In this supplement we review basic concepts and tools of financial analysis for OM. These include the types of cost (fixed, variable, sunk, opportunity, avoidable), risk and expected value, and depreciation (straight line, sum-of-the-years'-digits, declining balance, double-declining balance, and depreciation-by-use). We also discuss activity-based costing and cost-of-capital calculations. Our focus is on capital investment decisions.

CONCEPTS AND DEFINITIONS

We begin with some basic definitions.

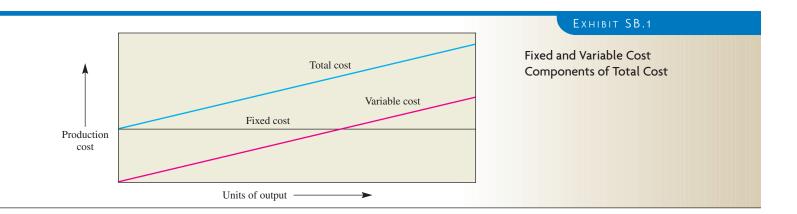
Fixed Costs A *fixed cost* is any expense that remains constant regardless of the level of output. Although no cost is truly fixed, many types of expense are virtually fixed over a wide range of output. Examples are rent, property taxes, most types of depreciation, insurance payments, and salaries of top management.

Variable Costs Variable costs are expenses that fluctuate directly with changes in the level of output. For example, each additional unit of sheet steel produced by USX requires a specific amount of material and labor. The incremental cost of this additional material and labor can be isolated and assigned to each unit of sheet steel produced. Many overhead expenses are also variable because utility bills, maintenance expense, and so forth vary with the production level.

Exhibit SB.1 illustrates the fixed and variable cost components of total cost. Note that total cost increases at the same rate as variable costs because fixed costs are constant.

Sunk Costs Sunk costs are past expenses or investments that have no salvage value and therefore should not be taken into account in considering investment alternatives. Sunk costs could also be current costs that are essentially fixed, such as rent on a building. For example, suppose an ice cream manufacturing firm occupies a rented building and is considering making sherbet in the same building. If the company enters sherbet production, its cost accountant will assign some of the rental expense to the sherbet operation. However, the building rent remains unchanged and therefore is not a relevant expense to be considered in making the decision. The rent is *sunk;* that is, it continues to exist and does not change in amount regardless of the decision.

Opportunity Costs *Opportunity cost* is the benefit *forgone*, or advantage *lost*, that results from choosing one action over the *best-known alternative* course of action.



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Suppose a firm has \$100,000 to invest, and two alternatives of comparable risk present themselves, each requiring a \$100,000 investment. Investment A will net \$25,000; Investment B will net \$23,000. Investment A is clearly the better choice, with a \$25,000 net return. If the decision is made to invest in B instead of A, the opportunity cost of B is \$2,000, which is the benefit forgone.

Avoidable Costs Avoidable costs include any expense that is *not* incurred if an investment is made but that *must* be incurred if the investment is *not* made. Suppose a company owns a metal lathe that is not in working condition but is needed for the firm's operations. Because the lathe must be repaired or replaced, the repair costs are avoidable if a new lathe is purchased. Avoidable costs reduce the cost of a new investment because they are not incurred if the investment is made. Avoidable costs are an example of how it is possible to "save" money by spending money.

Expected Value Risk is inherent in any investment because the future can never be predicted with absolute certainty. To deal with this uncertainty, mathematical techniques such as expected value can help. Expected value is the expected outcome multiplied by the probability of its occurrence. Recall that in the preceding example the expected outcome of Alternative A was \$25,000 and B, \$23,000. Suppose the probability of A's actual outcome is 80 percent while B's probability is 90 percent. The expected values of the alternatives are determined as follows:

Expected outcome \times Probability that actual outcome will be the expected outcome = Expected value Investment A: $$25,000 \times 0.80 = $20,000$ Investment B: $$23,000 \times 0.90 = $20,700$

Investment B is now seen to be the better choice, with a net advantage over A of \$700.

Economic Life and Obsolescence When a firm invests in an incomeproducing asset, the productive life of the asset is estimated. For accounting purposes, the asset is depreciated over this period. It is assumed that the asset will perform its function during this time and then be considered obsolete or worn out, and replacement will be required. This view of asset life rarely coincides with reality.

Assume that a machine expected to have a productive life of 10 years is purchased. If at any time during the ensuing 10 years a new machine is developed that can perform the same task more efficiently or economically, the old machine has become obsolete. Whether or not it is "worn out" is irrelevant.

The *economic life* of a machine is the period over which it provides the best method for performing its task. When a superior method is developed, the machine has become obsolete. Thus, the stated *book value* of a machine can be a meaningless figure.

Depreciation Depreciation is a method for allocating costs of capital equipment. The value of any capital asset—buildings, machinery, and so forth—decreases as its useful life is expended. *Amortization* and *depreciation* are often used interchangeably. Through convention, however, *depreciation* refers to the allocation of cost due to the physical or functional deterioration of *tangible* (physical) assets, such as buildings or equipment, whereas *amortization* refers to the allocation of cost over the useful life of *intangible* assets, such as patents, leases, franchises, and goodwill.

Depreciation procedures may not reflect an asset's true value at any point in its life because obsolescence may at any time cause a large difference between true value and book value. Also, because depreciation rates significantly affect taxes, a firm may choose a particular method from the several alternatives with more consideration for its effect on taxes than its ability to make the book value of an asset reflect the true resale value.

Next we describe five commonly used methods of depreciation.

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STRAIGHT-LINE METHOD

Under this method, an asset's value is reduced in uniform annual amounts over its estimated useful life. The general formula is

Annual amount to be depreciated $= \frac{\text{Cost} - \text{Salvage value}}{\text{Estimated useful life}}$

A machine costing \$10,000, with an estimated salvage value of \$0 and an estimated life of 10 years, would be depreciated at the rate of \$1,000 per year for each of the 10 years. If its estimated salvage value at the end of the 10 years is \$1,000, the annual depreciation charge is

$$\frac{\$10,000 - \$1,000}{10} = \$900$$

SUM-OF-THE-YEARS'-DIGITS (SYD) METHOD

The purpose of the SYD method is to reduce the book value of an asset rapidly in early years and at a lower rate in the later years of its life.

Suppose that the estimated useful life is five years. The numbers add up to 15: 1 + 2 + 3 + 4 + 5 = 15. Therefore, we depreciate the asset by $5 \div 15$ after the first year, $4 \div 15$ after the second year, and so on, down to $1 \div 15$ in the last year.

DECLINING-BALANCE METHOD

This method also achieves an accelerated depreciation. The asset's value is decreased by reducing its book value by a constant percentage each year. The percentage rate selected is often the one that just reduces book value to salvage value at the end of the asset's estimated life. In any case, the asset should never be reduced below estimated salvage value. Use of the declining-balance method and allowable rates are controlled by Internal Revenue Service regulations. As a simplified illustration, the preceding example is used in the next table with an arbitrarily selected rate of 40 percent. Note that depreciation is based on full cost, *not* cost minus salvage value.

Year	Depreciation Rate	Beginning Book Value	DEPRECIATION CHARGE	Accumulated Depreciation	Ending Book Value
1	0.40	\$17,000	\$6,800	\$ 6,800	\$10,200
2	0.40	10,200	4,080	10,880	6,120
3	0.40	6,120	2,448	13,328	3,672
4	0.40	3,672	1,469	14,797	2,203
5		2,203	203	15,000	2,000

In the fifth year, reducing book value by 40 percent would have caused it to drop below salvage value. Consequently, the asset was depreciated by only \$203, which decreased book value to salvage value.

DOUBLE-DECLINING-BALANCE METHOD

Again, for tax advantages, the double-declining-balance method offers higher depreciation early in the life span. This method uses a percentage twice the straight line for the life span of the item but applies this rate to the undepreciated original cost. The method is the same as the declining-balance method, but the term *double-declining balance* means double the straight-line rate. Thus, equipment with a 10-year life span would have a straight-line depreciated amount) of 20 percent per year.

DEPRECIATION-BY-USE METHOD

The purpose of this method is to depreciate a capital investment in proportion to its use. It is applicable, for example, to a machine that performs the same operation many times. The life of the machine is estimated not in years but rather in the total number of operations it

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may reasonably be expected to perform before wearing out. Suppose that a metal-stamping press has an estimated life of one million stamps and costs \$100,000. The charge for depreciation per stamp is then \$100,000 \div 1,000,000, or \$0.10. Assuming a \$0 salvage value, the depreciation charges are as shown in the following table:

			YEARLY		_
Year	Total Yearly Stamps	Cost per Stamp	DEPRECIATION CHARGE	ACCUMULATED DEPRECIATION	Ending Book Value
1	150,000	0.10	\$ 15,000	\$ 15,000	\$85,000
2	300,000	0.10	30,000	45,000	55,000
3	200,000	0.10	20,000	65,000	35,000
4	200,000	0.10	20,000	85,000	15,000
5	100,000	0.10	10,000	95,000	5,000
6	50,000	0.10	5,000	100,000	0

The depreciation-by-use method is an attempt to gear depreciation charges to actual use and thereby coordinate expense charges with productive output more accurately. Also, because a machine's resale value is related to its remaining productive life, it is hoped that book value will approximate resale value. The danger, of course, is that technological improvements will render the machine obsolete, in which case book value will not reflect true value.

ACTIVITY-BASED COSTING

To know how much it costs to make a certain product or deliver a service, some method of allocating overhead costs to production activities must be applied. The traditional approach is to allocate overhead costs to products on the basis of direct labor dollars or hours. By dividing the total estimated overhead costs by total budgeted direct labor hours, an overhead rate can be established. The problem with this approach is that direct labor as a percentage of total costs has fallen dramatically over the past decade. For example, introduction of advanced manufacturing technology and other productivity improvements has driven direct labor to as low as 7 to 10 percent of total manufacturing costs in many industries. As a result, overhead rates of 600 percent or even 1,000 percent are found in some highly automated plants.

This traditional accounting practice of allocating overhead to direct labor can lead to questionable investment decisions; for example, automated processes may be chosen over labor-intensive processes based on a comparison of projected costs. Unfortunately, overhead does not disappear when the equipment is installed, and overall costs may actually be lower with the labor-intensive process. It can also lead to wasted effort because an inordinate amount of time is spent tracking direct labor hours. For example, one plant spent 65 percent of computer costs tracking information about direct labor transactions even though direct labor accounted for only 4 percent of total production costs.¹

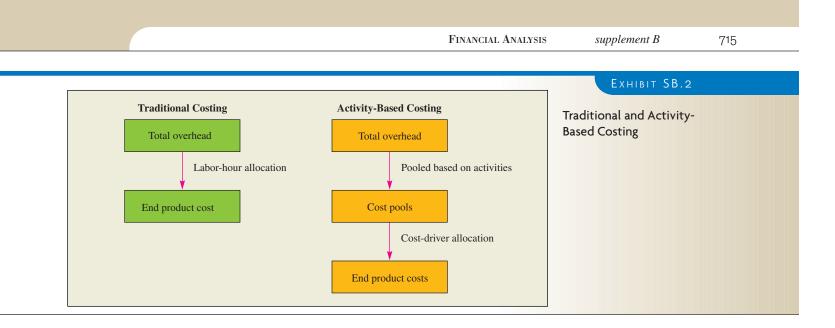
Activity-based costing techniques have been developed to alleviate these problems by refining the overhead allocation process to more directly reflect actual proportions of overhead consumed by the production activity. Causal factors, known as *cost drivers*, are identified and used as the means for allocating overhead. These factors might include machine hours, beds occupied, computer time, flight hours, or miles driven. The accuracy of overhead allocation, of course, depends on the selection of appropriate cost drivers.

Activity-based costing involves a two-stage allocation process, with the first stage assigning overhead costs to *cost activity pools*. These pools represent activities such as performing machine setups, issuing purchase orders, and inspecting parts. In the second stage, costs are assigned from these pools to activities based on the number or amount of pool-related activity required in their completion. Exhibit SB.2 compares traditional cost accounting and activity-based costing.

Consider the example of activity-based costing in Exhibit SB.3. Two products, A and B, are produced using the same number of direct labor hours. The same number of direct labor hours produces 5,000 units of Product A and 20,000 units of Product B. Applying traditional costing, identical overhead costs would be charged to each product. By applying



Material receipts



\$120/receipt

	BAS	IC DATA				
	TRACEABLE		Events of Transactions			
Αςτινιτγ	Costs	Total	Product A	PRODUCT B		
Machine setups	\$230,000	5,000	3,000	2,000		
Quality inspections	160,000	8,000	5,000	3,000		
Production orders	81,000	600	200	400		
Machine-hours worked	314,000	40,000	12,000	28,000		
Material receipts	90,000	750	150	600		
Number of units produced		25,000	5,000	20,000		
	\$875,000					
	Overhead R	ATES BY ACTIVITY				
Αςτινιτγ	(a) Traceabli Costs	-	(b) Total Events Transactions	(a) ÷ (b) Rate per Event or Transaction		
Machine setups	\$230,00	0	5,000	\$46/setups		
Quality inspections	160,00	0	8,000	\$20/inspection		
Production orders	81,00	0	600	\$135/order		
Machine-hours worked	314,00	0	40,000	\$7.85/hour		

Overhead Allocations by an Activity Approach

	PRODUCT	гA	PRODUCT B	
	Events or Transactions	Amount	Events or Transactions	Amount
Machine setups, at \$46/setup	3,000	\$138,000	2,000	\$ 92,000
Quality inspections, at \$20/inspection	5,000	100,000	3,000	60,000
Product orders, at \$135/order	200	27,000	400	54,000
Machine-hours worked, at \$7.85/hour	12,000	94,200	28,000	219,800
Material receipts, at \$120/receipt	150	18,000	600	72,000
Total overhead cost assigned		\$377,200		\$497,800
Number of units produced		5,000		20,000
Overhead cost per unit, Total overhead No. of units		\$75.44		\$24.89

750

90,000

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activity-based costing, traceable costs are assigned to specific activities. Because each product required a different amount of transactions, different overhead amounts are allocated to these products from the pools.

As stated earlier, activity-based costing overcomes the problem of cost distortion by creating a cost pool for each activity or transaction that can be identified as a cost driver, and by assigning overhead cost to products or jobs on a basis of the number of separate activities required for their completion. Thus, in the previous situation, the low-volume product would be assigned the bulk of the costs for machine setup, purchase orders, and quality inspections, thereby showing it to have high unit costs compared to the other product.

Finally, activity-based costing is sometimes referred to as *transactions costing*. This transactions focus gives rise to another major advantage over other costing methods: It improves the traceability of overhead costs and thus results in more accurate unit cost data for management.

THE EFFECTS OF TAXES

Tax rates and the methods of applying them occasionally change. When analysts evaluate investment proposals, tax considerations often prove to be the deciding factor because depreciation expenses directly affect taxable income and therefore profit. The ability to write off depreciation in early years provides an added source of funds for investment. Before 1986, firms could employ an investment tax credit, which allowed a direct reduction in tax liability. But tax laws change, so it is crucial to stay on top of current tax laws and try to predict future changes that may affect current investments and accounting procedures.

CHOOSING AMONG INVESTMENT PROPOSALS

The capital investment decision has become highly rationalized, as evidenced by the variety of techniques available for its solution. In contrast to pricing or marketing decisions, the capital investment decision can usually be made with a higher degree of confidence because the variables affecting the decision are relatively well known and can be quantified with fair accuracy.

Investment decisions may be grouped into six general categories:

- 1 Purchase of new equipment or facilities.
- 2 Replacement of existing equipment or facilities.
- 3 Make-or-buy decisions.
- 4 Lease-or-buy decisions.
- 5 Temporary shutdowns or plant abandonment decisions.
- 6 Addition or elimination of a product or product line.

Investment decisions are made with regard to the lowest acceptable rate of return on investment. As a starting point, the lowest acceptable rate of return may be considered to be the cost of investment capital needed to underwrite the expenditure. Certainly an investment will not be made if it does not return at least the cost of capital.

Investments are generally ranked according to the return they yield in excess of their cost of capital. In this way, a business with only limited investment funds can select investment alternatives that yield the highest net returns. (*Net return* is the earnings an investment yields after gross earnings have been reduced by the cost of the funds used to finance the investment.) In general, investments should not be made unless the return in funds exceeds the marginal cost of investment capital. (*Marginal cost* is the incremental cost of each new acquisition of funds from outside sources.)

DETERMINING THE COST OF CAPITAL

The cost of capital is calculated from a weighted average of debt and equity security costs. This average will vary depending on the financing strategy employed by the company. The most common sources of financing are short-term debt, long-term debt, and equity securities. A bank loan is an example of short-term debt. Bonds normally provide long-term debt. Finally, stock is a common form of equity financing. In the following, we give a short example of each

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form of financing, and then show how they are combined to find the weighted average cost of capital.

The cost of short-term debt depends on the interest rate on the loan and whether the loan is discounted. Remember that interest is a tax-deductible expense for a company.

Cost of short-term debt = $\frac{\text{Interest paid}}{\text{Proceeds received}}$

If a bank discounts a loan, interest is deducted from the face of the loan to get the proceeds. When a compensating balance is required (that is, a percentage of the face value of the loan is held by the bank as collateral), proceeds are also reduced. In either case, the effective or real interest rate on the loan is higher than the face interest rate owing to the proceeds received from the loan being less than the amount (face value) of the loan.

EXAMPLE OF SHORT-TERM DEBT

A company takes a \$150,000, one-year, 13 percent loan. The loan is discounted, and a 10 percent compensating balance is required. The effective interest rate is computed as follows:

$$\frac{13\% \times \$150,000}{\$115,500} = \frac{\$19,500}{\$115,500} = 16.89\%$$

Proceeds received equal

Face of loan	\$150,000
Less interest	(19,500)
Compensating balance (10% \times \$150,000)	(15,000)
Proceeds	\$115,500

Notice how the effective cost of the loan is significantly greater than the stated interest rate.

Long-term debt is normally provided through the sale of corporate bonds. The real cost of bonds is obtained by computing two types of yield: simple (face) yield and yield to maturity (effective interest rate). The first involves an easy approximation, but the second is more accurate. The nominal interest rate equals the interest paid on the face (maturity value) of the bond and is always stated on a per-annum basis. Bonds are generally issued in \$1,000 denominations and may be sold above face value (at a premium) or below (at a discount, termed original issue discount, or OID). A bond is sold at a discount when the interest rate is below the going market rate. In this case, the yield will be higher than the nominal interest rate. The opposite holds for bonds issued at a premium.

The issue price of a bond is the par (or face value) times the premium (or discount).

Simple yield =
$$\frac{\text{Nominal interest}}{\text{Issue price of bond}}$$

Yield to maturity = $\frac{\frac{\text{Nominal interest} + \frac{\text{Discount (or premium)}}{\text{Years}}}{\frac{\text{Issue price} + \text{Maturity value}}{2}}$

EXAMPLE OF LONG-TERM DEBT

A company issues a \$400,000, 12 percent, 10-year bond for 97 percent of face value. Yield computations are as follows:

Nominal annual payment = $12\% \times $400,000$ = \$48,000 Bond proceeds = $97\% \times $400,000$ = \$388,000

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Bond discount = $3\% \times $400,000$ = \$12,000 Simple yield = $\frac{12\% \times $400,000}{97\% \times $400,000} = \frac{$48,000}{$388,000} = 12.4\%$ Yield to maturity = $\frac{$48,000 + \frac{$12,000}{10}}{\frac{$388,000 + $400,000}{10}} = \frac{$48,000 + $1,200}{$394,000} = 12.5\%$

The actual cost of equity securities (stocks) comes in the form of dividends, which are not tax deductible to the corporation.

$$Cost of common stock = \frac{Dividends per share}{Value per share} + Growth rate of dividends$$

Here the value per share equals the market price per share minus flotation costs (that is, the cost of issuing securities, such as brokerage fees and printing costs). It should be noted that this valuation does not consider what the investor expects in market price appreciation. This expectation is based on the expected growth in earnings per share and the relative risk taking by purchasing the stock. The capital asset pricing model (CAPM) can be used to capture this impact.²

EXAMPLE OF THE COST OF COMMON STOCK

A company's dividend per share is \$10, net value is \$70 per share, and the dividend growth rate is 5 percent.

Cost of the stock =
$$\frac{\$10}{\$70} + 0.05 = 19.3\%$$

To compute the weighted average cost of capital, we consider the percentage of the total capital that is being provided by each financing alternative. We then calculate the after-tax cost of each financing alternative. Finally, we weight these costs in proportion to their use.

EXAMPLE OF CALCULATING THE WEIGHTED AVERAGE COST OF CAPITAL

Consider a company that shows the following figures in its financial statements:

Short-term bank loan (13%)	\$1 million
Bonds payable (16%)	\$4 million
Common stock (10%)	\$5 million

For our example, assume that each of the percentages given above represents the cost of the source of capital. In addition to the above, we need to consider the tax rate of the firm because the interest paid on the bonds and on the short-term loan is tax deductible. Assume a corporate tax rate of 40 percent.

	Percent	After-Tax Cost	WEIGHTED AVERAGE COST
Short-term bank loan	10%	13% × 60% = 7.8%	.78%
Bonds payable	40%	$16\% \times 60\% = 9.6\%$	3.84%
Common stock	50%	10%	5%
Total	100%		9.62%

Keep in mind that in developing this section we have made many assumptions in these calculations. When these ideas are applied to a specific company, many of these assumptions may change. The basic concepts, though, are the same; keep in mind that the goal is to

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simply calculate the after-tax cost of the capital used by the company. We have shown the cost of capital for the entire company, though often only the capital employed for a specific project is used in the calculation.

INTEREST RATE EFFECTS

There are two basic ways to account for the effects of interest accumulation. One is to compute the total amount created over the time period into the future as the *compound value*. The other is to remove the interest rate effect over time by reducing all future sums to presentday dollars, or the *present value*.

Compound Value of a Single Amount Albert Einstein was quoted as saying that compound interest is the eighth wonder of the world. After reviewing this section showing compound interest's dramatic growth effects over a long time, you might wish to propose a new government regulation: On the birth of a child, the parents must put, say, \$1,000 into a retirement fund for that child, available at age 65. This might reduce the pressure on Social Security and other state and federal pension plans. Although inflation would decrease the value significantly, there would still be a lot left over. At 14 percent return on investment, our \$1,000 would increase to \$500,000 after subtracting the \$4.5 million for inflation. That is still a 500-fold increase. (Many mutual funds today have long-term performances in excess of 14 percent per year.)

Spreadsheets and calculators make such computation easy. The box titled "Using a Spreadsheet" shows the most useful financial functions. However, many people still refer to tables for compound values. Using Appendix G, Table G.1 (compound sum of \$1), for example, we see that the value of \$1 at 10 percent interest after three years is \$1.331. Multiplying this figure by \$10 gives \$13.31.

Compound Value of an Annuity An *annuity* is the receipt of a constant sum each year for a specified number of years. Usually an annuity is received at the end of a period and does not earn interest during that period. Therefore, an annuity of \$10 for three years would bring in \$10 at the end of the first year (allowing the \$10 to earn interest if invested for the remaining two years), \$10 at the end of the second year (allowing the \$10 to earn interest if to earn interest for the remaining one year), and \$10 at the end of the third year (with no time to earn interest). If the annuity receipts were placed in a bank savings account at 5 percent interest, the total or compound value of the \$10 at 5 percent for the three years would be

Year	RECEIPT AT END OF YEAR		Compound Interest Factor $(1 + i)^n$		Value at End of Third Year
1	\$10.00	×	$(1 + 0.05)^2$	=	\$11.02
2	10.00	×	$(1 + 0.05)^1$	=	10.50
3	10.00	×	(1 + 0.05)°	=	10.00
					\$31.52

The general formula for finding the compound value of an annuity is

$$S_n = R[(1+i)^{n-1} + (1+i)^{n-2} + \dots + (1+i)^1 + 1]$$

where

 S_n = Compound value of an annuity

R = Periodic receipts in dollars

n = Length of the annuity in years

Applying this formula to the preceding example, we get

$$S_n = R[(1+i)^2 + (1+i) + 1]$$

= \$10[(1+0.05)^2 + (1+0.05) + 1] = \$31.52

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USING A SPREADSHEET

We hope that you are all doing these calculations using a spreadsheet program. Even though the computer makes these calculations simple, it is important that you understand what the computer is actually doing. Further, you should check your calculations manually to make sure that you have the formulas set up correctly in your spreadsheet. There are many stories of the terrible consequences of making a wrong decision based on a spreadsheet with errors!

For your quick reference, the following are the financial functions you will find most useful. These are from the Microsoft Excel help screens.

PV (rate, nper, pmt)-Returns the present value of an investment. The present value is the total amount that a series of future payments is worth now. For example, when you borrow money, the loan amount is the present value to the lender. Rate is the interest rate per period. For example, if you obtain an automobile loan at a 10 percent annual interest rate and make monthly payments, your interest rate per month is 10%/12, or .83%. You would enter 10%/12, or .83%, or .0083, in the formula as the rate. Nper is the total number of payment periods in an annuity. For example, if you get a four-year car loan and make monthly payments, your loan has 4*12 (or 48) periods. You would enter 48 into the formula for nper. Pmt is the payment made each period and cannot change over the life of the annuity. Typically, this includes principal and interest but no other fees or taxes. For example, the monthly payment on a \$10,000, four-year car loan at 12 percent is \$263.33. You would enter 263.33 into the formula as pmt.

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FV (rate, nper, pmt)—Returns the future value of an investment based on periodic, constant payments and a constant interest rate. Rate is the interest rate per period. Nper is the total number of payment periods in an annuity. Pmt is the payment made each period; it cannot change over the life of the annuity. Typically, pmt contains principal and interest but no other fees or taxes.

NPV (rate, value1, value2, ...)—Returns the net present value of an investment based on a series of periodic cash flows and a discount rate. The net present value of an investment is today's value of a series of future payments (negative values) and income (positive values). Rate is the rate of discount over the length of one period. Value1, value2, ... must be equally spaced in time and occur at the end of each period.

IRR (values)—Returns the internal rate of return for a series of cash flows represented by the numbers in values. (*Values* is defined below.) These cash flows do not have to be even, as they would be for an annuity. The internal rate of return is the interest rate received for an investment consisting of payments (negative values) and income (positive values) that occur at regular periods. *Values* is an array or a reference to cells that contain numbers for which you want to calculate the internal rate of return. Values must contain at least one positive value and one negative value to calculate the internal rate of return. IRR uses the order of values to interpret the order of cash flows. Be sure to enter your payment and income values in the sequence you want.

In Appendix G, Table G.2 lists the compound value factor of \$1 for 5 percent after three years as 3.152. Multiplying this factor by \$10 yields \$31.52.

In a fashion similar to our previous retirement investment example, consider the beneficial effects of investing \$2,000 each year, just starting at the age of 21. Assume investments in AAA-rated bonds are available today yielding 9 percent. From Table G.2 in Appendix G, after 30 years (at age 51) the investment is worth 136.3 times \$2,000, or \$272,600. Fourteen years later (at age 65) this would be worth \$963,044 (using a hand calculator, because the table goes only to 30 years, and assuming the \$2,000 is deposited at the end of each year)! But what 21-year-old thinks about retirement?

Present Value of a Future Single Payment Compound values are used to determine future value after a specific period has elapsed; present value (PV) procedures accomplish just the reverse. They are used to determine the current value of a sum or stream of receipts expected to be received in the future. Most investment decision techniques use present value concepts rather than compound values. Because decisions affecting the future are made in the present, it is better to convert future returns into their present value at the time the decision is being made. In this way, investment alternatives are placed in better perspective in terms of current dollars.

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An example makes this more apparent. If a rich uncle offers to make you a gift of \$100 today or \$250 after 10 years, which should you choose? You must determine whether the \$250 in 10 years will be worth more than the \$100 now. Suppose that you base your decision on the rate of inflation in the economy and believe that inflation averages 10 percent per year. By deflating the \$250, you can compare its relative purchasing power with \$100 received today. Procedurally, this is accomplished by solving the compound formula for the present sum, *P*, where *V* is the future amount of \$250 in 10 years at 10 percent. The compound value formula is

$$V = P(1+i)^n$$

Dividing both sides by $(1 + i)^n$ gives

$$P = \frac{V}{(1+i)^n}$$
$$= \frac{250}{(1+0.10)^{10}} = \$96.39$$

This shows that, at a 10 percent inflation rate, \$250 in 10 years will be worth \$96.39 today. The rational choice, then, is to take the \$100 now.

The use of tables is also standard practice in solving present value problems. With reference to Appendix G, Table G.3, the present value factor for \$1 received 10 years hence is 0.386. Multiplying this factor by \$250 yields \$96.50.

Present Value of an Annuity The present value of an annuity is the value of an annual amount to be received over a future period expressed in terms of the present. To find the value of an annuity of \$100 for three years at 10 percent, find the factor in the present value table that applies to 10 percent in *each* of the three years in which the amount is received and multiply each receipt by this factor. Then sum the resulting figures. Remember that annuities are usually received at the end of each period.

Year	Amount Received at End of Year		Present Value Factor at 10%		Present Value
1	\$100	×	0.909	=	\$ 90.90
2	100	×	0.826	=	82.60
3	100	×	0.751	=	75.10
Total receipts	\$300		Total present value	=	\$248.60

The general formula used to derive the present value of an annuity is

$$A_n = R\left[\frac{1}{(1+i)} + \frac{1}{(1+i)^2} + \dots + \frac{1}{(1+i)^n}\right]$$

where

 A_n = Present value of an annuity of *n* years

R = Periodic receipts

n = Length of the annuity in years

Applying the formula to the preceding example gives

$$A_n = \$100 \left[\frac{1}{(1+0.10)} + \frac{1}{(1+0.10)^2} + \frac{1}{(1+0.10)^3} \right]$$
$$= \$100(2.487) = \$248.70$$

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In Appendix G, Table G.4 contains present values of an annuity for varying maturities. The present value factor for an annuity of \$1 for three years at 10 percent (from Appendix G, Table G.4) is 2.487. Given that our sum is \$100 rather than \$1, we multiply this factor by \$100 to arrive at \$248.70.

When the stream of future receipts is uneven, the present value of each annual receipt must be calculated. The present values of the receipts for all years are then summed to arrive at total present value. This process can sometimes be tedious, but it is unavoidable.

Discounted Cash Flow The term *discounted cash flow* refers to the total stream of payments that an asset will generate in the future discounted to the present time. This is simply present value analysis that includes all flows: single payments, annuities, and all others.

METHODS OF RANKING INVESTMENTS

Net Present Value The net present value method is commonly used in business. With this method, decisions are based on the amount by which the present value of a projected income stream exceeds the cost of an investment.

A firm is considering two alternative investments. The first costs \$30,000 and the second, \$50,000. The expected yearly cash income streams are shown in this table:

	Cash	Cash Inflow			
Year	ALTERNATIVE A	ALTERNATIVE B			
1	\$10,000	\$15,000			
2	10,000	15,000			
3	10,000	15,000			
4	10,000	15,000			
5	10,000	15,000			

To choose between Alternatives A and B, find which has the highest net present value. Assume an 8 percent cost of capital.

ALTERNATIVE A	ALTERNATIVE B
3.993 (PV factor)	3.993 (PV factor)
\times \$10,000 = \$39,930	\times \$15,000 = \$59,895
Less cost of investment = $30,000$	Less cost of investment = $50,000$
Net present value $=$ \$ 9,930	Net present value $=$ \$ 9,895

Investment A is the better alternative. Its net present value exceeds that of Investment B by \$35 (\$9,930 - \$9,895 = \$35).

Payback Period The payback method ranks investments according to the time required for each investment to return earnings equal to the cost of the investment. The rationale is that the sooner the investment capital can be recovered, the sooner it can be reinvested in new revenue-producing projects. Thus, supposedly, a firm will be able to get the most benefit from its available investment funds.

Consider two alternatives requiring a \$1,000 investment each. The first will earn \$200 per year for six years; the second will earn \$300 per year for the first three years and \$100 per year for the next three years.

If the first alternative is selected, the initial investment of \$1,000 will be recovered at the end of the fifth year. The income produced by the second alternative will total \$1,000 after only four years. The second alternative will permit reinvestment of the full \$1,000 in new revenue-producing projects one year sooner than the first.

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Though the payback method is declining in popularity as the sole measure in investment decisions, it is still frequently used in conjunction with other methods to indicate the time commitment of funds. The major problems with payback are that it does not consider income beyond the payback period and it ignores the time value of money. A method that ignores the time value of money must be considered questionable.

Internal Rate of Return The internal rate of return may be defined as the interest rate that equates the present value of an income stream with the cost of an investment. There is no procedure or formula that may be used directly to compute the internal rate of return—it must be found by interpolation or iterative calculation.

Suppose we wish to find the internal rate of return for an investment costing \$12,000 that will yield a cash inflow of \$4,000 per year for four years. We see that the present value factor sought is

$$\frac{\$12,000}{\$4,000} = 3.000$$

and we seek the interest rate that will provide this factor over a four-year period. The interest rate must lie between 12 and 14 percent because 3.000 lies between 3.037 and 2.914 (in the fourth row of Appendix G, Table G.4). Linear interpolation between these values, according to the equation

$$I = 12 + (14 - 12) \frac{(3.037 - 3.000)}{(3.037 - 2.914)}$$
$$= 12 + 0.602 = 12.602\%$$

gives a good approximation to the actual internal rate of return.

When the income stream is discounted at 12.6 percent, the resulting present value closely approximates the cost of investment. Thus the internal rate of return for this investment is 12.6 percent. The cost of capital can be compared with the internal rate of return to determine the net rate of return on the investment. If, in this example, the cost of capital were 8 percent, the net rate of return on the investment would be 4.6 percent.

The net present value and internal rate of return methods involve procedures that are essentially the same. They differ in that the net present value method enables investment alternatives to be compared in terms of the dollar value in excess of cost, whereas the internal rate of return method permits comparison of rates of return on alternative investments. Moreover, the internal rate of return method occasionally encounters problems in calculation, as multiple rates frequently appear in the computation.

Ranking Investments with Uneven Lives When proposed investments have the same life expectancy, comparison among them, using the preceding methods, will give a reasonable picture of their relative value. When lives are unequal, however, there is the question of how to relate the two different time periods. Should replacements be considered the same as the original? Should productivity for the shorter-term unit that will be replaced earlier be considered higher? How should the cost of future units be estimated?

No estimate dealing with investments unforeseen at the time of decision can be expected to reflect a high degree of accuracy. Still, the problem must be dealt with, and some assumptions must be made in order to determine a ranking.

SAMPLE PROBLEMS: INVESTMENT DECISIONS

EXAMPLE SB.1: An Expansion Decision

William J. Wilson Ceramic Products, Inc., leases plant facilities in which firebrick is manufactured. Because of rising demand, Wilson could increase sales by investing in new equipment to expand output. The selling price of \$10 per brick will remain unchanged if output and sales increase. Based on engineering and cost estimates, the accounting department provides management with the following

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cost estimates based on an annual increased output of 100,000 bricks:

Cost of new equipment having an expected life of five years	\$500,000
Equipment installation cost	20,000
Expected salvage value	0
New operation's share of annual lease expense	40,000
Annual increase in utility expenses	40,000
Annual increase in labor costs	160,000
Annual additional cost for raw materials	400,000

The sum-of-the-years'-digits method of depreciation will be used, and taxes are paid at a rate of 40 percent. Wilson's policy is not to invest capital in projects earning less than a 20 percent rate of return. Should the proposed expansion be undertaken?

SOLUTION

Compute the cost of investment:

Acquisition cost of equipment	\$500,000
Equipment installation costs	20,000
Total cost of investment	\$520,000

Determine yearly cash flows throughout the life of the investment.

The lease expense is a sunk cost. It will be incurred whether or not the investment is made and is therefore irrelevant to the decision and should be disregarded. Annual production expenses to be considered are utility, labor, and raw materials. These total \$600,000 per year.

Annual sales revenue is $10 \times 100,000$ units of output, which totals 1,000,000. Yearly income before depreciation and taxes is thus 1,000,000 gross revenue, less 600,000 expenses, or 400,000.

Next, determine the depreciation charges to be deducted from the \$400,000 income each year using the SYD method (sum-of-years'-digits = 1 + 2 + 3 + 4 + 5 = 15):

Year	PROPORTION OF \$500,000 Year to Be Depreciated		DEPRECIATION CHARGE
1	5/15 × \$500,000	=	\$ 166,667
2	4/15 × 500,000	=	133,333
3	3/15 × 500,000	=	100,000
4	2/15 × 500,000	=	66,667
5	1/15 × 500,000	=	33,333
	Accumulated depreciation		\$500,000

Find each year's cash flow when taxes are 40 percent. Cash flow for only the first year is illustrated: Earnings before depreciation and taxes \$4

arnings before depreciation and taxes		\$400,000
Deduct: Taxes at 40% (40% × 400,000)	\$160,000	
Tax benefit of depreciation expense (0.4 $ imes$ 166,667)	66,667	93,333
Cash flow (first year)		\$306,667

Determine the present value of the cash flow. Because Wilson demands at least a 20 percent rate of return on investments, multiply the cash flows by the 20 percent present value factor for each year. The factor for each respective year must be used because the cash flows are not an annuity.

	PRESENT VALUE				
Year	Factor		Cash Flow		Present Value
1	0.833	×	\$306,667	=	\$255,454
2	0.694	×	293,333	=	203,573
3	0.579	×	280,000	=	162,120
4	0.482	×	266,667	=	128,533
5	0.402	×	253,334	=	101,840
	Total present value of cash flows (discounted at 20%) $=$				\$851,520

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Now find whether net present value is positive or negative:

Total present value of cash flows	\$851,520
Total cost of investment	520,000
Net present value	\$331,520

Net present value is positive when returns are discounted at 20 percent. Wilson will earn an amount in excess of 20 percent on the investment. The proposed expansion should be undertaken.

EXAMPLE SB.2: A Replacement Decision

For five years Bennie's Brewery has been using a machine that attaches labels to bottles. The machine was purchased for \$4,000 and is being depreciated over 10 years to a \$0 salvage value using straightline depreciation. The machine can be sold now for \$2,000. Bennie can buy a new labeling machine for \$6,000 that will have a useful life of five years and cut labor costs by \$1,200 annually. The old machine will require a major overhaul in the next few months at an estimated cost of \$300. If purchased, the new machine will be depreciated over five years to a \$500 salvage value using the straightline method. The company will invest in any project earning more than the 12 percent cost of capital. The tax rate is 40 percent. Should Bennie's Brewery invest in the new machine?

SOLUTION

Determine the cost of investment:

Price of the new machine		\$6,000
Less: Sale of old machine	\$2,000	
Avoidable overhaul costs	300	2,300
Effective cost of investment		\$3,700

Determine the increase in cash flow resulting from investment in the new machine:

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Yearly cost savings = $$1,200$		
Differential depreciation		
Annual depreciation on old machine:		
$\frac{\text{Cost} - \text{Salvage}}{\text{Expected life}} = \frac{\$4,000 - \$0}{10} = \400		
Annual depreciation on new machine:		
$\frac{\text{Cost} - \text{Salvage}}{\text{Expected life}} = \frac{\$6,000 - \$500}{5} = \$1,100$		
Differential depreciation = $$1,100 - $400 = 700		
Yearly net increase in cash flow into the firm:		
Cost savings		\$1,200
Deduct: Taxes at 40%	\$480	
Add: advantage of increase in		
depreciation (0.4 \times \$700)	280	200
Yearly increase in cash flow		\$1,000
Determine total present value of the investment:		
The five-year cash flow of \$1,000 per year is an annuity.		
Discounted at 12 percent, the cost of capital, the present	value is	
3.605 × \$1,000 = \$3,605		
The present value of the new machine, if sold at its salvage the fifth year, is	ge value of \$500 at	the end of
0.567 × \$500 = \$284		
Total present value of the expected cash flows is		
\$3,605 + \$284 = \$3,889		

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Determine whether net present value is positive:

Total present value	\$3,889
Cost of investment	3,700
Net present value	\$189

Bennie's Brewery should make the purchase because the investment will return slightly more than the cost of capital.

Note: The importance of depreciation has been shown in this example. The present value of the yearly cash flow resulting from operations is

 $(\text{Cost savings} - \text{Taxes}) \times (\text{Present value factor})$ $(\$1,200 - \$480) \times (3.605) = \$2,596$

This figure is \$1,104 less than the \$3,700 cost of the investment. Only a very large depreciation advantage makes this investment worthwhile. The total present value of the advantage is \$1,009:

(Tax rate \times Differential depreciation) \times (PV factor)

 $(0.4 \times \$700) \times (3.605) = \$1,009$

EXAMPLE SB.3: A Make-or-Buy Decision

The Triple X Company manufactures and sells refrigerators. It makes some of the parts for the refrigerators and purchases others. The engineering department believes it might be possible to cut costs by manufacturing one of the parts currently being purchased for \$8.25 each. The firm uses 100,000 of these parts each year. The accounting department compiles the following list of costs based on engineering estimates:

Fixed costs will increase by \$50,000.

Labor costs will increase by \$125,000.

Factory overhead, currently running \$500,000 per year, may be expected to increase 12 percent. Raw materials used to make the part will cost \$600,000.

Given the preceding estimates, should Triple X make the part or continue to buy it?

SOLUTION

Find the total cost incurred if the part were manufactured:

Additional fixed costs	\$ 50,000
Additional labor costs	125,000
Raw materials cost	600,000
Additional overhead costs = 0.12 \times \$500,000	60,000
Total cost to manufacturer	\$835,000

Find the cost per unit to manufacture:

$$\frac{\$835,000}{100,000} = \$8.35 \text{ per unit}$$

Triple X should continue to buy the part. Manufacturing costs exceed the present cost to purchase by \$0.10 per unit.

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FOOTNOTES

1 T. Johnson and R. Kaplan, *Relevance Lost: The Rise and Fall of Management Accounting* (Boston: Harvard Business School Press, 1987), p. 188.

2 A description of capital asset pricing is included in many finance textbooks; see, for example, Z. Bodie, A. Kane, and A. Marcus, *Investments*, 3rd ed. (Burr Ridge, IL: Richard D. Irwin, 1996), pp. 236–65.