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MATEMATIKA I.
Pot 3.

Příklad 1 - Řešte systém lineárních rovnic /proveďte zkoušku

$$\begin{aligned} x_1 - 2x_2 - 3x_3 &= 1 \\ -x_1 + 4x_2 - 5x_3 + 4x_2 &= -4 \\ -2x_1 + 4x_2 + x_3 + 2x_2 &= 4 \\ 3x_1 - 7x_2 - 5x_3 - 3x_2 &= -3 \end{aligned}$$

$$\left(\begin{array}{cccc|c} 1 & -2 & -3 & 0 & 1 \\ -1 & +4 & -5 & 4 & -4 \\ -2 & +4 & +1 & 2 & 4 \\ 3 & -7 & -5 & -3 & -3 \end{array}\right) \sim \left(\begin{array}{cccc|c} 1 & -2 & -3 & 0 & 1 \\ 0 & 2 & -8 & 4 & -3 \\ 0 & 0 & -5 & 2 & 6 \\ 0 & -1 & 4 & -3 & -6 \end{array}\right) \sim \left(\begin{array}{cccc|c} 1 & -2 & -3 & 0 & 1 \\ 0 & 2 & -8 & 4 & -3 \\ 0 & 0 & -5 & 2 & 6 \\ 0 & 0 & 0 & -2 & -15 \end{array}\right)$$

$x_1 + 2\frac{93}{10} - 3\frac{9}{5} = 1$ $\quad x_1 = 1 - \frac{93}{5} + \frac{27}{5}$ $\quad x_1 = -\frac{61}{5}$

$-2x_4 = -15$ $\quad x_4 = \frac{15}{2}$

$-5x_3 + 2\frac{15}{2} = 6$ $\quad -5x_3 = -9$ $\quad x_3 = \frac{9}{5}$

$2x_2 - 8\frac{9}{5} + 4\frac{15}{2} = -3$ $\quad 2x_2 = -3 - 30 + \frac{72}{5}$ $\quad 2x_2 = -\frac{93}{5}$ $\quad x_2 = -\frac{93}{10}$

Zkouška: $L_1 = -\frac{61}{5} + \frac{93}{5} - \frac{27}{5} = \frac{5}{5} = 1$ $\quad L_1 = P_1 \quad P_1 = 1$

$L_2 = \frac{61}{5} - \frac{186}{5} - 9 + 30 = -\frac{20}{5} = -4$ $\quad L_2 = P_2 \quad P_2 = -4$

$L_3 = \frac{122}{5} - \frac{186}{5} + \frac{9}{5} + 15 = \frac{20}{5} = 4$ $\quad L_3 = P_3 \quad P_3 = 4$

$L_4 = \frac{183}{5} + \frac{651}{10} - 9 - \frac{45}{2} = -\frac{30}{10} = -3$ $\quad L_4 = P_4 \quad P_4 = -3$

Příklad 2 - Určete hodnotu determinantu

$$\begin{vmatrix} 1 & 2 & 4 & -3 & 0 \\ 1 & 0 & 3 & -1 & 0 \\ 3 & 2 & 1 & 0 & 2 \\ -1 & 2 & -1 & 0 & 2 \\ -2 & 1 & 0 & 0 & 2 \end{vmatrix}$$

a) rozvojem podle některého řádku, nebo sloupce

b) pomocí elementárních transformací

rozvoj dle 4. sloupce

$$\begin{vmatrix} 1 & 2 & 4 & -3 & 0 \\ 1 & 0 & 3 & -1 & 0 \\ 3 & 2 & 1 & 0 & 2 \\ -1 & 2 & -1 & 0 & 2 \\ -2 & 1 & 0 & 0 & 2 \end{vmatrix} = -3(-1)^{1+4}\cdot \begin{vmatrix} 1 & 0 & 3 & 0 \\ 3 & 2 & 1 & 2 \\ -1 & 2 & -1 & 2 \\ -2 & 1 & 0 & 2 \end{vmatrix} + (-1)\cdot(-1)^{2+4}\cdot \begin{vmatrix} 1 & 2 & 4 & 0 \\ 3 & 2 & 1 & 2 \\ -1 & 2 & -1 & 2 \\ -2 & 1 & 0 & 2 \end{vmatrix} =$$

$$= 3\cdot 20 - 52 = 60 - 52 = \underline{\underline{8}}$$

podle 1 ř.

$$\begin{vmatrix} 1 & 0 & 3 & 0 \\ 3 & 2 & 1 & 2 \\ -1 & 2 & -1 & 2 \\ -2 & 1 & 0 & 2 \end{vmatrix} = 1\cdot(-1)^{1+1}\cdot \begin{vmatrix} 2 & 1 & 2 \\ 2 & -1 & 2 \\ 1 & 0 & 2 \end{vmatrix} + 3\cdot(-1)^{1+3}\cdot \begin{vmatrix} 3 & 2 & 2 \\ -1 & 2 & 2 \\ -2 & 1 & 2 \end{vmatrix} = -4 + 2 + 0 - (-2 + 4 + 0) +$$

$$+ 3\cdot\left[12 - 8 - 2 - (-8 - 4 + 6)\right] = -4 + 3\cdot 8 = -4 + 24 = \underline{\underline{20}}$$

podle 4. ř.

$$\begin{vmatrix} 1 & 2 & 4 & 0 \\ 3 & 2 & 1 & 2 \\ -1 & 2 & -1 & 2 \\ -2 & 1 & 0 & 2 \end{vmatrix} = (-2)\cdot(-1)^{4+1} \begin{vmatrix} 2 & 4 & 0 \\ 2 & 1 & 2 \\ 2 & -1 & 2 \end{vmatrix} + 1\cdot(-1)^{4+2}\cdot \begin{vmatrix} 1 & 4 & 0 \\ 3 & 1 & 2 \\ -1 & -1 & 2 \end{vmatrix} + 2\cdot(-1)^{4+4}\cdot \begin{vmatrix} 1 & 2 & 4 \\ 3 & 2 & 1 \\ -1 & 2 & -1 \end{vmatrix} =$$

$$= 2\left[4 + 16 + 0 - (0 + 16 - 4)\right] + 2 - 8 + 0 - (0 + 24 - 2) + 2\cdot\left[-2 - 2 + 24 - (-8 - 6 + 2)\right] = 16 - 28 + 64 =$$

$$= 80 - 28 = \underline{\underline{52}}$$

příklad 3 Je dána matice

$$A = \begin{pmatrix} 1 & 2 & 3 & 4 \\ -1 & 0 & 2 & 1 \\ 1 & 0 & -1 & 0 \\ -3 & 1 & 4 & 2 \end{pmatrix}$$

Najděte inverzní matici A^{-1}. Proveďte zkoušku

$$\begin{array}{l}\\ \\ \\ \\ \end{array}\left(\begin{array}{cccc|cccc} 1 & 2 & 3 & 4 & 1 & 0 & 0 & 0 \\ -1 & 0 & 2 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 & 1 & 0 \\ -3 & 1 & 4 & 2 & 0 & 0 & 0 & 1 \end{array}\right)$$

$$\begin{array}{l} \\ 1F+2F \\ 1F-3F \\ 3\cdot 1F+4F \end{array}\left(\begin{array}{cccc|cccc} 1 & 2 & 3 & 4 & 1 & 0 & 0 & 0 \\ 0 & 2 & 5 & 5 & 1 & 1 & 0 & 0 \\ 0 & 2 & 4 & 4 & 1 & 0 & -1 & 0 \\ 0 & 7 & 13 & 14 & 3 & 0 & 0 & 1 \end{array}\right)$$

$$\begin{array}{l} \\ \\ 2F-3F \\ 7\cdot 2F-(2)4F \end{array}\left(\begin{array}{cccc|cccc} 1 & 2 & 3 & 4 & 1 & 0 & 0 & 0 \\ 0 & 2 & 5 & 5 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 9 & 7 & 1 & 7 & 0 & -2 \end{array}\right)$$

$$\begin{array}{l} \\ \\ \\ 9\cdot 3F+4F \end{array}\left(\begin{array}{cccc|cccc} 1 & 2 & 3 & 4 & 1 & 0 & 0 & 0 \\ 0 & 2 & 5 & 5 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & -2 & 1 & -2 & -9 & -2 \end{array}\right)$$

$$\begin{array}{l} 1F-2F \\ 2F-5\cdot 3F \\ 3F-4F \\ \\ \end{array}\left(\begin{array}{cccc|cccc} 1 & 0 & -2 & -1 & 0 & -1 & 0 & 0 \\ 0 & 2 & 0 & 0 & 1 & -4 & -5 & 0 \\ 0 & 0 & 1 & 0 & \frac{1}{2} & 0 & -\frac{7}{2} & -1 \\ & & & & -\frac{1}{2} & 1 & \frac{9}{2} & 1 \end{array}\right)$$

$$\begin{array}{l} 1F+2\cdot 3F \\ \\ \\ \\ \end{array}\left(\begin{array}{cccc|cccc} 1 & 0 & 0 & -1 & 1 & -1 & -7 & -2 \\ 0 & 1 & 0 & 0 & \frac{1}{2} & -2 & -\frac{5}{2} & 0 \\ 0 & 0 & 1 & 0 & +\frac{1}{2} & 0 & -\frac{7}{2} & -1 \\ 0 & 0 & 0 & 1 & -\frac{1}{2} & 1 & \frac{9}{2} & 1 \end{array}\right)$$

$$\begin{array}{l} 1F+4F \\ \\ \\ \\ \end{array}\left(\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & \frac{1}{2} & 0 & -\frac{5}{2} & -1 \\ 0 & 1 & 0 & 0 & \frac{1}{2} & -2 & -\frac{5}{2} & 0 \\ 0 & 0 & 1 & 0 & \frac{1}{2} & 0 & -\frac{7}{2} & -1 \\ 0 & 0 & 0 & 1 & -\frac{1}{2} & 1 & \frac{9}{2} & 1 \end{array}\right)$$

Zk: $A \cdot A^{-1} = E$

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ -1 & 0 & 2 & 1 \\ 1 & 0 & -1 & 0 \\ -3 & 1 & 4 & 2 \end{pmatrix} \cdot \begin{pmatrix} \frac{1}{2} & 0 & -\frac{5}{2} & -1 \\ \frac{1}{2} & -2 & -\frac{5}{2} & 0 \\ \frac{1}{2} & 0 & -\frac{7}{2} & -1 \\ -\frac{1}{2} & 1 & \frac{9}{2} & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$