Technical Note Two

technical note two

LEARNING CURVES

technical note

APPLICATION OF LEARNING CURVES

A **learning curve** is a line displaying the relationship between unit production time and the cumulative number of units produced. Learning (or experience) curve theory has a wide range of application in the business world. In manufacturing, it can be used to estimate the time for product design and production, as well as costs. Learning curves are important and are sometimes overlooked as one of the trade-offs in just-in-time (JIT) systems, where sequencing and short runs achieve lower inventories by forfeiting some benefits of experience from long product runs. Learning curves are also an integral part in planning corporate strategy, such as decisions concerning pricing, capital investment, and operating costs based on experience curves.

Learning curve

Learning curves can be applied to individuals or organizations. **Individual learning** is improvement that results when people repeat a process and gain skill or efficiency from their own experience. That is, "practice makes perfect." **Organizational learning** results from practice as well, but it also comes from changes in administration, equipment, and product design. In organizational settings, we expect to see both kinds of learning occurring simultaneously and often describe the combined effect with a single learning curve.

Individual learning

Organizational learning

Learning curve theory is based on three assumptions:

1 The amount of time required to complete a given task or unit of a product will be less each time the task is undertaken.
2 The unit time will decrease at a decreasing rate.
3 The reduction in time will follow a predictable pattern.

Each of these assumptions was found to hold true in the airplane industry, where learning curves were first applied.[1] In this application, it was observed that, as output doubled, there was a 20 percent reduction in direct production worker-hours per unit between doubled units. Thus, if it took 100,000 hours for Plane 1, it would take 80,000 hours for Plane 2, 64,000 hours for Plane 4, and so forth. Because the 20 percent reduction meant that, say, Unit 4 took only 80 percent of the production time required for Unit 2, the line connecting the coordinates of output and time was referred to as an "80 percent learning curve." (By convention, the percentage learning rate is used to denote any given exponential learning curve.)

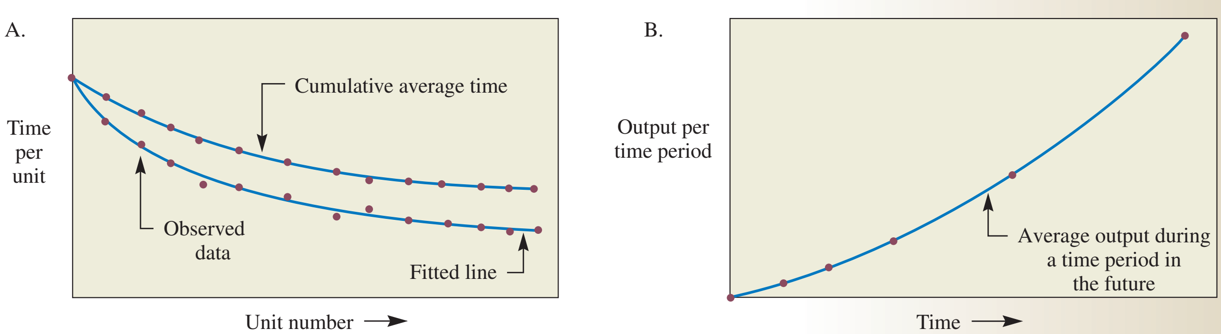
A learning curve may be developed from an arithmetic tabulation, by logarithms, or by some other curve-fitting method, depending on the amount and form of the available data.

There are two ways to think about the improved performance that comes with learning curves: time per unit (as in Exhibit TN2.1A) or units of output per time period (as in TN2.1B). *Time per unit* shows the decrease in time required for each successive unit. *Cumulative average time* shows the cumulative average performance times as the total

Interactive Operations Management

EXHIBIT TN2.1

Learning Curves Plotted as Times and Numbers of Units



A Progress Curve **Industrial Learning**

number of units increases. Time per unit and cumulative average times are also called *progress curves* or *product learning,* and are useful for complex products or products with a longer cycle time. *Units of output per time period* is also called *industry learning* and is generally applied to high-volume production (short cycle time).

Note in Exhibit TN2.1A that the cumulative average curve does not decrease as fast as the time per unit because the time is being averaged. For example, if the time for Units 1, 2, 3, and 4 were 100, 80, 70, and 64, they would be plotted that way on the time per unit graph, but would be plotted as 100, 90, 83.3, and 78.5 on the cumulative average time graph.

PLOTTING LEARNING CURVES

There are many ways to analyze past data to fit a useful trend line. We will use the simple exponential curve first as an arithmetic procedure and then by a logarithmic analysis. In an arithmetical tabulation approach, a column for units is created by doubling, row by row, as 1, 2, 4, 8, 16. . . The time for the first unit is multiplied by the learning percentage to obtain the time for the second unit. The second unit is multiplied by the learning percentage for the fourth unit, and so on. Thus, if we are developing an 80 percent learning curve, we would arrive at the figures listed in column 2 of Exhibit TN2.2. Because it is often desirable for planning purposes to know the cumulative direct labor hours, column 4, which lists this information, is also provided. The calculation of these figures is straightforward; for example, for Unit 4, cumulative average direct labor hours would be found by dividing cumulative direct labor hours by 4, yielding the figure given in column 4.

Exhibit TN2.3 shows three curves with different learning rates: 90 percent, 80 percent, and 70 percent. Note that if the cost of the first unit was $100, the 30th unit would cost $59.63 at the 90 percent rate and $17.37 at the 70 percent rate. Differences in learning rates can have dramatic effects.

In practice, learning curves are plotted using a graph with logarithmic scales. The unit curves become linear throughout their entire range and the cumulative curve becomes linear after the first few units. The property of linearity is desirable because it facilitates extrapolation and permits a more accurate reading of the cumulative curve. This type of scale is an option in Microsoft Excel. Simply generate a regular scatter plot in your spreadsheet and then select each axis and format the axis with the logarithmic option. Exhibit TN2.4 shows the 80 percent unit cost curve and average cost curve on a logarithmic scale. Note that the cumulative average cost is essentially linear after the eighth unit.

Although the arithmetic tabulation approach is useful, direct logarithmic analysis of learning curve problems is generally more efficient because it does not require a complete enumeration of successive time–output combinations. Moreover, where such data are not available, an analytical model that uses logarithms may be the most convenient way of obtaining output estimates.

EXHIBIT TN2.2

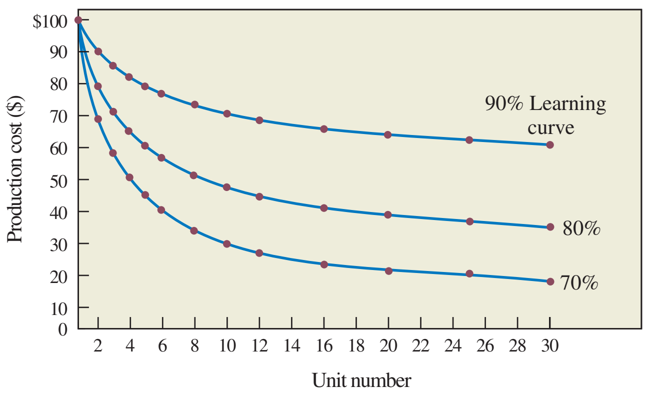
Unit, Cumulative, and Cumulative Average Direct Labor Worker-Hours Required for an 80 Percent Learning Curve

Excel: Learning Curves

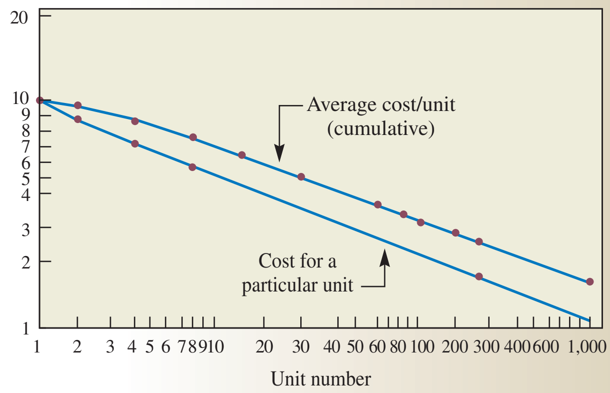
(1) Unit Number	(2) Unit Direct Labor Hours	(3) Cumulative Direct Labor Hours	(4) Cumulative Average Direct Labor Hours
1	100,000	100,000	100,000
2	80,000	180,000	90,000
4	64,000	314,210	78,553
8	51,200	534,591	66,824
16	40,960	892,014	55,751
32	32,768	1,467,862	45,871
64	26,214	2,392,453	37,382
128	20,972	3,874,395	30,269
256	16,777	6,247,318	24,404

EXHIBITS TN2.3 & TN2.4

TN2.3—Arithmetic Plot of 70, 80, and 90 Percent Learning Curves



TN2.4—Logarithmic Plot of an 80 Percent Learning Curve



LOGARITHMIC ANALYSIS

The normal form of the learning curve equation is[2]

[TN2.1] $$Y_x = Kx^n$$

where

x = Unit number
Y_x = Number of direct labor hours required to produce the xth unit
K = Number of direct labor hours required to produce the first unit
$n = \log b/\log 2$ where b = Learning percentage

We can solve this mathematically or by using a table, as shown in the next section. Mathematically, to find the labor-hour requirement for the eighth unit in our example (Exhibit TN2.2), we would substitute as follows:

$$Y_8 = (100{,}000)(8)^n$$

Using logarithms:

$$Y_8 = 100{,}000(8)^{\log 0.8/\log 2}$$

$$= 100{,}000(8)^{-0.322} = \frac{100{,}000}{(8)^{0.322}}$$

$$= \frac{100{,}000}{1.9535} = 51{,}192$$

Therefore, it would take 51,192 hours to make the eighth unit. (See the spreadsheet "Learning Curves.")

LEARNING CURVE TABLES

When the learning percentage is known, Exhibits TN2.5 and TN2.6 can be easily used to calculate estimated labor hours for a specific unit or for cumulative groups of units. We need only multiply the initial unit labor hour figure by the appropriate tabled value.

To illustrate, suppose we want to double-check the figures in Exhibit TN2.2 for unit and cumulative labor hours for Unit 16. From Exhibit TN2.5, the unit improvement factor for Unit 16 at 80 percent is .4096. This multiplied by 100,000 (the hours for Unit 1) gives 40,960, the same as in Exhibit TN2.2. From Exhibit TN2.6, the cumulative improvement

EXHIBIT TN2.5

Improvement Curves: Table of Unit Values

	Unit Improvement Factor							
Unit	60%	65%	70%	75%	80%	85%	90%	95%
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	.6000	.6500	.7000	.7500	.8000	.8500	.9000	.9500
3	.4450	.5052	.5682	.6338	.7021	.7729	.8462	.9219
4	.3600	.4225	.4900	.5625	.6400	.7225	.8100	.9025
5	.3054	.3678	.4368	.5127	.5956	.6857	.7830	.8877
6	.2670	.3284	.3977	.4754	.5617	.6570	.7616	.8758
7	.2383	.2984	.3674	.4459	.5345	.6337	.7439	.8659
8	.2160	.2746	.3430	.4219	.5120	.6141	.7290	.8574
9	.1980	.2552	.3228	.4017	.4930	.5974	.7161	.8499
10	.1832	.2391	.3058	.3846	.4765	.5828	.7047	.8433
12	.1602	.2135	.2784	.3565	.4493	.5584	.6854	.8320
14	.1430	.1940	.2572	.3344	.4276	.5386	.6696	.8226
16	.1290	.1785	.2401	.3164	.4096	.5220	.6561	.8145
18	.1188	.1659	.2260	.3013	.3944	.5078	.6445	.8074
20	.1099	.1554	.2141	.2884	.3812	.4954	.6342	.8012
22	.1025	.1465	.2038	.2772	.3697	.4844	.6251	.7955
24	.0961	.1387	.1949	.2674	.3595	.4747	.6169	.7904
25	.0933	.1353	.1908	.2629	.3548	.4701	.6131	.7880
30	.0815	.1208	.1737	.2437	.3346	.4505	.5963	.7775
35	.0728	.1097	.1605	.2286	.3184	.4345	.5825	.7687
40	.0660	.1010	.1498	.2163	.3050	.4211	.5708	.7611
45	.0605	.0939	.1410	.2060	.2936	.4096	.5607	.7545
50	.0560	.0879	.1336	.1972	.2838	.3996	.5518	.7486
60	.0489	.0785	.1216	.1828	.2676	.3829	.5367	.7386
70	.0437	.0713	.1123	.1715	.2547	.3693	.5243	.7302
80	.0396	.0657	.1049	.1622	.2440	.3579	.5137	.7231
90	.0363	.0610	.0987	.1545	.2349	.3482	.5046	.7168
100	.0336	.0572	.0935	.1479	.2271	.3397	.4966	.7112
120	.0294	.0510	.0851	.1371	.2141	.3255	.4830	.7017
140	.0262	.0464	.0786	.1287	.2038	.3139	.4718	.6937
160	.0237	.0427	.0734	.1217	.1952	.3042	.4623	.6869
180	.0218	.0397	.0691	.1159	.1879	.2959	.4541	.6809
200	.0201	.0371	.0655	.1109	.1816	.2887	.4469	.6757
250	.0171	.0323	.0584	.1011	.1691	.2740	.4320	.6646
300	.0149	.0289	.0531	.0937	.1594	.2625	.4202	.6557
350	.0133	.0262	.0491	.0879	.1517	.2532	.4105	.6482
400	.0121	.0241	.0458	.0832	.1453	.2454	.4022	.6419
450	.0111	.0224	.0431	.0792	.1399	.2387	.3951	.6363
500	.0103	.0210	.0408	.0758	.1352	.2329	.3888	.6314
600	.0090	.0188	.0372	.0703	.1275	.2232	.3782	.6229
700	.0080	.0171	.0344	.0659	.1214	.2152	.3694	.6158
800	.0073	.0157	.0321	.0624	.1163	.2086	.3620	.6098
900	.0067	.0146	.0302	.0594	.1119	.2029	.3556	.6045
1,000	.0062	.0137	.0286	.0569	.1082	.1980	.3499	.5998
1,200	.0054	.0122	.0260	.0527	.1020	.1897	.3404	.5918
1,400	.0048	.0111	.0240	.0495	.0971	.1830	.3325	.5850
1,600	.0044	.0102	.0225	.0468	.0930	.1773	.3258	.5793
1,800	.0040	.0095	.0211	.0446	.0895	.1725	.3200	.5743
2,000	.0037	.0089	.0200	.0427	.0866	.1683	.3149	.5698
2,500	.0031	.0077	.0178	.0389	.0806	.1597	.3044	.5605
3,000	.0027	.0069	.0162	.0360	.0760	.1530	.2961	.5530

EXHIBIT TN2.6

Improvement Curves: Table of Cumulative Values

Excel: Learning Curves

CUMULATIVE IMPROVEMENT FACTOR

UNIT	60%	65%	70%	75%	80%	85%	90%	95%
1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2	1.600	1.650	1.700	1.750	1.800	1.850	1.900	1.950
3	2.045	2.155	2.268	2.384	2.502	2.623	2.746	2.872
4	2.405	2.578	2.758	2.946	3.142	3.345	3.556	3.774
5	2.710	2.946	3.195	3.459	3.738	4.031	4.339	4.662
6	2.977	3.274	3.593	3.934	4.299	4.688	5.101	5.538
7	3.216	3.572	3.960	4.380	4.834	5.322	5.845	6.404
8	3.432	3.847	4.303	4.802	5.346	5.936	6.574	7.261
9	3.630	4.102	4.626	5.204	5.839	6.533	7.290	8.111
10	3.813	4.341	4.931	5.589	6.315	7.116	7.994	8.955
12	4.144	4.780	5.501	6.315	7.227	8.244	9.374	10.62
14	4.438	5.177	6.026	6.994	8.092	9.331	10.72	12.27
16	4.704	5.541	6.514	7.635	8.920	10.38	12.04	13.91
18	4.946	5.879	6.972	8.245	9.716	11.41	13.33	15.52
20	5.171	6.195	7.407	8.828	10.48	12.40	14.61	17.13
22	5.379	6.492	7.819	9.388	11.23	13.38	15.86	18.72
24	5.574	6.773	8.213	9.928	11.95	14.33	17.10	20.31
25	5.668	6.909	8.404	10.19	12.31	14.80	17.71	21.10
30	6.097	7.540	9.305	11.45	14.02	17.09	20.73	25.00
35	6.478	8.109	10.13	12.72	15.64	19.29	23.67	28.86
40	6.821	8.631	10.90	13.72	17.19	21.43	26.54	32.68
45	7.134	9.114	11.62	14.77	18.68	23.50	29.37	36.47
50	7.422	9.565	12.31	15.78	20.12	25.51	32.14	40.22
60	7.941	10.39	13.57	17.67	22.87	29.41	37.57	47.65
70	8.401	11.13	14.74	19.43	25.47	33.17	42.87	54.99
80	8.814	11.82	15.82	21.09	27.96	36.80	48.05	62.25
90	9.191	12.45	16.83	22.67	30.35	40.32	53.14	69.45
100	9.539	13.03	17.79	24.18	32.65	43.75	58.14	76.59
120	10.16	14.11	19.57	27.02	37.05	50.39	67.93	90.71
140	10.72	15.08	21.20	29.67	41.22	56.78	77.46	104.7
160	11.21	15.97	22.72	32.17	45.20	62.95	86.80	118.5
180	11.67	16.79	24.14	34.54	49.03	68.95	95.96	132.1
200	12.09	17.55	25.48	36.80	52.72	74.79	105.0	145.7
250	13.01	19.28	28.56	42.05	61.47	88.83	126.9	179.2
300	13.81	20.81	31.34	46.94	69.66	102.2	148.2	212.2
350	14.51	22.18	33.89	51.48	77.43	115.1	169.0	244.8
400	15.14	23.44	36.26	55.75	84.85	127.6	189.3	277.0
450	15.72	24.60	38.48	59.80	91.97	139.7	209.2	309.0
500	16.26	25.68	40.58	63.68	98.85	151.5	228.8	340.6
600	17.21	27.67	44.47	70.97	112.0	174.2	267.1	403.3
700	18.06	29.45	48.04	77.77	124.4	196.1	304.5	465.3
800	18.82	31.09	51.36	84.18	136.3	217.3	341.0	526.5
900	19.51	32.60	54.46	90.26	147.7	237.9	376.9	587.2
1,000	20.15	31.01	57.40	96.07	158.7	257.9	412.2	647.4
1,200	21.30	36.59	62.85	107.0	179.7	296.6	481.2	766.6
1,400	22.32	38.92	67.85	117.2	199.6	333.9	548.4	884.2
1,600	23.23	41.04	72.49	126.8	218.6	369.9	614.2	1001
1,800	24.06	43.00	76.85	135.9	236.8	404.9	678.8	1116
2,000	24.83	44.84	80.96	144.7	254.4	438.9	742.3	1230
2,500	26.53	48.97	90.39	165.0	296.1	520.8	897.0	1513
3,000	27.99	52.62	98.90	183.7	335.2	598.9	1047	1791

factor for cumulative hours for the first 16 units is 8.920. When multiplied by 100,000, this gives 892,000, which is reasonably close to the exact value of 892,014 shown in Exhibit TN2.2.

The following is a more involved example of the application of a learning curve to a production problem.

EXAMPLE TN2.1: Sample Learning Curve Problem

Captain Nemo, owner of the Suboptimum Underwater Boat Company (SUB), is puzzled. He has a contract for 11 boats and has completed 4 of them. He has observed that his production manager, young Mr. Overick, has been reassigning more and more people to torpedo assembly after the construction of the first four boats. The first boat, for example, required 225 workers, each working a 40-hour week, while 45 fewer workers were required for the second boat. Overick has told them that "this is just the beginning" and that he will complete the last boat in the current contract with only 100 workers!

Overick is banking on the learning curve, but has he gone overboard?

SOLUTION

Because the second boat required 180 workers, a simple exponential curve shows that the learning percentage is 80 percent (180 ÷ 225). To find out how many workers are required for the 11th boat, we look up unit 11 for an 80 percent improvement ratio in Exhibit TN2.5 and multiply this value by the number required for the first sub. By interpolating between Unit 10 and Unit 12 we find the improvement ratio is equal to .4629. This yields 104.15 workers (.4269 interpolated from table × 225). Thus, Overick's estimate missed the boat by four people.

EXAMPLE TN2.2: Estimating Cost Using Learning Curves

SUB has produced the first unit of a new line of minisubs at a cost of $500,000—$200,000 for materials and $300,000 for labor. It has agreed to accept a 10 percent profit, based on cost, and it is willing to contract on the basis of a 70 percent learning curve. What will be the contract price for three minisubs?

SOLUTION

Cost of first sub		$ 500,000
Cost of second sub		
Materials	$200,000	
Labor: $300,000 × .70	210,000	410,000
Cost of third sub		
Materials	200,000	
Labor: $300,000 × .5682	170,460	370,460
Total cost		1,280,460
Markup: $1,280,460 × .10		128,046
Selling price		$1,408,506

If the operation is interrupted, then some relearning must occur. How far to go back up the learning curve can be estimated in some cases. ●

ESTIMATING THE LEARNING PERCENTAGE

If production has been under way for some time, the learning percentage is easily obtained from production records. Generally speaking, the longer the production history, the more accurate the estimate. Because a variety of other problems can occur during the early stages of production, most companies do not begin to collect data for learning curve analysis until some units have been completed.

If production has not started, estimating the learning percentage becomes enlightened guesswork. In such cases the analyst has these options:

1 Assume that the learning percentage will be the same as it has been for previous applications within the same industry.

2 Assume that it will be the same as it has been for the same or similar products.
3 Analyze the similarities and differences between the proposed startup and previous startups and develop a revised learning percentage that appears to best fit the situation.

The following guidelines are useful for estimating the impact of learning on manufacturing tasks.[3] These guidelines use estimates of the percentage of time spent on manual work (i.e., hand assembly) versus the time spent on machine-controlled work (i.e., machining).

- 75 percent hand assembly/25 percent machining = 80 percent learning
- 50 percent hand assembly/50 percent machining = 85 percent
- 25 percent hand assembly/75 percent machining = 90 percent

Another set of guidelines based on what is seen in specific industries is the following:

- Aerospace, 85 percent
- Shipbuilding, 80–85 percent
- Complex machine tools for new models, 75–85 percent
- Repetitive electronics manufacturing, 90–95 percent
- Repetitive machining or punch-press operations, 90–95 percent
- Repetitive electrical operations (wiring and circuit board fabrication), 75–85 percent
- Repetitive welding operations, 90 percent
- Raw materials manufacturing, 93–96 percent
- Purchased parts fabrication, 85–88 percent

There are two reasons for disparities between a firm's learning rate and that of its industry. First, there are the inevitable differences in operating characteristics between any two firms, stemming from the equipment, methods, product design, plant organization, and so forth. Second, procedural differences are manifested in the development of the learning percentage itself, such as whether the industry rate is based on a single product or on a product line, and the manner in which the data were aggregated.

How Long Does Learning Go On?

Does output stabilize, or is there continual improvement? Some areas can be shown to improve continually even over decades (radios, computers, and other electronic devices; and, if we allow for the effects of inflation, also automobiles, washing machines, refrigerators, and most other manufactured goods). If the learning curve has been valid for several hundreds or thousands of units, it will probably be valid for several hundreds or thousands more. On the other hand, highly automated systems may have a near-zero learning curve because, after installation, they quickly reach a constant volume.

GENERAL GUIDELINES FOR LEARNING

In this section we offer guidelines for two categories of "learners": individuals and organizations.

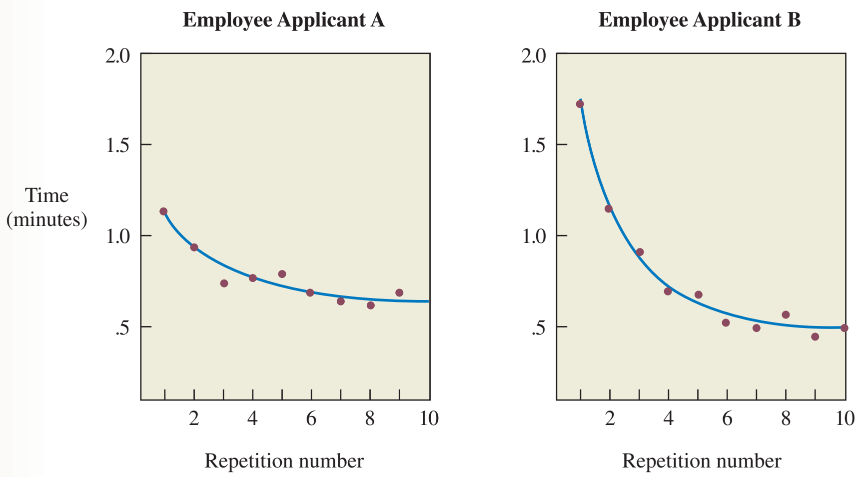
Individual Learning

A number of factors affect an individual's performance and rate of learning. Remember that there are two elements involved: the rate of learning and the initial starting level. To explain this more clearly, compare the two learning curves in Exhibit TN2.7. Suppose these were the times for two individuals who performed a simple mechanical test administered by the personnel department as part of their application for employment in the assembly area of manufacturing.

Which applicant would you hire? Applicant A had a much lower starting point but a slower learning rate. Applicant B, although starting at a much higher point, is clearly the

EXHIBIT TN2.7

Test Results of Two Job Applicants



better choice. This points out that performance times are important—not just the learning rate by itself.

Some general guidelines to improve individual performance based on learning curves include the following:

1 **Proper selection of workers.** A test should be administered to help choose the workers. These tests should be representative of the planned work: a dexterity test for assembly work, a mental ability test for mental work, tests for interaction with customers for front office work, and so on.
2 **Proper training.** The more effective the training, the faster the learning rate.
3 **Motivation.** Productivity gains based on learning curves are not achieved unless there is a reward. Rewards can be money (individual or group incentive plans) or nonmonetary (employee of the month awards, etc.).
4 **Work specialization.** As a general rule, the simpler the task, the faster the learning. Be careful that boredom doesn't interfere; if it does, redesign the task.
5 **Do one or very few jobs at a time.** Learning is faster on each job if completed one at a time, rather than working on all jobs simultaneously.
6 **Use tools or equipment that assists or supports performance.**
7 **Provide quick and easy access for help.** The benefits from training are realized and continue when assistance is available.
8 **Allow workers to help redesign their tasks.** Taking more performance factors into the scope of the learning curve can, in effect, shift the curve downward.

ORGANIZATIONAL LEARNING

Organizations learn as well. It has been argued that organizational learning is critical to sustaining a competitive advantage. For the individual, it is easy to conceptualize how knowledge is acquired and retained and how this results in an individual learning effect. Certainly, a main source of organizational learning is the individual learning of the employees. An organization also acquires knowledge in its technology, its structure, documents that it retains, and standard operating procedures.[4] For example, as a manufacturing unit becomes experienced, knowledge is embedded in software and in tooling used for production. Knowledge can also be embedded in the organization's structure. For example, when an organization shifts its industrial engineering group from a functional organization centralized in one area to a decentralized organization where individuals are deployed to particular parts of the plant floor, knowledge about how to become more productive is embedded in the organization's structure.

Knowledge can depreciate if individuals leave the organization. When Lockheed had problems in the production of the L-1011, it was blamed on the fact that the company hired 2,000 inexperienced employees to quickly ramp up production. These employees were put through a four-week training program in aircraft construction. Initial costs rose rather than fell during the initial production of the plane due to the inexperienced workers.

Knowledge can also depreciate if technologies become inaccessible or difficult to use. An example of this is the difficulty in accessing data collected by Landsat, an earth surveillance program. Ninety percent of the data collected before 1979 is now inaccessible because the data were recorded by equipment that no longer exists or cannot be operated. Knowledge can also depreciate if a company's records and routine processes are lost. When Steinway Piano Company decided to put a discontinued piano back into production, the plant discovered it no longer had records or blueprints for the piano.

LEARNING CURVES APPLIED TO HEART TRANSPLANT MORTALITY

Service

Learning curves provide an excellent means to examine performance. The best comparison for one's performance would be the learning rates for competitors in the industry. Even when a standard or expected level is unknown, much can still be learned by simply using and plotting data in a learning curve fashion. As an illustration of this ability to learn about one's performance, we present the experience of a heart transplant facility in a hospital.[5]

The learning curve model in the heart transplant analysis was of the form

$$Y_i = B_0 + B_1 x^{-B_2}$$

where Y_i is the cumulative average resource consumption (the total number of deaths, costs, and so on divided by the number of transplants), B_0 is the asymptote (the minimum), B_1 is the maximum possible reduction (the difference between the first unit and minimum B_0), x is the total number of units produced, and B_2 is the rate of change for each successive unit as it moves toward the lower bound.

Exhibit TN2.8 shows the coefficients that were obtained for the model. Exhibit TN2.9 shows the cumulative death rate. This seems to follow an industrial learning curve with a rate just over 80 percent. Seven of the first 23 transplant patients died within a year after transplant surgery. Only 4 of the next 39 patients died within a year. For the cumulative average length of stay, shown in Exhibit TN2.10, the reduction rate is approximately 9 percent.

The least sloping curve (the lowest learning rate) is the cost of heart transplants. Exhibit TN2.11 shows that the initial costs were in the vicinity of $150,000. After 51 surviving patients (62 procedures, 11 died), the average cost was still close to $100,000. (A learning rate of 80 percent would result in an average cost of $40,000; a 90 percent rate would result in a cost of $80,000.)

Why are learning rates high in death rate reduction and low in average length of stay, with the lowest rate in cost reduction? Smith and Larsson question whether the low learning rates may be related to conservatism in dealing with human lives. Or could it be due to the power and insulation of the heart transplant team from pressure to reduce cost? The purpose of this study on learning curves was to make institutions and administrators aware of learning. Institutions need to behave according to learning curve logic—that is, in pricing as well as in motivation for continuous improvement.

EXHIBIT TN2.8

Consumption Coefficients for Heart Transplant Learning Model

	B_0 (ASYMPTOTE)	B_1 (RANGE)	B_2 (RATE)	PERCENTAGE DECREASE
Death rate	.2329	.8815	.2362	21.04%
Length of stay	28.26	23.76	.0943	9.00
Units of service	1,282.84	592.311	.0763	7.35
Adjusted charges	$96,465.90	$53,015.80	.0667	6.45

EXHIBIT TN2.9

Death Rates, Less than One Year Survival

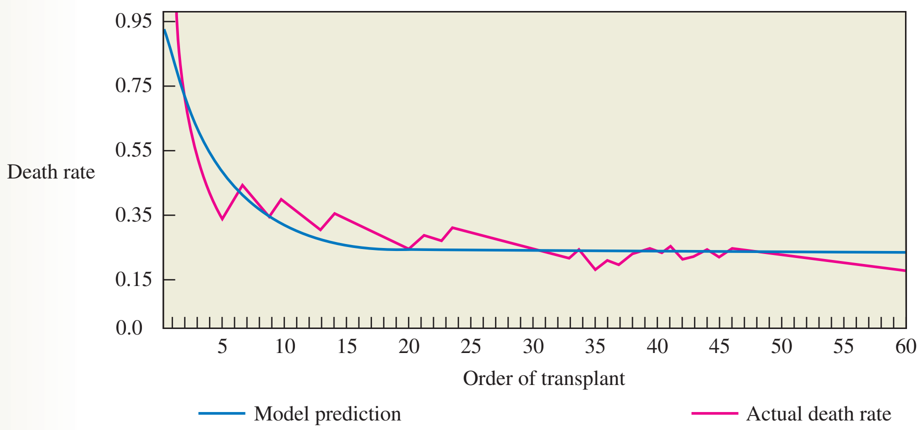


EXHIBIT TN2.10

Average Length of Stay (ALOS) for Heart Transplant Survivors

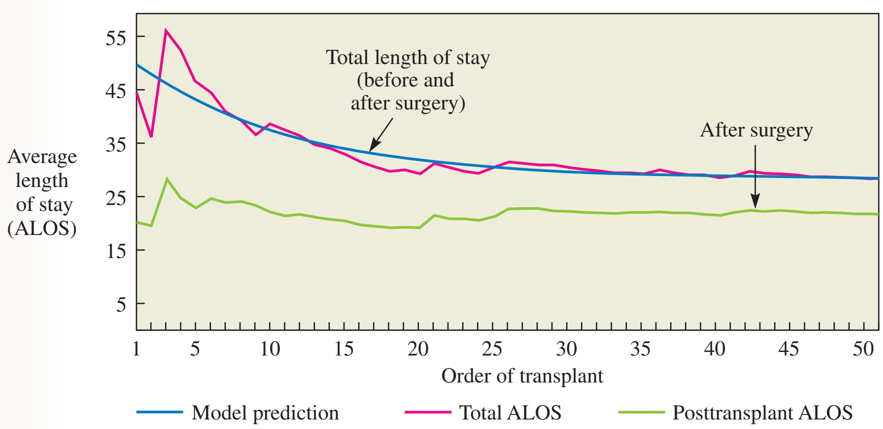
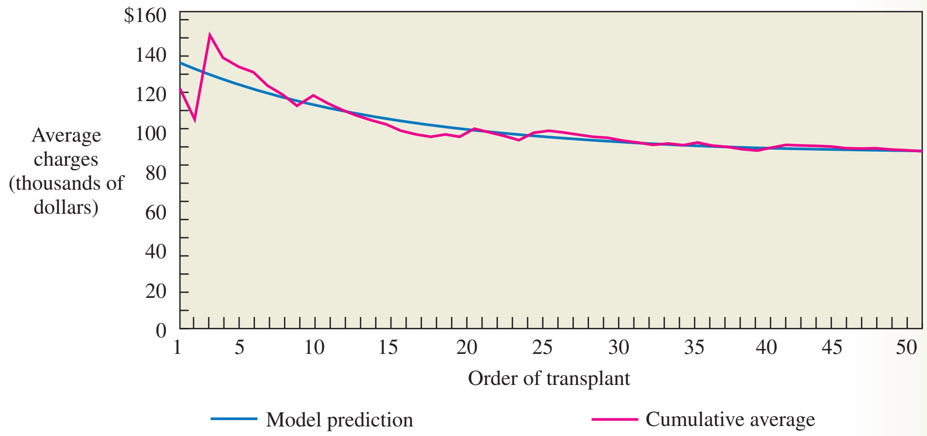


EXHIBIT TN2.11

Cost for Heart Transplant Survivors



NOTE: FOR TRANSPLANT ADMISSION ONLY, ACTUAL COSTS ARE APPROXIMATELY 50 PERCENT OF CHARGES.

KEY TERMS

Learning curve A line displaying the relationship between unit production time and the cumulative number of units produced.

Individual learning Improvement that results when people repeat a process and gain skill or efficiency from their own experience.

Organizational learning Improvement that comes both from experience and from changes in administration, equipment, and product design.

FORMULA REVIEW

Logarithmic curve:

[TN2.1] $$Y_x = Kx^n$$

SOLVED PROBLEMS

SOLVED PROBLEM 1

A job applicant is being tested for an assembly line position. Management feels that steady-state times have been approximately reached after 1,000 performances. Regular assembly line workers are expected to perform the task within four minutes.

a. If the job applicant performed the first test operation in 10 minutes and the second one in 9 minutes, should this applicant be hired?
b. What is the expected time that the job applicant would take to finish the 10th unit?
c. What is a significant limitation of this analysis?

Solution

a. Learning rate = 9 minutes/10 minutes = 90%
From Exhibit TN2.5, the time for the 1,000th unit is .3499 × 10 minutes = 3.499 minutes. Yes, hire the person.
b. From Exhibit TN2.5, unit 10 at 90% is .7047. Therefore, the time for the 10th unit = .7047 × 10 = 7.047 minutes.
c. More data should be collected on the job applicant's performance.

SOLVED PROBLEM 2

Boeing Aircraft collected the following cost data on the first 8 units of their new business jet.

UNIT NUMBER	COST ($ MILLIONS)	UNIT NUMBER	COST ($ MILLIONS)
1	$100	5	60
2	83	6	57
3	73	7	53
4	62	8	51

a. Estimate the learning curve for the new business jet.
b. Estimate the average cost for the first 1,000 units of the jet.
c. Estimate the cost to produce the 1,000th jet.

Solution

a. First, estimate the learning curve rate by calculating the average learning rate with each doubling of production.

Units 1 to 2 = 83/100 = 83%

Units 2 to 4 = 62/83 = 74.7%

Units 4 to 8 = 51/62 = 82.26%

Average = (83 + 74.4 + 82.6)/3 = 80%

b. The average cost of the first 1,000 units can be estimated using Exhibit TN2.6. The cumulative improvement factor for the 1,000th unit at 80 percent learning is 158.7. The cost to produce the first 1,000 units is

$$\$100M \times 158.7 = \$15,870M$$

The average cost for each of the first 1,000 units is

$$\$15,870M/1,000 = \$15.8M$$

c. To estimate the cost to produce the 1,000th unit use Exhibit TN2.5.
The unit improvement factor for the 1,000th unit at 80 percent is .1082.
The cost to produce the 1,000th unit is

$$\$100M \times .1082 = \$10.82M$$

Review and Discussion Questions

1 If you kept any of your old exam grades from last semester, get them out and write down the grades. Use Exhibits TN2.5 and TN2.6, use log-log graph paper, or use a spreadsheet to find whether the exponential curve fits showing that you experienced learning over the semester (insofar as your exam performance is concerned). If not, can you give some reasons why not?
2 How might the following business specialists use learning curves: accountants, marketers, financial analysts, personnel managers, and computer programmers?
3 As a manager, which learning percentage would you prefer (other things being equal), 110 percent or 60 percent? Explain.
4 What difference does it make if a customer wants a 10,000-unit order produced and delivered all at one time or in 2,500-unit batches?

Problems

1 A time standard was set as 0.20 hour per unit based on the 50th unit produced. If the task has a 90 percent learning curve, what would be the expected time of the 100th, 200th, and 400th unit?
2 You have just received 10 units of a special subassembly from an electronics manufacturer at a price of $250 per unit. A new order has also just come in for your company's product that uses these subassemblies, and you wish to purchase 40 more to be shipped in lots of 10 units each. (The subassemblies are bulky, and you need only 10 a month to fill your new order.)
a. Assuming a 70 percent learning curve by your supplier on a similar product last year, how much should you pay for each lot? Assume that the learning rate of 70 percent applies to each lot of 10 units, not each unit.
b. Suppose you are the supplier and can produce 20 units now but cannot start production on the second 20 units for two months. What price would you negotiate for the second 20 units?
3 Johnson Industries received a contract to develop and produce four high-intensity long-distance receiver/transmitters for cellular telephones. The first took 2,000 labor hours and $39,000 worth of purchased and manufactured parts; the second took 1,500 labor hours and

1. 100th = 0.18 hr.
200th = 0.16 hr.
400th = 0.15 hr.

2. *a.*

1st 10 units =	$2,500.00
2nd 10 units =	1,750.00
3rd 10 units =	1,420.50
4th 10 units =	1,225.00
5th 10 units =	1,092.00

b. Between max. of $4,250 and min. of $2,645.50.

$37,050 in parts; the third took 1,450 labor hours and $31,000 in parts; and the fourth took 1,275 labor hours and $31,492 in parts.

Johnson was asked to bid on a follow-on contract for another dozen receiver/transmitter units. Ignoring any forgetting factor effects, what should Johnson estimate time and parts costs to be for the dozen units? (Hint: There are two learning curves—one for labor and one for parts.)

3. LR parts, 90%, LR labor, 80%
Labor: 11,556 hours.
Parts: $330,876.

4 Lambda Computer Products competed for and won a contract to produce two prototype units of a new type of computer that is based on laser optics rather than on electronic binary bits.

The first unit produced by Lambda took 5,000 hours to produce and required $250,000 worth of material, equipment usage, and supplies. The second unit took 3,500 hours and used $200,000 worth of materials, equipment usage, and supplies. Labor is $30 per hour.

a. Lambda was asked to present a bid for 10 additional units as soon as the second unit was completed. Production would start immediately. What would this bid be?

b. Suppose there was a significant delay between the contracts. During this time, personnel and equipment were reassigned to other projects. Explain how this would affect the subsequent bid.

4. *a.* Labor, $570,150. Materials, 1,356,750 plus something for profit.
b. Need to consider forgetting and relearning. Time and cost could be much higher.

5 You've just completed a pilot run of 10 units of a major product and found the processing time for each unit was as follows:

UNIT NUMBER	TIME (HOURS)
1	970
2	640
3	420
4	380
5	320
6	250
7	220
8	207
9	190
10	190

a. According to the pilot run, what would you estimate the learning rate to be?

b. Based on *a*, how much time would it take for the next 190 units, assuming no loss of learning?

c. How much time would it take to make the 1,000th unit?

5. *a.* From log-log plot, LR = 60%
b. About 8,029 hours.
c. 6.0 hours.

6 Lazer Technologies Inc. (LTI) has produced a total of 20 high-power laser systems that could be used to destroy any approaching enemy missiles or aircraft. The 20 units have been produced, funded in part as private research within the research and development arm of LTI, but the bulk of the funding came from a contract with the U.S. Department of Defense (DoD).

Testing of the laser units has shown that they are effective defense weapons, and through redesign to add portability and easier field maintenance, the units could be truck-mounted.

DoD has asked LTI to submit a bid for 100 units.

The 20 units that LTI has built so far cost the following amounts and are listed in the order in which they were produced:

UNIT NUMBER	COST ($ MILLIONS)	UNIT NUMBER	COST ($ MILLIONS)
1	$12	11	$3.9
2	10	12	3.5
3	6	13	3.0
4	6.5	14	2.8
5	5.8	15	2.7
6	6	16	2.7
7	5	17	2.3
8	3.6	18	3.0
9	3.6	19	2.9
10	4.1	20	2.6

6. *a.* 70%
b. $145,956,000.
c. Cost $.0851 \times \$12 = \$1{,}021{,}200$.

a. Based on past experience, what is the learning rate?
b. What bid should LTI submit for the total order of 100 units, assuming that learning continues?
c. What is the cost expected to be for the last unit under the learning rate you estimated?

7. 4,710 hours.

7 Jack Simpson, contract negotiator for Nebula Airframe Company, is currently involved in bidding on a follow-up government contract. In gathering cost data from the first three units, which Nebula produced under a research and development contract, he found that the first unit took 2,000 labor hours, the second took 1,800 labor hours, and the third took 1,692 hours.

In a contract for three more units, how many labor hours should Simpson plan for?

8. Learning rate = 70%; unreasonable to ask for 4.5 hours. After 25, average repetitions time is about 3 hours.

8 Honda Motor Company has discovered a problem in the exhaust system of one of its automobile lines and has voluntarily agreed to make the necessary modifications to conform with government safety requirements. Standard procedure is for the firm to pay a flat fee to dealers for each modification completed.

Honda is trying to establish a fair amount of compensation to pay dealers and has decided to choose a number of randomly selected mechanics and observe their performance and learning rate. Analysis demonstrated that the average learning rate was 90 percent, and Honda then decided to pay a $60 fee for each repair (3 hours × $20 per flat-rate hour).

Southwest Honda, Inc., has complained to Honda Motor Company about the fee. Six mechanics, working independently, have completed two modifications each. All took 9 hours on the average to do the first unit and 6.3 hours to do the second. Southwest refuses to do any more unless Honda allows at least 4.5 hours. The dealership expects to perform the modification to approximately 300 vehicles.

What is your opinion of Honda's allowed rate and the mechanics' performance?

9. *a.* Cost of 22nd unit = $32,732.40.
b. 1,886 hours.
c. Average cost = $43,126.50.

9 United Research Associates (URA) had received a contract to produce two units of a new cruise missile guidance control. The first unit took 4,000 hours to complete and cost $30,000 in materials and equipment usage. The second took 3,200 hours and cost $21,000 in materials and equipment usage. Labor cost is charged at $18 per hour.

The prime contractor has now approached URA and asked to submit a bid for the cost of producing another 20 guidance controls.

a. What will the last unit cost to build?
b. What will be the average time for the 20 missile guidance controls?
c. What will the average cost be for guidance control for the 20 in the contract?

10. *a.* If first unit is 100 minutes, the learning rate needs to be 75%, not 80% (80/100). Do not hire.
b. See ISM.

10 United Assembly Products (UAP) has a personnel screening process for job applicants to test their ability to perform at the department's long-term average rate. UAP has asked you to modify the test by incorporating learning theory. From the company's data, you discovered that if people can perform a given task in 30 minutes or less on the 20th unit, they achieve the group long-run average. Obviously, all job applicants cannot be subjected to 20 performances of such a task, so you are to determine whether they will likely achieve the desired rate based on only 2 performances.

a. Suppose a person took 100 minutes on the first unit and 80 minutes on the second. Should this person be hired?
b. What procedure might you establish for hiring (i.e., how to evaluate the job applicant's two performances)?
c. What is a significant limitation of this analysis?

11. *a.* 3rd = 35.1 hrs.
b. Average = 7.9 hrs. each; well worth it.

11 A potentially large customer offered to subcontract assembly work that is profitable only if you can perform the operations at an average time of less than 20 hours each. The contract is for 1,000 units.

You run a test and do the first one in 50 hours and the second one in 40 hours.

a. How long would you expect the third one to take?
b. Would you take the contract? Explain.

12. 11th 2.4476/.9 = $2.7196 million
12th 2.3953/.9 = $2.6615 million
$5.3811 million total

12 Western Turbine, Inc., has just completed the production of the 10th unit of a new high-efficiency turbine/generator. Its analysis showed that a learning rate of 85 percent existed over the production of the 10 units. If the 10th unit contained labor costs of $2.5 million, what price should Western Turbine charge for labor on the units 11 and 12 to make a profit of 10 percent of the selling price?

13. *a.* 42.165 minutes.
b. 35.56 minutes.

13 FES Auto has recently hired Meg the mechanic to specialize in front-end alignments. Although she is a trained auto mechanic, she has not used FES's brand of equipment before taking this job. The standard time allocated for a front-end alignment is 30 minutes. Her first front-end alignment took 50 minutes and her second 47.5 minutes.

a. What is the expected time for Meg's 10th front-end alignment?
b. What is the expected time for Meg's 100th front-end alignment?

14 An initial pilot run of 10 units produces the following times:

UNIT NUMBER	TIME (MINUTES)
1	39
2	29
3	23
4	19
5	17
6	16
7	15
8	13
9	13
10	12

a. According to this pilot run, what is your estimate of the learning rate?
b. How much time will it take for the next 90 units?
c. How much time will it take to make the 2,000th unit?

14. *a.* 70%
b. 501.5 minutes.
c. .78 minute.

15 A new bank clerk needed an hour to encode his first 500 checks, 51 minutes for the second 500, and 46 minutes for the third 500. After how many batches of 500 checks will he be able to work at the standard rate of 1,000 checks per hour?

15. 85% learning, .4930 hour after 20 batches.

16 A fast-food trainee takes an hour to prepare his first 20 sandwiches, 45 minutes for the second 20, and 38 minutes for the third 20. What will his production rate be after 24 hours of experience?

16. 75% learning, 100 batches in 24 hours, .1479 hour/batch, or 135 sandwiches per hour.

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FOOTNOTES

1 See the classic paper by T. P. Wright, "Factors Affecting the Cost of Airplanes," *Journal of the Aeronautical Sciences,* February 1936, pp. 122–28.

2 This equation says that the number of direct labor hours required for any given unit is reduced exponentially as more units are produced.

3 Rodney D. Stewart, Richard M. Wyskida, and James D. Johannes (eds.), *Cost Estimator's Reference Manual,* 2nd ed. (New York: John Wiley & Sons, 1995).

4 See L. Argote, "Organizational Learning Curves: Persistence, Transfer and Turnover," *International Journal of Technology Management* 11, no. 7, 8 (1996), pp. 759–69.

5 D. B. Smith, and J. L. Larsson, "The Impact of Learning on Cost: The Case of Heart Transplantation," *Hospital and Health Sciences Administration* 34, no. 1 (Spring 1989), pp. 85–97.