Markéta Švábová
tutor: M. Matulová
POT 3 Matematika I.
datum: 2.2.2008
UČO: 253837

1)
$$\left(\begin{array}{cccc|c} 1 & -2 & -3 & 0 & 1 \\ -1 & 4 & -5 & 4 & -4 \\ -2 & 4 & 1 & 2 & 4 \\ 3 & -7 & -5 & -3 & -3 \end{array}\right) \sim \left(\begin{array}{cccc|c} 1 & -2 & -3 & 0 & 1 \\ 0 & 2 & -8 & 4 & -3 \\ 0 & 0 & -5 & 2 & 6 \\ 0 & +1 & -4 & +3 & 6 \end{array}\right) \sim \left(\begin{array}{cccc|c} 1 & -2 & -3 & 0 & 1 \\ 0 & 2 & -8 & 4 & -3 \\ 0 & 0 & -5 & 2 & 6 \\ 0 & 0 & 0 & -2 & -15 \end{array}\right)$$

$$x_1 - 2x_2 - 3x_3 = 1$$
$$x_2 - 4x_3 + 3x_4 = 6$$
$$-5x_3 + 2x_4 = 6$$
$$-2x_4 = -15$$

$$x_4 = \frac{15}{2}$$

$$5x_3 + 15 = 6$$
$$-5x_3 = -9$$
$$x_3 = \frac{9}{5}$$

$$x_2 - \frac{36}{5} + \frac{45}{2} = 6 \quad /\cdot 10$$
$$10x_x{}^2 - 72 + 225 = 60$$
$$10x_2 = -93$$
$$x_2 = \frac{-93}{10}$$

$$x_1 + \frac{93}{5} - \frac{25}{5} = 1 \quad /\cdot 5$$
$$5x_1 + 66 = 5$$
$$5x_1 = -61$$
$$x_1 = -\frac{61}{5}$$

$$-\frac{61}{5} + \frac{93}{5} - \frac{27}{5} = 1$$
$$1 = 1$$

$$\frac{61}{5} - \frac{186}{5} - \frac{45}{5} + 30 = -4 \quad /\cdot 5$$
$$-170 + 150 = -20$$
$$-20 = -20$$

$$\frac{122}{5} - \frac{186}{5} + \frac{9}{5} + 15 = 4 \quad /\cdot 5$$
$$-55 + 77 = 20$$
$$20 = 20$$

$$-\frac{183}{5} + \frac{651}{10} - 9 - \frac{45}{2} = -3 \quad /\cdot 10$$
$$-366 + 651 - 90 - 225 = -30$$
$$-30 = 30$$

2)

a) $\begin{vmatrix} 1 & 2 & 4 & -3 & 0 \\ 1 & 0 & 3 & -1 & 0 \\ 3 & 2 & 1 & 0 & 2 \\ -1 & 2 & -1 & 0 & 2 \\ -2 & 1 & 0 & 0 & 2 \end{vmatrix} = (-1)^{1+4} \cdot (-3) \cdot \begin{vmatrix} 1 & 0 & 3 & 0 \\ 3 & 2 & 1 & 2 \\ -1 & 2 & -1 & 2 \\ -2 & 1 & 0 & 2 \end{vmatrix} + (-1)^{2+4} \cdot (-1) \begin{vmatrix} 1 & 2 & 4 & 0 \\ 3 & 2 & 1 & 2 \\ -1 & 2 & -1 & 2 \\ -2 & 1 & 0 & 2 \end{vmatrix} + 0 + 0 + 0 =$

$= 3 \left[(-1)^{1+1} \cdot (1) \cdot \begin{vmatrix} 2 & 1 & 2 \\ 2 & -1 & 2 \\ 1 & 0 & 2 \end{vmatrix} + 0 + (-1)^{1+3} \cdot 3 \begin{vmatrix} 3 & 2 & 2 \\ -1 & 2 & 2 \\ -2 & 1 & 2 \end{vmatrix} + 0 \right] -$

$- \left[0 + (-1)^{2+4} 2 \begin{vmatrix} 1 & 2 & 4 \\ -1 & 2 & -1 \\ -2 & 1 & 0 \end{vmatrix} + (-1)^{3+4} \cdot 2 \begin{vmatrix} 1 & 2 & 4 \\ 3 & 2 & 1 \\ -2 & 1 & 0 \end{vmatrix} + (-1)^{4+4} \cdot 2 \cdot \begin{vmatrix} 1 & 2 & 4 \\ 3 & 2 & 1 \\ -1 & 2 & -1 \end{vmatrix} \right] =$

$= 3[-4 + 0 + 2 + 2 - 0 - 4 + 3(12 - 2 - 8 - 8 - 6 + 4)] - [2 \cdot (0 - 4 + 4 + 16 + 1 - 0) -$

$- (0 + 12 - 4 + 16 - 1 - 0) + 2(-2 + 24 - 2 + 8 - 2 + 6)] = 3[-4 + 24] -$

$- [34 - 46 + 64] = 60 - 52 = \underline{\underline{8}}$

2)
b)

$$\begin{vmatrix} 1 & 2 & 4 & -3 & 0 \\ 1 & 0 & 3 & -1 & 0 \\ 3 & 2 & 1 & 0 & 2 \\ -1 & 2 & -1 & 0 & 2 \\ -2 & 1 & 0 & 0 & 2 \end{vmatrix} \sim \begin{vmatrix} 1 & 2 & 4 & -3 & 0 \\ 0 & -2 & -1 & 2 & 0 \\ 0 & -4 & -11 & 9 & 2 \\ 0 & 4 & 3 & -3 & 2 \\ 0 & 5 & 8 & -6 & 2 \end{vmatrix} \sim (-2) \begin{vmatrix} 1 & 2 & 4 & -3 & 0 \\ 0 & 1 & \frac{1}{2} & -1 & 0 \\ 0 & -4 & -11 & 9 & 2 \\ 0 & 4 & 3 & -3 & 2 \\ 0 & 5 & 8 & -6 & 2 \end{vmatrix} \sim$$

$$\sim \begin{vmatrix} 1 & 2 & 4 & -3 & 0 \\ 0 & 1 & \frac{1}{2} & -1 & 0 \\ 0 & 0 & -9 & 5 & 2 \\ 0 & 0 & 1 & 1 & 2 \\ 0 & 0 & \frac{11}{2} & -1 & 2 \end{vmatrix} \sim (-2)\cdot(-1) \begin{vmatrix} 1 & 2 & 4 & -3 & 0 \\ 0 & 1 & \frac{1}{2} & -1 & 0 \\ 0 & 0 & 1 & 1 & 2 \\ 0 & 0 & -9 & 5 & 2 \\ 0 & 0 & \frac{11}{2} & -1 & 2 \end{vmatrix} \sim 2\cdot \begin{vmatrix} 1 & 2 & 4 & -3 & 0 \\ 0 & 1 & \frac{1}{2} & -1 & 0 \\ 0 & 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 14 & 20 \\ 0 & 0 & 0 & -\frac{13}{2} & -9 \end{vmatrix} \sim$$

$$\sim 2\cdot 14 \begin{vmatrix} 1 & 2 & 4 & -3 & 0 \\ 0 & 1 & \frac{1}{2} & -1 & 0 \\ 0 & 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 1 & \frac{10}{7} \\ 0 & 0 & 0 & -\frac{13}{2} & -9 \end{vmatrix} \sim 28 \begin{vmatrix} 1 & 2 & 4 & -3 & 0 \\ 0 & 1 & \frac{1}{2} & -1 & 0 \\ 0 & 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 1 & \frac{10}{7} \\ 0 & 0 & 0 & 0 & \frac{2}{7} \end{vmatrix} = 28\cdot 1\cdot 1\cdot 1\cdot 1\cdot \frac{2}{7} = \underline{\underline{8}}$$

3) $A = \begin{pmatrix} 1 & 2 & 3 & 4 \\ -1 & 0 & 2 & 1 \\ 1 & 0 & -1 & 0 \\ -3 & 1 & 4 & 2 \end{pmatrix} \qquad A^T = \begin{pmatrix} 1 & -1 & 1 & -3 \\ 2 & 0 & 0 & 1 \\ 3 & 2 & -1 & 4 \\ 4 & 1 & 0 & 2 \end{pmatrix}$

$$A = |A| = (-1)^{3+1}\, 1 \cdot \begin{pmatrix} 2 & 3 & 4 \\ 0 & 2 & 1 \\ 1 & 4 & 2 \end{pmatrix} + 0 + (-1)^{3+3} \cdot (-1) \cdot \begin{pmatrix} 1 & 2 & 4 \\ -1 & 0 & 1 \\ -3 & 1 & 2 \end{pmatrix} + 0 =$$

$$= 8 + 0 + 3 - 8 - 8 - 0 - (0 - 4 - 6 - 0 - 1 + 4) = -5 + 7 = 2 = \underline{\underline{|A|}}$$

$$Adj \begin{pmatrix} \begin{vmatrix} 0 & 0 & 1 \\ 2 & -1 & 4 \\ 1 & 0 & 2 \end{vmatrix} & -\begin{vmatrix} 2 & 0 & 1 \\ 3 & -1 & 4 \\ 4 & 0 & 2 \end{vmatrix} & \begin{vmatrix} 2 & 0 & 1 \\ 3 & 2 & 4 \\ 4 & 1 & 2 \end{vmatrix} & -\begin{vmatrix} 2 & 0 & 0 \\ 3 & 2 & -1 \\ 4 & 1 & 0 \end{vmatrix} \\ -\begin{vmatrix} -1 & 1 & -3 \\ 2 & -1 & 4 \\ 1 & 0 & 2 \end{vmatrix} & \begin{vmatrix} 1 & 1 & -3 \\ 3 & -1 & 4 \\ 4 & 0 & 2 \end{vmatrix} & -\begin{vmatrix} 1 & -1 & -3 \\ 3 & 2 & 4 \\ 4 & 1 & 2 \end{vmatrix} & \begin{vmatrix} 1 & -1 & 1 \\ 3 & 2 & -1 \\ 4 & 1 & 0 \end{vmatrix} \\ \begin{vmatrix} -1 & 1 & -3 \\ 0 & 0 & 1 \\ 1 & 0 & 2 \end{vmatrix} & -\begin{vmatrix} 1 & 1 & -3 \\ 2 & 0 & 1 \\ 4 & 0 & 2 \end{vmatrix} & \begin{vmatrix} 1 & -1 & -3 \\ 2 & 0 & 1 \\ 4 & 1 & 2 \end{vmatrix} & -\begin{vmatrix} 1 & -1 & 1 \\ 2 & 0 & 0 \\ 4 & 1 & 0 \end{vmatrix} \\ -\begin{vmatrix} -1 & 1 & -3 \\ 0 & 0 & 1 \\ 2 & -1 & 4 \end{vmatrix} & \begin{vmatrix} 1 & 1 & -3 \\ 2 & 0 & 1 \\ 3 & -1 & 4 \end{vmatrix} & -\begin{vmatrix} 1 & -1 & -3 \\ 2 & 0 & 1 \\ 3 & 2 & 4 \end{vmatrix} & \begin{vmatrix} 1 & -1 & 1 \\ 2 & 0 & 0 \\ 3 & 2 & -1 \end{vmatrix} \end{pmatrix} = \begin{pmatrix} 1 & 0 & -5 & -2 \\ 1 & -4 & -5 & 0 \\ 1 & 0 & -7 & -2 \\ -1 & 2 & 9 & 2 \end{pmatrix}$$

$$A^{-1} = \frac{1}{|A|}\, Adj = \frac{1}{2} \cdot \begin{pmatrix} 1 & 0 & -5 & -2 \\ 1 & -4 & -5 & 0 \\ 1 & 0 & -7 & -2 \\ -1 & 2 & 9 & 2 \end{pmatrix} = \underline{\underline{\begin{pmatrix} \frac{1}{2} & 0 & -\frac{5}{2} & -1 \\ \frac{1}{2} & -2 & -\frac{5}{2} & 0 \\ \frac{1}{2} & 0 & -\frac{7}{2} & -1 \\ -\frac{1}{2} & 1 & \frac{9}{2} & 1 \end{pmatrix}}} = A^{-1}$$

$$A \cdot A^{-1} = \begin{pmatrix} 1 & 2 & 3 & 4 \\ -1 & 0 & 2 & 1 \\ 1 & 0 & -1 & 0 \\ -3 & 1 & 4 & 2 \end{pmatrix} \cdot \begin{pmatrix} \frac{1}{2} & 0 & -\frac{5}{2} & -1 \\ \frac{1}{2} & -2 & -\frac{5}{2} & 0 \\ \frac{1}{2} & 0 & -\frac{7}{2} & -1 \\ -\frac{1}{2} & 1 & \frac{9}{2} & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = E$$

$$A^{-1} \cdot A = \begin{pmatrix} \frac{1}{2} & 0 & -\frac{5}{2} & -1 \\ \frac{1}{2} & -2 & -\frac{5}{2} & 0 \\ \frac{1}{2} & 0 & -\frac{7}{2} & -1 \\ \frac{1}{2} & 1 & \frac{9}{2} & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 & 3 & 4 \\ -1 & 0 & 2 & 1 \\ 1 & 0 & -1 & 0 \\ -3 & 1 & 4 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = E$$