whereas the membership condition involves a discrete change. When all members are identical, each utilizes the same amount of the shared good and, therefore, both the utilization and the membership decisions are the same discrete decision. All members then pay the same toll *or* membership fee. In contrast, as shown by the general model, a heterogeneous club's members pay the same rates of toll, but their total toll payments differ based upon tastes. Another difference between homogeneous and heterogeneous club models is the appearance of a separate membership span (i.e., time in club) condition when a time dimension is introduced into the analysis (see *Section* VII-B). If members are different, the membership spans differ between members and are no longer identical to the discrete membership decision in which membership implies a specified span.

Other differences associated with heterogeneity concern discrimination possibilities, transaction costs, and the stability question. Discrimination against another member's characteristics can be practiced only when members differ in some way.[19] Hence, heterogeneity permits an important aspect, commonly observed in clubs, to be examined (see *Section* IV-C). If members are heterogeneous, then the provision, toll, and membership decisions are more difficult to reach owing to the aggregation of diverse preferences. The transaction costs associated with these decisions are expected to be greater than in homogeneous clubs. Both types of clubs are voluntary in the sense that members would not join (or remain in the club) unless a net gain resulted from membership. In heterogeneous clubs, members' goals

[17] In the literature some researchers included the crowding term on the benefit side in the utility function; others included it on the cost side; and still others included it on both sides. When crowding is included in the utility function, it reflects crowding externalities in the club (Sandler, 1979). In contrast, its inclusion on the cost side indicates either cost-sharing benefits or resource costs (Artle and Averous, 1973). Finally, its inclusion on both sides can reflect all three influences (Berglas and Pines, 1978).

[18] Some heterogeneous membership models do not derive both a toll and membership condition, since all members are forced by assumption to consume the same amount of the good (e.g., see Berglas [1976a], Ng [1973; 1974; 1978]). This assumption appears to conflict with heterogeneity, unless monitoring of utilization is impossible.

[19] Price discrimination can always be exercised (Pauly, 1967).

differ, which can necessitate a centralized authority to enforce decisions and to determine memberships. Another difference concerns whether the membership is stable or unstable. Instability occurs when coalitions can entice members to leave by offering them net improvements in their club payoff. This issue is discussed in *Section* V.

A basic controversy related to the heterogeneity question concerns whether mixed clubs are Pareto optimal. This issue has far-reaching implications for such social problems as: are segregated neighborhoods desirable? should school children be bused to achieve racial balance? and is exclusionary zoning optimal? The literature appears split on the mixed club issue with one group of researchers depicting models where heterogeneous memberships are optimal (e.g., DeSerpa [1977], Ng [1973], Oakland [1972], Rothenberg [1970a; 1970b]) and another demonstrating models where mixed clubs are usually not preferred to homogeneous ones (e.g., Berglas [1976a, p. 120], Berglas and Pines [1978, p. 12], Helpman [1979], McGuire [1974a, pp. 121–25], Porter [1978], and Joseph Stiglitz [1977]). This latter group *has recognized* that mixed clubs may be desirable when strong scale economies require a larger membership than possible with homogeneity, when multiproducts are being provided, or when members have different skills (see especially Berglas [1976b], and Berglas and Pines [1978, p. 27]).

This is an interesting controversy, since a careful examination of all of the models in both camps indicates that the solutions are correctly derived and are internally consistent with respect to their assumptions! The apparent resolution involves a "second-best" constraint invoked by all researchers arguing *against* mixed clubs. This constraint requires all members to share equally club costs and can result in a lower welfare level as compared with its absence (see Peter Diamond [1973, p. 532] on this type of second-best constraint).[20] Thus, mixed clubs, where members utilize the good by different amounts yet pay the same total fees, can always be shown to be less desirable than homogeneous ones. In the general model of *Section* III, this second-best constraint is not imposed and, consequently, mixed clubs are desirable and optimal whenever utilization rates can be monitored and individual cost shares assigned. The optimality of mixed clubs hinges on the amount of transaction costs; i.e., are monitoring costs prohibitive so that second-best constraints must be imposed?

B. *The Membership Size Question: What are Pareto-optimal Conditions?*

The issue concerning Pareto optimality was first raised by Oakland (1972, p. 340) and Ng (1973; 1974) with respect to Buchanan's derivation of the membership condition. For example, Ng argued that by maximizing the net benefits of a representative individual, Buchanan maximized *average* net benefits instead of total net benefits (Ng, 1973, p. 294) and, thereby, failed to give proper Pareto-optimal conditions. In contrast, Berglas defended Buchanan and employed Buchanan's methodology (Berglas, 1976a, p. 117). Helpman and Hillman (1976; 1977) stated that Ng's membership condition is incorrect and provided an alternative. The debate still continues with recent exchanges between Ng (1978) and Hillman and Swan (1979a).

The resolution of this debate hinges upon a recognition of the different types of club problems analyzed (Helpman and Hillman, 1977, p. 295). Buchanan examined a decentralized consumer choice problem in which the number of clubs was

[20] The form of this constraint is as follows: $I = y^i + C(X,s)/s$, where C is the cost of the club, I is income, and the price of the private good is one. Other researchers imposed another second-best constraint requiring all members to utilize the club equally, irrespective of taste (e.g., McGuire [1974a] and Porter [1978]).

unspecified and a within-club point of view was taken. This point of view implies that *average* net benefits for the members are maximized.[21] Berglas has used the same approach and explicitly allowed for multiple (or replicable) clubs. In contrast, Ng examined the membership condition for one club from the point of view of the entire economy and, in so doing, he maximized *total* net benefits for the entire economy (also see footnotes 41 and 42). However, Helpman and Hillman (1977, pp. 293–94) correctly criticized Ng's use of Leibnitz's rule for differentiation of a sum or an integral; i.e., Ng stated the correct problem but failed in its execution when he dropped a $(y^s - \tilde{y}^s)$ term, which need not be zero. If the true one-club, Pareto-optimal membership condition *for the economy* is analyzed, the proper procedure is that of Helpman and Hillman (1977), Sandler (1979), and the general model of *Section* III wherein both members and non-members are considered. The *first* model that derived a membership condition from an economy's point of view was that of Artle and Averous (1973).

Neither the within-club, nor the entire economy, one-club point of view ensures Pareto optimality under all situations. The former may fail when the membership size is large relative to the entire population; the latter will fail when multiple clubs are desirable. Further discussion of these failures is deferred to *Section* V-C where they are related to the game-theoretic core concept.

The four-graph representation for the total economy point of view is presented in *Figure* 2 and contrasts with *Figure* 1. The graphs in *Figure* 2 assume that each member utilizes the entire provision of the shared good. On the ordinate in quadrants I and II, the net aggregate benefits of the club are measured, while on the abscissas, output and membership are measured. The shape of the net benefit curves, $NB(\cdot)$, in quadrant I indicates that net benefits from increased facility size will eventually reach a maximum due to increasing marginal costs and diminishing marginal benefit whenever membership size is fixed (see Ng [1973, pp. 295–96]). In quadrant II, net benefits from expanding membership peak out owing to crowding when the facility size is held fixed. The optimal points in quadrant I correspond to the points on the X_{opt} curve, and those in quadrant II correspond to the s_{opt} curve. As before, equilibrium occurs at point E where the s_{opt} and X_{opt} curves intersect. The interaction of the two conditions is easily seen.

C. *Discrimination and Clubs*

In the non-game-theory literature, discriminatory clubs refer to those sharing arrangements in which members consume both the shared good *and* the characteristics or attributes (e.g., race, religion, appearance, presence) of the other members (DeSerpa, 1977; DeSerpa and Stephen Happel, 1978; Ng and Tollison, 1974; Tollison, 1972). Each club member has a fixed vector of characteristics that is made available to the other members according to his utilization rate of the club (DeSerpa, 1977).[22] Heavier users provide the club with a larger amount of their attributes than less heavy users. The total available quantity of any membership attribute de-

[21] McGuire (1974a) followed Buchanan's approach when he solved the problem:

$$\max U^i(y^i, X)$$
$$\text{subject to } I = y^i + C(X, s)/s.$$

McGuire's provision condition is identical to the Samuelson condition, and his membership condition requires cost per person to be a minimum. This latter requirement implies that the average cost of adding a member equals the marginal cost of adding a member. That maximizing average net benefits is not the same as maximizing total net benefits for the club can be shown by comparing the membership conditions when $U^i(\cdot)$ and $sU^i(\cdot)$ are maximized, respectively. The former requires $s(\partial C/\partial s) - C(\cdot) = 0$, and the latter requires $U^i(\cdot)/s = (\partial U^i/\partial y^i)[s(\partial C/\partial s) - C]/s^2$.

[22] The discussion in the text is based on the DeSerpa model (1977), since his discriminatory model is the most complete one in the literature.

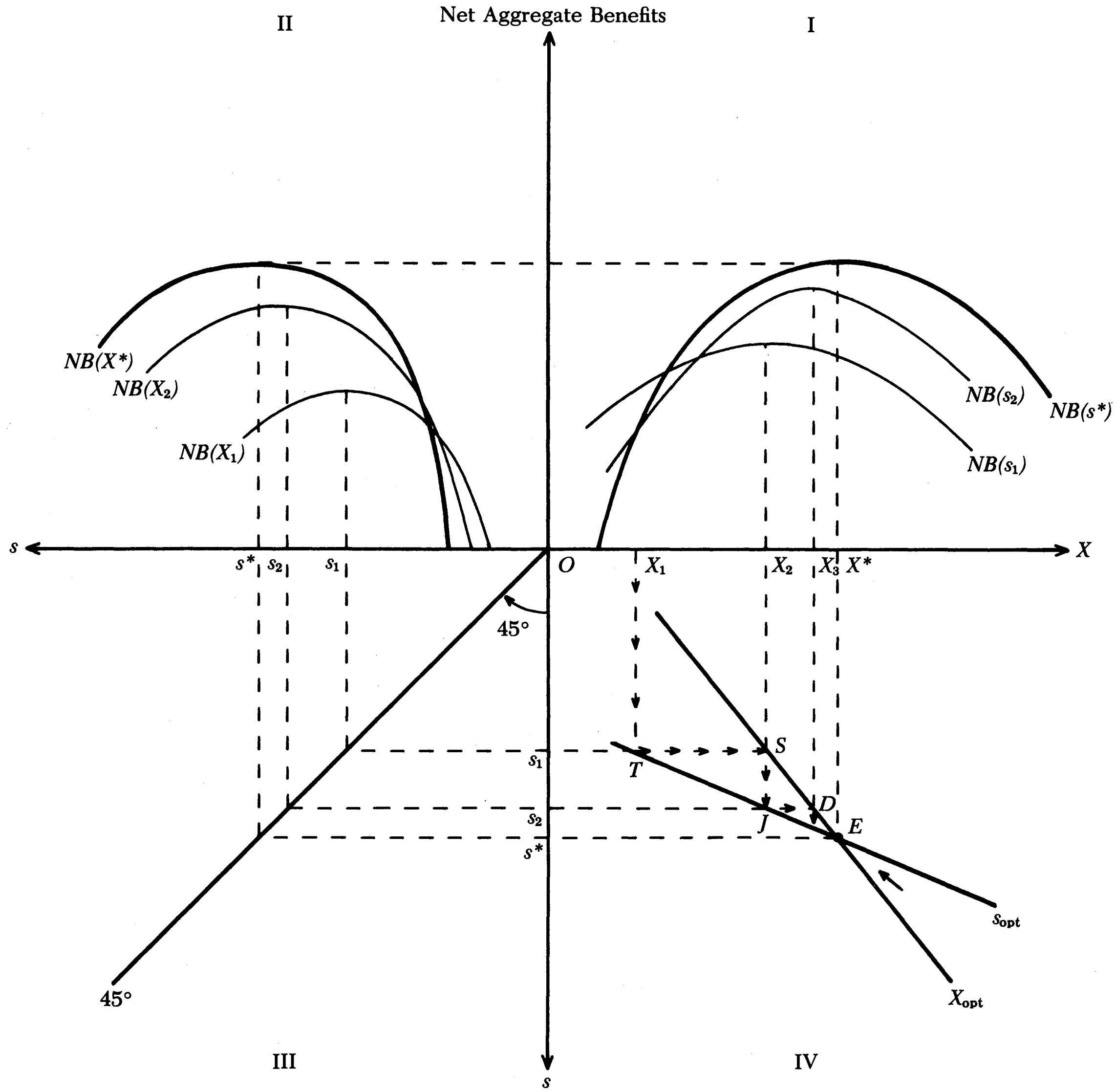

Figure 2

pends upon the aggregate utilization rates of all members and the amount of that attribute associated with each member. By varying the utilization rate, a member determines his consumption of the other members' characteristics as well as that of the shared good.

Members' characteristics may be viewed by the other members as generating either an increase (e.g., beauty, congeniality) or a decrease (e.g., race, rudeness) in utility. Crowding can be treated as one characteristic of the membership that arises from the mere presence of the other members. Thus, the general model of *Section* III can be extended to include discrimination, provided that a vector of membership characteristics is generated by the utilization rates and one characteristic corresponds to congestion.[23]

The provision condition of the discriminatory club model remains essentially unchanged from that of the general model; however, the utilization and membership conditions change as each member consumes a *club package* consisting of the shared good and the members' characteristics (DeSerpa, 1977, pp. 34, 37–38). A member utilizes the club until the marginal benefits that he derives from the club package equal the associated marginal costs (or value) that the club experiences from the member's utilization.[24] The membership condition must also include both aspects of the club package. Some members with desirable traits may be paid to join, since they generate enough positive characteristics to offset any crowding caused by their presence. The analysis of discriminatory clubs allows for a study of the pure taste for association rationale for club formation.

D. *Exclusion Costs*

A number of investigators[25] have examined exclusion costs associated with the erection, maintenance, or existence of a mechanism to limit club utilization or membership. At least three different approaches to exclusion costs exist in the literature. In one approach, waiting costs can result from the existence of the exclusion mechanism, and these waiting costs are a *preutilization* crowding or *queuing* cost, which depends on the number of users (Barzel, 1974; Nichols, Smolensky, and Tideman, 1971; Porter, 1977).[26] In this case, the general club model remains basically unchanged, except for the interpretation of crowding costs, which now include preutilization and utilization crowding.

A second approach concerns the resource costs of the exclusion mechanism and is best illustrated by Oakland's treatment (1972, pp. 351–55). Oakland assumed that these exclusion costs rise as the quantity of the shared good increases, but they *fall* as the average utilization rate increases. As k increases, a smaller number of potential users must be excluded. In order to represent these exclusion costs, the general model's transformation function must include k as an additional argument with a negative influence on resource costs.

When exclusion costs are introduced in this manner, the optimal provision amount is reduced, whereas optimal utilization and membership are increased (Oakland, 1972, pp. 353–54). For provision, the marginal benefits from reduced crowding must be equal to the *sum* of marginal production costs and marginal exclusion costs. Since this sum is greater than marginal production costs alone, marginal benefits from provision must be

[23] One important difference between the general model and that of DeSerpa (1977) concerns his introduction of a time dimension for sharing; i.e., the proportion of a club's hours utilized by a member. The difference requires a slight alteration to the general model, since congestion now depends on the utilization rate and is not experienced equally by all members. On the different sharing dimensions, see DeSerpa (1978b).

[24] Unlike the general model of *Section* III, the tolls differ between members.

[25] For example, see Yoram Barzel (1974); Otto Davis and Whinston (1967); Kamien and Schwartz (1970); Kamien, Schwartz, and Roberts (1973); R. Millward (1970); Gene Mumy and Steve Hanke (1975, pp. 716–17); Donald Nichols, Eugene Smolensky, and T. Nicolaus Tideman (1971); Oakland (1972); Sandler and Cauley (1976); and Klaus Spremann (1978).

[26] Nichols, Smolensky, and Tideman (1971) and Roger Feldman (1978) have demonstrated that queues can determine membership size according to the opportunity cost of time. Furthermore, Nichols *et al.* have argued that queues are efficient allocating devices, unless alternative means are less costly than the deadweight loss of the queue. Rationing by congestion charges differs from that by queuing because the first collects a toll, while the queue collects nothing (Porter, 1977).

correspondingly larger for optimality (see equation (6)); optimal provision falls. Furthermore, utilization rates increase because the resulting marginal net damage must equal the marginal exclusion costs of utilization. This marginal net damage is the difference between marginal crowding costs and the user's marginal benefits from utilization. Without exclusion costs, optimal utilization is reached when marginal net damage is zero (see equation (7))—all users fully compensate for damages inflicted on the club. With exclusion costs, utilization is expanded even when marginal net damages are positive, provided that these marginal net damages are *less* than marginal exclusion costs. In this situation, exclusion is more costly than the uncompensated damage. These exclusion costs can increase utilization rates and, in an analogous fashion, membership size.

A third method of handling exclusion costs is that of Kamien, Schwartz, and Roberts (1973, pp. 225–29). These authors have determined the optimal degree of exclusion by including the degree of exclusion in the transformation function. Their optimal exclusion condition compared the marginal benefits of utilization (i.e., nonexclusion) with the marginal exclusion costs.[27]

E. *Self-Financing and Efficient Tolls*

An important question, "can efficient tolls self-finance optimal provision of the shared good?" has been often asked (e.g., see DeSerpa [1978a], Mohring and Harwitz [1962], Timothy Muzondo [1978], Oakland [1972], Sandler [1975b; 1979], Sandler and Cauley [1976], and David Winch [1973]). An examination of conditions (6) and (7) of the general model can begin to answer this question. When the per-unit toll *(T)* is set equal to the right-hand side of (7) and the result is substituted into the provision condition, the finance condition is derived.

$$kT = MRT_{xy} \qquad \text{(Finance Condition)} \quad (9)$$

This condition states that the sum of the tolls paid on a unit of the shared good suffices to finance *marginal* costs of provision; the toll is multiplied by the average utilization rate when determining toll revenues on a unit of the shared good because each unit can be utilized more than once. Even though an efficient per-unit toll serves to finance marginal costs, the toll fails to self-finance the shared good whenever average costs exceed marginal costs —i.e., increasing returns are present.[28]

Deviations from this financing result are usually traceable to the form of the congestion function.[29] If crowding depends upon k *and* the provision amount, the new financing condition requires that $(1-Xc_X/kc_k)kT$ equals MRT_{xy} (Oakland, 1972, p. 342). When congestion costs are homogeneous of degree zero in utilization and provision, marginal costs are financed because c_X is zero and the new financing condition degenerates to (9). Otherwise self-financing is not necessarily assured whenever c depends on k and X. In an interesting paper, DeSerpa analyzed three alternative crowding functions and showed how self-financing is responsive to the form of this function (1978a).

When increasing returns characterize the production of the shared good, a two-part tariff can levy a fixed membership

[27] Kamien, Schwartz, and Roberts's condition is important because it distinguished between publicness in consumption models (1973), where the degree of rivalry is exogenous to the problem (Alan Evans, 1970; Mohring and J. Hayden Boyd, 1971; Oakland, 1969), and publicness in production, where the degree of rivalry is an endogenous variable.

[28] This financing condition assumes that no individual consumes the entire amount of the shared good (i.e., $x^i < X$ for all i). When this is not the case, financing problems can arise from free-riding behavior by the group of full utilizers (Oakland, 1972, pp. 348–49).

[29] Another deviation can result from the sharing of joint products associated with the shared good (Muzondo, 1978; Sandler and Cauley, 1976).

charge *and* a utilization fee. The fixed charge attempts to recover the deficient revenue required to self-finance the provision of the shared good (see *Section* VI-A). Some form of outside intervention (e.g., government subsidies) is needed for financing purposes when a fixed charge is not assigned.

F. *The Institutional Forms of Clubs*

Buchanan stated that myriad institutional forms exist for clubs: e.g., a club can be privately owned by a cooperative membership, operated for a profit by a firm, or publicly controlled by the government (Buchanan, 1965, p. 7; 1968, chap. 9). Berglas (1976a, p. 118) and Berglas and Pines (1978, p. 11) demonstrated that a perfectly competitive industry with identical firms supplying the shared good would achieve the *same* efficiency conditions as those of a private cooperative. In this case, each firm operates as a club, and no nonmarket intervention is needed. For clubs consisting of overlapping generations of members, Sandler showed that adequate incentives exist for privately owned clubs to act with foresight, but do not exist for government-operated ones owing to short-run political objectives (1979, pp. 18–22). Nevertheless, privately owned clubs must be operated by a firm rather than by its members if free-rider problems with respect to the good's maintenance are to be avoided.

In analyzing alternative institutional forms, Hillman compared clubs operated by a profit-maximizing monopoly with those of a nonprofit cooperative (1978a). He found that the nondiscriminating monopolist provides a smaller output, charges higher prices, and operates more crowded facilities than the nonprofit cooperative. By using Berglas's results, perfect competition is seen to be preferable to nondiscriminating monopoly whenever a firm operates the club; however, Hillman and Swan have shown that a discriminating monopolist will always achieve an efficient market solution (1979a, p. 9).

In contrast, Ng argued that centralization or government intervention is necessary to achieve efficiency, since members will maximize average rather than total net benefits (1973; 1974). However, there is no reason why the members cannot form their own centralized authority to collect the proper tolls because efficiency gains will result.

With exclusion and monitoring capabilities,[30] nonprofit cooperatives and competitive firms can usually operate the club efficiently. The existence of a club situation does not automatically indicate market failure. A complete analysis of institutional form must await a fuller treatment of transaction costs associated with alternative institutional forms.

G. *Other Issues in Club Theory*

1. *Convexity of the Crowding Function.* Although most club models examining partially rival goods employ a crowding function,[31] relatively few researchers have studied the actual form and convexity of this function (see Anthony Boardman and Lester Lave [1977], Färe and Grosskopf [1979a; 1979b], Hillman [1978], Robert Inman [1978], and Alan A. Walters [1961]). This is an important issue, since the existence of a solution depends upon this convexity. By studying the properties of the crowding function, Färe and Grosskopf (1979a, pp. 3–6) have shown under what situations Hillman's (1978) nonprofit clubs exist. Furthermore, Inman has generalized the congestion studies of highways

[30] Oakland analyzed a firm's provision of a nonrival good that is excludable and found that inefficiency resulted because preferences could not be monitored (1974).

[31] The use of a crowding function in the utility function is actually the introduction of a technology of consumption. Club models that explicitly use the new theory of consumption include Muzondo (1978), Sandler and Cauley (1976), and Agnar Sandmo (1973).

by providing a congestion function that includes the major functions previously used (1978). More work on the form of the crowding function appears necessary and is currently underway by Färe and Grosskopf.

2. *Price and Income Effects in Club Theory.* Adams and Royer derived Slutsky equations to analyze price effects on club membership and provision (1977, pp. 155–57). Their model assumes identical individuals who equally share costs and who utilize identical amounts of the shared good. As is typical for Slutsky representations, the income effect and the total effect of price changes on membership and provision *are indeterminant* and depend upon the form of the crowding functions, the interaction between the membership and provision conditions, and the nature of the shared good (i.e., is it normal or inferior?). Similar results for income effects were derived by Heavey and Gunzenhauser (1978).

In the special case where the quantity of the shared good is fixed, Adams and Royer demonstrated that an increase in income reduces club membership (1977, pp. 148–49). Therefore, a tendency towards an inverse relationship between membership size and income is predicted.

3. *Relationship between Congestion, Common Property, and Pollution.* A number of distinctions and similarities can be drawn between the externalities associated with congestion, common property, and pollution. Rothenberg contrasted congestion and pollution by indicating that congestion has a symmetric effect on users and abusers, whereas pollution has an asymmetric effect (1970a, pp. 114–15). That is, polluters may not be users of the environment that they pollute. Haveman then demonstrated that this asymmetry implies that pollution is not self-limiting, since the benefits to the abuser are not exhausted as the quality of the environment deteriorates (1973, p. 285). For congestion and common property problems, overuse results as participants maximize their net benefits rather than total net benefits (Weitzman, 1974; Robert Dorfman, 1974, p. 14). This overuse is self-limiting because the surplus from overuse is eventually exhausted as the quality of the good deteriorates (or the rent from exploitation falls to zero).

V. *Game Theory Models of Club Formation*

A. *The Game Theory Formulation*

Game theory has helped to determine the optimum number of clubs and their stability; although, as pointed out in *Section* V-C, some generality is sacrificed in the formulation. By defining the set of players in a game as the entire population, both club and non-club members are considered for all possible partitions of the population into multiple clubs. The concept of the core is used to examine the stability of the various partitions. The core is a less general solution than a Pareto optimum, since a partition may be Pareto optimal but not in the core, whereas a partition in the core is also Pareto optimal.

N-person game theory, developed by John von Neumann and Oskar Morgenstern (1944), provides a natural formulation of club problems. In game theory, players have an incentive to form coalitions because of the payoffs available from playing the game, while in club theory members form clubs to partake of the available benefits. The characteristic function used in game theory indicates the total net benefit available to all potential coalitions of players and can be used in club theory with some modification. If $\tilde{S} = \{1, \ldots, \tilde{s}\}$ is the set consisting of the entire population, and $S = \{1, \ldots, s\}$ is some subset of the population, then the characteristic function is denoted $v(S)$ for all $S \subseteq \tilde{S}$.[32] This function represents the total net benefit available to a club consisting of the members in S.[33] If the entire population forms a single club, the total net benefit is $v(\tilde{S})$.

Each individual in the population is interested in the net benefit or payoff available to him from possible club participation. If $b = (b_1, \ldots, b_s)$ is any vector of these net benefits available to the population, then individual rationality implies $b_i \geqq v(\{i\})$ for $i = 1, \ldots, \tilde{s}$. This holds because if individual i is in a club where $b_i < v(\{i\})$, he can always do better by dropping out and forming a club of one. The rationality concept can be carried over to any set of individuals including the entire population. Thus,

$$\Sigma_{i \varepsilon S}\, b_i \geqq v(S) \tag{10}$$

for all $S \subseteq \tilde{S}$. Net benefit vectors that satisfy (10) are said to be in the core. As a solution, the core implies that no individual or set of individuals can improve upon their situation by forming a different partition. Since the entire population is one of these sets, a core solution implies a Pareto optimum. If a core solution is obtained for a club consisting of the entire population, then this is a stable club, since no member or set of members will have an incentive to drop out. Also, if a core solution is obtained for a collection of m clubs, $(S_1, \ldots, S_m)$, then this is a stable collection, since no member or set of members will have an incentive to transfer among clubs or to drop out and form new clubs.

B. *Optimum Club Size and the Optimum Number of Clubs*

With a homogeneous population, the v function depends only on the number of club members and not on the composition of the membership. In this framework, a number of interesting results regarding club size and the number of clubs have been obtained by Pauly (1967; 1970b). TABLE 1 will be useful in discussing these results. For each membership size, the total and average net benefits for the club are listed. The average increases up to $s = 4$ because of scale economies, camaraderie, and/or the sharing of an impure public good, but after $s = 4$ the average falls as these benefits give way to increased congestion and decision costs. The net benefits are zero for clubs of size 9 or greater.

The optimum club size, s^*, was defined by Pauly as the one for which average net benefits are maximized; i.e., $v(S^*)/s^*$ is a maximum over all s (1967, p. 315; 1970b, p. 56). This follows Buchanan's (1965) approach. This is not the same as maximizing total net benefits for the economy[34] and is at variance with the nongame formula-

[32] In game theory, the characteristic function is defined to be *super-additive*, that is $v(S' \cup S'') \geqq v(S') + v(S'')$ for all mutually exclusive sets S', $S'' \subseteq \tilde{S}$. The term $v(S' \cup S'')$ does not mean necessarily that the players in $S' \cup S''$ are acting as a single decision unit, for if the two sets can do better apart than together, they will stay apart and $v(S' \cup S'') = v(S') + v(S'')$. In club theory, $v(S' \cup S'')$ implies that the two sets do act as a single decision unit (i.e., one club). If the two sets could do better forming separate clubs rather than a single club, it must be the case that $v(S' \cup S'') < v(S') + v(S'')$. Thus, club characteristic functions may be subadditive to allow for this possibility (Pauly, 1970b, p. 55).

[33] The authors who have applied game theory to clubs have assumed transferable utility and have used a scalar-valued characteristic function (see R. Duncan Luce and Howard Raiffa [1957] for a description of transferable utility). Pauly defined $v(S)$ as the difference between gross benefit and gross cost functions (1970b). Both are measured in terms of a *numéraire* good. Total benefits are the summation of individual benefits, and total costs include resource costs, decision costs, and congestion costs. Stephen Littlechild (1975) defined $v(S)$ as the profit available from full-time operation of the club, while Sorenson, Tschirhart, and Whinston (1978b) equated consumers' surplus with the characteristic function.

[34] Pauly stated that maximizing average net benefit is the same as setting the marginal gross benefit equal to the marginal gross cost of adding a member (1967, p. 316). But this latter approach maximizes total net benefit for the club, which will differ from maximizing the average unless the total function is linear in s. This approach also differs from maximizing the total net benefit for the economy. See footnote 21 on this point and also the discussion in *Section* IV-B.

TABLE 1
NET BENEFITS AND CLUB SIZE

CLUB SIZE (s)	1	2	3	4	5	6	7	8	9	10	. . .
TOTAL NET BENEFITS [$v(S)$]	.4	1.50	9	16	17.5	18	14	8	0	0	. . .
AVERAGE NET BENEFITS [$v(S)/s$]	.4	.75	3	4	3.5	3	2	1	0	0	. . .

tions of Ng (1973), Helpman and Hillman (1977), and that of the general model. This apparent contradiction is reconciled in *Section* V-C.

Consider the case where $s^* = \tilde{s}$, so that the optimum club size is the entire population. The core exists, and one solution in the core is where each club member receives $v(S^*)/s^*$. From TABLE 1, $s^* = \tilde{s} = 4$ and each member receives $v(\{1,2,3,4\})/4 = 4$. With these payoffs, no subset of members can do better by abandoning the club.

There are, however, other core solutions that involve unequal payoffs. In that event, price discrimination is being practiced, although the basis for discrimination is not the members' characteristics as in *Section* IV-C. In a world of equals, characteristics do not vary. The core places bounds on the extent of this price discrimination. For example, suppose member one is being discriminated against, and the payoffs are $b = (.4, 5.2, 5.2, 5.2)$. Member one could do no better by dropping out of the club, since $b_1 = .4$ is the most he could attain alone. Nor could he persuade any other member or two members to join him in a club of two or three because their net benefits would be less than they currently receive. These payoffs are in the core. But if two members were discriminated against in a like manner so that $b = (.4, .4, 7.6, 7.6)$, members one and two could either abandon the club and form a new club of two, or they could be joined by either member three or four in a new club of three. Their situation improves in both cases. In general, a core solution must satisfy the condition that if members in subset S are being discriminated against, they must receive at least $v(S)$.

When $s^* < \tilde{s}$, the existence of the core will depend on whether $\tilde{s}/s^*$ is an integer and whether equal sharing is enforced. If $\tilde{s}/s^*$ is an integer, then the core is attained only if the entire population is divided into clubs of size s^* where each member receives the equal share $v(S^*)/s^*$. When clubs are not all of size s^*, there will be incentives for non-members or members in oversized clubs to join undersized clubs. If all clubs are oversized, some members will break off into smaller clubs. Suppose $\tilde{s} = 8$ in TABLE 1, and two clubs form with three and five members each. At least one of the members in the club of five can transfer to the three-member club. This will be an improvement for the club of three and possibly for the remaining members in the five-member club depending on whether there is discrimination and to what extent. Furthermore, only equal-sharing payoffs are in the core because any club that practices discrimination will find that either the members being discriminated against are abandoning the club or the members in the discriminating group are replaced by members of other clubs. If two clubs have payoffs of (.4, 5.2, 5.2, 5.2) and (4, 4, 4, 4), then member one from the first club could move to the second club, or any member of the second club could replace any one of the last three members in the first club. These results suggest that a multiclub world provides safeguards against discrimination, since those being discriminated against have the option of transferring to another club (Pauly, 1970b, p. 59).

When $\tilde{s}/s^*$ exceeds one but is not an integer, the core does not exist.[35] An individual omitted from all clubs of size s^* because of the integer problem can always bid his way into a club by offering to accept a payoff lower than a club member. The rejected club member is then on the outside and free to bid his way into a club. This shuffling never ends.

With a heterogeneous population, the optimum number of clubs is considerably more elusive. Pauly worked with a heterogeneous population divided into homogeneous groups (1970b). Each group is divided into multiple clubs where average net benefits are maximized and there is no integer problem. Even in this restrictive model, however, little can be said. The main result is that a core exists if the clubs consist of identical members with equal payoffs and that clubs with higher average payoffs have fewer members than those with lower average payoffs. This ensures that members of larger clubs have no incentive to transfer to smaller clubs or accept members from smaller clubs (Pauly, 1970b, pp. 60–64).

Sorenson, Tschirhart, and Whinston have examined heterogeneous clubs where the only impetus for formation is decreasing production costs (1978b). Unlike the homogeneous case, total net benefits to the club do not depend simply on the *number* of members, but rather on the *number* and *identity* of members. This means that maximizing average net benefits is not a useful determinant of optimum club size. In the homogeneous case, members can evaluate their own payoff by comparing it to the average benefit because their contribution to the club is the same as everyone else's contribution; in the heterogeneous case, this same evaluation is meaningless, since a member may contribute more or less than others contribute. When a club consists of the entire population, a core can exist for a membership greater than where the average benefit is maximized. A sufficient condition for core existence is that the provision level that maximizes consumers' surplus is in the range of decreasing average costs. Depending on the members' demands, core existence is also possible beyond minimum average cost; but there is the possibility that for any output beyond minimum average cost a single club is not optimal.[36] Determining the optimum number of clubs is similar to determining the optimum number of firms that have U-shaped average cost curves. And mixed clubs in this situation may very well be desirable.

C. *A Comparison of the Game and Non-Game Formulations*

A direct comparison of the game and nongame formulations of club theory is complicated by the different assumptions used. The game formulation does not admit a well-defined interaction between the provision and toll conditions. The nongame formulations have a provision condition that is separate from the membership condition, and the toll condition requires that marginal cost pricing be adopted. For

[35] An exception is given by Pauly's Theorem 1 (1970b, p. 56). If equal sharing is enforced and all clubs are of size s^*, the core exists. An omitted member cannot bid into a club, since he cannot bargain for a payoff lower than the members are receiving.

[36] The relationship between the average cost curve and the number of clubs is related to the definition of a single-product monopoly. An often used definition is that the firm's cost function be subadditive up to the quantity of output produced and, therefore, a single supplier is optimum (William Baumol, 1977). If $v(S)$ is the maximum consumers' surplus available to S, a subadditive cost function is not sufficient for a super-additive v function (Sorenson, Tschirhart, and Whinston, 1978b, p. 94).

a homogeneous population, each club member's total payment is identical and consistent with his demands, since provision and toll conditions are found simultaneously. In the game formulation, there is unlimited bargaining among the population regarding payments to support the clubs.[37] The payoff, b_i, represents a benefit and charge to individual *i*, where the charge is a lump-sum payment. The *v* function implies nothing regarding the distribution of these charges among the members. For a particular *b* vector, a particular level of provision follows. But another *b* vector may lead to different demands and another level of provision. Provision is relative to the charges. But if $v(S)$ is understood to be associated with an optimum provision for membership *S*, then the members' demands must be insensitive to the charges levied against them. Otherwise, as bargaining proceeded, provision would change.

Essentially, the game formulation allows the membership, provision, and finance conditions to be solved simultaneously, while the toll condition is solved afterwards. There is latitude in solving the toll condition, since different tolls are compatible with a single solution to the other three conditions. This is clearly different from the nongame formulation.[38] Buchanan stated that, ". . . the quantity of the good, the size of the club sharing in its consumption, and the cost-sharing arrangements must be determined simultaneously" (1965, p. 12). Also, the finance condition in the game formulation stipulates that total resource cost of the club will be covered. A result from *Section* IV-E is that in the nongame formulation full financing is not always accomplished, for example, when there are increasing returns to scale in production. Thus, the game formulation implicitly utilizes a second-best pricing approach.

The authors who have used a game-theoretic formulation have approached the provision question in different ways. Pauly avoided the issue and simply stated that $v(S)$ reflects net benefit from the quantity of good chosen (1970b, p. 55). Littlechild abstracted from the level of provision and was concerned only with whether the good is produced at all (1975, p. 118). Basically, demands are perfectly inelastic and insensitive to charges, provided that the charge to a member does not exceed his benefit. In an approach that used elastic demands, Sorenson, Tschirhart, and Whinston defined $v(S)$ as the maximum consumers' surplus available to *S* (1978b). Provision was determined by the intersection of the aggregate downward sloping demand curve and the marginal cost curve. This yields a unique provision level for each membership size, which corresponds to the level attained under marginal cost pricing. However, the membership lump-sum charges may be greater or less than the amount that would be collected if marginal cost pricing is used. To ensure that these charges do not alter aggregate demand, zero income elasticity must be assumed. Finally, Ellickson maximized the sum of individual utilities subject to a transformation function with one crowded public good and one private good (1973). This is very similar to the nongame approach. The solution produced provision conditions for the public good and the total amount of the private

[37] Arrow divided the approaches to imperfectly competitive situations into those using per-unit pricing and those using unlimited bargaining (1970). The nongame and game formulations correspond to the former and latter, respectively. He went on to indicate how unrestricted bargaining can yield ambiguous results, such as Pareto-optimal allocations that are unstable and therefore not in the core.

[38] This simplification regarding the toll condition allows the use of a game with transferable utility and a scalar-valued characteristic function. The generality of the nongame formulation can be captured in a game without transferable utility. Vincente Salas and Whinston followed this procedure in a study of regulated industries with multiple outputs (1978), a subject that may prove useful in analyzing clubs with multiple outputs (see *Section* VII-A).

good. He then solved each member's utility maximization problem to find demand functions and tolls that support these provision levels. A problem arises, however, when more than one distribution of the sum of utilities are considered. Each different distribution must be associated with different tolls to satisfy the derived demand functions. But different tolls imply different demands and provision levels, and the sum of utilities will change. The sum cannot reflect a constant provision level with changing toll conditions and at the same time remain compatible with the demand functions.[39]

In spite of the difference between the game and nongame formulations, there are parallels between them regarding the optimum number of clubs for the homogeneous membership case. Both Ng (1973) and Helpman and Hillman (1976; 1977) implicitly assumed that only one club forms when they maximize total net benefits in their nongame formulations. For the one-club solution with part of the population omitted from the club, the core is empty owing to the integer problem. If the club's membership is greater than that which maximizes average net benefits, there is an incentive to eject members until the average net benefits of those remaining are maximized. If the club's membership is equal to that which maximizes average net benefits, nonmembers are in a position to bid their way into the club. One exception to the empty core would be the case where average net benefits are maximized when the entire population is in one club (e.g., $\tilde{s} \leqq 4$ in TABLE 1 and no integer problem).

The one-club constraint may yield a Pareto optimum in some cases. Referring again to TABLE 1, suppose $\tilde{s} = 6$. The core does not exist because there is an integer problem. Maximizing total net benefits yields a Pareto-optimal club of 6, while maximizing average net benefits yields a Pareto-inferior club of 4. The integer problem brings into question the definition of the optimum club size as that which maximizes average net benefits (Tisdell, 1977, p. 452; Sorenson, Tschirhart, and Whinston, 1978b, p. 86). As Ng has pointed out, for Pareto optimality an integer problem may require the club to go beyond the membership that maximizes average net benefit (1973, p. 298). Unfortunately, there seems to be no single correct answer to the definition question, since there are two different points of view: the within-club point of view (maximizing average) versus the total economy point of view (maximizing total).

For $\tilde{s} > 6$, requiring one club becomes too restrictive for Pareto optimality.[40] If $\tilde{s} = 8$, a one-club constraint yields a size of 6. But two clubs of 4 each are Pareto superior and in the core. If $\tilde{s} = 12$, maximizing total net benefits in a one-club framework again yields a club of 6. Two clubs of 6 each are Pareto inferior to three clubs of 4 each.[41] Thus, to obtain an econ-

[39] Members of Ellickson's sharing groups were characterized by the utility function $U^i = y^i X$. Maximizing utility subject to a budget constraint yielded specific demand functions. By maximizing the sum of utilities subject to an economy-wide resource constraint, the utility vector $(U^1, U^2, U^3) = {}^{15}\!/\!_8, {}^{15}\!/\!_8, {}^{15}\!/\!_{16})$ was attained for $X = {}^{15}\!/\!_8$ and $(y^1, y^2, y^3) = (1, 1, \frac{1}{2})$. Then using the demand functions, prices were found that cleared the market and yielded the utility vector. But a second vector, $(U^1, U^2, U^3) = (2, 2, {}^{11}\!/\!_{16})$, was then assumed to be a possibility. This vector is not consistent with the same amount of public and private goods, and the same demand functions.

[40] For a different set of numbers in TABLE 1, requiring one club can be too restrictive for Pareto optimality even when $\tilde{s} \leqq 6$. For example, suppose $v(S) =$ 3 instead of 1.5 for $s = 2$. Then with $\tilde{s} = 6$, one club of 6 members is Pareto inferior to two clubs of sizes 4 and 2. In this situation, the integer problem requires for Pareto optimality that no club be of greater size than that which maximizes average net benefits.

[41] Ng has pointed out that in this case his optimality conditions are satisfied for both two clubs of 6 or three clubs of 4 (1973). This is true, but the number of clubs must be *stipulated at the outset.* His one-club formulation yields one club of 6.

omy-wide Pareto optimum, the number of clubs must be treated as a variable. Comparisons must be made for each alternative partition of the population to achieve a Pareto optimum and possibly a core solution (Ellickson, 1973).[42] When this is done and the core exists, maximizing total and maximizing average net benefits converge to the same result: a Pareto-optimum solution in the core. Total net benefits are maximized when each club is maximizing average net benefits and everyone is in a club (e.g., $\bar{s} = 12$ with three clubs of 4 each). Because Buchanan (1965) and Berglas (1976a) have maximized average net benefits, their solution along with Pauly's will be identical to Ng's, Helpman and Hillman's, and that of the general model, *provided* that the number of clubs is a variable in the latter three.

VI. *Applications of the Theory of Clubs*

The wide range of problems that has been addressed by the analysis of club theory underscores the usefulness of the theory in applied work. A sampling of applied work in six areas is now provided to illustrate this usefulness.

A. *Public Utilities*

1. *Two-part Tariffs and the Club Principle.* A long-standing problem in public utility economics concerns pricing under conditions of decreasing average cost. Pareto optimality requires prices equal to marginal cost of provision, but this results in a deficit.[43] The remedies for covering this deficit constitute the marginal cost controversy, which was summarized by Nancy Ruggles (1950).

One possible remedy is to use a nonlinear price structure: in particular, a two-part tariff.[44] The two-part tariff consists of a lump-sum license fee for each consumer and a toll per-unit of utilization. The license fees are designed to cover any deficit caused by tolls set below average cost and can be interpreted as club entrance fees, since they must be paid before any utilization occurs. In this public utility club, the members share a private good with their only incentive for membership being the decreasing costs of provision. The license fees must be large enough to ensure full financing, yet not so large as to drive away too many potential club members. That some potential members may be driven away is an old point (Lewis, 1941); however, only recently have the conditions for an optimal toll and license fee been made clear by Ng and Weisser (1974). They considered a uniform license fee across consumers and derived the Pareto-optimal toll, license fee, and number of consumers or membership size.[45] The

[42] This applies to all models of club theory, including our general model, that constrain the number of clubs to a fixed number. Ng stated that the number of clubs could be made a variable (1973); but as an alternative he suggested that additional clubs could be treated as if they were consuming a different good. Helpman and Hillman (1977), and Hillman and Swan (1979a) indicated that their solution is Pareto optimal when there is an *explicit* one-club constraint. Such a constraint essentially defines institutional limits on club formations. These latter researchers' argument for Pareto optimality is relevant within the confines of these limits (Buchanan, 1962).

[43] A price or toll equal to marginal cost of provision for a Pareto optimum results because the shared good is private. For an impure public good depicted in the general model, the toll per-unit equals marginal congestion cost for a Pareto optimum. The finance condition (9) illustrates that these latter tolls equal marginal cost of provision, but will not cover total cost of provision if there are increasing returns to scale.

[44] The literature on two-part tariffs is extensive. Some early papers include those by W. Arthur Lewis (1941) and Ronald Coase (1946), while more recent papers are by Walter Oi (1971), Ng and Mendel Weisser (1974), Spremann (1978), and Bridger Mitchell (1978).

[45] Wiseman was critical of the club principle in public utility pricing (1957). He argued that two-part tariffs do not avoid interpersonal comparisons of utility, since there is no "best" way to set license fees. The uniform license fee is one way, although nonuniform fees may allow for welfare improvements. Another way is to use game-theoretic solutions, which determine a unique set of fees by accounting for the opportunities available to all subsets of consum-

toll may deviate from marginal cost of provision, and the license fee may even be negative.

2. *Transportation and Congestion Functions.* Roger Sherman has drawn attention to a bias in transportation (1967). Consumers pay marginal cost for private transportation and average cost for public transportation and, therefore, favor the former. To alleviate the bias, Sherman has suggested the use of a transportation club. Consumers would pay a license fee for the privilege of using a public transportation system, and then a toll per trip equal to marginal cost.

Other work has dwelt on the congestion costs inherent in transportation systems. Early studies of highway congestion were done by Walters (1961), Mohring and Harwitz (1962), Clifford Sharp (1966), and Johnson (1964). Mohring and Harwitz maximized the net benefit of highway travel, where the costs include both capital costs of highway construction and congestion or travel costs given by the general function $c(X,s)$.[46] Their conditions for a maximum required: (1) that highway size be increased to the point where the marginal cost of an increment in highway size equals the marginal congestion cost saved from that increment (provision); and (2) a level of traffic such that the driver's cost of making a trip is equal to his travel time cost plus the marginal travel cost other drivers must bear because of the increased congestion caused by this driver (membership). The optimum toll is set equal to the marginal travel cost imposed on other drivers.[47] William Vickrey emphasized that these provision, membership, and toll conditions must be solved simultaneously for highways by arguing that congestion tolls are not only useful in the short-run, but should be part of any long-run expansion plan (1969). Congestion is not something that must be completely eliminated; rather, an optimum level of congestion must be found (Davis and Whinstón, 1967). Mohring and Harwitz also pointed out that the tolls will fully finance the highway if there are constant or decreasing returns to scale in production and if $c(X,s)$ is homogeneous of degree zero (see *Section* IV-E). This last condition implies that a doubling of both traffic and highway size does not change total congestion cost.

Congestion tolls also have been examined for air travel (e.g., Michael Levine, 1969; Alan Carlin and Rolla Park, 1970; James Likens, 1976). Likens studied the three major airports serving Washington, D.C.; and he noted that National is considerably more congested than Dulles or Baltimore. A congestion function identical to the one used in the general model of *Section* III was estimated based on regressions against inbound and outbound, commercial and general aviation traffic. Optimal tolls for all three airports were found simultaneously, since the demand at any one airport depends on the tolls at the other two. This is essentially a problem where the number of clubs is fixed at three and optimum membership is determined for each.

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ers (Littlechild, 1970; Sorenson, Tschirhart, and Whinston, 1978a). Gerald Faulhaber used the game-theoretic core to place bounds on the tolls when license fees are not used (1975).

[46] Empirical work requires that the exact form of $c(X,s)$ be specified. This is difficult for highway travel because of the complex relationship between traffic speed and density. See Anthony Boardman and Lester Lave (1977) and Inman (1978) for estimates of this function used in deriving optimum tolls. The complexity of the function can also cause ambiguous results regarding the effect of demand shifts on optimum tolls (F. J. Anderson and N. C. Bonsor, 1974b). A more complicated situation is when there are heterogeneous highway users who contribute to congestion in varying amounts. Sherman derived tolls for bus and auto travelers that depend, in part, on the substitutability between the two modes (1971).

[47] Total congestion costs are $sc(X,s)$. Therefore, marginal congestion cost is $\partial sc(X,s)/\partial s = c(X,s) + s(\partial c(X,s)/\partial s)$. The first term is the travel cost the individual driver experiences. The second term is the travel cost imposed on other drivers and must be set equal to the toll. The analogue to both terms is the left-hand side of equation (7).

3. *Communications.* The telephone has received considerable attention regarding the size of the sharing group. Artle and Averous have referred to telephone subscribers as members of a telephone club (1973). They maximized the net benefits of both subscribers and non-subscribers and, in so doing, were the first to derive the correct Pareto-optimum membership size condition for one club (see *Section* IV-B). The benefit enjoyed by a subscriber includes both making and receiving calls. This was noted by Lyn Squire who used a benefit function that depends on utilization of the phone and the number of other subscribers (1973). He then derived provision, membership, and toll conditions based on these benefits. The growth of the telephone system was discussed in a dynamic Artle and Averous model by von Rabenau and Stahl (1974). Jeffrey Rohlfs also discussed growth by examining in more detail the demand for service (1974). Demand depends on the number of subscribers and also on their identity. This is like a club where the characteristics of the members are important. Rohlfs showed that starting the telephone club could be difficult under these conditions, since there is no incentive for the first members to join.

B. *Community Size, City Size, and Local Public Goods*

In his celebrated article, Tiebout specified a model where "voting with the feet" for the various public good packages, as represented by the different communities, circumvents the preference revelation problem and, in so doing, achieves a Pareto-optimal allocation (1956). Hence, a decentralized decision process can reach allocative efficiency for local public goods. To reach this result, Tiebout invoked some limiting assumptions regarding the *continuity* of choice, public good provision, income, and costs (1956, p. 419). He assumed that a "sufficiently" large number of communities exists, each with its own *fixed* public good package. Additionally, all individuals live on dividend income, have full knowledge of available public good packages, and can move costlessly between communities. For each community, *an optimal size* exists in which the *cost per person* for the public good package is at a minimum.[48] Communities below this size will attract residents, while those above will repel residents, since public good costs are assumed to be shared equally. Finally, all intercommunity spillovers are zero; perfect exclusion for the community club is possible.

The Tiebout community differs in some important respects from the Buchanan club. Tiebout's communities are multiproduct clubs, whereas Buchanan clubs are single-product ones. Public good provision is held fixed in the Tiebout model but not in the Buchanan model. Thus, the provision and membership decisions are not simultaneously determined in the Tiebout model; instead, a population chooses among the alternative packages. Membership (or community) size is solely based on cost sharing in the Tiebout model, while size is also based on camaraderie in the Buchanan club. A final difference concerns the infinite number of individuals in the Tiebout population and the finite number in the Buchanan population (see especially Truman Bewley [1979]). With infinite populations being partitioned into a large number of communities (an infinite number in some models), no one needs to be left out and, thus, the integer problem has been assumed away. On the other hand, Tiebout communities have at least two important similarities with Buchanan clubs: both share costs equally between homogeneous members and both take a within-club

[48] The determination of optimal community size via a club model must be distinguished from the related, but different question, concerning optimal jurisdictional size. This latter issue refers to a comparison between political and economic domains associated with a public good. See Albert Breton (1965; 1966), Oates (1972), and Olson (1969).

point of view (i.e., what is optimal from the member's point of view).

For Pareto optimality, the number of communities in Tiebout's model must be sufficiently large so that all individuals can find a public good package ideally suited to their tastes (Stiglitz, 1977, pp. 308–09; Ellickson, 1979a, p. 276).[49] The entire heterogeneous population must be partitioned into homogeneous groups, one group in each community, such that the cost per person for the public good package is a minimum in each community (see Mueller [1976, p. 413]). Under these assumptions, no individual can change location and be better off (see *Section* V and Pauly [1970b]). Mixed communities (i.e., those containing people with differing tastes) are not Pareto optimal owing to the equal cost-sharing assumption (see *Section* IV-A). With all of these requirements, Pareto optimality is an unlikely real-world solution to the Tiebout model, although it is a theoretical possibility.

Numerous investigators have relaxed one or more of Tiebout's assumptions in an attempt to achieve a more realistic representation of community formation and, consequently, have raised questions concerning the Pareto optimality of resource allocation in their modified Tiebout model.[50] Buchanan and Goetz (1972) allowed for nondividend incomes, tax shares, and crowding (also see Buchanan and Wagner [1970] and Flatters, Henderson, and Mieszkowski [1974]). Their model showed that an entrant to a community confers positive externalities in terms of reduced cost shares for the residents, and negative externalities in terms of increased crowding (1972). Non-optimality is predicted because the entrant equates marginal private *benefits* with marginal private costs rather than equating marginal *social* benefits with marginal social costs. Consequently, the marginal influence on others is not included in the entrant's migration decision; i.e., individuals' actions equate the averages between the communities rather than equating the margins.

Other extensions considered the introduction of migration policy costs and different tastes and skills. Arad and Hillman have shown that the inclusion of immigration policy costs, whereby residents pay to relocate entrants, changes the condition for an optimal community size (1979). Minimum average cost per person is no longer desirable in the presence of these costs, since the marginal costs of inducing migration must be weighed against the marginal savings in spreading costs over more residents. In another extension, Berglas demonstrated that economies of scale and the costs of forming smaller communities explain the existence of mixed communities (1976b, p. 416). He also indicated that variations in skills among people determine community patterns, especially because these variations affect marginal products which, in turn, determine income and tax shares. Resulting differences in tax shares will certainly influence locational choice.

In their formal extensions of Tiebout's model, most researchers did not account for the multiproduct assumption of Tiebout and, therefore, formulated single-product club models (e.g., see Buchanan and Goetz [1972]; Flatters, Henderson, and Mieszkowski [1974]; and McGuire [1974a]). These researchers also lost sight of Tiebout's fixed provision assumption

[49] Failure to have a sufficiently large number of communities results in a nonconvexity in the form of holes in the production or consumption set (Ellickson, 1979a, p. 276). This nonconvexity gives rise to a discontinuity in the choice set. On Tiebout and the existence of a solution, see Bewley (1979), Ellickson (1971; 1973; 1977; 1979a; 1979b), Donald Richter (1978), and Frank Westhoff (1977).

[50] A representative list includes the works of Ruth Arad and Hillman (1979); Berglas (1976b); Berglas and Pines (1978); Buchanan and Charles Goetz (1972); Buchanan and Richard Wagner (1970); Frank Flatters, Vernon Henderson, and Peter Mieszkowski (1974); Bruce Hamilton (1975); Helpman (1979); Litvack and Oates (1970); McGuire (1974a); Oates (1968; 1969; 1972); Polinsky (1974); Rothenberg (1970b); and William Wheaton (1975).

when they simultaneously derived provision and membership conditions. A most interesting paper by Berglas and Pines examined the determination of community size when *two* congestion-prone public goods are shared (1978). Further work on this issue should produce more accurate representations of the Tiebout process and a better understanding of multiproduct clubs (see *Section* VII-A).

Helpman and Pines formulated a type of Tiebout model to examine the optimal dispersion of a fixed population over a number of cities, each of which contains a unique package of public goods (1980). Their analysis depicted how investment policies and income redistribution programs could bring about an optimal population dispersion pattern. Oscar Fisch presented club models that can be directly applied to the determination of optimal city size (1975). He specified a model where an increase in a city's population raises income due to scale economies of production in the central business district, but it also increases rents due to crowding. Optimal city size is reached when the marginal increase in income equals the marginal increase in rents. In a different approach, researchers employed a congestion model to determine the optimal land-use pattern in cities (Avinash Dixit, 1973; Yitzhak Oron, Pines, and Eytan Sheshinski, 1973; Robert Solow, 1973; Solow and Vickrey, 1971). These researchers derived the optimal division between land used for roads and residential areas within a city. The land-use models for cities are similar to the general model of *Section* III, since the traffic congestion function depends upon either the traffic density, which is identical to the average utilization rate *(k)*, or the volume of vehicles.

C. *Political Coalitions*

Coalition formation in politics shares both similarities and differences with club formation. In politics, the objective is to form a winning coalition, or one that will be victorious over any opposing coalitions. A winning coalition reaps all the potential rewards, while the losing coalition receives zero or even negative rewards. In fact, the political contest is often assumed to be a zero-sum game so that the victors win whatever the losers lose. Under this assumption, the size of the winning coalition is determined by the size principle (William Riker, 1962; Riker and Peter Ordeshook, 1973): The equilibrium size of the winning coalition is always the minimum required for victory, i.e., 50 percent of the relevant population plus one under majority rule.

In club theory, there is no analogous concept to winning or losing. Any club that forms and provides benefits to its members is essentially a winner. And multiple clubs need not be opposing one another, except that they may compete for members when there is no core. A club that maximizes average net benefits is using the size principle, since both provide the maximum per member payoff.

The key difference between political coalitions and clubs is that in clubs optimum provision changes with membership size (Pauly, 1967, p. 321), while in political coalitions optimal provision is fixed and does not change with membership size. Fixed provision is equivalent to the zero-sum game assumption, and political coalitions would want to minimize membership size in order to maximize net average benefits. However, Thomas McCaleb investigated political coalitions where the payoff is in terms of an impure public good instead of a private good (1974). This is essentially a relaxation of the zero-sum assumption, and as McCaleb showed, a minimum winning coalition may no longer be optimal.

D. *Recreation*

The use of national and state parks, forests, and wilderness areas in the United States has increased dramatically over the

TABLE 2
SYNOPTIC COMPARISON OF HIGHWAY AND RECREATION AREAS

	HIGHWAY	RECREATIONAL AREA
Measure of Congestion	Time required to complete a trip of fixed length	The number of trail and camp encounters with other hikers (George Stankey, 1972)
Interaction between measure and membership size	A technical function relating number of vehicles and their speed (Boardman and Lave, 1977)	A technical function relating number of hikers and their likelihood of encounters (Smith and Krutilla, 1974; Cicchetti and Smith, 1976b)
Valuation of measure	Estimate value of travel time	Estimate willingness to pay to avoid encounters

past two decades. Increased usage means greater congestion at these sites and a concomitant reduction in the quality of the recreational experience. This has led researchers to study optimum congestion and provision levels for these areas by using analytical techniques similar to those used in club theory.

The problem was conceptualized geometrically by Anthony Fisher and John Krutilla (1972). They constructed total benefit and total cost curves as a function of user days, where these days can be thought of as membership size. Both of these curves depend on the capacity of the recreational site, and a different set of curves was constructed for each capacity level. Their construction is similar to the second quadrant of *Figure* 2 where net benefit curves are constructed, one for each provision level, as a function of membership. Fisher and Krutilla proceeded to determine an optimum membership size and provision level in a like manner to *Figure* 2. Their definition of total cost also included damage to the ecological environment. This is essentially a depreciation cost, or an intertemporal cost due to utilization, which is important for clubs that have multiple generations of members (see *Section* VII-B).

The congestion applications to recreation parallel those in the highway studies discussed earlier in this *Section*. In both cases there are three main aspects in determining the relationship between membership size and congestion costs. First, a measure of congestion is defined that links crowding with decreased utility; second, the physical relationship between this measure and total membership is derived; and third, the value members place on the congestion measure is established.[51] TABLE 2 provides a comparison between the highway congestion problem and one approach to the congestion problem in a low-density recreational area.

In a series of works, Charles Cicchetti and V. Kerry Smith estimated optimal membership size for a low-density recreation area of fixed capacity (1973; 1976a; 1976b). They first estimated a representative individual's willingness-to-pay function that depends on trail and camp en-

[51] A popular method of estimating the demand curve for recreational sites is to use travel costs to and from the site as a proxy for price. Consumers' surplus can then be measured from the estimated curve. However, this measure will underestimate the true surplus if the site is characterized by excess demand (Kenneth McConnell and Virginia Duff, 1976).

counters. Questionaires were distributed to hikers to obtain the information. By differentiating the aggregate willingness to pay function with respect to the number of hikers, they illustrated analytically how the optimum membership size is determined where the marginal congestion cost imposed by an additional hiker is equal to the average congestion cost to all hikers. This is identical to the membership condition of McGuire (see [1974a] and footnote 21) and assumes that congestion cost falls equally on all users. When this is not the case, the optimum membership may be different depending on the distribution of these costs (Freeman and Haveman, 1977).

The possibility of multiple recreation sites administered as clubs was considered by Anderson and Bonsor in a theoretical framework (1974a). Each club charges a toll based on congestion costs, and in equilibrium visitors will distribute themselves over the clubs according to their tastes for crowding. Clubs with high (low) tolls experience low (high) congestion. Mixed clubs can be optimal in this situation, because both sharing arrangements and tolls may vary (see *Section* IV-A).[52] McConnell (1977) and Timothy Deyak and Smith (1978) both applied this concept by investigating optimum congestion patterns among multiple clubs or recreational areas. Deyak and Smith contrasted remote camping to developed camping. Their results showed that congestion is an important determinant of demand for the former, but that the same claim could not be made for the latter. McConnell studied patterns of beach use in Rhode Island, and through the use of questionaires he determined optimum congestion levels among diverse beaches that range from 50 users per acre to over 5000 users per acre. The conclusion was that varying congestion standards across beaches could increase social value.

E. *International Organizations and Alliances*

Beginning with Olson's study of collective action (1965), a great deal of interest was and continues to be shown in applying public good theory to an analysis of international organizations, especially alliances.[53] Olson and Zeckhauser provided a theoretical basis for the observed disproportionate burden sharing by large NATO allies (e.g., U.S., U.K.) during the 1950's and early 1960's (1966). Although they recognized the impurely public aspects of general defense expenditures, Olson and Zeckhauser presented a pure public good examination of deterrence expenditures within an alliance. Using a Cournot reaction process,[54] they showed that a stable, suboptimal independent-adjustment equilibrium results as allies adjust for deterrence "spill-ins," but not for spillovers; hence, allies equate *their* marginal benefits and marginal costs *rather than* those for the *entire* alliance when deciding expenditures. The full range of spillover benefits is consequently unaccounted for, and suboptimality results. Olson and Zeckhauser also indicated that their analysis applies to other interna-

[52] Tisdell discussed various ownership possibilities for a wilderness area (1979). One possibility that he criticized is club ownership, since the tendency would be to maximize average instead of total net benefits and, therefore, a Pareto optimum is not attained. But if multiple clubs are desirable, which would seem likely for wilderness areas, this criticism is not valid (*Section* V).

[53] A few representative articles from this literature are the following: Olson (1971), Olson and Richard Zeckhauser (1966), John Ruggie (1972), Bruce Russett (1970), Russett and John Sullivan (1971), Sandler (1975a; 1977), Sandler and Cauley (1975), Sandler and William Schulze (1980), and Raimo Väyrynen (1976). For a more complete bibliography of the collective-goods literature of alliances, see Sandler and Forbes (1980).

[54] The Cournot reaction process for public goods was discussed by Breton (1970), McGuire (1974b), Ng (1971), Pauly (1970a), and Sandler and Robert Shelton (1972).

tional organizations such as the U.N. (1966, p. 275).

Sandler recently reexamined alliance behavior and stressed the club aspects of defense expenditures by separating these expenditures into deterrent and protective (i.e., damage-limiting) components (1977). Deterrence, as provided by Trident submarines and nuclear missiles, relies on the threat of punishment and is nonrival between allies; i.e., additional allies can join the alliance without diminishing the amount of deterrence provided to the existing allies. In contrast, many of the benefits of protective weapons (e.g., antiballistic missles, antitank guided weapons) are impurely public owing to elements of rivalry and exclusion. When an arsenal of protective weapons is required to protect a larger front or perimeter as a new ally joins, a *thinning of forces* results from a spatial rivalry, which detracts from the protection of other allies. Since damage-limiting forces can be withdrawn and deployed elsewhere, many benefits of protective weapons are excludable at the will of the provider.

By specifying a club model for alliances similar to the general model of *Section* III, Sandler derived membership, toll, and provision conditions (1977, pp. 450–55). Unlike Olson and Zeckhauser, Sandler showed that membership size restrictions must be placed on alliances due to the rival aspects of thinning. He also demonstrated that self-financing and stability are promoted by a toll scheme that charges for the excludable benefits of the alliance. In a subsequent study, suboptimality was shown to depend upon the ratio of excludable to total alliance benefits (Sandler and Forbes, 1980). As this ratio approaches one, suboptimality falls to zero, since club and market arrangements can allocate defense costs according to utilization rates. Sandler and Forbes tested their model for 1960–75 and found that defense payments of the allies more closely correspond to the benefit proxies during the 1970's when NATO increased its relative expenditures on protective weapons whose benefits usually are excludable.

Other international organizations (e.g., the World Health Organization, International Telecommunications Union) can be analyzed with a club model owing to rivalry aspects associated with the benefits they provide. Devolution can also be explained with a club model in which one or more regional participants no longer sees net benefits from membership in the "national club."

F. *Cost Allocation and Game Theory*

Increasing returns to scale in production are an impetus for the formation of clubs. Members can aggregate their demands and attain a given provision level at less cost than if they obtained the good on their own. Aggregation, however, gives rise to problems of cost allocation: tolls equal to marginal production cost result in a deficit. Therefore, if members are to cover total cost, another pricing arrangement is required. If a member has a disproportionate share of the cost allocated to him, he may choose to abandon the facility and go it alone. This would be a loss to the remaining members, since they are less able to take advantage of the scale economies. The same concept also applies to any set of members. Thus, any member or set of members must not be overcharged if scale economies are to be fully exploited. The concept of coalitions of members being charged acceptable sums suggests game theory and the core as a working framework.

As mentioned in *Section* V-C, using a game-theoretic characteristic function with transferable utility implies that demands must be insensitive to tolls if a unique provision level is associated with each membership size. The assumption used in the game-theoretic applied work is that demands are perfectly inelastic over some range of tolls. The characteristic function is defined as

$$v(S) = \Sigma_{i=1}^{s} C(x^i) - C(\Sigma_{i=1}^{s} x^i), \quad (11)$$

where $C(x^i)$ is the cost of serving member i alone and $C(\Sigma_{i=1}^{s} x^i)$ is the cost of serving all members in S together.[55] Equation (11) is the net benefit of cost savings available to S. Provided that the cost function is subadditive, $v(S)$ is positive. Once $v(S)$ is calculated for all relevent subcoalitions, game-theoretic solutions can be used to determine payoffs in the core.[56]

Dermot Gately studied the problem of electric power production in India (1974). The club members were different regions within India that could enjoy savings through cooperation in planning investments. Some of the solutions that were explored include the Shapley value, nucleolus, the von Neumann–Morgenstern, and payoffs that minimize the tendency to disrupt the club. The nucleolus was used by Mitsuo Suzuki and Mikio Nakayama to allocate the costs of water projects in Japan where the club membership included cities and agricultural associations (1976). The capital cost of airport runways at Birmingham Airport, England, was allocated to aircraft types in a study by Littlechild and Thompson (1977). They examined the Shapley value and variations of the nucleolus and pointed out that the charges, which include a price per landing and takeoff and a lump-sum fee for common costs, are an example of the club principle used in public utility pricing (Wiseman, 1957). One final study in this area involved cost allocation of a waste-water treatment system in Missouri (Loehman *et al.*, 1979). Club members included communities and industrial plants located along a river. A generalized Shapley value was used as an allocation mechanism.[57] In all of these papers, the size of the club consisted of the entire relevant population; i.e., all of the aircraft types, all of the communities and industries on the river, and so on. The inherent economies of scale in each problem implied that optimality is achieved in one club with everyone included (Littlechild, 1975, p. 122; Sorenson, Tschirhart, and Whinston, 1978b, p. 83).

VII. *Current and Future Directions for Research on Clubs*

A. *Multiproduct Clubs*

With few exceptions,[58] analyses of clubs have examined collectives sharing *only* one impure public good; however, clubs often share and provide multiple services or products. Country clubs provide a golf course, a swimming pool, tennis courts, and rooms for social events. Analysis of multiproduct clubs must answer a host of questions. Should the entire membership

[55] A variation of (11) was used by Littlechild (1975) and Littlechild and G. F. Thompson (1977), where a measure of benefits replaced $C(x^i)$ for each member. If the measure of benefits is consumers' surplus, the resulting characteristic function is strategically equivalent to equation (11) for inelastic demands (Sorenson, Tschirhart, and Whinston, 1976, p. 505).

[56] Two of the most often used solutions are the Shapley value (Lloyd Shapley, 1953) and the nucleolus (David Schmeidler, 1969). Martin Shubik was the first to propose using the Shapley value as a method of cost allocation (1962). Littlechild (1970) and Edna Loehman and Whinston (1971) suggested using the value in the realm of public utilities, and Susan Hamlen, William Hamlen, and Tschirhart (1977) applied it to accounting problems.

[57] See Guillermo Owen (1968), Loehman and Whinston (1976), and Hamlen, Hamlen, and Tschirhart (1980), for a discussion of the generalized Shapley value.

[58] Articles that specifically referred to multiproduct clubs include Berglas and Pines (1978, pp. 20–23), Sandler (1977, p. 455), and Tiebout (1956). Other investigators examined joint products in a mixed framework (e.g., see Barzel [1971], Boyd [1971], Ellickson [1978], Lawrence Leuzzi and Richard Pollock [1976], E. J. Mishan [1969; 1971], Mohring and Boyd [1971], Muzondo [1978], Pauly [1970a], Samuelson [1969], Sandler [1978], Sandler and Cauley [1976], and Neil Singer [1971]); however, these joint products typically included a private and an impure or pure public good, rather than multiple impure public goods. Consequently, the questions raised here were not examined.

share in all of the services offered, or should an optimal sharing group be determined for each service, even though members are excluded from some services? Will some clubs attempt to attract members by offering just one or a few services, while others offer a wide range of services? Will self-financing of multiproduct clubs require cross subsidization between products with some products paying for others? Is complementarity in the production of multiple outputs a sufficient justification for multiproduct clubs, or must complementarity in consumption be required as well? These questions demonstrate that membership, stability, self-financing, and provision issues are more complex for multiproduct clubs.

Since communities, cities, and local governments share multiple services, an adequate study of multiproduct clubs will provide a better theoretical foundation for local public good analyses. A similar statement was made by Berglas and Pines who presented a multiproduct club model; but they introduced a constraint requiring the entire membership to share in each service (1978, p. 21). Furthermore, they did not examine cross subsidization or complementarity in production.

In a most interesting series of papers, Baumol and others presented an analysis of multiproduct firms producing private goods (see Baumol [1977]; Baumol, Elizabeth Bailey, and Robert Willig [1977], Baumol and Dietrich Fischer [1978]; Jarusz Ordover and Willig [1978]; John Panzar and Willig [1977a; 1977b] and game-theoretic approaches by Faulhaber [1975] and Salas and Whinston [1978]). These articles studied the sustainability of monopoly, cross subsidization, economies of scope (i.e., complementarity in production), and the optimal number of firms: all these issues are closely related to the above questions. A marriage between their analysis and that of club theory should produce a useful theory of multiproduct clubs.[59] In so doing, the sharing group size *for each product* must be introduced into the cost functions, and the partial indivisibility of the multiple products must be accounted for.

B. *Intergenerational Clubs*

Another extension of club analyses concerns clubs with multiple over-lapping generations of members[60] (e.g., communities, fraternities, professional associations, international organizations). Decisions regarding national park management, highway maintenance, wilderness area provision, and school district design can be examined with the principles of intergenerational clubs. In contrast to previous studies of clubs, an intertemporal model is required when analyzing intergenerational clubs, since costs imposed upon club members by an entrant or user may be *either* atemporal (i.e., crowding) or intertemporal (i.e., depreciation due to utilization) and, additionally, may involve both present and future members. Depreciation is the intertemporal analogue of crowding and can be best described as a wearing down or detraction in the good's quality due to utilization. The depreciation phenomenon may appear as reduced attractiveness, loss in operative efficiency, loss in regenerative ability, and reduced usable area.

Sandler presented an intergenerational club model, which included depreciation and which derived Pareto-optimal membership, toll, and provision conditions (1979). These conditions included both crowding and depreciation costs and, therefore, justified a smaller membership and larger tolls and provision than in the absence of depreciation costs. Sandler also

[59] Our current research is pursuing this method for multiproduct clubs.

[60] Earlier treatments that mentioned this problem, but did not analyze it in an intertemporal framework, include Davis and Whinston (1967), and Fisher and Krutilla (1972).

demonstrated optimal requirements for maintaining the shared goods and for determining the membership periods of the participants: both considerations are new to club modeling. His analysis showed that the *sequence* of members must be considered for all club decisions, and myopia is strongly related to the institutional form of clubs. All club issues, especially self-financing, increased in complexity for intergenerational clubs. More research on intergenerational clubs is necessary, since many questions remain unresolved (e.g., the stability question, the optimal provision paths).

C. *Other Possible Directions for Future Research on Clubs*

As mentioned in *Section* IV-F, transaction costs must be more fully incorporated into club theory if the institutional form question is to be resolved. This incorporation represents an important future research direction. Another direction involves the stability issue for heterogeneous clubs—to date, little has been done on this question (however, see Pauly [1970b] and Sorenson, Tschirhart, and Whinston [1978b]). Still another direction for future research concerns a dynamic representation of clubs investigating such issues as innovation and growth in demand. Smith examined innovations in production and delivery systems for clubs (1976). Although his study is a worthwhile start, he did not go far enough, since a static framework was used. Two studies on the telephone system presented perhaps the only dynamic model on clubs; however, these studies made many restrictive assumptions (e.g., stationary populations, absence of crowding costs) limiting their dynamic analysis of clubs (Artle and Averous, 1973; von Rabenau and Stahl, 1974).

Another suggested research direction concerns the inclusion of a spatial dimension to club analysis. A previous attempt ignored crowding when the location of members was considered (Jon Harford, 1979). Currently, the most complete specifications of a spatial club with crowding are those concerned with optimal city size (see Dixit [1973]; Fisch [1975]; Oron, Pines, and Sheshinski [1973]). More work on spatial clubs appears warranted, since *no general* analysis of spatial clubs exists. A final area of research that may prove fruitful is to allow for uncertainty. Hillman and Swan introduced the possibility that club membership is uncertain, and individuals maximize expected utility over two states of the world: membership and nonmembership (1979b). Another possibility is to consider congestion as random in nature, and to explore how various forms of risk-averse behavior effect club participation.

VIII. *Summary and Conclusions*

Although the formal study of clubs in economics is a relatively recent occurrence, some of the essential aspects of the subject have been studied for over sixty years. This long-term interest is not surprising given the fundamental nature of the subject, viz., people voluntarily cooperating for mutual advantage. Cooperation can be observed among industries, communities, and countries, and it forms, in part, the subject matter of welfare economics, public finance, and game theory.

The recent emphasis on club theory has generated a number of controversies. In *Sections* IV and V, these controversies have been analyzed, and in some cases resolved. The necessary conditions for a Pareto-optimal membership size were shown to depend on whether: (1) the club consists of homogeneous or heterogeneous members; (2) the point of view is that of the club members or the total economy; and (3) the number of clubs is a variable. A variable number of clubs requires that the optimal number and size of clubs be determined simultaneously. For heterogeneous membership, mixed clubs were shown to be efficient when there are no second-best constraints imposed. Self-financing of clubs hinged upon scale economies and the form of the congestion function utilized, and various institutional forms of club ownership could achieve optimal results, provided exclusion capabilities exist. The links between the game and nongame formulations were also discussed, although the former were shown to be less general.

While old controversies are being settled, new ones will surely emerge.

Nevertheless, the theory of clubs is applicable now to numerous issues. This is evident from the wide variety of topics covered by the selected applications in *Section* VI. For some cases, the applicability is immediately apparent (e.g., community size), while for other cases it is less so (e.g., public utilities). In every case, one or more of the basic elements in club analysis contributed to the topic.

In this evaluative survey, the efforts of many researchers have been synthesized for the purpose of resolving controversies, demonstrating applications, and suggesting areas of future work. If this survey has accomplished these goals, it should stimulate further interest in the theory of clubs.

References

ADAMS, ROY D. AND ROYER, JEFFREY S. "Income and Price Effects in the Economic Theory of Clubs," *Public Finance,* 1977, *32*(2), pp. 141–58.

ALLEN, LARRY; AMACHER, RYAN C. AND TOLLISON, ROBERT D. "The Economic Theory of Clubs: A Geometric Exposition," *Public Finance,* 1974, *29*(3–4), pp. 386–91.

ANDERSON, F. J. AND BONSOR, N. C. "Allocation, Congestion, and the Valuation of Recreational Resources," *Land Econ.,* Feb. 1974[a], *50*(1), pp. 51–57.

——— AND BONSOR, N. C. "Pricing and Valuation of Transport Facilities in the Presence of Congestion," *Economica,* Nov. 1974[b], *41*(164), pp. 424–31.

ANDERSON, RICHARD K.; PORTER, PHILIP K. AND MAURICE, S. CHARLES. "The Economics of Consumer-Managed Firms," *Southern Econ. J.,* July 1979, *46*(1), pp. 119–30.

ARAD, RUTH W. AND HILLMAN, ARYE L. "The Collective Good Motive for Immigration Policy," *Australian Econ. Pap.,* Dec. 1979, *18*(33), pp. 243–57.

ARROW, KENNETH J. "The Organization of Economic Activity: Issues Pertinent to the Choice of Market versus Non-Market Allocation," in *Public expenditures and policy analysis.* Edited by ROBERT H. HAVEMEN AND JULIUS MARGOLIS. Chicago: Markham, 1970, pp. 59–73.

ARTLE, ROLAND AND AVEROUS, CHRISTIAN P. "The Telephone System as a Public Good: Static and Dynamic Aspects," *Bell J. Econ.,* Spring 1973, *4*(1), pp. 89–100.

BADELT, CHRISTOPH. "The Socio-Economic Impact of Self-Organization." Mimeographed. Vienna: University of Economics, 1978.

———. "The Provision of Goods and Services by Community Groups—A Theoretical Approach." Mimeographed. Vienna: University of Economics, 1979.

BARZEL, YORAM. "The Market for a Semipublic Good: The Case of the *American Economic Review,*" *Amer. Econ. Rev.,* Sept. 1971, *61*(4), pp. 665–74.

———. "A Theory of Rationing by Waiting," *J. Law Econ.,* April 1974, *17*(1), pp. 73–95.

BAUMOL, WILLIAM J. "On the Proper Cost Tests for Natural Monopoly in a Multiproduct Industry," *Amer. Econ. Rev.,* Dec. 1977, *67*(5), pp. 809–22.

———; BAILEY, ELIZABETH E. AND WILLIG, ROBERT D. "Weak Invisible Hand Theorems on the Sustainability of Prices in a Multiproduct Natural Monopoly," *Amer. Econ. Rev.,* June 1977, *67*(3), pp. 350–65.

——— AND FISCHER, DIETRICH. "Cost-Minimizing Number of Firms and Determination of Industry Structure," *Quart J. Econ.,* August 1978, *92*(3), pp. 439–67.

BERGLAS, EITAN. "On the Theory of Clubs," *Amer. Econ. Rev.,* May 1976[a], *66*(2), pp. 116–121.

———. "Distribution of Tastes and Skills and the Provision of Local Public Goods," *J. Public Econ.,* Nov. 1976[b], *6*(4), pp. 409–23.

——— AND PINES, DAVID. "Clubs, Local Public Goods and Transportation Models: A Synthesis." Working Paper Number 32/78, Foerder Institute for Economic Research, Ramat-Aviv, Israel, Oct. 1978.

BEWLEY, TRUMAN. "A Critique of Tiebout's Theory of Local Public Expenditures." Discussion Paper No. 370, Northwestern University, Feb. 1979.

BOARDMAN, ANTHONY E. AND LAVE, LESTER B. "Highway Congestion and Congestion Tolls," *J. Urban Econ.,* July 1977, *4*(3), pp. 340–59.

BOYD, J. HAYDEN. "Joint Products, Collective Goods, and External Effects: Comment," *J. Polit. Econ.*, Sept./Oct. 1971, *79*(5), pp. 1138–40.

BRETON, ALBERT. "A Theory of Government Grants," *Can. J. Econ. Polit. Sci.*, May 1965, *31*(2), pp. 175–87.

______. "A Theory of the Demand for Public Goods," *Can. J. Econ. Polit. Sci.*, Nov. 1966, *32*(4), pp. 455–67.

______. "Public Goods and the Stability of Federalism," *Kyklos*, 1970, *23*(4), pp. 882–902.

BUCHANAN, JAMES M. "The Relevance of Pareto Optimality," *J. Conflict Resolution*, Dec. 1962, *6*(4), pp. 341–54.

______. "An Economic Theory of Clubs," *Economica*, Feb. 1965, *32*(125), pp. 1–14.

______. *The demand and supply of public goods.* Chicago: Rand McNally, 1968.

______ AND GOETZ, CHARLES J. "Efficiency Limits of Fiscal Mobility: An Assessment of the Tiebout Model," *J. Public Econ.*, April 1972, *1*(1), pp. 25–43.

______ AND WAGNER, RICHARD E. "An Efficiency Basis for Federal Fiscal Equalization," in *The analysis of public output.* Edited by JULIUS MARGOLIS. New York: National Bureau of Economic Research; distributed by Columbia University Press, 1970, pp. 139–62.

CARLIN, ALAN AND PARK, ROLLA E. "Marginal Cost Pricing of Airport Runway Capacity," *Amer. Econ. Rev.*, June 1970, *60*(3), pp. 310–19.

CEBULA, RICHARD J. AND GATONS, PAUL K. "A Note on Public, Private and Ambiguous Goods," *Public Finance*, March 1972, *27*(1), pp. 69–72.

CHAMBERLIN, JOHN. "Provision of Collective Goods as a Function of Group Size," *Amer. Polit. Sci. Rev.*, June 1974, *68*(2), pp. 707–16.

CICCHETTI, CHARLES J. AND SMITH, V. KERRY. "Congestion, Quality Deterioration, and Optimal Use: Wilderness Recreation in the Spanish Peaks Primitive Area," *Soc. Sci. Res.*, March 1973, *2*(1), pp. 15–30.

______ AND SMITH, V. KERRY. "The Measurement of Individual Congestion Costs: An Economic Application to Wilderness Recreation." in *Theory and measurement of economic externalities.* Edited by STEVEN A. Y. LIN. New York: Academic Press, 1976[a], pp. 183–200.

______ AND SMITH, V. KERRY. *The costs of congestion: An econometric analysis of wilderness.* Cambridge, Mass.: Ballinger, 1976[b].

COASE, RONALD H. "The Marginal Cost Controversy," *Economica*, August 1946, *13*(51), pp. 169–82.

DAVIS, OTTO A. AND WHINSTON, ANDREW B. "On the Distinction between Public and Private Goods," *Amer. Econ. Rev.*, May 1967, *57*(2), pp. 360–73.

DE SERPA, ALLAN C. "A Theory of Discriminatory Clubs," *Scot. J. Polit. Econ.*, Feb. 1977, *24*(1), pp. 33–41.

______. "Congestion, Pollution, and Impure Public Goods," *Public Finance*, 1978[a], *33*(1–2), pp. 68–83.

______. "Multidimensional Public Goods." Mimeographed. Tempe: Arizona State University, 1978[b].

______ AND HAPPEL, STEPHEN K. "The Economics of the Olympic Games: An Application of the Economic Theory of Clubs," in *Public goods and public policy.* Edited by WILLIAM LOEHR AND TODD SANDLER. Beverly Hills, Calif.: Sage, 1978, pp. 99–121.

DEYAK, TIMOTHY A. AND SMITH, V. KERRY. "Congestion and Participation in Outdoor Recreation: A Household Production Function Approach," *J. Environ. Econ. Manage.*, March 1978, *5*(1), pp. 63–80.

DIAMOND, PETER A. "Consumption Externalities and Imperfect Corrective Pricing," *Bell J. Econ.*, Autumn 1973, *4*(2), pp. 526–38.

DIXIT, AVINASH K. "The Optimum Factory Town," *Bell J. Econ.*, Autumn 1973, *4*(2), pp. 637–51.

DORFMAN, ROBERT. "The Technical Basis for Decision Making," in *The governance of common property resources.* Edited by EDWIN T. HAEFELE. Baltimore: Johns Hopkins University Press for Resources for the Future, 1974, pp. 5–25.

EDELSON, NOEL M. "Congestion Tolls Under Monopoly," *Amer. Econ. Rev.*, Dec. 1971, *61*(5), pp. 873–82.

ELLICKSON, BRYAN. "Jurisdictional Fragmentation and Residential Choice," *Amer. Econ. Rev.*, May 1971, *61*(2), pp. 334–39.

______. "A Generalization of the Pure Theory of Public Goods," *Amer. Econ. Rev.*, June 1973, *63*(3), pp. 417–32.

______. "The Politics and Economics of Decentralization," *J. Urban Econ.*, April 1977, *4*(2), pp. 135–49.

______. "Public Goods and Joint Supply," *J. Public Econ.*, June 1978, *9*(3), pp. 373–82.

______. "Local Public Goods and the Market of Neighborhoods," in *The economics of neighborhood.* Edited by DAVID SEGAL. New York: Academic Press, 1979[a], pp. 263–94.

______. "Competitive Equilibrium with Local Public Goods," *J. Econ. Theory*, August 1979[b], *21*(1), pp. 46–61.

EVANS, ALAN W. "Private Good, Externality, Public Good," *Scot. J. Polit. Econ.*, Feb. 1970, *17*(1), pp. 79–89.

FÄRE, ROLF AND GROSSKOPF, SHAWNA. "Existence of Nonprofit Clubs." Mimeographed. Carbondale: Southern Illinois University, 1979[a].

——— AND GROSSKOPF, SHAWNA. "Congested Public Goods: An Example." Mimeographed, Southern Illinois University, 1979[b].

FAULHABER, GERALD R. "Cross-Subsidization: Pricing in Public Enterprises," *Amer. Econ. Rev.*, Dec. 1975, *65*(5), pp. 966–77.

FELDMAN, ROGER D. "Rationing Congested Goods by Preferences on Quality," *Public Finance*, 1978, *33*(3), pp. 225–31.

FISCH, OSCAR. "Optimal City Size, the Economic Theory of Clubs and Exclusionary Zoning," *Public Choice*, Winter 1975, *24*, pp. 59–70.

FISHER, ANTHONY C. AND KRUTILLA, JOHN V. "Determination of Optimal Capacity of Resource-Based Recreational Facilities," *Natural Res. J.*, July 1972, *12*(3), pp. 417–44.

FLATTERS, FRANK; HENDERSON, VERNON AND MIESZKOWSKI, PETER. "Public Goods, Efficiency, and Regional Fiscal Equalization," *J. Public Econ.*, May 1974, *3*(2), pp. 99–112.

FREEMAN, A. MYRICK, III AND HAVEMAN, ROBERT H. "Congestion, Quality Deterioration, and Heterogeneous Tastes," *J. Public Econ.*, Oct. 1977, *8*(2), pp. 225–32.

GATELY, DERMOT. "Sharing the Gains from Regional Cooperation: A Game Theoretic Application to Planning Investment in Electric Power," *Int. Econ. Rev.*, Feb. 1974, *15*(1), pp. 195–208.

HAMILTON, BRUCE W. "Zoning and Property Taxation in a System of Local Governments," *Urban Stud.*, June 1975, *12*(2), pp. 205–11.

HAMLEN, SUSAN S.; HAMLEN, WILLIAM A., JR. AND TSCHIRHART, JOHN T. "The Use of Core Theory in Evaluating Joint Cost Allocation Schemes," *Accounting Rev.*, July 1977, *52*(3), pp. 616–27.

———; HAMLEN, WILLIAM A., JR. AND TSCHIRHART, JOHN T. "The Use of the Generalized Shapley Allocation in Joint Cost Allocation," *Accounting Rev.*, April 1980, *55*(2), pp. 269–87.

HARFORD, JON D. "The Spatial Aspects of Local Public Goods: A Note," *Public Finance Quart.*, Jan. 1979, *7*(1), pp. 122–28.

HAVEMAN, ROBERT H. "Common Property, Congestion, and Environmental Pollution," *Quart. J. Econ.*, May 1973, *87*(2), pp. 278–87.

HEAD, JOHN G. AND SHOUP, CARL S. "Public Goods, Private Goods, and Ambiguous Goods," *Econ. J.*, Sept. 1969, *79*(315), pp. 567–72.

HEAVEY, JEROME F. AND GUNZENHAUSER, MILES K. "Income Effects and Club Size," *Public Finance*, 1978, *33*(1–2), pp. 84–89.

HELPMAN, ELHANAN. "On Optimal Community Formation," *Economic Letters*, 1979, *1*(3), pp. 289–93.

——— AND HILLMAN, ARYE L. "On Optimal Club Size." Working Paper No. 91 (Rev.), Foerder Institute for Economic Research, Ramat Aviv, Israel, 1976.

——— AND HILLMAN, ARYE L. "Two Remarks on Optimal Club Size," *Economica*, August 1977, *44*(175), pp. 293–96.

——— AND PINES, DAVID. "Optimal Public Investment and Dispersion Policy in a System of Open Cities," *Amer. Econ. Rev.*, June 1980, *70*(3), pp. 507–14.

HILLMAN, ARYE L. "The Theory of Clubs: A Technological Formulation," in *Essays in public economics: The Kiryat Anavim papers*. Edited by AGNAR SANDMO. Lexington, Mass.: Heath, Lexington Books, 1978[a], pp. 29–47.

———. "Symmetries and Asymmetries between Public Input and Public Good Equilibria," *Public Finance*, 1978[b], *33*(3), pp. 269–79.

——— AND SWAN, PETER L. "Participation Rules for Profit-Maximizing, Pareto-Efficient, and Not-For-Profit Clubs." Mimeographed. November 1979[a].

——— AND SWAN, PETER L. "Club Participation under Uncertainty." Mimeographed. November 1979[b].

INMAN, ROBERT P. "A Generalized Congestion Function for Highway Travel," *J. Urban Econ.*, Jan. 1978, *5*(1), pp. 21–34.

JOHNSON, M. BRUCE. "On the Economics of Road Congestion," *Econometrica*, Jan./April 1964, *32* (1–2), pp. 137–50.

KAMIEN, MORTON I. AND SCHWARTZ, NANCY L. "Revelation of Preference for a Public Good with Imperfect Exclusion," *Public Choice*, Fall 1970, *9*, pp. 19–30.

———; SCHWARTZ, NANCY L. AND ROBERTS, DONALD J. "Exclusion, Externalities, and Public Goods," *J. Public Econ.*, July 1973, *2*(3), pp. 217–30.

KNIGHT, FRANK H. "Some Fallacies in the Interpretation of Social Cost," *Quart. J. Econ.*, August 1924, *38*(4), pp. 582–606.

LEUZZI, LAWRENCE AND POLLOCK, RICHARD. "Option Demand: A Mixed Goods Case," *Public Finance*, 1976, *31*(3), pp. 396–405.

LEVINE, MICHAEL E. "Landing Fees and the Airport Congestion Problem," *J. Law Econ.*, April 1969, *12*(1), pp. 79–108.

LEWIS, W. ARTHUR. "The Two-Part Tariff," *Economica*, August 1941, *8*(31), pp. 249–70.

LIKENS, JAMES D. "The Welfare Costs of Nonoptimal Airport Utilization," *J. Public Econ.*, Jan./Feb. 1976, *5*(1–2), pp. 81–102.

LITTLECHILD, STEPHEN C. "A Game-Theoretic Approach to Public Utility Pricing," *Western Econ. J.*, June 1970, *8*(2), pp. 162–66.

———. "Common Costs, Fixed Charges, Clubs and Games," *Rev. Econ. Stud.*, Jan. 1975, *42*(129), pp. 117–24.

——— AND THOMPSON, G. F. "Airport Landing

Fees: A Game Theory Approach," *Bell J. Econ.*, Spring 1977, *8*(1), pp. 186–204.

LITVACK, JAMES M. AND OATES, WALLACE E. "Group Size and the Output of Public Goods: Theory and Application to State-Local Finance in the United States," *Public Finance*, 1970, *25*(1), pp. 42–62.

LOEHMAN, EDNA T. AND WHINSTON, ANDREW B. "A New Theory of Pricing and Decision Making for Public Investment," *Bell J. Econ.*, Autumn 1971, *2*(2), pp. 606–25.

——— AND WHINSTON, ANDREW B. "A Generalized Cost Allocation Scheme," in *Theory and measurement of economic externalities.* Edited by STEVEN A. Y. LIN. New York: Academic Press, 1976, pp. 87–101.

———. ET AL. "Cost Allocation for a Regional Wastewater Treatment System," *Water Resources Res.*, April 1979, *15*(2), pp. 193–202.

LOEHR, WILLIAM AND SANDLER, TODD. "On the Public Character of Goods," in *Public goods and public policy.* Edited by WILLIAM LOEHR AND TODD SANDLER. Beverly Hills, Calif.: Sage, 1978, pp. 11–37.

LUCE, R. DUNCAN AND RAIFFA, HOWARD. *Games and decisions.* New York: Wiley, 1957.

MCCALEB, THOMAS S. "The Size Principle and Collective-Consumption Payoffs to Political Coalitions," *Public Choice*, Spring 1974, *17*, pp. 107–09.

MCCONNELL, KENNETH E. "Congestion and Willingness to Pay: A Study of Beach Use," *Land Econ.*, May 1977, *53*(2), pp. 185–95.

——— AND DUFF, VIRGINIA A. "Estimating Net Benefits of Recreation Under Conditions of Excess Demand," *J. Environ. Econ. Manage.*, Feb. 1976, *2*(3), pp. 224–30.

MCGUIRE, MARTIN. "Private Good Clubs and Public Good Clubs: Economic Models of Group Formation," *Swedish J. Econ.*, March 1972, *74*(1), pp. 84–99.

———. "Group Segregation and Optimal Jurisdictions," *J. Polit. Econ.*, Jan./Feb. 1974[a], *82*(1), pp. 112–32.

———. "Group Size, Group Homogeneity, and the Aggregate Provision of a Pure Public Good under Cournot Behavior," *Public Choice*, Summer 1974[b], *18*, pp. 107–26.

MILLWARD, R. "Exclusion Costs, External Economies, and Market Failure," *Oxford Econ. Pap.*, March 1970, *22*(1), pp. 24–38.

MISHAN, E. J. "The Relationship between Joint Products, Collective Goods, and External Effects," *J. Polit. Econ.*, May/June 1969, *77*(3), pp. 329–48.

———. "The Postwar Literature on Externalities: An Interpretative Essay," *J. Econ. Lit.*, March 1971, *9*(1), pp. 1–28.

———. "Joint Products, Collective Goods, and External Effects: Reply," *J. Polit. Econ.*, Sept./Oct. 1971, *79*(5), pp. 1141–50.

MITCHELL, BRIDGER M. "Optimal Pricing of Local Telephone Service," *Amer. Econ. Rev.*, Sept. 1978, *68*(4), pp. 517–37.

MOHRING, HERBERT AND HARWITZ, MITCHELL. *Highway benefits: An analytical framework.* Evanston, Ill.: Northwestern University Press, 1962.

——— AND BOYD, J. HAYDEN. "Analyzing 'Externalities': 'Direct Interaction' vs 'Asset Utilization' Frameworks," *Economica*, Nov. 1971, *38*(152), pp. 347–61.

MUELLER, DENNIS C. "Public Choice: A Survey," *J. Econ. Lit.*, June 1976, *14*(2), pp. 395–433.

MUMY, GENE E. AND HANKE, STEVE H. "Public Investment Criteria for Underpriced Public Products," *Amer. Econ. Rev.*, Sept. 1975, *65*(4), pp. 712–20.

MUSGRAVE, RICHARD A. AND MUSGRAVE, PEGGY B. *Public finance in theory and practice.* Second edition. New York: McGraw-Hill, [1973] 1976.

MUZONDO, TIMOTHY R. "Mixed and Pure Public Goods, User Charges, and Welfare," *Public Finance*, 1978, *33*(3), pp. 314–30.

VON NEUMANN, JOHN AND MORGENSTERN, OSKAR. *Theory of games and economic behavior.* Princeton, N.J.: Princeton University Press, 1944.

NG, YEW-KWANG. "Public Goods and the Stability of Federalism: An Extension," *Kyklos*, 1971, *24*(3), pp. 562–63.

———. "The Economic Theory of Clubs: Pareto Optimality Conditions," *Economica*, August 1973, *40*(159), pp. 291–98.

———. "The Economic Theory of Clubs: Optimal Tax/Subsidy," *Economica*, August 1974, *41*(163), pp. 308–21.

———. "Optimal Club Size: A Reply," *Economica*, Nov. 1978, *45*(180), pp. 407–10.

———. *Welfare economics: Introduction and development of basic concepts.* New York: Halsted Press, 1980.

——— AND TOLLISON, ROBERT D. "A Note on Consumption Sharing and Non-Exclusion Rules," *Economica*, Nov. 1974, *41*(164), pp. 446–50.

——— AND WEISSER, MENDEL. "Optimal Pricing with a Budget Constraint—The Case of the Two-part Tariff," *Rev. Econ. Stud.*, July 1974, *41*(127), pp. 337–45.

NICHOLS, DONALD; SMOLENSKY, EUGENE AND TIDEMAN, T. NICOLAUS. "Discrimination by Waiting Time in Merit Goods," *Amer. Econ. Rev.*, June 1971, *61*(3), pp. 312–23.

OAKLAND, WILLIAM H. "Joint Goods," *Economica*, August 1969, *36*(143), pp. 253–68.

———. "Congestion, Public Goods and Welfare," *J. Public Econ.*, Nov. 1972, *1*(3–4), pp. 339–57.

———. "Public Goods, Perfect Competition, and Underproduction," *J. Polit. Econ.*, Sept./Oct. 1974, *82*(5), pp. 927–39.

OATES, WALLACE E. "The Theory of Public Finance in a Federal System," *Can. J. Econ.*, Feb. 1968, *1*(1), pp. 37–54.

———. "The Effects of Property Taxes and Local Public Spending on Property Values: An Empirical Study of Tax Capitalization and the Tiebout Hypothesis." *J. Polit. Econ.*, Nov./Dec. 1969, *77*(6), pp. 957–71.

———. *Fiscal federalism*. New York: Harcourt, Brace, Jovanovich, 1972.

OI, WALTER Y. "A Disneyland Dilemma: Two-Part Tariffs for a Mickey Mouse Monopoly," *Quart. J. Econ.*, Feb. 1971, *85*(1), pp. 77–96.

OLSON, MANCUR, JR. *The logic of collective action: Public goods and the theory of groups*. Cambridge, Mass.: Harvard University Press, 1965.

———. "The Principle of 'Fiscal Equivalence': The Division of Responsibilities among Different Levels of Government," *Amer. Econ. Rev.*, May 1969, *59*(2), pp. 479–87.

———. "Increasing Incentives for International Cooperation," *Int. Organization*, Autumn 1971, *25*(4), pp. 866–74.

———. "Evaluating Performance in the Public Sector," in *The measurement of economic and social performance*. Edited by MILTON MOSS. New York: National Bureau of Economic Research; distributed by Columbia University Press, 1973, pp. 355–409.

——— AND ZECKHAUSER, RICHARD. "An Economic Theory of Alliances," *Rev. Econ. Statist.*, Aug. 1966, *48*(3), pp. 266–79.

ORDOVER, JARUSZ A. AND WILLIG, ROBERT D. "On the Optimal Provision of Journals qua Sometimes Shared Goods," *Amer. Econ. Rev.*, June 1978, *68*(3), pp. 324–38.

ORON, YITZHAK; PINES, DAVID AND SHESHINSKI, EYTAN. "Optimal vs. Equilibrium Land Use Patterns and Congestion Toll," *Bell J. Econ.*, Autumn 1973, *4*(2), pp. 619–36.

OWEN, GUILLERMO. *Game theory*. Philadelphia: Saunders, 1968.

PANZAR, JOHN C. AND WILLIG, ROBERT D. "Free Entry and the Sustainability of Natural Monopoly," *Bell J. Econ.*, Spring 1977[a], *8*(1), pp. 1–22.

——— AND WILLIG, ROBERT D. "Economies of Scale in Multi-Output Production," *Quart J. Econ.*, August 1977[b], *91*(3), pp. 481–93.

PAULY, MARK V. "Clubs, Commonality, and the Core: An Integration of Game Theory and the Theory of Public Goods," *Economica*, August 1967, *34*(135), pp. 314–24.

———. "Optimality, 'Public' Goods, and Local Governments: A General Theoretical Analysis," *J. Polit. Econ.*, May/June 1970[a], *78*(3), pp. 572–85.

———. "Cores and Clubs," *Public Choice*, Fall 1970[b], *9*, pp. 53–65.

PIGOU, A. C. *The economics of welfare*. London: Macmillan, 1920.

POLINSKY, A. MITCHELL. *Collective consumption goods and local public finance theory: A suggested analytical framework*. Urban Institute Reprint 168-1207-8. Washington, D.C.: Urban Institute, 1974.

PORTER, RICHARD C. "On the Optimal Size of Underpriced Facilities," *Amer., Econ. Rev.*, Sept. 1977, *67*(4), pp. 753–60.

———. "The Economics of Congestion: A Geometric Review," *Public Finance Quart.*, Jan. 1978, *6*(1), pp. 23–52.

VON RABENAU, BURCKHARD AND STAHL, KONRAD. "Dynamic Aspects of Public Goods: A Further Analysis of the Telephone System," *Bell J. Econ.*, Autumn 1974, *5*(2), pp. 651–69.

RICHTER, DONALD K. "Existence and Computation of a Tiebout General Equilibrium," *Econometrica*, July 1978, *46*(4), pp. 779–805.

RIKER, WILLIAM H. *The theory of political coalitions*. New Haven, Conn.: Yale University Press, 1962.

——— AND ORDESHOOK, PETER C. *An introduction to positive political theory*. Englewood Cliffs, N.J.: Prentice Hall, 1973.

ROHLFS, JEFFREY. "A Theory of Interdependent Demand for a Communication Service," *Bell J. Econ.*, Spring 1974, *5*(1), pp. 16–37.

ROTHENBERG, JEROME. "The Economics of Congestion and Pollution: An Integrated View," *Amer. Econ. Rev.*, May 1970[a], *60*(2), pp. 114–21.

———. "Local Decentralization and the Theory of Optimal Government," in *The analysis of public output*. Edited by JULIUS MARGOLIS. New York: National Bureau of Economic Research; distributed by Columbia University Press, 1970[b], pp. 31–64.

RUGGIE, JOHN G. "Collective Goods and Future International Collaboration," *Amer. Polit. Sci. Rev.*, Sept. 1972, *66*(3), pp. 874–93.

RUGGLES, NANCY D. "Recent Developments in the Theory of Marginal Cost Pricing," *Rev. Econ. Stud.*, 1950, *17*(43), pp. 107–26.

RUSSETT, BRUCE M. *What price vigilance? The burden of national defense*. New Haven, Conn.: Yale University Press, 1970.

——— AND SULLIVAN, JOHN D. "Collective Goods and International Organization," *Int. Organization*, Autumn 1971, *25*(4), pp. 845–65.

SALAS, VINCENTE AND WHINSTON, ANDREW B. "Subsidy Free Welfare Games." Mimeographed. Lafayette, Ind.: Purdue University, May 1978.

SAMUELSON, PAUL A. "The Pure Theory of Public

Expenditure," *Rev. Econ. Statist.*, Nov. 1954, *36*(4), pp. 387–89.

______. "A Diagrammatic Exposition of a Theory of Public Expenditure," *Rev. Econ. Statist.*, Nov. 1955, *37*(4), pp. 350–56.

______. "Contrast between Welfare Conditions for Joint Supply and for Public Goods," *Rev. Econ. Statist.*, Feb. 1969, *51*(1), pp. 26–30.

SANDLER, TODD. "The Economic Theory of Alliances: Realigned," in *Comparative public policy: Issues, theories and methods.* Edited by CRAIG LISKE, WILLIAM LOEHR AND JOHN MCCAMANT. Beverly Hills, Calif.: Sage distributed by Wiley, Halsted Press, New York, 1975[a], pp. 223–39.

______. "Pareto Optimality, Pure Public Goods, Impure Public Goods and Multiregional Spillovers," *Scot. J. Polit. Econ.*, Feb. 1975[b], *22*(1), pp. 25–38.

______. "Impurity of Defense: An Application to the Economics of Alliances," *Kyklos*, 1977, *30*(3), pp. 443–60.

______. "Public Goods and the Theory of Second Best," *Public Finance*, 1978, *33*(3), pp. 331–44.

______. "A Theory of Intergenerational Clubs." Mimeographed. Laramie: University of Wyoming, 1979.

______ AND CAULEY, JON. "On the Economic Theory of Alliances," *J. Conflict Resolution*, June 1975, *19*(2), pp. 335–48.

______ AND CAULEY JON. "Multiregional Public Goods, Spillovers, and the New Theory of Consumption," *Public Finance*, 1976, *31*(3), pp. 376–95.

______ AND FORBES, JOHN F. "Burden Sharing, Strategy, and the Design of NATO," *Econ. Inquiry*, July 1980, *18*(3), pp. 425–44.

______ AND SCHULZE, WILLIAM D. "Explorations in the Economics of Outer Space," in *The theory and structures of international political economy.* Edited by TODD SANDLER. Boulder, Colo.: Westview Press, 1980.

______ AND SHELTON, ROBERT, B. "Fiscal Federalism, Spillovers, and the Export of Taxes," *Kyklos*, 1972, *25*(4), pp. 736–53.

SANDMO, AGNAR. "Public Goods and the Technology of Consumption," *Rev. Econ. Stud.*, Oct. 1973, *40*(4), pp. 517–28.

SCHMEIDLER, DAVID. "The Nucleolus of a Characteristic Function Game," *SIAM J. Applied Math.*, Nov. 1969, *17*(6), pp. 1163–70.

SCHELLING, THOMAS C. "Models of Segregation," *Amer. Econ. Rev.*, May 1969, *59*(2), pp. 488–93.

SHAPLEY, LLOYD. "Value for *n*-Person Games," in *Contributions to the theory of games.* Volume II. Edited by HAROLD W. KUHN AND ALBERT W. TUCKER. Princeton, N.J.: Princeton University Press, 1953, pp. 307–17.

SHARP, CLIFFORD, "Congestion and Welfare: An Examination of the Case for a Congestion Tax," *Econ. J.*, Dec. 1966, *76*(304), pp. 806–17.

SHERMAN, ROGER. "Club Subscriptions for Public Transport Passengers," *J. Transp. Econ. Policy*, Sept. 1967, *1*(3), pp. 237–42.

______. "Congestion Interdependence and Urban Transit Fares," *Econometrica*, May 1971, *39*(3), pp. 565–76.

SHUBIK, MARTIN. "Incentives, Decentralized Control, the Assignment of Joint Costs and Internal Pricing," *Manage. Sci.*, April 1962, *8*(3), pp. 325–43.

SINGER, NEIL M. "Joint Products, Collective Goods, and External Effects: Comment," *J. Polit. Econ.*, Sept./Oct. 1971, *79*(5), pp. 1136–37.

SMITH, JAN. "Communities, Associations, and the Supply of Collective Goods," *Amer. J. Sociology*, Sept. 1976, *82*(2), pp. 291–308.

SMITH, V. KERRY. "The Role of Innovation in the Provision of Local Public Goods," *Public Finance Quart.*, July 1976, *4*(3), pp. 285–94.

______ AND KRUTILLA, JOHN V. "A Simulation Model for the Management of Low Density Recreation Areas," *J. Environ. Econ. Manage.*, Nov. 1974, *1*(3), pp. 187–201.

SOLOW, ROBERT M. "Congestion Cost and the Use of Land for Streets," *Bell J. Econ.*, Autumn 1973, *4*(2), pp. 602–18.

______ AND VICKREY, WILLIAM S. "Land Use in a Long Narrow City," *J. Econ. Theory*, Dec. 1971, *3*(4), pp. 430–47.

SORENSON, JOHN R.; TSCHIRHART, JOHN T. AND WHINSTON, ANDREW B. "A Game Theoretic Approach to Peak Load Pricing," *Bell J. Econ.*, Autumn 1976, *7*(2), pp. 497–520.

______; TSCHIRHART, JOHN T. AND WHINSTON, ANDREW B. "A Theory of Pricing under Decreasing Costs," *Amer. Econ. Rev.*, Sept. 1978[a], *68*(4), pp. 614–24.

______; TSCHIRHART, JOHN T. AND WHINSTON, ANDREW B. "Private Good Clubs and the Core," *J. Public Econ.*, August 1978[b], *10*(1), pp. 77–95.

SPREMANN, KLAUS. "On Welfare Implications and Efficiency of Entrance Fee Pricing," *Z. Nationalökon.*, 1978, *38*(3–4), pp. 231–52.

SQUIRE, LYN. "Some Aspects of Optimal Pricing for Telecommunications," *Bell J. Econ.*, Autumn 1973, *4*(2), pp. 515–25.

STANKEY, GEORGE H. "A Strategy for the Definition and Management of Wilderness Quality," in *Natural environments: Studies in theoretical and applied analysis.* Edited by JOHN V. KRUTILLA. Baltimore: Johns Hopkins University Press for Resources for the Future, 1972, pp. 88–114.

STIGLITZ, JOSEPH E. "The Theory of Local Public Goods," in *The economics of public services.* Edited by MARTIN S. FELDSTEIN AND ROBERT P. INMAN. London: Macmillan; New York: Wiley, Halsted Press, 1977, pp. 274–333.

SUZUKI, MITSUO AND NAKAYAMA, MIKIO. "The Cost Assignment of Cooperative Water Resource Development: A Game Theoretic Approach," *Manage. Sci.*, June 1976, *22*(10), pp. 1081–86.

TIEBOUT, CHARLES M. "A Pure Theory of Local Expenditures," *J. Polit. Econ.*, Oct. 1956, *64*(5), pp. 416–24.

TISDELL, CLEM A. "Coalitions and Club-Aspects of Their Formation and Economic Efficiency," in *Mathematical economics and game theory: Essays in honor of Oskar Morgenstern.* Edited by RUDOLF HENN AND OTTO MOESCHLIN. New York and Berlin: Springer-Verlag, 1977, pp. 446–56.

———. "Economics of Wilderness," in *Wilderness Australia: Delineation, management, and prospects.* Edited by B. ROBERTSON. Canberra, Australia: Canberra College of Advanced Education, 1979, forthcoming.

TOLLISON, ROBERT D. "Consumption Sharing and Non-Exclusion Rules," *Economica,* August 1972, *39*(155), pp. 276–91.

VÄYRYNEN, RAIMO. "The Theory of Collective Goods, Military Alliances, and International Security," *Int. Soc. Sci. J.*, June 1976, *38*(2), pp. 288–305.

VICKREY, WILLIAM S. "Congestion Theory and Transport Investment," *Amer. Econ. Rev.*, May 1969, *59*(2), pp. 251–60.

WALTERS, ALAN A. "The Theory and Measurement of Private and Social Cost of Highway Congestion," *Econometrica,* Oct. 1961, *29*(4), pp. 676–99.

WEITZMAN, MARTIN L. "Free Access vs Private Ownership as Alternative Systems for Managing Common Property," *J. Econ. Theory,* June 1974, *8*(2), pp. 225–34.

WESTHOFF, FRANK. "Existence of Equilibria in Economies with a Local Public Good," *J. Econ. Theory,* Feb. 1977, *14*(1), pp. 84–112.

WHEATON, WILLIAM C. "Consumer Mobility and Community Tax Bases: The Financing of Local Public Goods," *J. Public Econ.*, Nov. 1975, *4*(4), pp. 377–84.

WINCH, DAVID M. "The Pure Theory of Non-Pure Goods," *Can. J. Econ.*, May 1973, *6*(2), pp. 149–63.

WISEMAN, JACK. "The Theory of Public Utility Price—An Empty Box," *Oxford Econ. Pap.*, Feb. 1957, *9*(1), pp. 56–74.