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Treasury Management

Lessons in Finance and Investment at

Part 2

Masarykova univerzita

Ekonomicko-správní fakulta

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Trilingual index

English-German-Czech index of
key terms in investment

English	German	Czech
annual equivalent	Äquivalente Annuität	ekvivalentní anuita

Any volunteers?

Treasury Management

Survey

1 Basics (units 1–5, 10)

2 Cash flow from financial activities (units 6-8)

3 Cash flow from operations (unit 9)

4 Cash flow from investment activities (units 11-16)

3 CF's

- Complete account of an investment
- Dominance
- Net present value
- Annual equivalent
- Internal rate of return
- Payback period

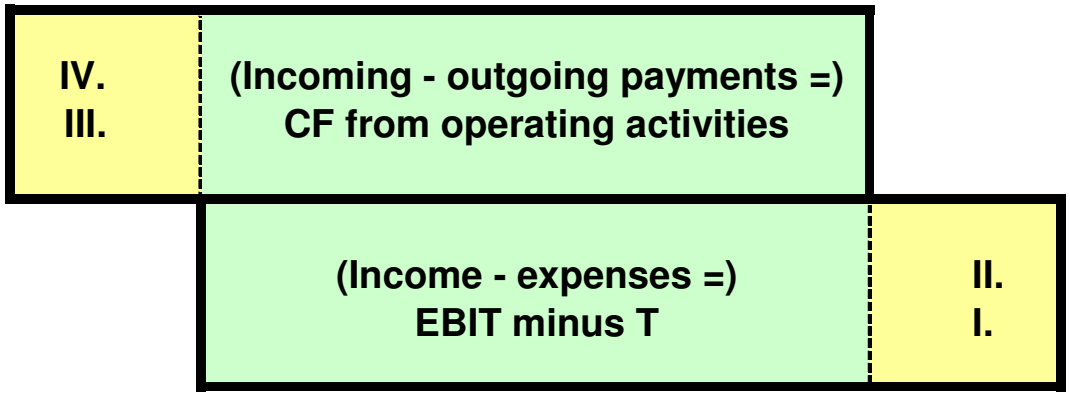
6 D's

Exercise 7-2

Make up a source income statement for the Brněnské Marcipánové a Nugátové Kontor by restructuring the 2025 income statement according to the following form!

Source income statement (form)		
	2025	2024
Revenues		
Changes in inventories of finished goods and work in progress		
Production for own fixed assets capitalized		
Cost of purchased materials and services		
Personnel expenses		
Depreciation and amortization on tangible and intangible assets		
Core operating profit		
Other operating income		
Other operating expenses		
Other operating profit		
Operating profit ("EBIT")		
Income from participations		
Income from other financial assets		
Other interest income		
Depreciation and amortization on financial assets and financial current assets		
Interest expenses		
Financial profit ("I")		
Extraordinary income		
Extraordinary expenses		
Extraordinary items		
Earnings before tax ("EBT")		
Income tax, Other taxes } **("T")**		
Earnings after tax ("EAT")		

Schmalenbach's Bar Graph revisited (Fig. 7-1)



within operations !!!

	EBIT minus T	
+	expenses that do not affect cash & cash equivalents (correction type I)	within operations!!!
-	income that does not affect cash & cash equivalents (correction type II)	
-	non-expense applications of cash & cash equivalents (correction type III)	
+	non-expense originations of cash & cash equivalents (correction type IV)	
=	Cash flow from operating activities	

Exercise 7-3

Starting point is exercise 4-4. As you can easily check, this is CFP's source income statement for the fiscal year 2025.

Bridge the gap between EBIT minus T and cash flow from operating activities by making up a calculation considering for the four different types of corrections!

Source income statement for CFP v.o.s., Brno, for the time period from January 01, 2025, to December 31, 2025 (TCZK)		
		2025
Revenues		700
Changes in inventories of finished goods and work in progress		0
Production for own fixed assets capitalized		0
Cost of purchased materials and services		400
Personnel expenses		180
Depreciation and amortization on tangible and intangible assets		5
	Core operating profit	115
Other operating income		20
Other operating expenses		0
	Other operating profit	20
	Operating profit ("EBIT")	135
Income from participations		0
Income from other financial assets		0
Other interest income		0
Depreciation and amortization on financial assets and financial current assets		0
Interest expenses		35
	Financial profit ("I")	-35
	Earnings before tax ("EBT")	100
Income tax } ("T")		0
Other taxes }		0
	Earnings after tax ("EAT")	100

Exercise 7-1

After receipt of a corresponding purchase order by fax as of September 01, 2026, the Brněnské Marcipánové a Nugátové Kontor still on the same day delivers 3 kg of crude marzipan amounting to CZK 450 to Jemná Čokoláda. As the Kontor grants September 30, 2026, as time of payment, Jemná Čokoláda pays the amount due only at the end of the month of September.

Assume Jemná Čokoláda's perspective and translate the transactions between buyer and supplier into the symbolism known to you from exercise 2-2! To this end, first register the transactions on a "gross"-level by means of two exchange contracts and then on a "net"-level by only one contract!

Management

Cybernetics

System-Theoretic Approach (Hans Ulrich)

(1) Steering of economic **systems**

(2) ... by means of a close-meshed network ... of **directives** or ...

(3) ... by granting extensive **freedom of decision** ... to lower hierarchy levels.

(4) possible instruments: **internal accounting prices**, **profit centers, intrapreneurship**

(5) dynamic version: **feedback control system**

Hans Ulrich, 1919-1997

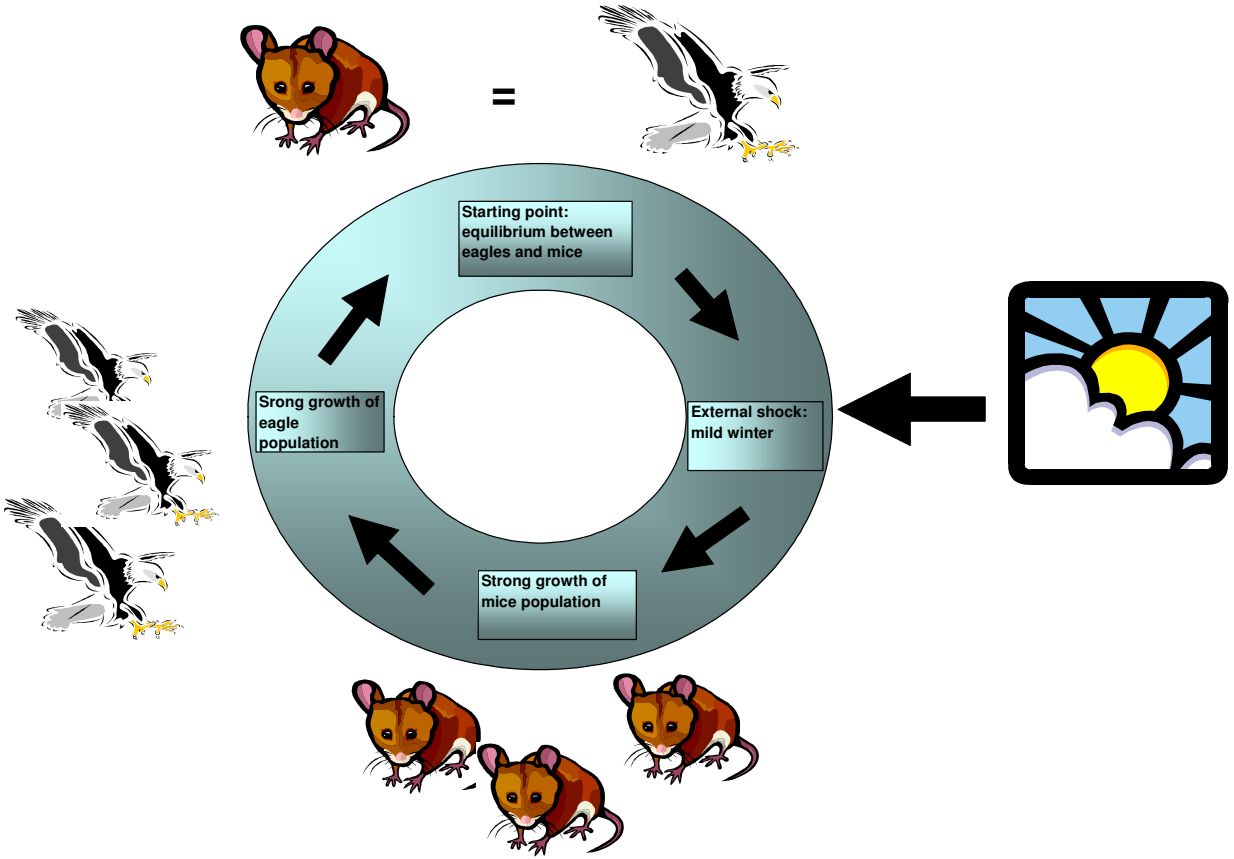
PO(C)-Concept

Erich Gutenberg

Management is ...

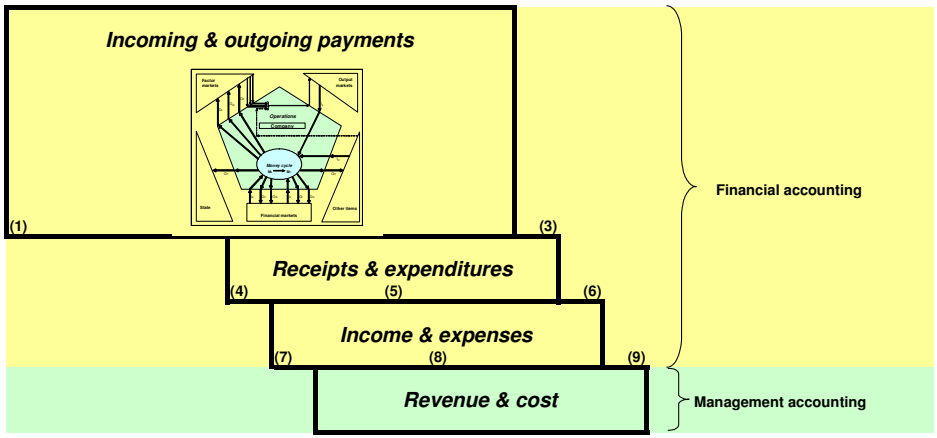
... planning, organisation and control.

Feedback Control Systems: An Example



Treasury Management (Financial Management)

- ✓ Steering of the money cycle
- ✓ Management of the cash flow statement

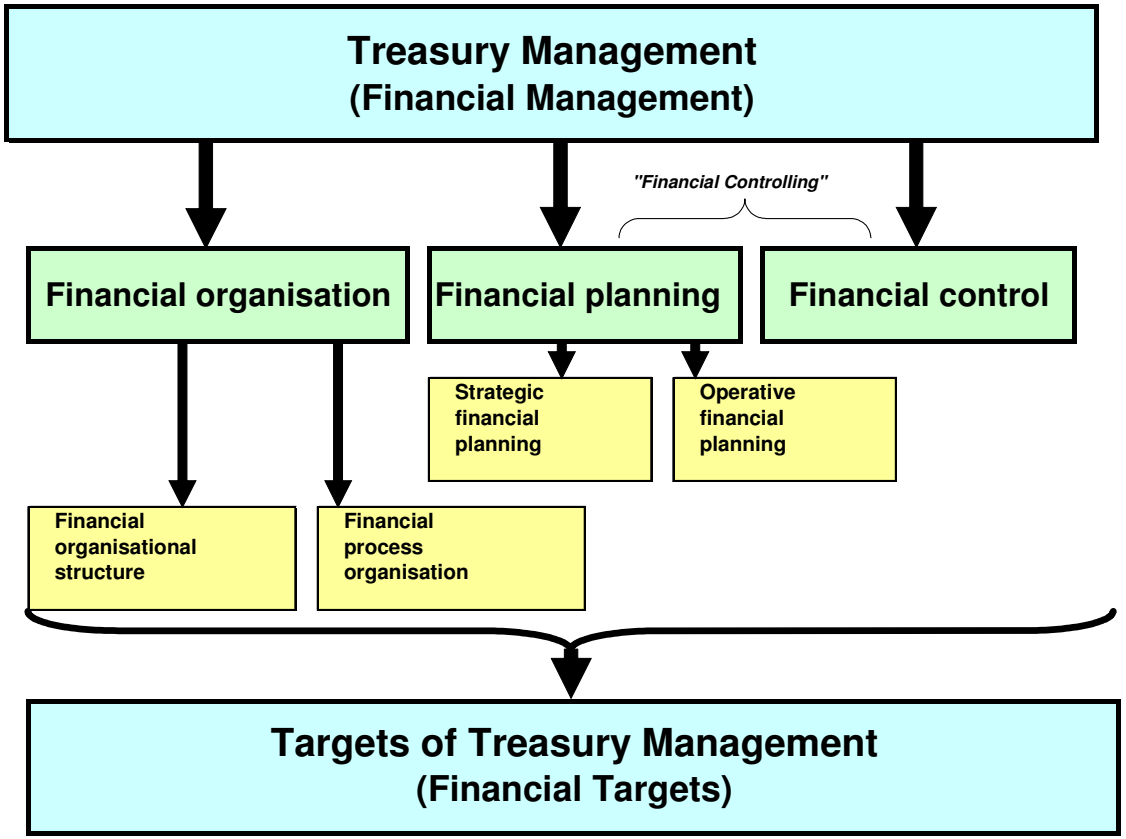


Consolidated cash flow statement (€ million)		2006	2005
EBIT		1.983	1.738
Depreciation and amortization of tangible and intangible assets		1.250	1.200
Change in provisions for pensions and other provisions		273	-19
Change in net working capital		1.137	66
Income taxes paid		-543	-499
Elimination of negative difference first-time consolidation		-410	0
Other		-427	-452
Cash flow from operating activities of continuing operations		3.263	2.034
Cash flow from operating activities of discontinued operations		0	150
Total cash flow from operating activities	$(+I_S+I_O-O_M-O_L-O_O-O_T)$	**3.263**	**2.184**
First-time consolidation		108	19
Company acquisitions		-205	0
Investments in tangible assets (excl. Finance leases)		-1.824	-1.922
Other investments		-268	-253
Company divestments		0	48
Divestment of stores		484	670
Disposals of fixed assets		403	313
Cash flow from investing activities of continuing operations		-1.302	-1.125
Cash flow from investing activities of discontinued operations		0	-43
Total cash flow from investing activities	$-O_F+I_F-O_E+I_P+I_W-O_D+I_F+I_R$	**-1.302**	**-1.168**
Profit distribution			
- to parent company stockholders		-334	-334
- to other stockholders		-122	-72
Raising of financial liabilities		1.423	935
Redemption/repayment of financial liabilities		-1.571	-1.415
Interest paid		-610	-637
Interest received		169	137
Profit and loss transfers and other financing activities		50	-6
Cash flow from financing activities of continuing operations		-995	-1.392
Cash flow from financing activities of discontinued operations			23
Total cash flow from financing activities	$+I_E-O_P-O_W+I_D-O_T-O_R$	**-995**	**-1.369**
Total cash flows		**966**	**-353**
Exchange rate effects on cash and cash equivalents		-1	13
Overal change in cash and cash equivalents		965	-340
Cash and cash equivalents on January 1	M_0	**1.767**	**2.107**
Cash and cash equivalets on December 31	M_1	**2.732**	**1.767**

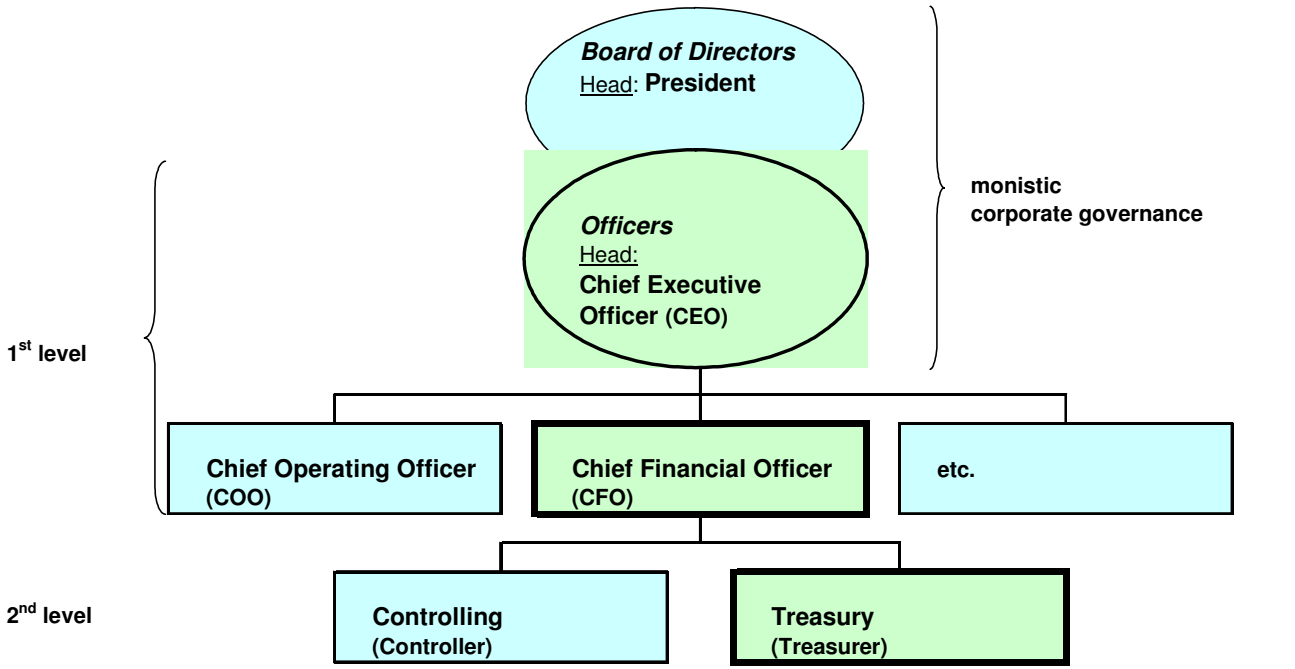
Objectives of Treasury Management

- ✓ Assuring the company's ability to pay (⇒ sufficient liquidity)
- ✓ Little annoyance or (even better) strong support of operations (⇒ no affluent liquidity)
- ✓ Great contribution to the company's rentability (⇒ efficient use of liquidity reserves)
- ✓ Efficient risk management (⇒ implementation of hedging, insurance contracts, derivatives etc.)
- ✓ Little restriction of entrepreneurial freedom (⇒ avoidance of too many covenants)

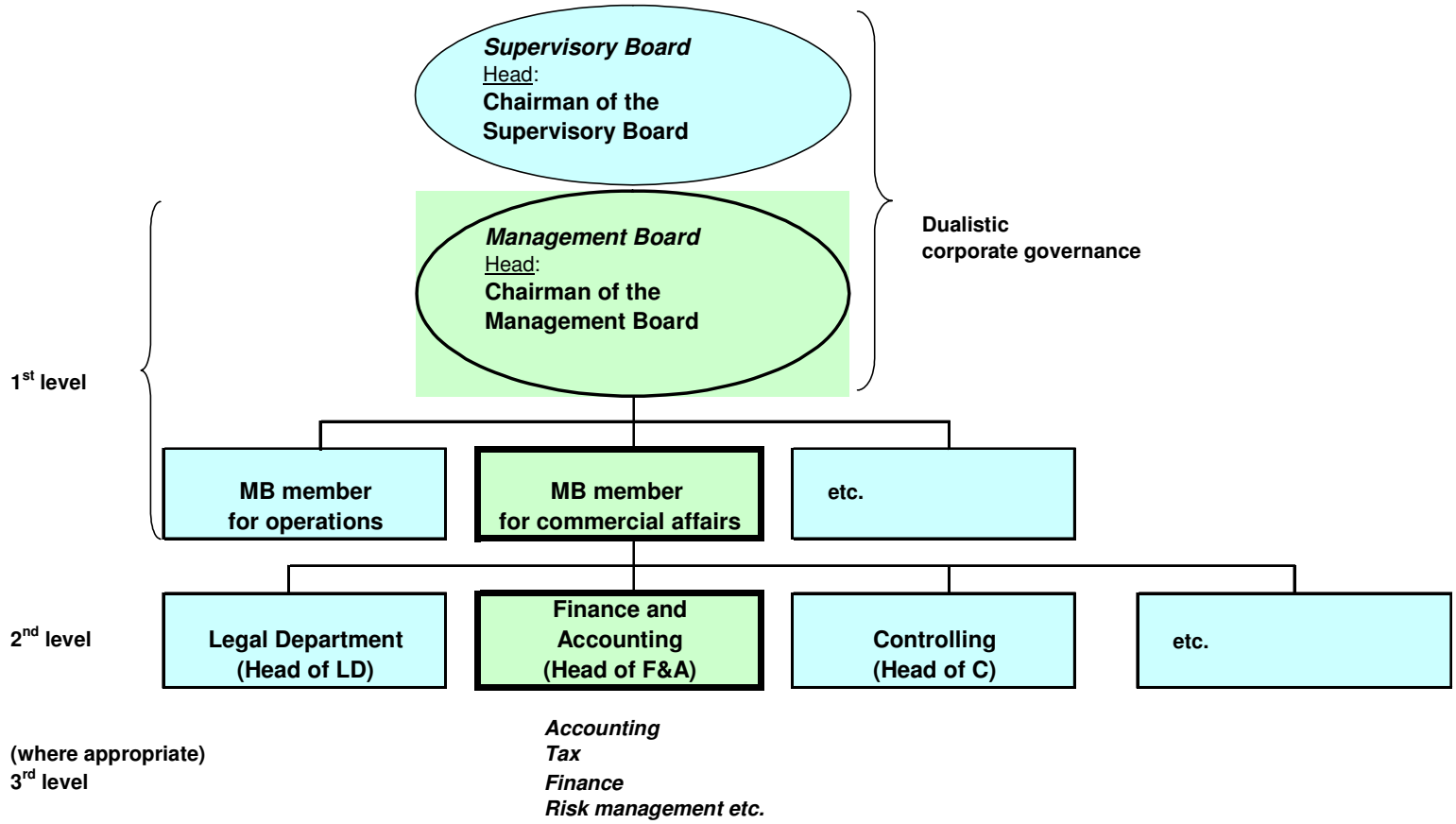
The POC-Structure of Treasury Management



Financial Organisational Structure: The US Model (here: Inc.)



Financial Organisational Structure: The European Model (here: AG)



Questions of Strategic Financial Planning

- ✓ How many main banking relationships?
  Which banks?
- ✓ Securitization and going public
- ✓ (Debt:) Fix-floating mix
- ✓ Target rating
- ✓ Three years budget
- ✓ Etc.

Exercise 13-1

The Inovatívny obchod a.s. from Brno is listed on the stock exchange and with 21,000 employees and 150 locations one of the leading department store companies in Europe. Within the management board, Mrs. Alice Babičková is in charge of the entire commercial affairs of the company. Among other organisational units, the treasury department directed by Mr. František Kohut belongs to her area of responsibility.

i) Is the corporate governance of the company monistic or dualistic? Does the organisation of the financial sphere of Inovatívny obchod follow the European or the US-Model?

The accumulated lines of credit of the company amount to CZK 7,000,000. As of December 31, 2025, the company reports the subsequent data (consolidated balance sheet, consolidated income statement, consolidated cash flow statement).

Consolidated balance sheet, Inovatívny obchod a.s., Brno, as of December 31, 2025									
Assets	**25, CZK mio.**	**%**	**24, CZK mio.**	**%**	**Liabilities**	**25, CZK mio.**	**%**	**24, CZK mio.**	**%**
Fixed assets					**Equity**				
Intangible assets	*20,0*	*1,4%*	*18,0*	*1,3%*	*Capital stock*	*240,0*	*16,9%*	*240,0*	*17,2%*
Tangible assets	*315,0*	*22,2%*	*300,0*	*21,5%*	*Additional paid-in capital*	*20,3*	*1,4%*	*20,3*	*1,5%*
Financial assets	*125,0*	*8,8%*	*115,0*	*8,2%*	*Reserves from retained earnings*	*103,3*	*7,3%*	*97,3*	*7,0%*
	460,0	**32,4%**	**433,0**	**31,0%**	*Net profit*	*59,1*	*4,2%*	*86,0*	*6,2%*
						422,7	**29,8%**	**443,6**	**31,8%**
Current assets									
Inventories	810,3	*57,1%*	805,2	*57,7%*	**Provisions**				
Accounts receivable	*10,9*	*0,8%*	*12,3*	*0,9%*	*Provisions for pensions & similar commitments*	*198,5*	*14,0%*	*154,2*	*11,1%*
Other receivables	*62,4*	*4,4%*	*69,5*	*5,0%*	*Other provisions*	*125,9*	*8,9%*	*55,2*	*4,0%*
Cash, cash equivalents etc.	*70,8*	*5,0%*	*70,8*	*5,1%*		**324,4**	**22,8%**	**209,4**	**15,0%**
1.Cash, cheques	*4,1*	0,3%	*6,2*	0,4%	**Liabilities**				
2. Bank deposits	*61,4*	4,3%	*59,8*	4,3%	*Financial liabilities*	*257,0*	*18,1%*	*261,2*	*18,7%*
3. Securities	*5,3*	0,4%	*4,8*	0,3%	*Accounts payable*	*256,3*	*18,0%*	*310,1*	*22,2%*
	954,4	**67,2%**	**957,8**	**68,7%**	*Other liabilities*	*155,6*	*11,0%*	*165,8*	*11,9%*
						668,9	**47,1%**	**737,1**	**52,8%**
Deferred tax assets	**2,2**	**0,2%**	**1,8**	**0,1%**	**Deferred tax liabilities**	**1,3**	**0,1%**	**1,9**	**0,1%**
Prepaid expenses & deferred charges	**3,4**	***0,2%***	**2,6**	***0,2%***	**Prepayments & deferred income**	**2,7**	**0,2%**	**3,2**	**0,2%**
	1420,0	**100,0%**	**1395,2**	**100,0%**		**1420,0**	**100,0%**	**1395,2**	**100,0%**

Consolidated cash flow statement, Inovatívny obchod a.s., Brno, 2025, CZK mio.		
	2025	**2024**
EBIT	153,4	221,7
Depreciation and amortization of tangible and intangible assets	89,6	84,3
Change in provisions for pensions and other provisions	50,0	45,0
Chenge in net working capital	-52,4	-60,3
Income taxes paid	10,3	40,4
Other	-20,3	-60,3
Cash flow from operating activities	**230,6**	**270,8**
Investments in tangible assets (excl. Finance leases)	-100,0	-105,9
Other investments	-16,2	-15,8
Company acquisitions & divestments	-10,2	-30,8
Cash flow from investing activities	**-126,4**	**-152,5**
Profit distributions		
- to parent company stockholders	-80,0	-80,0
- to other stockholders	-1,0	-1,0
Raising of financial liabilities	301,2	350,4
Redemption/repayment of financial liabilities	-281,6	-299,1
Interest paid	-70,5	-87,4
Interest received	25,1	24,3
Profit & loss transfers and other financing activities	2,1	-10,5
Cash flow from financing activities	**-104,7**	**-103,3**
Total cash flows	**-0,5**	**15,0**
Cash and cash equivalents as of January 1	66,0	51,0
Cash and cash equivalents as of December 31	65,5	66,0

Consolidated income statement, Inovatívny obchod a.s., Brno, CZK mio.										
	2028B	%	% change	2027B	%	% change	2026B	%	% change	2025A
Gross sales	**4813,0**	**115,1%**	**2,2%**	**4710,0**	**115,1%**	**2,4%**	**4600,0**	**115,1%**	**2,7%**	**4477,4**
Sales tax	633,0	15,1%	2,2%	619,5	15,1%	2,4%	605,0	15,1%	3,0%	587,4
Net sales	**4180,0**	**100,0%**	**2,2%**	**4090,5**	**100,0%**	**2,4%**	**3995,0**	**100,0%**	**2,7%**	**3890,0**
Cost of sales	2435,0	58,3%	2,3%	2380,0	58,2%	2,1%	2330,0	58,3%	2,2%	2279,5
Gross profit on sales	**1745,0**	**41,7%**	**2,0%**	**1710,5**	**41,8%**	**2,7%**	**1665,0**	**41,7%**	**3,4%**	**1610,5**
Selling expenses	1705,0	40,8%	2,1%	1670,0	40,8%	1,8%	1640,0	41,1%	2,6%	1598,8
General administrative expenses	114,0	2,7%	3,6%	110,0	2,7%	2,8%	107,0	2,7%	5,8%	101,1
Other operating income	298,0	7,1%	1,0%	295,0	7,2%	0,7%	293,0	7,3%	0,4%	291,8
Other operating expenses	35,0	0,8%	-22,2%	45,0	1,1%	12,5%	40,0	1,0%	2,8%	38,9
EBITA	**189,0**	**4,5%**	**4,7%**	**180,5**	**4,4%**	**5,6%**	**171,0**	**4,3%**	**4,7%**	**163,4**
Amortization good will	10,0	0,2%	0,0%	10,0	0,2%	0,0%	10,0	0,3%	0,0%	10,0
EBIT	**179,0**	**4,3%**	**5,0%**	**170,5**	**4,2%**	**5,9%**	**161,0**	**4,0%**	**5,0%**	**153,4**
Result from associated companies	-1,0	0,0%	50,0%	-1,5	0,0%	40,0%	-2,1	-0,1%	-5,0%	-2,0
Interest result	-49,0	-1,2%	-4,1%	-47,0	-1,1%	-4,3%	-45,0	-1,1%	0,0%	-45,0
Other financial result	1,3	0,0%	0,0%	1,3	0,0%	-7,1%	1,4	0,0%	0,0%	1,4
Financial profit	**-48,7**	**-1,2%**	**-3,1%**	**-47,2**	**-1,2%**	**-3,2%**	**-45,7**	**-1,1%**	**-0,2%**	**-45,6**
EBT	**130,3**	**3,1%**	**5,7%**	**123,3**	**3,0%**	**6,9%**	**115,3**	**2,9%**	**7,1%**	**107,7**
Income taxes	51,5	1,2%	5,7%	48,7	1,2%	6,9%	45,5	1,1%	6,9%	42,6
Net profit for the period	**78,8**	**1,9%**	**5,7%**	**74,6**	**1,8%**	**6,9%**	**69,8**	**1,7%**	**7,0%**	**65,2**
Allocable to minorities	9,1	0,2%	3,4%	8,8	0,2%	25,7%	7,0	0,2%	14,8%	6,1
Net profit	**69,7**	**1,7%**	**6,0%**	**65,8**	**1,6%**	**4,8%**	**62,8**	**1,6%**	**6,2%**	**59,1**

ii) Make yourself familiar with the data by elaborating the three immediately recognisable links between the three calculations!

As can be scrutinized here, Mr. Kohut's treasury department has made up the internal budgeting and reporting form similar to the cash flow statement according to IAS/IFRS:

Consolidated Cash Flow Statement (Budget), Inovatívny obchod a.s., Brno, CZK mio.			
	2028	2027	2026
Net sales			
Costs of goods sold			
Wages			
Overheads			
Other incoming payments from operations			
Other outgoing payments from operations			
Income taxes			
Cash flow operations			
Cash flow investment			
Profit distribution			
Increase in capital			
Debt finance			
Redemption			
Interest result			
Other			
Cash flow finance			
Total cash flows			
Cash & cash equivalents as of January 1			
Cash & cash equivalents as of December 31			

iii) Does the treasury department for internal purposes calculate the cash flow from operating activities in a direct or in an indirect manner?

For budgeting purposes 2026-2028, the treasury department deems the subsequent projections to be valid:

- ⇒ incoming payments from net sales will be like in the consolidated budget income statement
- ⇒ outgoing payments for cogs will be like in the consolidated budget income statement
- ⇒ outgoing payments for wages per capita will equal CZK 30,000 in 2026; from then onwards yearly increase of 2.3%
- ⇒ outgoing payments for overheads will amount to 49% of consolidated total assets in 2026; from then onwards yearly increase of 2.0%
- ⇒ other incoming payments from operations amount to 10% of other operating income in the consolidated budget income statement
- ⇒ other outgoing payments from operations amount to 60% of other operating expenses in the consolidated budget income statement
- ⇒ income taxes paid will equal the expenses for income taxes in the consolidated budget income statement
- ⇒ profit distribution remains on the 2025 level visible in the consolidated cash flow statement

⇒ cash flow investment: CZK 230 mio. in 2026, CZK 240 mio. in 2027, CZK 250 mio. in 2028

⇒ debt finance and redemption, respectively, will in 2026, 2027 and 2028 equal the amounts of the year 2025 visible in the consolidated cash flow statement rounded to CZK 10 mio.

⇒ interest results on the payment level will equal the interest results in the consolidated budget income statement

⇒ other financial items will equal the total of result from associated companies and other financial result in the consolidated budget income statement

iv) Make up a first draft of the strategic treasury budget of Inovatívny obchod by using the internal form and taking into consideration the aforementioned projections! Then comment on your result!

Series of Payment

Cash effectiveness

e

+

Timely difference

$t = 0,1,\ldots,\bar{t}$

=

Series of payment

$e_t \quad ; \quad t = 0,1,\ldots,\bar{t}$

Investment Projects

An investment project (sometimes called "investment" in short) is a series of payment

- **that begins with an outgoing payment,**
- **that features at least one change in sign**
- **and whose realisation still has to be decided on.**

Types:

- **Real investments**
- **Financial investments**

Exercise 5-1

The Jemná Čokoláda a.s. wants to acquire a machine for the production of chocolate bars. The price of the machine due for immediate payment in t=0 amounts to CZK 100,000. The machine would for three years allow the production of 5,000 chocolate bars per year. Each bar could be sold for instant incoming payment of CZK 20. On the other hand, ingredients (secret recipe!) and other production factors would require outgoing payments of CZK 10 per bar. At the end of the physical life of the machine, disassembly cost would induce outgoing payments of CZK 40,000.

Assume Jemná Čokoláda's point of view and determine the series of payment of the project "chocolate bar machine"!

Financing projects

A financing project is a series of payment

- **that begins with an incoming payment,**
- **that features at least one change in sign**
- **and whose realisation still has to be decided on.**

⇒ ***Many of the concepts developed subsequently for investment projects can be transferred mutatis mutandis to financing projects.***

Exercise 6-1

The financial contract known from exercise 4-1 is again taken into consideration.

From whose perspective is the signing of this contract a financing project, from Candice's or Quentin's?

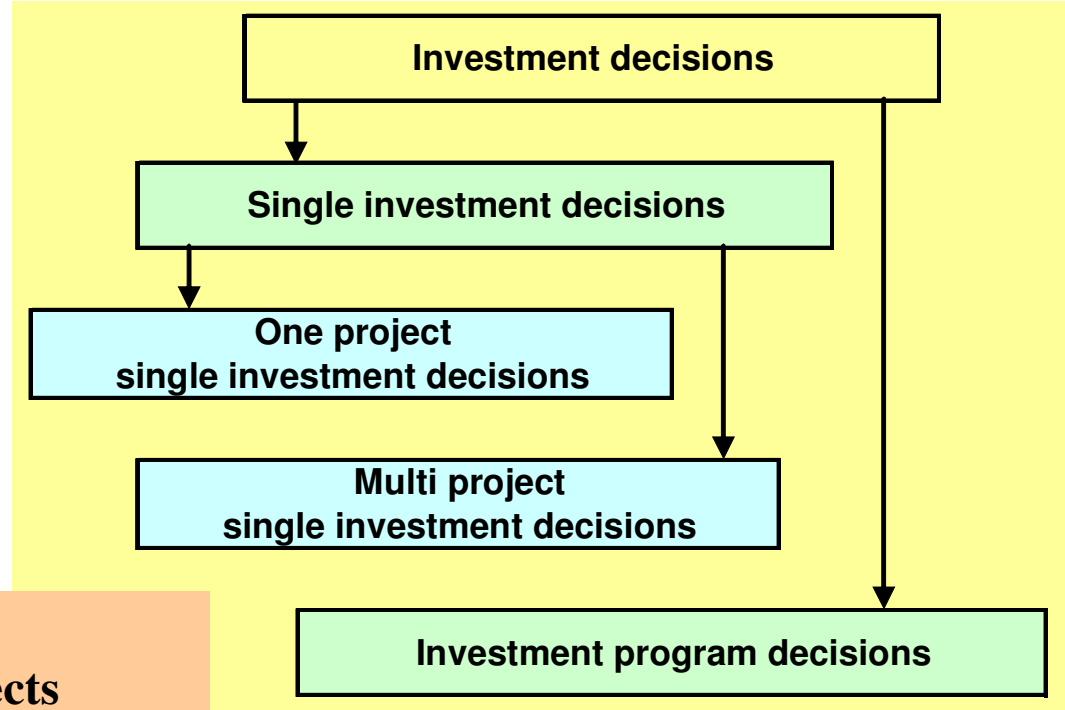
Exercise 5-2

The Jemná Čokoláda a.s. has entered the stage of a more intense scrutiny of the chocolate bar machine known from exercise 5-1. To this end, it is to be compared with the status quo.

Determine the series of payment of the status quo for this scenario!

Investment Decisions (Fig. 5-1)

An investment decision is a selection from a catalogue of investment projects and the status quo.



Typical of TREASURY MANAGEMENT:
Selection from a catalogue of investment projects
or
from a catalogue of financing projects,
but not
from a catalogue of investment *and* financing projects.

Investment calculus

Investment calculus offers procedures for investment decisions (i. e.: for the comparison of certain series of payment).

(Economic calculus is the more general concept and offers procedures for investment desisions *and* financing decisions.)

⇒ 6 D's

- Complete account
- Dominance
- Net present value
- Annual equivalent
- Internal rate of return
- Payback period

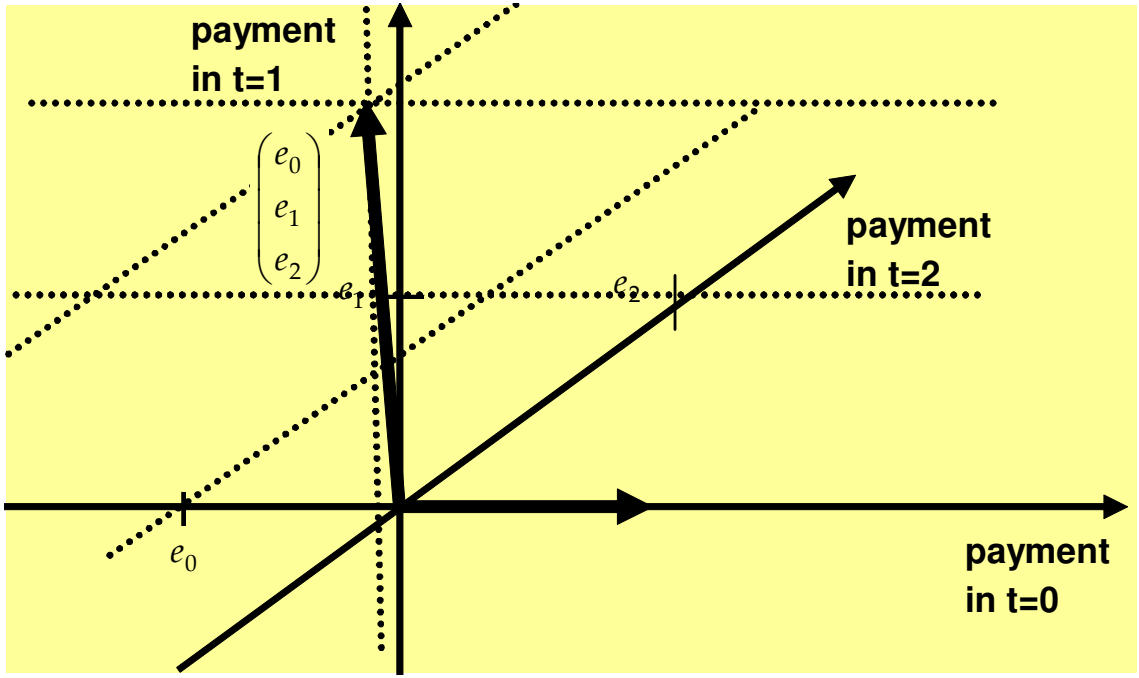
…Dynamic Investment Calculus…

DYNAMIC INVESTMENT CALCULUS
(1) starts off from incoming and outgoing payments (upmost level of Schmalenbach's bar graph) and
(2) takes the timely differences between different payments explicitly into consideration.

(In other words: Dynamic investment calculus is based on series of payment.)

If one of the two aforementioned criteria is not fulfilled, it is STATIC INVESTMENT CALCULUS.

Scalars and Vectors, Norm of a Vector



SCALAR:
onedimensional figure in vector space
VECTOR:
multidemsional figure in vector space

Norm of a vector ε:

$$\|\varepsilon\| = \sqrt{e_0^2 + e_1^2 + \ldots + e_{\bar{t}}^2}$$

Mathematically speaking, investment calculus requires the comparison of vectors, i. e. of multidemsionsal figures in the vector space.

Exercise 8-1

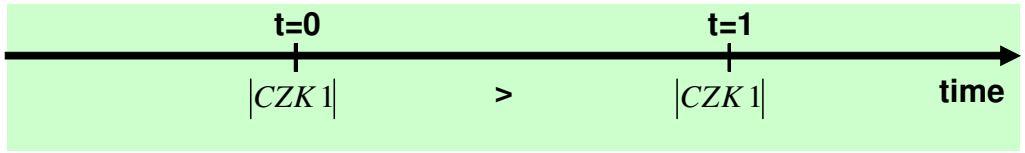
The chocolate bar machine known from exercises 5-1 and 5-2 is again taken into consideration.

Determine the norm of the series of payment of this investment project and interpret on your result from an economic point of view!

Our Catalogue of Assumptions for Investment Calculus

1. Payments do only occur at points in time like t=0 and t=1 and not inbetween (discrete time)
2. Payments relating to a period of time (like interest) are realised at the end of the relevant period and not at its beginning
   (payments in arrear, no payments in advance)
3. Deterministic payments and interest rates
   (no stochastic)
4. Timely invariant interest rates
   (homogenous term structure of interest rates)
5. No taxes

The Relevance of the Interest Rate



r>0

Usual scenario: $r_B > r_L$; borrowing rate exceeds lending rate (a market imperfection)

It is better to receive CZK 1 today than tomorrow.
It is worse to be obliged to pay CZK 1 today than tomorrow.

After transformation to a fixed point in time by means of the interest rate, the absolute value of CZK 1 paid in the present is greater than the absolute value of CZK 1 paid in the future.

Complete Account of an Investment Project (preliminary definition)

Explicit clearing of a series of payment by means of a simple account complying with the following conventions:

i. Except for the final point in time, the account may never be overdrafted. To this end, possible deficits have to covered by a separate credit at borrowing rate r_B.

(Please turn over.)

ii. Except for the final point in time, the account may never display a positive balance. To this end, possible surpluses have to be invested in a term money at lending rate r_L.

iii. Credits and term moneys each have a duration of one period.

iv. At the final point in time, no credits may be raised and no term money investments are possible anymore. The final balance of the account is represented by the symbol FW_P, i. e. the final wealth generated by the project in consideration

v. For the time being, the initial wealth of the holder of the account (or the decision taking entity, respectively) equals zero: $IW_S = 0$

Exercise 8-2

The chocolate bar machine known from exercises 5-1, 5-2 and 8-1 is again taken into consideration. The borrowing rate is $r_B = 0.05$ per period, the lending rate $r_L = 0.01$.

Determine by means of a complete account complying with the aforementioned conventions i. to v. the final wealth FW_P of the project (and thus the increase in final wealth compared to the status quo!)

The Objective of Investment Calculus

(Or more general: The Objective of Economic Action in a Dynamic Context)

Final Wealth Maximization

Complete Account: Decision Rule

a) For **ONE PROJECT SINGLE INVESTMENT DECISIONS**, the project taken into consideration is favourable *if it generates an increase in final wealth compared to the status quo*.

b) For **MULTI PROJECT SINGLE INVESTMENT DECISIONS**, the project *that maximizes the increase in final wealth compared to the status quo* is favourable. (If none of the projects taken into consideration generates an increase, at all, the status quo is favourable.)

Equivalent Initial Wealth of a Project

Fictitious back-calculation of the final wealth caused by a project that complies with the following two conventions:

i. After the point in time of decision ($t = 0$), for $\bar{t} \geq 1$ no further payments occur in $t = 1,...,\bar{t}$. This means that in $t = 0$ there is one single investment of a term money or one single raising of credit, respectively.

ii. If $FW_P > 0$, the equivalent initial wealth IW_P can only be generated by means of an investment in a term money in $t = 0$; r_L is the relevant interest rate then. For $FW_P < 0$, the equivalent initial wealth IW_P can only go back to an initial credit raising; in this case, r_B is the relevant interest rate.

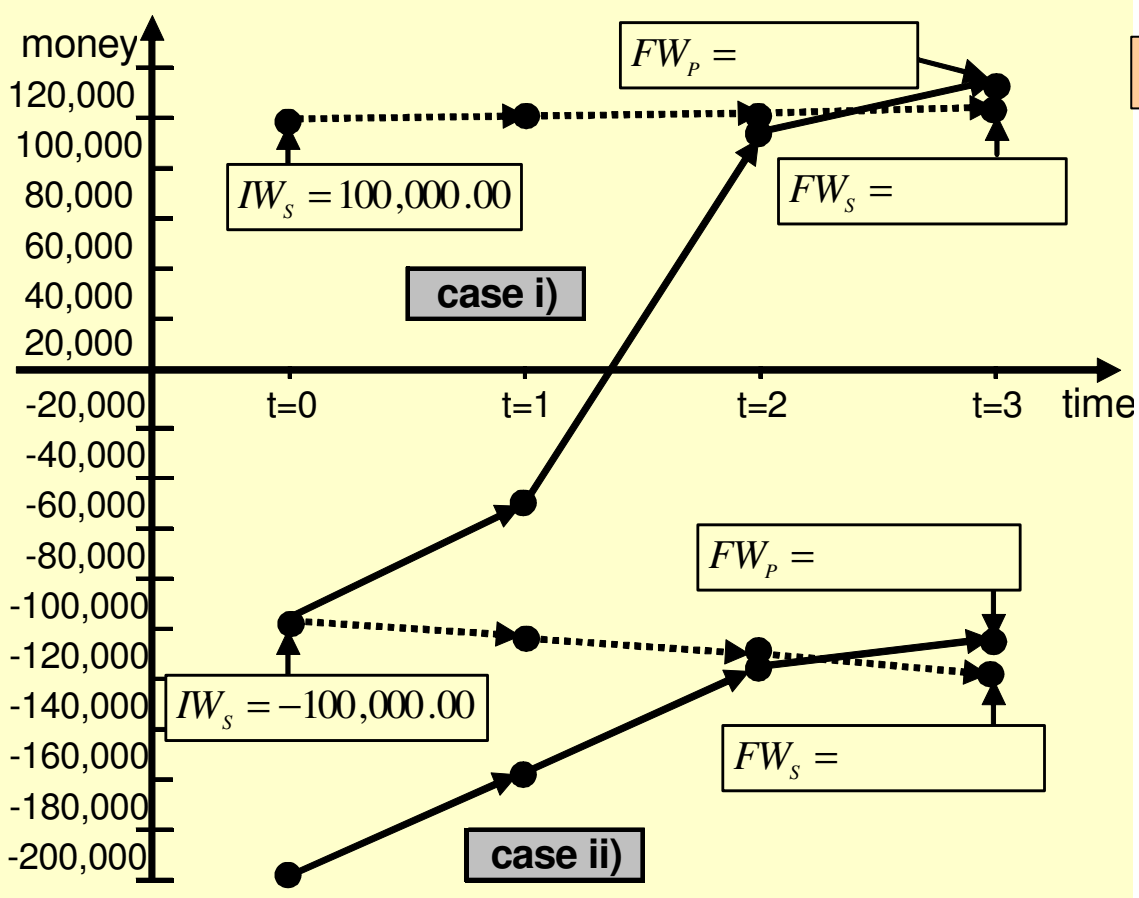
Exercise 8-4a

The chocolate bar machine known from exercises 5-1, 5-2, 8-1 and 8-2 is again taken into consideration. Like before, the borrowing rate is $r_B = 0.05$ per period, the lending rate $r_L = 0.01$.

Determine by means of a complete account following the aforementioned conditions i. and ii. the initial wealth IW_p that is equivalent with the final wealth FW_P calculated in exercise 8-2!

Non-Zero Initial Wealth

(Fig. 8-3)



In general, the increase in final wealth generated by an investment project is not independent from the level of the initial wealth of the decision taking entity.

Complete Account of an Investment Project (final definition)

Explicit clearing of a series of payment by means of a simple account complying with the following conventions:

i. Except for the final point in time, the account may never be overdrafted. To this end, possible deficits have to covered by a separate credit at borrowing rate r_B.

ii. Except for the final point in time, the account may never display a positive balance either. To this end,

possible surpluses have to be invested in a term money at lending rate r_L.

iii. Credits and term moneys each have a duration of one period.

iv. At the final point in time, no credits may be raised and no term money investments are possible anymore. The final balance of the account is represented by the symbol FW_P, i. e. the final wealth generated by the project in consideration

Compared to the preliminary definition of the complete account, the decision rule remains unchanged. (See separate slide.)

Exercise 8-5

The chocolate bar machine known from exercises 5-1, 5-2, 8-1, 8-2 and 8-4a is again taken in consideration. In contrast to exercise 8-2, the initial wealth of the decision-taking entity IW_S is different from zero and amounts to i) CZK 100,000 and ii) CZK -100,000.

Determine the increase in final wealth caused by the machine in case i) and in case ii), respectively, transfer your results to figure 8-3 and comment on them!

Jakob Bernoulli, 1655-1705
(with a view to stochastics)

FACHHOCHSCHULE BOCHUM
UNIVERSITY OF APPLIED SCIENCES

Dominance: Definition and Decision Rule

Let ε and ε' be non-identical series of payments (or, speaking differently, vectors of dimension $\bar{t}+1$).
ε DOMINATES ε' if and only if in pairwise comparison every element e_t of ε is greater or equal the corresponding element e'_t of ε'. Formally:

$$(D) \quad e_t \geq e'_t \qquad \forall \; t = 0,1,\ldots,\bar{t}$$

If a series of payments DOMINATES another one, it is also PREFERABLE to the other one in the sense of final value maximization.

Exercise 8-6

In addition to the well established brands hazelnut and brittle, the Jemná Čokoláda a.s. is currently having marzipan chocolate in contemplation. An investment in the marzipan project would have the following consequences (all data in CZK and as change compared to the status quo):

t=0

(1) Acquisitions amounting to 10,000,000 (immediately payment effective)

(2) Employee Lissi, who has always dreamt of marzipan chocolate when she was working at the conveyor belt and proposed the idea

to the staff suggestion scheme, is to receive an immediately payment effective premium amounting to 10,000.

t=1, 2,… , 10 (figures per year)

(3) Incoming payments caused by increased sale of chocolate amounting to 3,000,000

(4) Additional outgoing payments for (a) crude chocolate amounting to 1,000,000, (b) marzipan amounting to 300,000, (c) wages amounting to 200,000

(5) Lissi would be much more content with her work.

(6) Writeoffs of the new machinery in the financial accounting amounting to 600,000

(7) Writeoffs of the new machinery and the good will in the management accounting amounting to 400,000

(8) The supplier Brněnské Marcipánové a Nugátové Kontor would encounter a payment effective surplus amounting to 260,000.

i) *Determine the series of payment of project marzipan!*

Being deeply impressed by Lissi's idea, the marketing department of the Jemná Čokoláda a.s. has instantaneously calculated the projects nougat, walnut, strawberry yoghurt, raisin nut and peanut. The following series' of payment are presented to the board of directors:

brand	t=0	t=1, 2, …, 10 (per year)
Nougat	-10,010,000	1,400,000
Walnut	-10,010,000	1,300,000
Strawberry yoghurt	-10,300,000	1,500,000
Raisin nut	-5,000,000	600,000
Peanut	-2,000,000	300,000

ii) ***Preselect efficiently those chocolate brands that are under no circumstances consistent with the objective of final wealth maximization!***

Implicit Consideration of Accompanying Financial Activities

Discounting

$$g_0 = g_{\bar{t}} \cdot (1+r_L)^{-\bar{t}} \qquad if \qquad g_{\bar{t}} > 0$$

$$g_0 = g_{\bar{t}} \cdot (1+r_B)^{-\bar{t}} \qquad if \qquad g_{\bar{t}} < 0$$

Compounding

$$g_{\bar{t}} = g_0 \cdot (1+r_L)^{\bar{t}} \qquad if \qquad g_{\bar{t}} > 0$$

$$g_{\bar{t}} = g_0 \cdot (1+r_B)^{\bar{t}} \qquad if \qquad g_{\bar{t}} < 0$$

Exercise 8-7

The initial wealth of the decision taking entity be zero, i. e.: $IV_S = 0$**. Consider an investment project with the following series of payment:**

$$(-980.00,\ 1{,}100.00,\ -10.00)$$

As it is shown in the draft (margin number 161, table 8-5), the final wealth of the project for $r_L = 0.01$ **and** $r_B = 0.05$ **per period is 71.81, i. e.** $FV_P = 71.81$**.**

Check whether it is possible to calculate the final wealth of the project by means of compounding! To this end, apply (a) the lending rate, (b) the borrowing rate and (c) a more subtle strategy!

Deviation Analysis for Exercise 8-7 (and thus for More Complex Series of Payment)

Final wealth of the project

$$FV_P = [e_o \cdot (1+r_B) + e_1] \cdot (1+r_L) + e_2$$
$$= e_0 \cdot (1+r_B) \cdot (1+r_L) + e_1 \cdot (1+r_L) + e_2$$

What interest rate is relevant? The answer to this question in general requires a complete account!!!

Equivalent initial wealth of the project

$$IW_P = \frac{FW_P}{(1+r_L)^2} = \frac{e_0 \cdot (1+r_B)}{1+r_L} + \frac{e_1}{1+r_L} + \frac{e_2}{(1+r_L)^2}$$

Exercise 8-8

The investment project known from exercise 8-7 is again taken into consideration. The initial wealth of the decision taking entity remains at zero (i. e.: $IW_S = 0$), lending rate and borrowing rate remain at $r_L = 0.01$ and $r_B = 0.05$ per period, respectively.

Check whether it is basically possible to calculate the final wealth and the equivalent initial wealth of the project implicitly on the basis of the preceeding deviation analysis!

Perfect Financial Markets (PERFIMA)

Perfect financial markets comply in particular with the following three criteria:

i. There are no quantitative restrictions on accompanying financial activities (ABSENCE OF RATIONING).

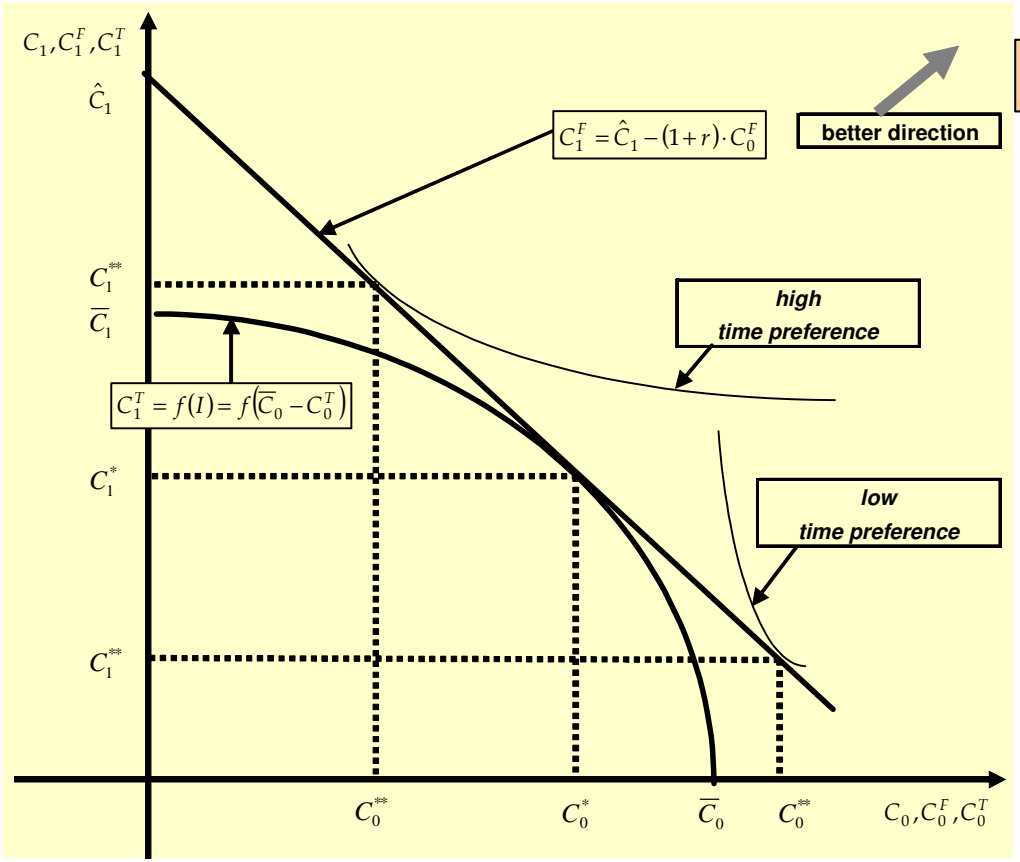
ii. The same interest rates are relevant for the raising of credit and the investment in term money (BORROWING RATE EQUALS LENDING RATE).

iii. The one-period investment in term money as well as the one-period raising of credit are possible. (FINEST TIMELY SCALING OF FINANCIAL CONTRACTS).

PERFIMA

⇒ Fisher-Separation

(Fig 8-4)



Irving Fisher, 1867-1947

On perfect financial markets, the real investment decision ("firms") is independent of the time preference and the intertemporal consumption decision ("households"). The opposite does not hold true.

PERFIMA ⇒ Implicit Calculus Even of More Complex Series of Payment

Final wealth

$$FV_P = e_0 \cdot (1+r_B) \cdot (1+r_L) + e_1 \cdot (1+r_L) + e_2$$

$$\overset{r_B = r_l = r}{=} e_0 \cdot (1+r)^2 + e_1 \cdot (1+r) + e_2$$

$$= \sum_{t=1}^{\bar{t}} e_t \cdot (1+r)^{\bar{t}-t} \equiv FW$$

Net present value

$$IW_P = \frac{e_0 \cdot (1+r_B)}{1+r_L} + \frac{e_1}{1+r_L} + \frac{e_2}{(1+r_L)^2}$$

$$\overset{r_B = r_L = r}{=} e_0 + e_1 \cdot (1+r)^{-1} + e_2 \cdot (1+r)^{-2} = \sum_{t=0}^{\bar{t}} e_t \cdot (1+r)^{-t} \equiv NPV$$

Net Present Value and Final Wealth Maximization

Final wealth maximization: The difference between the final wealth of the project and the final wealth in the status quo case has to be maximized. For arbitrary initial wealth of the decision taking unit IW_S, the final wealth of the project FW_P may inspite of the aforementioned problems be formulated in the following manner:

$$FW_P = \sum_{t=0}^{\bar{t}} \hat{e}_t \cdot \prod_{\tau=0}^{\bar{t}-t} \left(1 + r_{B,L}(\tau)\right)$$

where

$$\hat{e}_t = \begin{cases} e_0 + IW_S & for \;\; t = 0 \\ e_t & for \;\; t = 1,2,\ldots,\bar{t} \end{cases}$$

Continuing on this approach, the final wealth in the status quo case is given by:

$$FW_S = IW_S \cdot \prod_{t=0}^{\bar{t}} \left(1 + r_{B,L}(t)\right)$$

As we have reduced the timely accrual of the initial wealth in the status quo case on t=0, this more subtle terminology is apparently redundant in the last equation. (In this case, the borrowing rate *or* the lending rate has to be applied, but not a combination of them. On the other hand, the terminology allows to prove the result even for whole timely sequences of initial wealth.)

Now: PERFIMA

$$FW_P = \sum_{t=0}^{\bar{t}} \hat{e}_t \cdot \prod_{\tau=0}^{\bar{t}-t} \left(1 + r_{B,L}(\tau)\right)$$

$$\overset{r_B = r_L = r}{=} \sum_{t=0}^{\bar{t}} \hat{e}_t \cdot (1+r)^{\bar{t}-t}$$

As this expression is more simple now, a back-transformation from $\hat{e}_t$ to e_t is possible for the final wealth of the project:

$$FW_P = IW_S \cdot (1+r)^{\bar{t}} + \sum_{t=0}^{\bar{t}} e_t \cdot (1+r)^{\bar{t}-t}$$

Accordingly, the final wealth in the case of the status quo may for PERFIMA be formulated like follows:

$$FW_S = IW_S \cdot \prod_{t=0}^{\bar{t}} \left(1 + r_{B,L}(t)\right)$$

$$\overset{r_B = r_L = r}{=} IW_S \cdot (1+r)^{\bar{t}}$$

Now let us take the difference of both expressions:

$$FW_P - FW_S = IW_S \cdot (1+r)^{\bar{t}} + \sum_{t=0}^{\bar{t}} e_t \cdot (1+r)^{\bar{t}-t} - IW_S \cdot (1+r)^{\bar{t}}$$

$$= \sum_{t=0}^{\bar{t}} e_t \cdot (1+r)^{\bar{t}-t} = FV$$

The resulting expression was already defined as the final value FV of a project. If we now multiply by $(1+r)^{-\bar{t}}$, we obtain the following expression which is exactly the net present value of a project:

$$NPV = \sum_{t=0}^{\bar{t}} e_t \cdot (1+r)^{-t} = FV \cdot (1+r)^{-\bar{t}} = (FW_P - FW_S) \cdot (1+r)^{-\bar{t}}$$

In other words:

(i) The PERFIMA-assumption is crucial for the NPV-concept.

(ii) The NPV-concept is in full accordance with the objective of final wealth maximization.

(iii) The NPV is completely independent of the initial wealth of the decision-taking entity in the status quo case.

Exercise 9-1

The initial wealth in the case of the status quo be zero, i. e.: $IW_s = 0$. **The chocolate bar machine known from exercises 5-1, 8-1 and 8-2 is again taken into consideration. The financial market has become perfect now and the interest rates for borrowing and lending both equal 4% now, i. e.:** $r = 0.04$.

i) Calculate the final value as well as the net present value of the project!

ii) Now check the preceeding statement as to which the net present value equals the increase in final wealth discounted to the present!

Explicit Definition of the Net Present Value and Corresponding Decision Rule

Simon Stevin, 1548/49-1620 (the concept of discounting)

Gottfried Wilhelm Leibniz, 1646-1716 (dto.)

The net present value of a series of payment $e_0, e_1, ..., e_{\bar{t}}$ equals the sum of all its discounted future payments and the initial payment, i. e.:

$$(NPV1) \quad K = \sum_{t=0}^{\bar{t}} e_t \cdot (1+r)^{-t} = \sum_{t=0}^{\bar{t}} e_t \cdot q^{-t}$$

a) For one project single investment decisions, the project considered is PREFERABLE if and only if it has a POSITIVE NET PRESENT VALUE.

b) For multi project single investment decisions, the project featuring the MAXIMUM NET PRESENT VALUE is PREFERABLE if the latter one is POSITIVE. (If no project features a positive net present value, at all, the status quo is preferable.)

Special Computational Methods for the Net Present Value

Annuity

$$(NPV2) \qquad K^{Annuity} = e_0 + e \cdot \frac{1 - q^{-\bar{t}}}{q - 1} = e_0 + e \cdot Q(\bar{t}, r)$$

(Q: annuity factor)

Perpetuity

$$(NPV3) \qquad K^{Perpetuity} = e_0 + \frac{e}{r}$$

($1/r$: multiplier)

Exercise 9-2

Determine efficiently the net present values of those new chocolate brands kown from exercise 8-6 that cannot be eliminated by means of the dominance criterion! To this end, start off from an interest rate amounting to 5% per period, i. e.: $r = 0.05$***!***

Annual Equivalent: Definition and Decision Rule

The ANNUAL EQUIVALENT $\bar{e}$ of a project featuring the series of payment $e_0, e_1, ..., e_{\bar{t}}$ is a constant payment occurring in $t = 1,2,...,\bar{t}$ the net present value of which equals the net present value of the project:

$$(AE1) \qquad \bar{e} = \frac{1}{Q(r,\bar{t})} \cdot K(r)$$

a) For one project single investment decisions, the project considered is PREFERABLE if it has a POSITIVE ANNUAL EQUIVALENT.

b) For multi project single investment decisions, the annual equivalent criterion is ON PRINCIPLE NOT APPLICABLE (inconsistent with the objective of final wealth maximization)

Exercise 9-4

Determine the annual equivalent of the chocolate bar machine by going back to the results of exercises 5-1 and 9-1 and starting off from an unchanged interest rate amounting to 4% per perod, i. e.: $r = 0.04$***!***

Special Computational Methods for the Annual Equivalent

Annuity

$$(AE2) \qquad \bar{e}^{\,Annuity} = e + \frac{e_0}{Q(r,\bar{t})}$$

($1/Q$: reciprocal of the annuity factor)

Perpetuity

$$(AE3) \qquad \bar{e}^{\,Perpetuity} = e + r \cdot e_0$$

Exercise 9-5

Interest rate per period: 5%, i. e.: $r = 0.05$

i) Determine efficiently the annual equivalent of project marzipan (which is known from exercises 8-6 and 9-2)!

ii) Would the annual equivalent criterion in the context of exercise 8-6 be apt to select the optimal chocolate brand?

iii) Now assume project marzipan had an infinite duration and determine once more its annual equivalent!

Exercise 9-6

Starting point is the chocolate bar machine known from exercise 5-1.

i) Make up a table of values for the project by calculating the net present values for the following interest rates: a) 0%; b) 2%; c) 4%; d) 6%; e) 8%; f) 10%; g) 12%; h) 15%; i) 20%! Round up or down to even CZK amounts!

ii) Draw the net present value function of the project for positive interest rates!

The Net Present Value Function

Face value

$$K(r=0\%) \quad = \quad \sum_{t=0}^{\bar{t}} e_t \cdot (1+0)^{-t} \quad = \quad \sum_{t=0}^{\bar{t}} e_t$$

(the sum of all the elements of the series of payment)

Asymptotic behavior

$$\lim_{r\to\infty} K(r) \quad = \quad \lim_{r\to\infty} \sum_{t=0}^{\bar{t}} e_t \cdot (1+r)^{-t} \quad = \lim_{r\to\infty} \left[e_0 + \sum_{t=1}^{\bar{t}} e_t \right] \quad = \quad e_0$$

(converges asymptotically against the initial payment)

Slope and curvature

$$\frac{\delta K}{\delta r} < 0 \quad ; \quad \frac{\delta^2 K}{\delta r^2} > 0$$

(strictly monotonously decreasing and strictly convex for standard investment projects)

Standard Investment Projects

STANDARD PROJECTS are projects the series of payments of which features exactly one change of sign (be they standard financing projects or standard investment projects).

The series of payment of a STANDARD INVESTMENT PROJECT begins with an outgoing payment $e_0 > 0$ which is then followed by incoming payments only, i. e.: $e_t \geq 0 \ \forall \ t = 1, 2, ..., \bar{t}$, where at least one of these payments is strictly positive, i. e.: $\exists\, t : \ e_t > 0$.

Exercise 9-8

Start off from the following series of payment (all elements in CZK):

$$(-100{,}000,\ 158{,}900,\ 20{,}000,\ -80{,}010)$$

Make up a table of values for the project by calculating the net present values for the following interest rates: a) 0%; b) 2%; c) 4%; d) 6%; e) 8%; f) 10%; g) 12%; h) 15%; i) 20% and rounding to even CZK amounts and then draw the net present value function of the project for positive interest rates!

Eugen von Böhm-Bawerk, 1851 (Brno) – 1914 (Vienna)

The Internal Rate of Return: Definition and Decision Rule

The INTERNAL RATE OF RETURN r^* of a project featuring the series of payment $e_0, e_1, ..., e_{\bar{t}}$ is given by the interest rate that makes the net present value equal to 0, i. e.:

$$(IRR1) \qquad K(r^*) = \sum_{t=0}^{\bar{t}} e_t \cdot (1+r^*)^{-t} = 0$$

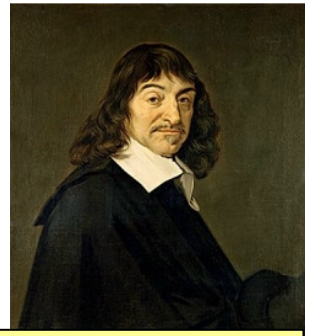
a) For one project single investment decisions, a standard investment project is PREFERABLE if ITS INTERNAL RATE OF RETURN EXCEEDS THE MARKET RATE ($r^* > r$). If the internal rate of return is lower than the market rate ($r^* < r$), it is disadvantageous compared to the status quo. For non-standard investment projects, the IRR-criterion is on principle NOT APPLICABLE.

b) For multi project single investment decisions, the IRR-criterion is on principle not applicable.

The Internal Rate of Return for Standard Investment Projects (SIP's)

Descartes' Rule of Signs
(applied to investment calculus)**:**

The NUMBER OF INTERNAL RATES OF RETURN of a project is either equal to the number of sign changes of its series of payment or less than it by a multiple of 2.



René Descartes, 1596-1650

Conclusion: A standard investment project with a strictly positive face value has exactly one internal rate of return.

Special Computational Methods for the Internal Rate of Return

SIP, series of payment with only 2 elements

$$(IRR2) \qquad r^* = \sqrt[\bar{t}]{-\frac{e_{\bar{t}}}{e_0}} - 1$$

SIP, finite annuity part immediately after initial outgoing payment

$$(IRR3) \qquad Q(r^*, \bar{t}) = -\frac{e_0}{e} \qquad \Rightarrow \qquad Table\ \ III$$

SIP, perpetuity immediately after initial outgoing payment

$$(IRR4) \qquad r^* = -\frac{e}{e_0}$$

SIP, outgoing payment, then fixed interest, then redemption in grand total

$$(IRR5a) \qquad r* \quad = \quad \frac{i + \frac{z-a}{\bar{t}}}{a}$$

i: interest rate; z: pay back rate; a: payout rate

SIP, outgoing payment, then fixed interest and instalment redemption

$$(IRR5b) \qquad r* \quad = \quad \frac{i + \frac{z-a}{T}}{a} \quad ; \qquad T \quad \equiv \quad f + \frac{\hat{t}+1}{2} \; ; \quad \hat{t} \quad = \bar{t} - f$$

f: years free of redemption; $\hat{t}$: years with redemption; T: "medium" term

Base formula for regula falsi

$$(IRR8) \qquad \hat{r}_{(1)} \quad = \quad \frac{r_L \cdot K(r_R) - r_R \cdot K(r_L)}{K(r_R) - K(r_L)}$$

L: *left of the zero*: R: *right of the zero*

Regula Falsi-Algorithm

i. Find interest rates r_L and r_R that comply with the conditions $(a)\ r_L < r_R$ (as close as possible) and $(b)\ K(r_L) \cdot K(r_R) < 0$.

ii. Determine $\hat{r}_{(1)}$ by means of formula $(IRR8)$.

iii. If $K(\hat{r}_{(1)}) = 0$, the procedure ends. Otherwise substitute r_L or r_R, respectively, by $\hat{r}_{(1)}$ so that again $(b)\ K(r_L) \cdot K(r_R) < 0$ is valid.

iv. Go back to steps ii. and iii. and apply the rules mutatis mutandis to determine $\hat{r}_{(2)}$ ($\hat{r}_{(3)}$,$\hat{r}_{(4)}$ and so on).

Exercise 9-9

(All payments in CZK.) Consider an investment project whose series of payment consists of an outgoing payment in t=0 amounting to -10,000.00 and an incoming payment in the amount of 11,576.25 in t=3.

i) Determine the internal rate of return of the project!

ii) Which payment in t=3 would instead result in an internal rate of return of 6%?

Exercise 9-10

Determine efficiently the internal rates of the return of the different chocolate brands known from exercise 8-6!

Exercise 9-11

Approximate the internal rate of return of project marzipan known from exercise 8-6 by assuming its annuity payment would cover an infinite time horizon and compare your result with the one from exercise 9-10!

Exercise 9-12

A fixed income credit contract that is redeemed in grand total after a maturity of $\bar{t} = 2$ years is considered. The interest rate is $i = 0.05$ per period, the payout rate $a = 0.95$ and the payback rate $z = 1.05$.

Approximate the internal rate of return of this financial contract!

Exercise 9-13

A fixed income credit contract with a face value amounting to CZK 100,000 that is paid out in the amount of CZK 94,714.62 is taken into consideration. The interest per year amounts to CZK 5,000. Repayment will be at face value.

Determine the internal rate of return of this financial contract for the following repayment patterns: (a) redemption in grand total, (b) instalment redemption (no free years), (c) annuity redemption and (d) zerobond! Then comment on your result!

Exercise 9-14

The chocolate bar machine known from exercises 9-6 and 5-1 is again taken into consideration.

i) ***Make a first linear estimate*** $\hat{r}_{(1)}$ ***for its internal rate of return by implementing the regula falsi-algorithm and choosing (intentionally in a suboptimal manner)*** $r_L = 0.04$ ***and*** $r_R = 0.08$***!***

ii) ***Now round*** $\hat{r}_{(1)}$ ***from part i) to entire percent and make a second linear estimate*** $\hat{r}_{(2)}$***!***

Payback period: Definition and Decision Rule

The PAYBACK PERIOD $[0, t^*]$ of a project featuring the series of payment $e_0, e_1, ..., e_{\bar{t}}$ is given by the point in time $t^* < \bar{t}$ at which its net present value becomes positive for the first time, i. e.:

$$(PBP1) \qquad \sum_{t=0}^{t^*-1} e_t \cdot (1+r)^{-t} < 0 \leq \sum_{t=0}^{t^*} e_t \cdot (1+r)^{-t}$$

a) **For one project single investment decisions, a standard investment project is PREFERABLE if it has a payback period. For non-standard investment projects, the PBP-criterion is on principle not applicable.**

b) **For multi project single investment decisions, the PBP-criterion is on principle not applicable.**

A Special Computational Method for the Payback Period

Annuity

$$(PBP2) \qquad Q(t^*-1, r) \; < \; -\frac{e_0}{e} \; \leq \; Q(t^*, r)$$

$$\Rightarrow \qquad table \;\; III$$

Exercise 9-15

Consider an investment project that is characterized by the subsequent series of payment (all figures in CZK):

$$e_0 = -500, \quad e_1 = 200, \quad e_2 = 100, \quad e_3 = 60$$

i) ***Start off from a market rate amounting to*** $r = 0.05$ ***and determine the payback period of the project!***

ii) ***Determine efficiently the payback period of project marzipan known from exercises 8-6, 9-2 and 9-5 for*** $r = 0.05$***!***