Vypracování:

1. Řešte systém lineárních rovnic

$$\begin{array}{rcrcrcrcr} x_1 & - & 2x_2 & - & 3x_3 & & & = & 1 \\ -x_1 & + & 4x_2 & - & 5x_3 & + & 4x_4 & = & -4 \\ -2x_1 & + & 4x_2 & + & x_3 & + & 2x_4 & = & 4 \\ 3x_1 & - & 7x_2 & - & 5x_3 & - & 3x_4 & = & -3 \end{array}$$

$$\begin{pmatrix} 1 & -2 & -3 & 0 & 1 \\ -1 & 4 & -5 & 4 & -4 \\ -2 & 4 & 1 & 2 & 4 \\ 3 & -7 & -5 & -3 & -3 \end{pmatrix} = \begin{pmatrix} 1 & -2 & -3 & 0 & 1 \\ 0 & 2 & -8 & 4 & -3 \\ 0 & 0 & -5 & 2 & 6 \\ 0 & -1 & 4 & -3 & -6 \end{pmatrix} = \begin{pmatrix} 1 & -2 & -3 & 0 & 1 \\ 0 & 2 & -8 & 4 & -3 \\ 0 & 0 & -5 & 2 & 6 \\ 0 & 0 & 0 & -2 & -15 \end{pmatrix}$$

$$-2 \cdot x_4 = -15$$

$$\underline{\underline{x_4 = \frac{15}{2} = 7{,}5}}$$

$$-5 \cdot x_3 + 2 \cdot \frac{15}{2} = 6$$

$$\underline{\underline{x_3 = \frac{9}{5} = 1{,}8}}$$

$$2 \cdot x_2 - 8 \cdot \frac{9}{5} + 4 \cdot \frac{15}{2} = -3$$

$$\underline{\underline{x_2 = -\frac{93}{10} = -9{,}3}}$$

$$x_1 - 2 \cdot \left(-\frac{93}{10}\right) - 3 \cdot \frac{9}{5} + 0 = 1$$

$$\underline{\underline{x_1 = -\frac{61}{5} = -12{,}2}}$$

Zk.:

$$3 \cdot (-12{,}2) - 7 \cdot (-9{,}3) - 5 \cdot 1{,}8 - 3 \cdot 7{,}5 = -3$$
$$-36{,}6 + 65{,}1 - 9 - 22{,}5 = -3$$
$$-3 = -3$$

$$-2 \cdot (-12{,}2) + 4 \cdot (-9{,}3) + 1{,}8 + 2 \cdot 7{,}5 = 4$$
$$24{,}4 - 37{,}2 + 1{,}8 + 15 = 4$$
$$4 = 4$$

$$12{,}2 + 4 \cdot (-9{,}3) - 5 \cdot 1{,}8 + 4 \cdot 7{,}5 = -4$$
$$12{,}2 - 37{,}2 - 9 + 30 = -4$$
$$-4 = -4$$

$$-12{,}2 - 2 \cdot (-9{,}3) - 3 \cdot 1{,}8 = 1$$
$$-12{,}2 + 18{,}6 - 5{,}4 = 1$$
$$1 = 1$$

2. Určete hodnotu determinantu
a) rozvojem podle řádku či sloupce
b) pomocí elementárních transformací

a)

$$\begin{vmatrix} 1 & 2 & 4 & -3 & 0 \\ 1 & 0 & 3 & -1 & 0 \\ 3 & 2 & 1 & 0 & 2 \\ -1 & 2 & -1 & 0 & 2 \\ -2 & 1 & 0 & 0 & 2 \end{vmatrix} = (-1)(-3)\begin{vmatrix} 1 & 0 & 3 & 0 \\ 3 & 2 & 1 & 2 \\ -1 & 2 & -1 & 2 \\ -2 & 1 & 0 & 2 \end{vmatrix} + (-1)\begin{vmatrix} 1 & 2 & 4 & 0 \\ 3 & 2 & 1 & 2 \\ -1 & 2 & -1 & 2 \\ -2 & 1 & 0 & 2 \end{vmatrix} =$$

$$3\left(\begin{vmatrix} 2 & 1 & 2 \\ 2 & -1 & 2 \\ 1 & 0 & 2 \end{vmatrix} + 3\begin{vmatrix} 3 & 2 & 2 \\ -1 & 2 & 2 \\ -2 & 1 & 2 \end{vmatrix}\right) - \left(\begin{vmatrix} 2 & 1 & 2 \\ 2 & -1 & 2 \\ 1 & 0 & 2 \end{vmatrix} - 2\begin{vmatrix} 3 & 1 & 2 \\ -1 & -1 & 2 \\ -2 & 0 & 2 \end{vmatrix} + 4\begin{vmatrix} 3 & 2 & 2 \\ -1 & 2 & 2 \\ -2 & 1 & 2 \end{vmatrix}\right) =$$

$$= 3(-4+3\cdot 8) - [-4+4\cdot 8 - 2(-6-4-4+2)] = 60-(28+24) = 60-52 = \underline{\underline{8}}$$

b)

$$\begin{vmatrix} 1 & 2 & 4 & -3 & 0 \\ 1 & 0 & 3 & -1 & 0 \\ 3 & 2 & 1 & 0 & 2 \\ -1 & 2 & -1 & 0 & 2 \\ -2 & 1 & 0 & 0 & 2 \end{vmatrix} \sim \begin{vmatrix} 1 & 2 & 4 & -3 & 0 \\ 0 & -2 & -1 & 2 & 0 \\ 0 & -4 & -11 & 9 & 2 \\ 0 & 4 & 3 & -3 & 2 \\ 0 & 5 & 8 & -6 & 2 \end{vmatrix} \sim \frac{1}{2}\cdot \begin{vmatrix} 1 & 2 & 4 & -3 & 0 \\ 0 & -2 & -1 & 2 & 0 \\ 0 & 0 & -9 & 5 & 2 \\ 0 & 0 & 1 & 1 & 2 \\ 0 & 0 & 11 & -2 & 4 \end{vmatrix} \sim$$

$$\sim \frac{1}{2}\cdot\frac{1}{9}\cdot\frac{1}{9}\cdot \begin{vmatrix} 1 & 2 & 4 & -3 & 0 \\ 0 & -2 & -1 & 2 & 0 \\ 0 & 0 & -9 & 5 & 2 \\ 0 & 0 & 0 & 14 & 20 \\ 0 & 0 & 0 & 37 & 58 \end{vmatrix} \sim \frac{1}{2}\cdot\frac{1}{9}\cdot\frac{1}{9}\cdot\frac{1}{14}\cdot \begin{vmatrix} 1 & 2 & 4 & -3 & 0 \\ 0 & -2 & -1 & 2 & 0 \\ 0 & 0 & -9 & 5 & 2 \\ 0 & 0 & 0 & 14 & 20 \\ 0 & 0 & 0 & 0 & 72 \end{vmatrix} =$$

$$= \frac{1}{2}\cdot\frac{1}{9}\cdot\frac{1}{9}\cdot\frac{1}{14}\cdot 1(-2)(-9)14\cdot 72 = \underline{\underline{8}}$$

2. Najděte inverzní matici A^{-1}. Proveďte zkoušku.

$$\mathbf{A} = \begin{pmatrix} 1 & 2 & 3 & 4 \\ -1 & 0 & 2 & 1 \\ 1 & 0 & -1 & 0 \\ -3 & 1 & 4 & 2 \end{pmatrix}$$

A^{-1} = ?
A . A^{-1} = E
B = (A | B)

$$\mathbf{B} = \left(\begin{array}{cccc|cccc} 1 & 2 & 3 & 4 & 1 & 0 & 0 & 0 \\ -1 & 0 & 2 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 & 1 & 0 \\ -3 & 1 & 4 & 2 & 0 & 0 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{cccc|cccc} 1 & 2 & 3 & 4 & 1 & 0 & 0 & 0 \\ 0 & 2 & 5 & 5 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 2 & 0 & 0 & 3 & 1 \end{array}\right) \sim$$

$$\sim \left(\begin{array}{cccc|cccc} 1 & 2 & 3 & 4 & 1 & 0 & 0 & 0 \\ 0 & 2 & 5 & 5 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 3 & 1 & 1 & 1 & -6 & -2 \end{array}\right) \sim \left(\begin{array}{cccc|cccc} 1 & 2 & 3 & 4 & 1 & 0 & 0 & 0 \\ 0 & 2 & 5 & 5 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & -2 & 1 & -2 & -9 & -2 \end{array}\right) \sim$$

$$\sim \left(\begin{array}{cccc|cccc} 1 & 0 & -2 & -1 & 0 & -1 & 0 & 0 \\ 0 & 2 & 0 & 0 & 1 & -4 & -5 & 0 \\ 0 & 0 & 2 & 0 & 1 & 0 & -7 & -2 \\ 0 & 0 & 0 & -2 & 1 & -2 & -9 & -2 \end{array}\right) \sim \left(\begin{array}{cccc|cccc} 1 & 0 & 0 & -1 & 1 & -1 & -7 & -2 \\ 0 & 2 & 0 & 0 & 1 & -4 & -5 & 0 \\ 0 & 0 & 2 & 0 & 1 & 0 & -7 & -2 \\ 0 & 0 & 0 & -2 & 1 & -2 & -9 & -2 \end{array}\right) \sim$$

$$\sim \left(\begin{array}{cccc|cccc} -2 & 0 & 0 & 0 & -1 & 0 & 5 & 2 \\ 0 & 2 & 0 & 0 & 1 & -4 & -5 & 0 \\ 0 & 0 & 2 & 0 & 1 & 0 & -7 & -2 \\ 0 & 0 & 0 & -2 & 1 & -2 & -9 & -2 \end{array}\right) \sim \left(\begin{array}{cccc|cccc} 2 & 0 & 0 & 0 & 1 & 0 & -5 & -2 \\ 0 & 2 & 0 & 0 & 1 & -4 & -5 & 0 \\ 0 & 0 & 2 & 0 & 1 & 0 & -7 & -2 \\ 0 & 0 & 0 & 2 & -1 & 2 & 9 & 2 \end{array}\right)$$

$$\underline{\underline{\mathbf{A}^{-1} = \begin{pmatrix} \frac{1}{2} & 0 & -\frac{5}{2} & -1 \\ \frac{1}{2} & -2 & -\frac{5}{2} & 0 \\ \frac{1}{2} & 0 & -\frac{7}{2} & -1 \\ -\frac{1}{2} & 1 & \frac{9}{2} & 1 \end{pmatrix}}}$$

Zkouška

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ -1 & 0 & 2 & 1 \\ 1 & 0 & -1 & 0 \\ -3 & 1 & 4 & 2 \end{pmatrix} \bullet \begin{pmatrix} \frac{1}{2} & 0 & -\frac{5}{2} & -1 \\ \frac{1}{2} & -2 & -\frac{5}{2} & 0 \\ \frac{1}{2} & 0 & -\frac{7}{2} & -1 \\ -\frac{1}{2} & 1 & \frac{9}{2} & 1 \end{pmatrix} = \mathbf{E}$$

$$\begin{pmatrix} \left(\frac{1}{2}+1+\frac{3}{2}-2\right) & (0-4+0+4) & \left(-\frac{5}{2}-5-\frac{21}{2}+18\right) & (-1+0-3+4) \\ \left(-\frac{1}{2}+0+1-\frac{1}{2}\right) & (0+0+0+1) & \left(\frac{5}{2}+0-7+\frac{9}{2}\right) & (1+0-2+1) \\ \left(\frac{1}{2}+0-\frac{1}{2}+0\right) & (0+0+0+0) & \left(-\frac{5}{2}+0+\frac{7}{2}+0\right) & (-1+0+1+0) \\ \left(-\frac{3}{2}+\frac{1}{2}+2-1\right) & (0-2+0+2) & \left(\frac{15}{2}-\frac{5}{2}-14+9\right) & (3+0-4+2) \end{pmatrix} = \mathbf{E}$$

$$\underline{\underline{\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \mathbf{E}}}$$