

technical note

technical note seven
**PROCESS CAPABILITY AND
 STATISTICAL QUALITY
 CONTROL**

TECHNICAL NOTE SEVEN

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IN MONITORING A PROCESS USING SQC, WORKERS TAKE A SAMPLE WHERE THE DIAMETERS ARE MEASURED AND THE SAMPLE MEAN IS CALCULATED AND PLOTTED. INVESTMENTS IN MACHINERY, TECHNOLOGY, AND EDUCATION ARE DESIGNED TO REDUCE THE NUMBER OF DEFECTS THAT THE PROCESS PRODUCES.



Vol. I

"Quality"

Vol. VII

"Manufacturing Quality at Honda"

"A Day in the Life of Quality at Honda"

"SPC at Honda"



This technical note on statistical quality control (SQC) covers the quantitative aspects of quality management. In general, SQC is a number of different techniques designed to evaluate quality from a conformance view. That is, how well are we doing at meeting the specifications that have been set during the design of the parts or services that we are providing? Managing quality performance using SQC techniques usually involves periodic sampling of a process and analysis of these data using statistically derived performance criteria.

As you will see, SQC can be applied to both manufacturing and service processes. Here are some examples of the types of situations where SQC can be applied:

- How many paint defects are there in the finish of a car? Have we improved our painting process by installing a new sprayer?
- How long does it take to execute market orders in our Web-based trading system? Has the installation of a new server improved the service? Does the performance of the system vary over the trading day?
- How well are we able to maintain the dimensional tolerance on our three-inch ball bearing assembly? Given the variability of our process for making this ball bearing, how many defects would we expect to produce per million bearings that we make?
- How long does it take for customers to be served from our drive-through window during the busy lunch period?



Processes that provide goods and services usually exhibit some variation in their output. This variation can be caused by many factors, some of which we can control and others that are inherent in the process. Variation that is caused by factors that can be clearly identified and possibly even managed is called **assignable variation**. For example, variation caused by workers not being equally trained or by improper machine adjustment is assignable variation. Variation that is inherent in the process itself is called **common variation**. Common variation is often referred to as *random variation* and may be the result of the type of equipment used to complete a process, for example.

As the title of this technical note implies, this material requires an understanding of very basic statistics. Recall from your study of statistics involving numbers that are normally distributed the definition of the mean and standard deviation. The mean is just the average value of a set of numbers. Mathematically this is

[TN7.1]

$$\bar{X} = \sum_{i=1}^N x_i / N$$

Assignable variation

Common variation

where:

x_i = Observed value

N = Total number of observed values

The standard deviation is

$$[\text{TN7.2}] \quad \sigma = \sqrt{\frac{\sum_{i=1}^N (x_i - \bar{X})^2}{N}}$$

In monitoring a process using SQC, samples of the process output would be taken, and sample statistics calculated. The distribution associated with the samples should exhibit the same kind of variability as the actual distribution of the process, although the actual variance of the sampling distribution would be less. This is good because it allows the quick detection of changes in the actual distribution of the process. The purpose of sampling is to find when the process has changed in some nonrandom way, so that the reason for the change can be quickly determined.

In SQC terminology, *sigma* is often used to refer to the sample standard deviation. As you will see in the examples, sigma is calculated in a few different ways, depending on the underlying theoretical distribution (i.e., a normal distribution or a Poisson distribution).

VARIATION AROUND US

● ● ● It is generally accepted that as variation is reduced, quality is improved. Sometimes that knowledge is intuitive. If a train is always on time, schedules can be planned more precisely. If clothing sizes are consistent, time can be saved by ordering from a catalog. But rarely are such things thought about in terms of the value of low variability. With engineers, the knowledge is better defined. Pistons must fit cylinders, doors must fit openings, electrical components must be compatible, and boxes of cereal must have the right amount of raisins—otherwise quality will be unacceptable and customers will be dissatisfied.

However, engineers also know that it is impossible to have zero variability. For this reason, designers establish specifications that define not only the target value of something but also acceptable limits about the target. For example, if the aim value of a dimension is 10 inches, the design specifications might then be 10.00 inches ± 0.02 inch. This would tell the manufacturing department that, while it should aim for exactly 10 inches, anything between 9.98 and 10.02 inches is OK. These design limits are often referred to as the **upper and lower specification limits** or the **upper and lower tolerance limits**.

Upper and lower specification or tolerance limits

A traditional way of interpreting such a specification is that any part that falls within the allowed range is equally good, whereas any part falling outside the range is totally bad. This is illustrated in Exhibit TN7.1. (Note that the cost is zero over the entire specification range, and then there is a quantum leap in cost once the limit is violated.)

Genichi Taguchi, a noted quality expert from Japan, has pointed out that the traditional view illustrated in Exhibit TN7.1 is nonsense for two reasons:

- 1 From the customer's view, there is often practically no difference between a product just inside specifications and a product just outside. Conversely, there is a far greater difference in the quality of a product that is the target and the quality of one that is near a limit.
- 2 As customers get more demanding, there is pressure to reduce variability. However, Exhibit TN7.1 does not reflect this logic.

Taguchi suggests that a more correct picture of the loss is shown in Exhibit TN7.2. Notice that in this graph the cost is represented by a smooth curve. There are dozens of

EXHIBIT TN7.1

A Traditional View of the Cost of Variability

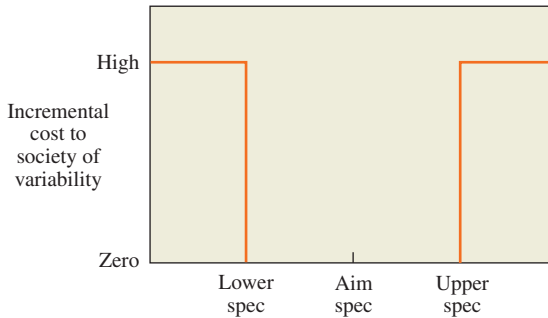
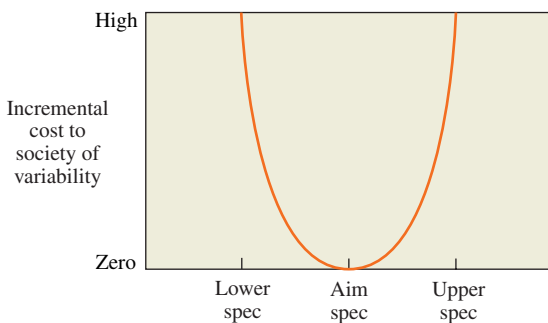


EXHIBIT TN7.2

Taguchi's View of the Cost of Variability



illustrations of this notion: the meshing of gears in a transmission, the speed of photographic film, the temperature in a workplace or department store. In nearly anything that can be measured, the customer sees not a sharp line, but a gradation of acceptability away from the “Aim” specification. Customers see the loss function as Exhibit TN7.2 rather than Exhibit TN7.1.

Of course, if products are consistently scrapped when they are outside specifications, the loss curve flattens out in most cases at a value equivalent to scrap cost in the ranges outside specifications. This is because such products, theoretically at least, will never be sold so there is no external cost to society. However, in many practical situations, either the process is capable of producing a very high percentage of product within specifications and 100 percent checking is not done, or if the process is not capable of producing within specifications, 100 percent checking is done and out-of-spec products can be reworked to bring them within specs. In any of these situations, the parabolic loss function is usually a reasonable assumption.

PROCESS CAPABILITY

● ● ● Taguchi argues that being within tolerance is not a yes/no decision, but rather a continuous function. The Motorola quality experts, on the other hand, argue that the process used to produce a good or deliver a service should be so good that the probability of generating a defect should be very, very low. Motorola made process capability and product design famous by adopting six-sigma limits. When we design a part, we specify that certain dimensions should be within the upper and lower tolerance limits.

As a simple example, assume that we are designing a bearing for a rotating shaft—say an axle for the wheel of a car. There are many variables involved for both the bearing and the axle—for example, the width of the bearing, the size of the rollers, the size of the axle, the length of the axle, how it is supported, and so on. The designer specifies tolerances for each of these variables to ensure that the parts will fit properly. Suppose that initially a

design is selected and the diameter of the bearing is set at 1.250 inches ± 0.005 inch. This means that acceptable parts may have a diameter that varies between 1.245 and 1.255 inches (which are the lower and upper tolerance limits).

Next, consider the process in which the bearing will be made. Let's say that by running some tests, we determine the machine output to have a standard deviation or sigma equal to 0.002 inch. What this means is that our process does not make each bearing exactly the same size. Assume that we are monitoring the process such that any bearings that are more than three standard deviations (± 0.006 inch) above or below 1.250 inches are rejected. This means that we will produce parts that vary between 1.244 and 1.256 inches. As we can see, our process limits are greater than the tolerance limits specified by our designer. This is not good, because we will produce some parts that do not meet specifications.

Motorola insists that a process making a part must be capable of operating so that the design tolerances are six standard deviations away from the process mean. For our bearing, this would mean that our process variation would need to be less than or equal to 0.00083 inch (remember our tolerance was ± 0.005 , which, when divided by 6, is 0.00083). To reduce the variation in the process, we would need to find some better method for controlling the formation of the bearing. Of course, another option would be to redesign the axle assembly so that such perfect bearings are not needed.

We can show the six-sigma limits using an exhibit. Assume that we have changed the process to produce with 0.00083 variation. Now, the design limits and the process limits are acceptable according to Motorola standards. Let's assume that the bearing diameter follows a bell-shaped normal distribution as in Exhibit TN7.3. From our knowledge of the normal distribution, we know that 99.7 percent of the bell-shaped curve falls within ± 3 sigma. We would expect only about three parts in 1,000 to fall outside of the three-sigma limits. The tolerance limits are another three sigma out from these control limits! In this case, the actual number of parts we would expect to produce outside the tolerance limits is only two parts per *billion*!

Suppose the central value of the process output shifts away from the mean. Exhibit TN7.4 shows the mean shifted one standard deviation closer to the upper specification limit. This

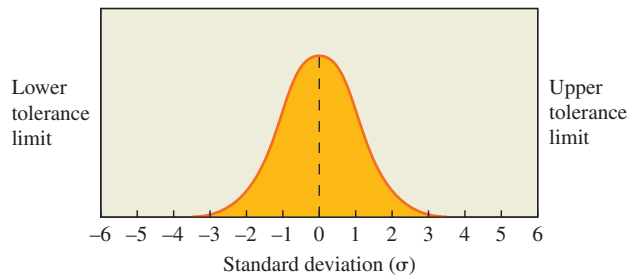
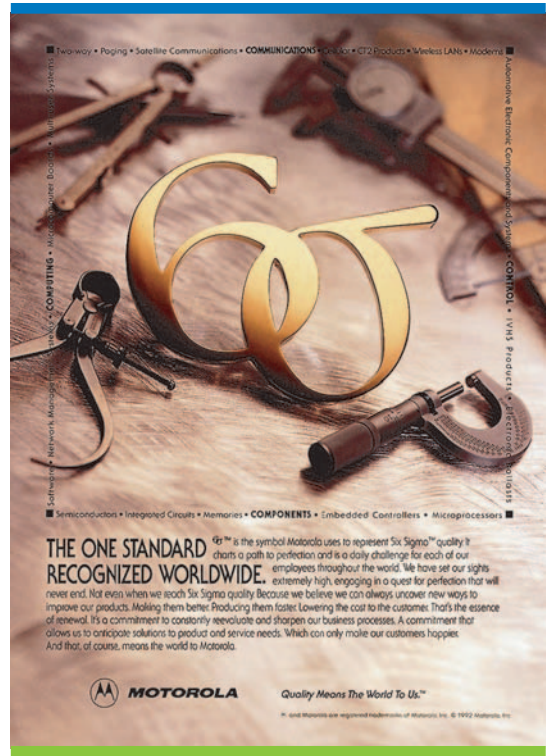


EXHIBIT TN7.3

Process Capability

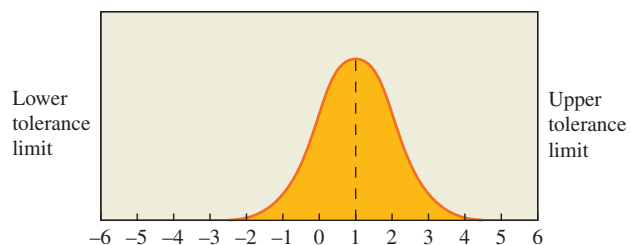


EXHIBIT TN7.4

Process Capability with a Shift in the Process Mean

causes a slightly higher number of expected defects, about four parts per million. This is still pretty good, by most people's standards. We use a calculation called the *capability index* to measure how well our process is capable of producing relative to the design tolerances. We describe how to calculate this index in the next section.

Capability index (C_{pk})

CAPABILITY INDEX (C_{pk})

The **capability index** (C_{pk}) shows how well the parts being produced fit into the range specified by the design limits. If the design limits are larger than the three sigma allowed in the process, then the mean of the process can be allowed to drift off-center before readjustment, and a high percentage of good parts will still be produced.

Referring to Exhibits TN7.3 and TN7.4, the capability index (C_{pk}) is the position of the mean and tails of the process relative to design specifications. The more off-center, the greater the chance to produce defective parts.

Because the process mean can shift in either direction, the direction of shift and its distance from the design specification set the limit on the process capability. The direction of shift is toward the smaller number.

Formally stated, the capability index (C_{pk}) is calculated as the smaller number as follows:

$$[TN7.3] \quad C_{pk} = \min \left[\frac{\bar{X} - LTL}{3\sigma} \quad \text{or} \quad \frac{UTL - \bar{X}}{3\sigma} \right]$$

For simplicity, let's assume our process mean is one inch and $\sigma = .001$ (σ is the symbol for standard deviation). Further, the process mean is exactly in the center as in Exhibit TN7.3. Then for $\bar{X} = 1.000$

$$C_{pk} = \min \left[\frac{1.000 - .994}{3(.001)} \quad \text{or} \quad \frac{1.006 - 1.000}{3(.001)} \right] = \\ = \min \left[\frac{.006}{.003} = 2 \quad \text{or} \quad \frac{.006}{.003} = 2 \right]$$

Because the mean is in the center, the two calculations are the same and equal to 2. If the mean shifted to $+1.5\sigma$ or 1.0015, then for $\bar{X} = 1.0015$

$$C_{pk} = \min \left[\frac{1.0015 - .994}{3(.001)} \quad \text{or} \quad \frac{1.006 - 1.0015}{3(.001)} \right] = \\ = \min [2.5 \quad \text{or} \quad 1.5]$$

$C_{pk} = 1.5$, which is the smaller number.

This tells us that the process mean has shifted to the right similar to Exhibit TN7.4, but parts are still well within design limits.

Assuming that the process is producing with a consistent standard deviation and the process is centered exactly between the design limits, as in Exhibit TN7.3, Birch calculated the fraction of defective units that would fall outside various design limits as follows:¹

DESIGN LIMITS	DEFECTIVE PARTS	FRACTION DEFECTIVE
$\pm 1\sigma$	317 per thousand	.3173
$\pm 2\sigma$	45 per thousand	.0455
$\pm 3\sigma$	2.7 per thousand	.0027
$\pm 4\sigma$	63 per million	.000063
$\pm 5\sigma$	574 per billion	.00000574
$\pm 6\sigma$	2 per billion	.00000002

Motorola's design limit of six sigma with a shift of the process off the mean by 1.5σ ($C_{pk} = 1.5$) gives 3.4 defects per million. If the mean is exactly in the center ($C_{pk} = 2$), then 2 defects per *billion* are expected, as the table above shows.

PROCESS CONTROL PROCEDURES

● ● ● Process control is concerned with monitoring quality *while the product or service is being produced*. Typical objectives of process control plans are to provide timely information on whether currently produced items are meeting design specifications and to detect shifts in the process that signal that future products may not meet specifications. **Statistical process control (SPC)** involves testing a random sample of output from a process to determine whether the process is producing items within a preselected range.

Statistical process control (SPC)

The examples given so far have all been based on quality characteristics (or *variables*) that are measurable, such as the diameter or weight of a part. **Attributes** are quality characteristics that are classified as either conforming or not conforming to specification. Goods or services may be observed to be either good or bad, or functioning or malfunctioning. For example, a lawnmower either runs or it doesn't; it attains a certain level of torque and horsepower or it doesn't. This type of measurement is known as sampling by attributes. Alternatively, a lawnmower's torque and horsepower can be measured as an amount of deviation from a set standard. This type of measurement is known as sampling by variables. The following section describes some standard approaches to controlling processes: first an approach useful for attribute measures and then an approach for variable measures. Both of these techniques result in the construction of control charts. Exhibit TN7.5 shows some examples for how control charts can be analyzed to understand how a process is operating.

Attributes



PROCESS CONTROL WITH ATTRIBUTE MEASUREMENTS: USING p CHARTS

Measurement by attributes means taking samples and using a single decision—the item is good, or it is bad. Because it is a yes or no decision, we can use simple statistics to create a p chart with an upper control limit (UCL) and a lower control limit (LCL). We can draw these control limits on a graph and then plot the fraction defective of each individual sample tested. The process is assumed to be working correctly when the samples, which are taken periodically during the day, continue to stay between the control limits.

$$[\text{TN}7.4] \quad \bar{p} = \frac{\text{Total number of defects from all samples}}{\text{Number of samples} \times \text{Sample size}}$$

$$[\text{TN}7.5] \quad s_p = \sqrt{\frac{\bar{p}(1 - \bar{p})}{n}}$$

$$[\text{TN}7.6] \quad \text{UCL} = \bar{p} + z s_p$$

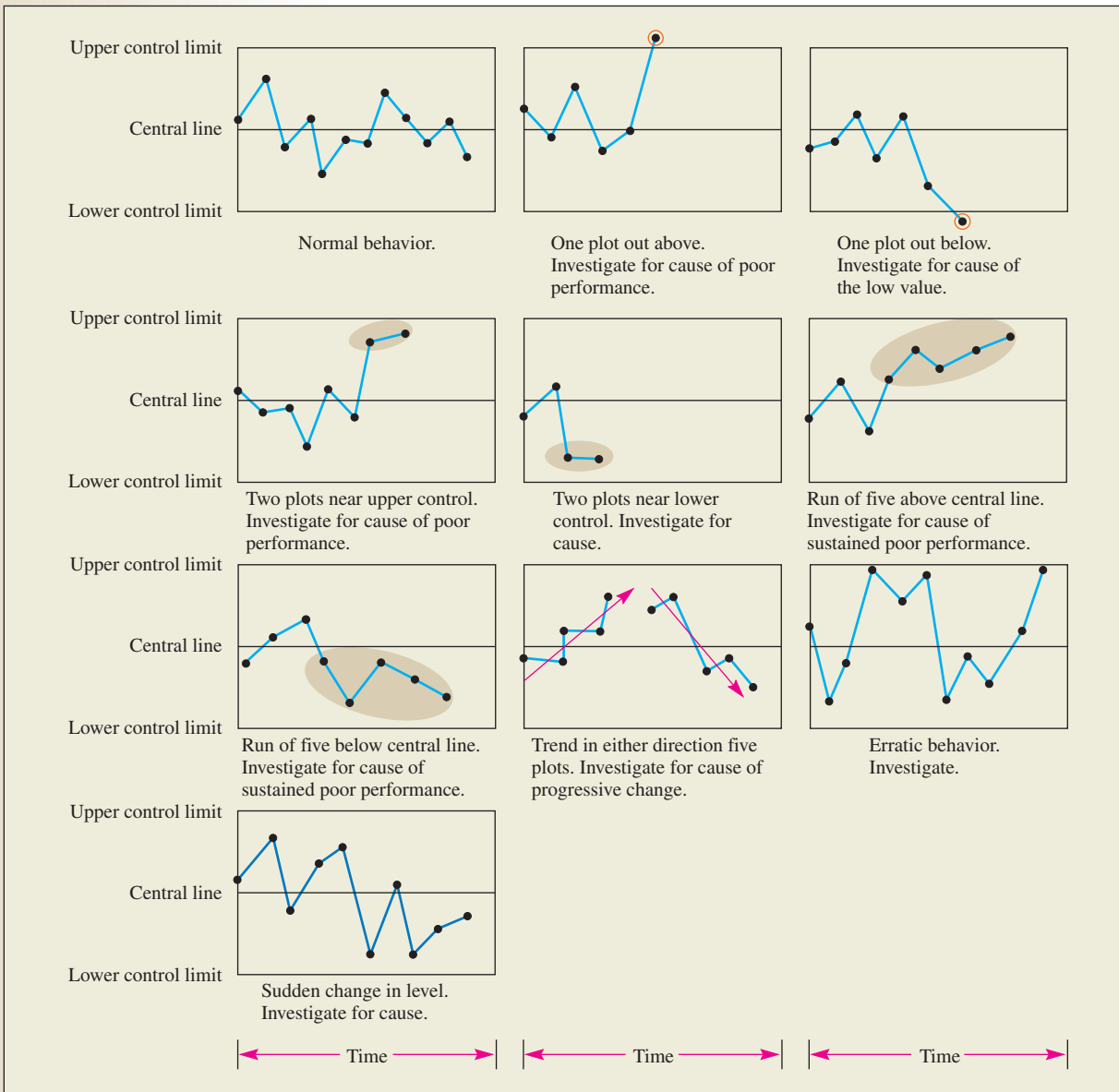
$$[\text{TN}7.7] \quad \text{LCL} = \bar{p} - z s_p$$

where \bar{p} is the fraction defective, s_p is the standard deviation, n is the sample size, and z is the number of standard deviations for a specific confidence. Typically, $z = 3$ (99.7 percent confidence) or $z = 2.58$ (99 percent confidence) is used.

Size of the Sample The size of the sample must be large enough to allow counting of the attribute. For example, if we know that a machine produces 1 percent defects, then a sample size of five would seldom capture a defect. A rule of thumb when setting up a p chart is to make the sample large enough to expect to count the attribute twice in each

EXHIBIT TN7.5

Control Chart Evidence for Investigation



sample. So an appropriate sample size if the defect rate were approximately 1 percent would be 200 units. One final note: In the calculations shown in equations TN7.4–7.7 the assumption is that the sample size is fixed. The calculation of the standard deviation depends on this assumption. If the sample size varies, the standard deviation and upper and lower control limits should be recalculated for each sample.

EXAMPLE TN7.1: Control Chart Design

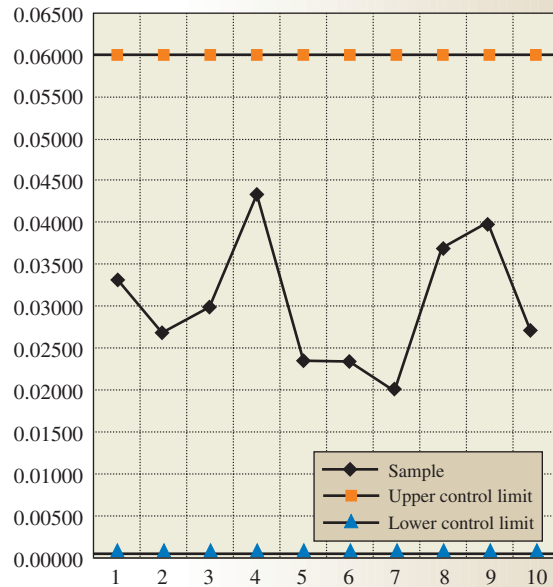


An insurance company wants to design a control chart to monitor whether insurance claim forms are being completed correctly. The company intends to use the chart to see if improvements in the design of the form are effective. To start the process the company collected data on the number of incorrectly completed claim forms over the past 10 days. The insurance company processes thousands of these forms each day, and due to the high cost of inspecting each form, only a small representative sample was collected each day. The data and analysis are shown in Exhibit TN7.6.

Insurance Company Claim Form

EXHIBIT TN7.6

SAMPLE	NUMBER INSPECTED	NUMBER OF FORMS COMPLETED INCORRECTLY	FRACTION DEFECTIVE
1	300	10	0.03333
2	300	8	0.02667
3	300	9	0.03000
4	300	13	0.04333
5	300	7	0.02333
6	300	7	0.02333
7	300	6	0.02000
8	300	11	0.03667
9	300	12	0.04000
10	300	8	0.02667
Totals	3000	91	0.03033
Sample standard deviation			0.00990



SOLUTION

To construct the control chart, first calculate the overall fraction defective from all samples. This sets the centerline for the control chart.

$$\bar{p} = \frac{\text{Total number of defects from all samples}}{\text{Number of samples} \times \text{Sample size}} = \frac{91}{3000} = .03033$$

Next calculate the sample standard deviation:

$$s_p = \sqrt{\frac{\bar{p}(1 - \bar{p})}{n}} = \sqrt{\frac{.03033(1 - .03033)}{300}} = .00990$$

Finally, calculate the upper and lower control limits. A z-value of 3 gives 99.7 percent confidence that the process is within these limits.

$$\text{UCL} = \bar{p} + 3s_p = .03033 + 3(.00990) = .06004$$

$$\text{LCL} = \bar{p} - 3s_p = .03033 - 3(.00990) = .00063$$

The calculations in Exhibit TN7.6, including the control chart, are included in the spreadsheet SPC.xls. ●



PROCESS CONTROL WITH VARIABLE MEASUREMENTS: USING \bar{X} AND R CHARTS

\bar{X} and R (range) charts are widely used in statistical process control.

In attribute sampling, we determine whether something is good or bad, fits or doesn't fit—it is a go/no-go situation. In **variables** sampling, however, we measure the actual weight, volume, number of inches, or other variable measurements, and we develop control charts to determine the acceptability or rejection of the process based on those measurements. For example, in attribute sampling we might decide that if something is over 10 pounds we will reject it and under 10 pounds we will accept it. In variable sampling we

Variables

PROCESS CONTROL CHARTS CAN BE GENERATED WITH COMPUTERS OR MANUALLY. SAMPLES ARE TAKEN FROM THE PROCESS AT STATED TIME INTERVALS AND THEIR AVERAGE PARAMETER VALUES ARE PLOTTED ON THE CHARTS. AS LONG AS THESE VALUES REMAIN WITHIN ESTABLISHED CONTROL LIMITS, THE PROCESS IS IN CONTROL.



measure a sample and may record weights of 9.8 pounds or 10.2 pounds. These values are used to create or modify control charts and to see whether they fall within the acceptable limits.

There are four main issues to address in creating a control chart: the size of the samples, number of samples, frequency of samples, and control limits.

Size of Samples For industrial applications in process control involving the measurement of variables, it is preferable to keep the sample size small. There are two main reasons. First, the sample needs to be taken within a reasonable length of time; otherwise, the process might change while the samples are taken. Second, the larger the sample, the more it costs to take.

Sample sizes of four or five units seem to be the preferred numbers. The *means* of samples of this size have an approximately normal distribution, no matter what the distribution of the parent population looks like. Sample sizes greater than five give narrower control limits and thus more sensitivity. For detecting finer variations of a process, it may be necessary, in fact, to use larger sample sizes. However, when sample sizes exceed 15 or so, it would be better to use \bar{X} charts with standard deviation σ rather than \bar{X} charts with the range R as we use in Example TN7.2.

Number of Samples Once the chart has been set up, each sample taken can be compared to the chart and a decision can be made about whether the process is acceptable. To set up the charts, however, prudence and statistics suggest that 25 or so samples be taken.

Frequency of Samples How often to take a sample is a trade-off between the cost of sampling (along with the cost of the unit if it is destroyed as part of the test) and the benefit of adjusting the system. Usually, it is best to start off with frequent sampling of a process and taper off as confidence in the process builds. For example, one might start with a sample of five units every half hour and end up feeling that one sample per day is adequate.

Control Limits Standard practice in statistical process control for variables is to set control limits three standard deviations above the mean and three standard deviations below. This means that 99.7 percent of the sample means are expected to fall within these control limits (that is, within a 99.7 percent confidence interval). Thus, if one sample mean falls outside this obviously wide band, we have strong evidence that the process is out of control.

HOW TO CONSTRUCT \bar{X} AND R CHARTS

If the standard deviation of the process distribution is known, the \bar{X} chart may be defined:

$$[\text{TN7.8}] \quad \text{UCL}_{\bar{X}} = \bar{\bar{X}} + z s_{\bar{X}} \quad \text{and} \quad \text{LCL}_{\bar{X}} = \bar{\bar{X}} - z s_{\bar{X}}$$

where

$s_{\bar{X}} = s/\sqrt{n}$ = Standard deviation of sample means

s = Standard deviation of the process distribution

n = Sample size

$\bar{\bar{X}}$ = Average of sample means or a target value set for the process

z = Number of standard deviations for a specific confidence level (typically, $z = 3$)

An \bar{X} chart is simply a plot of the means of the samples that were taken from a process. \bar{X} is the average of the means.

In practice, the standard deviation of the process is not known. For this reason, an approach that uses actual sample data is commonly used. This practical approach is described in the next section.

An R chart is a plot of the range within each sample. The range is the difference between the highest and the lowest numbers in that sample. R values provide an easily calculated measure of variation used like a standard deviation. An \bar{R} chart is the average of the range of each sample. More specifically defined, these are

$$[\text{Same as TN7.1}] \quad \bar{X} = \frac{\sum_{i=1}^n X_i}{n}$$

where

\bar{X} = Mean of the sample

i = Item number

n = Total number of items in the sample

$$[\text{TN7.9}] \quad \bar{\bar{X}} = \frac{\sum_{j=1}^m \bar{X}_j}{m}$$

where

$\bar{\bar{X}}$ = The average of the means of the samples

j = Sample number

m = Total number of samples

R_j = Difference between the highest and lowest measurement in the sample

\bar{R} = Average of the measurement differences R for all samples, or

$$[\text{TN7.10}] \quad \bar{R} = \frac{\sum_{j=1}^m R_j}{m}$$

E. L. Grant and R. Leavenworth computed a table (Exhibit TN7.7) that allows us to easily compute the upper and lower control limits for both the \bar{X} chart and the R chart.² These are defined as

$$[\text{TN7.11}] \quad \text{Upper control limit for } \bar{X} = \bar{\bar{X}} + A_2 \bar{R}$$

$$[\text{TN7.12}] \quad \text{Lower control limit for } \bar{X} = \bar{\bar{X}} - A_2 \bar{R}$$

$$[\text{TN7.13}] \quad \text{Upper control limit for } R = D_4 \bar{R}$$

$$[\text{TN7.14}] \quad \text{Lower control limit for } R = D_3 \bar{R}$$

EXHIBIT TN7.7

Factor for Determining from \bar{R} the Three-Sigma Control Limits for \bar{X} and R Charts

NUMBER OF OBSERVATIONS IN SUBGROUP <i>n</i>	FACTOR FOR \bar{X} CHART A_2	FACTORS FOR R CHART	
		LOWER CONTROL LIMIT D_3	UPPER CONTROL LIMIT D_4
2	1.88	0	3.27
3	1.02	0	2.57
4	0.73	0	2.28
5	0.58	0	2.11
6	0.48	0	2.00
7	0.42	0.08	1.92
8	0.37	0.14	1.86
9	0.34	0.18	1.82
10	0.31	0.22	1.78
11	0.29	0.26	1.74
12	0.27	0.28	1.72
13	0.25	0.31	1.69
14	0.24	0.33	1.67
15	0.22	0.35	1.65
16	0.21	0.36	1.64
17	0.20	0.38	1.62
18	0.19	0.39	1.61
19	0.19	0.40	1.60
20	0.18	0.41	1.59

Upper control limit for $\bar{X} = UCL_{\bar{X}} = \bar{\bar{X}} + A_2\bar{R}$

Lower control limit for $\bar{X} = LCL_{\bar{X}} = \bar{\bar{X}} - A_2\bar{R}$

Upper control limit for R = $UCL_R = D_4\bar{R}$

Lower control limit for R = $LCL_R = D_3\bar{R}$

Note: All factors are based on the normal distribution.

EXHIBIT TN7.8

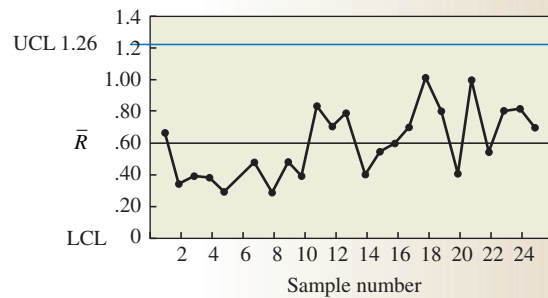
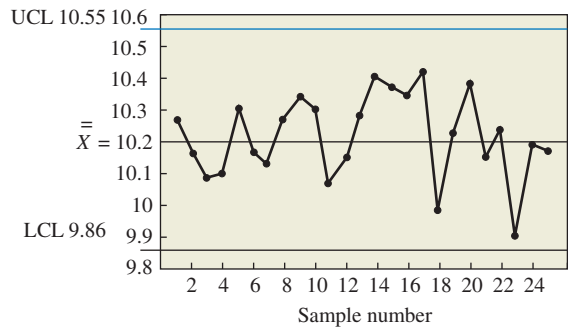
Measurements in Samples of Five from a Process

SAMPLE NUMBER	EACH UNIT IN SAMPLE					AVERAGE \bar{X}	RANGE R
1	10.60	10.40	10.30	9.90	10.20	10.28	.70
2	9.98	10.25	10.05	10.23	10.33	10.17	.35
3	9.85	9.90	10.20	10.25	10.15	10.07	.40
4	10.20	10.10	10.30	9.90	9.95	10.09	.40
5	10.30	10.20	10.24	10.50	10.30	10.31	.30
6	10.10	10.30	10.20	10.30	9.90	10.16	.40
7	9.98	9.90	10.20	10.40	10.10	10.12	.50
8	10.10	10.30	10.40	10.24	10.30	10.27	.30
9	10.30	10.20	10.60	10.50	10.10	10.34	.50
10	10.30	10.40	10.50	10.10	10.20	10.30	.40
11	9.90	9.50	10.20	10.30	10.35	10.05	.85
12	10.10	10.36	10.50	9.80	9.95	10.14	.70
13	10.20	10.50	10.70	10.10	9.90	10.28	.80
14	10.20	10.60	10.50	10.30	10.40	10.40	.40
15	10.54	10.30	10.40	10.55	10.00	10.36	.55
16	10.20	10.60	10.15	10.00	10.50	10.29	.60
17	10.20	10.40	10.60	10.80	10.10	10.42	.70
18	9.90	9.50	9.90	10.50	10.00	9.96	1.00
19	10.60	10.30	10.50	9.90	9.80	10.22	.80
20	10.60	10.40	10.30	10.40	10.20	10.38	.40
21	9.90	9.60	10.50	10.10	10.60	10.14	1.00
22	9.95	10.20	10.50	10.30	10.20	10.23	.55
23	10.20	9.50	9.60	9.80	10.30	9.88	.80
24	10.30	10.60	10.30	9.90	9.80	10.18	.80
25	9.90	10.30	10.60	9.90	10.10	10.16	.70

$\bar{\bar{X}} = 10.21$
 $\bar{R} = .60$

\bar{X} Chart and R Chart

EXHIBIT TN7.9

**EXAMPLE TN7.2: \bar{X} and R Charts**

We would like to create \bar{X} and R charts for a process. Exhibit TN7.8 shows measurements for all 25 samples. The last two columns show the average of the sample \bar{X} and the range R.

Values for A_2 , D_3 , and D_4 were obtained from Exhibit TN7.7.

$$\text{Upper control limit for } \bar{X} = \bar{\bar{X}} + A_2\bar{R} = 10.21 + .58(.60) = 10.56$$

$$\text{Lower control limit for } \bar{X} = \bar{\bar{X}} - A_2\bar{R} = 10.21 - .58(.60) = 9.86$$

$$\text{Upper control limit for } R = D_4\bar{R} = 2.11(.60) = 1.27$$

$$\text{Lower control limit for } R = D_3\bar{R} = 0(.60) = 0$$

SOLUTION

Exhibit TN7.9 shows the \bar{X} chart and R chart with a plot of all the sample means and ranges of the samples. All the points are well within the control limits, although sample 23 is close to the \bar{X} lower control limit. ●

ACCEPTANCE SAMPLING

DESIGN OF A SINGLE SAMPLING PLAN FOR ATTRIBUTES

Acceptance sampling is performed on goods that already exist to determine what percentage of products conform to specifications. These products may be items received from another company and evaluated by the receiving department, or they may be components that have passed through a processing step and are evaluated by company personnel either in production or later in the warehousing function. Whether inspection should be done at all is addressed in the following example.

Acceptance sampling is executed through a sampling plan. In this section we illustrate the planning procedures for a single sampling plan—that is, a plan in which the quality is determined from the evaluation of one sample. (Other plans may be developed using two or more samples. See J. M. Juran and F. M. Gryna's *Quality Planning and Analysis* for a discussion of these plans.)

EXAMPLE TN7.3: Costs to Justify Inspection

Total (100 percent) inspection is justified when the cost of a loss incurred by not inspecting is greater than the cost of inspection. For example, suppose a faulty item results in a \$10 loss and the average percentage defective of items in the lot is 3 percent.

SOLUTION

If the average percentage of defective items in a lot is 3 percent, the expected cost of faulty items is $0.03 \times \$10$, or \$0.30 each. Therefore, if the cost of inspecting each item is less than \$0.30, the economic decision is to perform 100 percent inspection. Not all defective items will be removed, however, because inspectors will pass some bad items and reject some good ones.

The purpose of a sampling plan is to test the lot to either (1) find its quality or (2) ensure that the quality is what it is supposed to be. Thus, if a quality control supervisor already knows the quality (such as the 0.03 given in the example), he or she does not sample for defects. Either all of them must be inspected to remove the defects or none of them should be inspected, and the rejects pass into the process. The choice simply depends on the cost to inspect and the cost incurred by passing a reject. ●

A single sampling plan is defined by n and c , where n is the number of units in the sample and c is the acceptance number. The size of n may vary from one up to all the items in the lot (usually denoted as N) from which it is drawn. The acceptance number c denotes the maximum number of defective items that can be found in the sample before the lot is rejected. Values for n and c are determined by the interaction of four factors (AQL, α , LTPD, and β) that quantify the objectives of the product's producer and its consumer. The objective of the producer is to ensure that the sampling plan has a low probability of rejecting good lots. Lots are defined as high quality if they contain no more than a specified level of defectives, termed the *acceptable quality level (AQL)*.³ The objective of the consumer is to ensure that the sampling plan has a low probability of accepting bad lots. Lots are defined as low quality if the percentage of defectives is greater than a specified amount, termed *lot tolerance percent defective (LTPD)*. The probability associated with rejecting a high-quality lot is denoted by the Greek letter alpha (α) and is termed the *producer's risk*. The probability associated with accepting a low-quality lot is denoted by the letter beta (β) and is termed the *consumer's risk*. The selection of particular values for AQL, α , LTPD, and β is an economic decision based on a cost trade-off or, more typically, on company policy or contractual requirements.

There is a humorous story supposedly about Hewlett-Packard during its first dealings with Japanese vendors, who place great emphasis on high-quality production. HP had insisted on 2 percent AQL in a purchase of 100 cables. During the purchase agreement, some heated discussion took place wherein the Japanese vendor did not want this AQL specification; HP insisted that they would not budge from the 2 percent AQL. The Japanese vendor finally agreed. Later, when the box arrived, there were two packages inside. One contained 100 good cables. The other package had 2 cables with a note stating: "We have sent

AN EMPLOYEE AT THE STELLENBOSCH WINERY BOTTLING PLANT CHECKS THE QUALITY OF NEWLY BOTTLED CHARDONNAY WINE FOR PROPER VOLUME AND CORK FIT.



you 100 good cables. Since you insisted on 2 percent AQL, we have enclosed 2 defective cables in this package, though we do not understand why you want them.”

The following example, using an excerpt from a standard acceptance sampling table, illustrates how the four parameters—AQL, α , LTPD, and β —are used in developing a sampling plan.

EXAMPLE TN7.4: Values of n and c

Hi-Tech Industries manufactures Z-Band radar scanners used to detect speed traps. The printed circuit boards in the scanners are purchased from an outside vendor. The vendor produces the boards to an AQL of 2 percent defectives and is willing to run a 5 percent risk (α) of having lots of this level or fewer defectives rejected. Hi-Tech considers lots of 8 percent or more defectives (LTPD) unacceptable and wants to ensure that it will accept such poor-quality lots no more than 10 percent of the time (β). A large shipment has just been delivered. What values of n and c should be selected to determine the quality of this lot?

SOLUTION

The parameters of the problem are AQL = 0.02, α = 0.05, LTPD = 0.08, and β = 0.10. We can use Exhibit TN7.10 to find c and n .

First, divide LTPD by AQL ($0.08 \div 0.02 = 4$). Then, find the ratio in column 2 that is equal to or just greater than that amount (4). This value is 4.057, which is associated with $c = 4$.

Finally, find the value in column 3 that is in the same row as $c = 4$, and divide that quantity by AQL to obtain n ($1.970 \div 0.02 = 98.5$).

The appropriate sampling plan is $c = 4$, $n = 99$. ●

OPERATING CHARACTERISTIC CURVES

While a sampling plan such as the one just described meets our requirements for the extreme values of good and bad quality, we cannot readily determine how well the plan discriminates between good and bad lots at intermediate values. For this reason, sampling plans are generally displayed graphically through the use of operating characteristic (OC) curves. These curves, which are unique for each combination of n and c , simply illustrate the probability of accepting lots with varying percentages of defectives. The procedure we have followed in developing the plan, in fact, specifies two points on an OC curve: one point defined by AQL and $1 - \alpha$, and the other point defined by LTPD and β . Curves for common values of n and c can be computed or obtained from available tables.⁴

Shaping the OC Curve A sampling plan discriminating perfectly between good and bad lots has an infinite slope (vertical) at the selected value of AQL. In Exhibit TN7.11, any percentage defective to the left of 2 percent would always be accepted, and those to the right, always rejected. However, such a curve is possible only with complete inspection of all units and thus is not a possibility with a true sampling plan.

An OC curve should be steep in the region of most interest (between the AQL and the LTPD), which is accomplished by varying n and c . If c remains constant, increasing the sample size n causes the OC curve to be more vertical. While holding n constant, decreasing c (the maximum number of defective units) also makes the slope more vertical, moving closer to the origin.

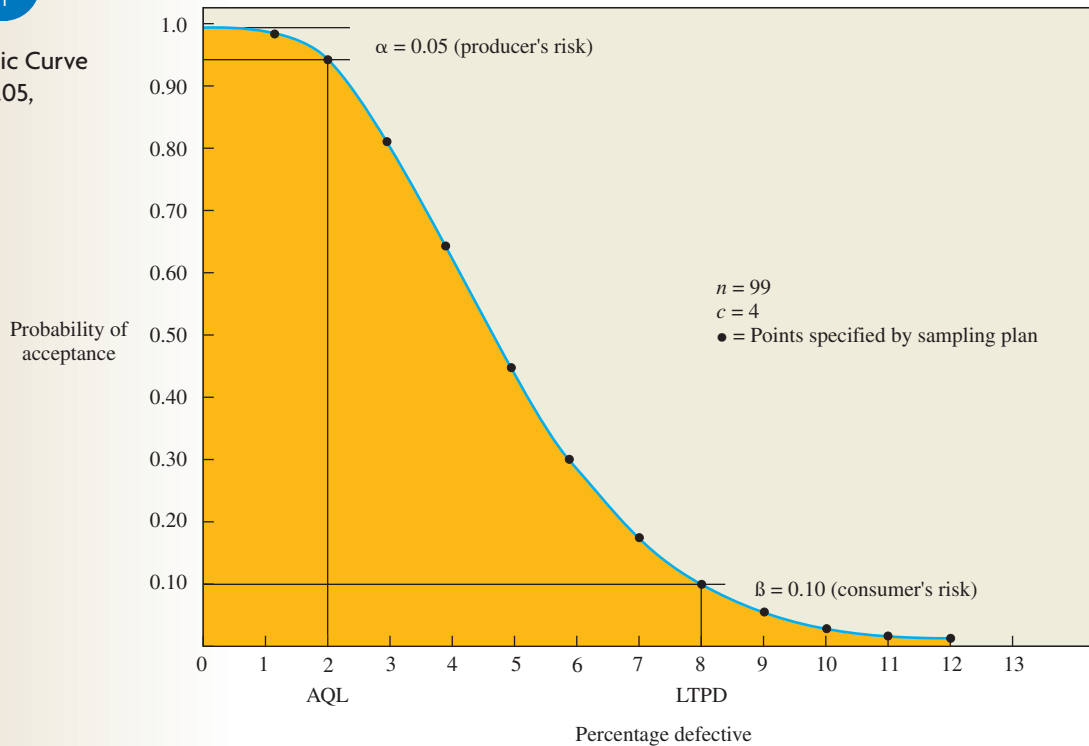
c	LTPD \div AQL	$n \cdot$ AQL	c	LTPD \div AQL	$n \cdot$ AQL
0	44.890	0.052	5	3.549	2.613
1	10.946	0.355	6	3.206	3.286
2	6.509	0.818	7	2.957	3.981
3	4.890	1.366	8	2.768	4.695
4	4.057	1.970	9	2.618	5.426

EXHIBIT TN7.10

Excerpt from a Sampling Plan
Table for $\alpha = 0.05$, $\beta = 0.10$

EXHIBIT TN7.11

Operating Characteristic Curve
for AQL = 0.02, $\alpha = 0.05$,
LTPD = 0.08, $\beta = 0.10$



The Effects of Lot Size The size of the lot that the sample is taken from has relatively little effect on the quality of protection. Consider, for example, that samples—all of the same size of 20 units—are taken from different lots ranging from a lot size of 200 units to a lot size of infinity. If each lot is known to have 5 percent defectives, the probability of accepting the lot based on the sample of 20 units ranges from about 0.34 to about 0.36. This means that as long as the lot size is several times the sample size, it makes little difference how large the lot is. It seems a bit difficult to accept, but statistically (on the average in the long run) whether we have a carload or box full, we'll get about the same answer. It just seems that a carload should have a larger sample size. Of course, this assumes that the lot is randomly chosen and that defects are randomly spread through the lot.

CONCLUSION

● ● ● Statistical quality control is a vital topic. Quality has become so important that statistical quality procedures are *expected* to be part of successful firms. Sampling plans and statistical process control are taken as given with the emphasis shifting to broader aspects (such as eliminating dockside acceptance sampling because of reliable supplier quality, and employee empowerment transforming much of the process control). World-class manufacturing companies expect people to understand the basic concepts of the material presented in this technical note.

KEY TERMS

Assignable variation Deviation in the output of a process that can be clearly identified and managed.

Common variation Deviation in the output of a process that is random and inherent in the process itself.

Upper and lower specification or tolerance limits The range of values in a measure associated with a process that are allowable given the intended use of the product or service.

Capability index (C_{pk}) The ratio of the range of values produced by a process divided by the range of values allowed by the design specification.

Statistical process control (SPC) Techniques for testing a random sample of output from a process to determine whether the process is producing items within a prescribed range.

Attributes Quality characteristics that are classified as either conforming or not conforming to specification.

Variables Quality characteristics that are measured in actual weight, volume, inches, centimeter, or other measure.

FORMULA REVIEW

Mean or average

$$[\text{TN7.1}] \quad \bar{X} = \sum_{i=1}^N x_i / N$$

Standard deviation

$$[\text{TN7.2}] \quad \sigma = \sqrt{\frac{\sum_{i=1}^N (x_i - \bar{X})^2}{N}}$$

Capability index

$$[\text{TN7.3}] \quad C_{pk} = \min \left[\frac{\bar{X} - \text{LTL}}{3\sigma}, \frac{\text{UTL} - \bar{X}}{3\sigma} \right]$$

Process control charts using attribute measurements

$$[\text{TN7.4}] \quad \bar{p} = \frac{\text{Total number of defects from all samples}}{\text{Number of samples} \times \text{Sample size}}$$

$$[\text{TN7.5}] \quad s_p = \sqrt{\frac{\bar{p}(1 - \bar{p})}{n}}$$

$$[\text{TN7.6}] \quad \text{UCL} = \bar{p} + z s_p$$

$$[\text{TN7.7}] \quad \text{LCL} = \bar{p} - z s_p$$

$$[\text{TN7.8}] \quad \text{UCL}_{\bar{X}} = \bar{\bar{X}} + z s_{\bar{X}} \quad \text{and} \quad \text{LCL}_{\bar{X}} = \bar{\bar{X}} - z s_{\bar{X}}$$

Process control \bar{X} and R charts

$$[\text{TN7.9}] \quad \bar{\bar{X}} = \frac{\sum_{j=1}^m \bar{X}_j}{m}$$

$$[\text{TN7.10}] \quad \bar{R} = \frac{\sum_{j=1}^m R_j}{m}$$

[TN7.11] Upper control limit for $\bar{X} = \bar{\bar{X}} + A_2\bar{R}$

[TN7.12] Lower control limit for $\bar{X} = \bar{\bar{X}} - A_2\bar{R}$

[TN7.13] Upper control limit for $R = D_4\bar{R}$

[TN7.14] Lower control limit for $R = D_3\bar{R}$

SOLVED PROBLEMS

SOLVED PROBLEM 1

Completed forms from a particular department of an insurance company were sampled daily to check the performance quality of that department. To establish a tentative norm for the department, one sample of 100 units was collected each day for 15 days, with these results:

SAMPLE	SAMPLE SIZE	NUMBER OF FORMS WITH ERRORS	SAMPLE	SAMPLE SIZE	NUMBER OF FORMS WITH ERRORS
1	100	4	9	100	4
2	100	3	10	100	2
3	100	5	11	100	7
4	100	0	12	100	2
5	100	2	13	100	1
6	100	8	14	100	3
7	100	1	15	100	1
8	100	3			

- Develop a p chart using a 95 percent confidence interval ($1.96s_p$).
- Plot the 15 samples collected.
- What comments can you make about the process?

Solution

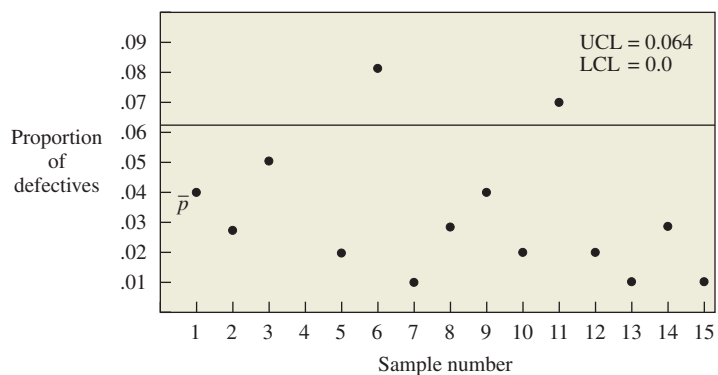
$$a. \bar{p} = \frac{46}{15(100)} = .0307$$

$$s_p = \sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = \sqrt{\frac{.0307(1-.0307)}{100}} = \sqrt{.0003} = .017$$

$$UCL = \bar{p} + 1.96s_p = .031 + 1.96(.017) = .064$$

$$LCL = \bar{p} - 1.96s_p = .031 - 1.96(.017) = -.003 \text{ or zero}$$

- The defectives are plotted below.



- c. Of the 15 samples, 2 were out of the control limits. Because the control limits were established as 95 percent, or 1 out of 20, we would say that the process is out of control. It needs to be examined to find the cause of such widespread variation.

SOLVED PROBLEM 2

Management is trying to decide whether Part A, which is produced with a consistent 3 percent defective rate, should be inspected. If it is not inspected, the 3 percent defectives will go through a product assembly phase and have to be replaced later. If all Part A's are inspected, one-third of the defectives will be found, thus raising the quality to 2 percent defectives.

- a. Should the inspection be done if the cost of inspecting is \$0.01 per unit and the cost of replacing a defective in the final assembly is \$4.00?
 b. Suppose the cost of inspecting is \$0.05 per unit rather than \$0.01. Would this change your answer in a?

Solution

Should Part A be inspected?

- .03 defective with no inspection.
- .02 defective with inspection.

- a. This problem can be solved simply by looking at the opportunity for 1 percent improvement.

$$\begin{aligned}\text{Benefit} &= .01(\$4.00) = \$0.04 \\ \text{Cost of inspection} &= \$0.01\end{aligned}$$

Therefore, inspect and save \$0.03 per unit.

- b. A cost of \$0.05 per unit to inspect would be \$0.01 greater than the savings, so inspection should not be performed.

REVIEW AND DISCUSSION QUESTIONS

- The capability index allows for some drifting of the process mean. Discuss what this means in terms of product quality output.
- Discuss the purposes of and differences between p charts and \bar{X} and R charts.
- In an agreement between a supplier and a customer, the supplier must ensure that all parts are within tolerance before shipment to the customer. What is the effect on the cost of quality to the customer?
- In the situation described in Question 3, what would be the effect on the cost of quality to the supplier?
- Discuss the trade-off between achieving a zero AQL (acceptable quality level) and a positive AQL (such as an AQL of 2 percent).

PROBLEMS

- A company currently using an inspection process in its material receiving department is trying to install an overall cost reduction program. One possible reduction is the elimination of one inspection position. This position tests material that has a defective content on the average of 0.04. By inspecting all items, the inspector is able to remove all defects. The inspector can inspect 50 units per hour. The hourly rate including fringe benefits for this position is \$9. If the inspection position is eliminated, defects will go into product assembly and will have to be replaced later at a cost of \$10 each when they are detected in final product testing.
 - Should this inspection position be eliminated?
 - What is the cost to inspect each unit?
 - Is there benefit (or loss) from the current inspection process? How much?
 - A metal fabricator produces connecting rods with an outer diameter that has a $1 \pm .01$ inch specification. A machine operator takes several sample measurements over time and determines the sample mean outer diameter to be 1.002 inches with a standard deviation of .003 inch.
 - Calculate the process capability index for this example.
 - What does this figure tell you about the process?
- Not inspecting
cost = \$20/hr. Cost to inspect = \$9/hr. Therefore, inspect.
 - \$.18/each.
 - \$.22 per unit.
 - $C_{pk} = .889$.
 - The process is capable but needs to adjust the mean downward to achieve the best quality.

3. a. $\bar{p} = .067$.
 UCL = .194.
 LCL = 0.
 b. Stop the process. Something is wrong. There is wide variation and two are out of limits.

4. a. Cost to inspect = \$.40/unit.
 Cost of defect = \$.50/unit.
 Inspect.
 b. Benefit is \$.10/unit.

5. Yes, inspecting is cheaper. Cost to inspect is \$.267/unit. Cost of defect is \$.30/unit.

6. $\bar{X} = 999.1$;
 UCL = 1014.965;
 LCL = 983.235;
 $\bar{R} = 21.733$;
 UCL = 49.551
 LCL = 0.
 See ISM for graph.

7. a. 87.1 sample size, round to 88.
 b. $c = 5$.

- 3 Ten samples of 15 parts each were taken from an ongoing process to establish a p chart for control. The samples and the number of defectives in each are shown in the following table:

SAMPLE	n	NUMBER OF DEFECTS IN		SAMPLE	n	NUMBER OF DEFECTS IN SAMPLE
		SAMPLE	SAMPLE			
1	15	3		6	15	2
2	15	1		7	15	0
3	15	0		8	15	3
4	15	0		9	15	1
5	15	0		10	15	0

- a. Develop a p chart for 95 percent confidence (1.96 standard deviations).
 b. Based on the plotted data points, what comments can you make?
- 4 Output from a process contains 0.02 defective units. Defective units that go undetected into final assemblies cost \$25 each to replace. An inspection process, which would detect and remove all defectives, can be established to test these units. However, the inspector, who can test 20 units per hour, is paid \$8 per hour, including fringe benefits. Should an inspection station be established to test all units?
 a. What is the cost to inspect each unit?
 b. What is the benefit (or loss) from the inspection process?
- 5 There is a 3 percent error rate at a specific point in a production process. If an inspector is placed at this point, all the errors can be detected and eliminated. However, the inspector is paid \$8 per hour and can inspect units in the process at the rate of 30 per hour.
 If no inspector is used and defects are allowed to pass this point, there is a cost of \$10 per unit to correct the defect later on.
 Should an inspector be hired?
- 6 Resistors for electronic circuits are manufactured on a high-speed automated machine. The machine is set up to produce a large run of resistors of 1,000 ohms each.
 To set up the machine and to create a control chart to be used throughout the run, 15 samples were taken with four resistors in each sample. The complete list of samples and their measured values are as follows:

SAMPLE NUMBER	READINGS (IN OHMS)			
1	1010	991	985	986
2	995	996	1009	994
3	990	1003	1015	1008
4	1015	1020	1009	998
5	1013	1019	1005	993
6	994	1001	994	1005
7	989	992	982	1020
8	1001	986	996	996
9	1006	989	1005	1007
10	992	1007	1006	979
11	996	1006	997	989
12	1019	996	991	1011
13	981	991	989	1003
14	999	993	988	984
15	1013	1002	1005	992

- Develop an \bar{X} chart and an R chart and plot the values. From the charts, what comments can you make about the process? (Use three-sigma control limits as in Exhibit TN7.7.)
- 7 In the past, Alpha Corporation has not performed incoming quality control inspections but has taken the word of its vendors. However, Alpha has been having some unsatisfactory experience recently with the quality of purchased items and wants to set up sampling plans for the receiving department to use.
 For a particular component, X, Alpha has a lot tolerance percentage defective of 10 percent. Zenon Corporation, from which Alpha purchases this component, has an acceptable

quality level in its production facility of 3 percent for component X. Alpha has a consumer's risk of 10 percent and Zenon has a producer's risk of 5 percent.

- a. When a shipment of Product X is received from Zenon Corporation, what sample size should the receiving department test?
 - b. What is the allowable number of defects in order to accept the shipment?
- 8 You are the newly appointed assistant administrator at a local hospital, and your first project is to investigate the quality of the patient meals put out by the food-service department. You conducted a 10-day survey by submitting a simple questionnaire to the 400 patients with each meal, asking that they simply check off that the meal was either satisfactory or unsatisfactory. For simplicity in this problem, assume that the response was 1,000 returned questionnaires from the 1,200 meals each day. The results are as follows:

	NUMBER OF UNSATISFACTORY MEALS	SAMPLE SIZE
December 1	74	1,000
December 2	42	1,000
December 3	64	1,000
December 4	80	1,000
December 5	40	1,000
December 6	50	1,000
December 7	65	1,000
December 8	70	1,000
December 9	40	1,000
December 10	<u>75</u>	<u>1,000</u>
	600	10,000

- a. Construct a p chart based on the questionnaire results, using a confidence interval of 95.5 percent, which is two standard deviations.
 - b. What comments can you make about the results of the survey?
- 9 Large-scale integrated (LSI) circuit chips are made in one department of an electronics firm. These chips are incorporated into analog devices that are then encased in epoxy. The yield is not particularly good for LSI manufacture, so the AQL specified by that department is 0.15 while the LTPD acceptable by the assembly department is 0.40.
- a. Develop a sampling plan.
 - b. Explain what the sampling plan means; that is, how would you tell someone to do the test?
- 10 The state and local police departments are trying to analyze crime rates so they can shift their patrols from decreasing-rate areas to areas where rates are increasing. The city and county have been geographically segmented into areas containing 5,000 residences. The police recognize that not all crimes and offenses are reported: people do not want to become involved, consider the offenses too small to report, are too embarrassed to make a police report, or do not take the time, among other reasons. Every month, because of this, the police are contacting by phone a random sample of 1,000 of the 5,000 residences for data on crime. (Respondents are guaranteed anonymity.) Here are the data collected for the past 12 months for one area:

MONTH	CRIME INCIDENCE	SAMPLE SIZE	CRIME RATE
January	7	1,000	0.007
February	9	1,000	0.009
March	7	1,000	0.007
April	7	1,000	0.007
May	7	1,000	0.007
June	9	1,000	0.009
July	7	1,000	0.007
August	10	1,000	0.010
September	8	1,000	0.008
October	11	1,000	0.011
November	10	1,000	0.010
December	8	1,000	0.008

- 8. a. $\bar{p} = .06$;
 $s_p = .0075$;
UCL = .075;
LCL = .045.
- b. Process is erratic and out of control.

- 9. a. $n = 31.3$. (Round sample size to 32.)
- b. Random sample 32; reject if more than 8 are defective.

- 10. $\bar{p} = .0083$;
 $s_p = .00287$;
UCL = .01396;
LCL = .00270;
Based on p chart, crime rate has not increased.

Construct a p chart for 95 percent confidence (1.96) and plot each of the months. If the next three months show crime incidences in this area as

January = 10 (out of 1,000 sampled)

February = 12 (out of 1,000 sampled)

March = 11 (out of 1,000 sampled)

what comments can you make regarding the crime rate?

- 11. $\bar{p} = 0.015$;
- $s_p = 0.00384$;
- UCL = 0.0225;
- LCL = 0.0075;
- Three areas outside UCL warrant further investigation; four areas below LCL warrant investigation.

- 11 Some citizens complained to city council members that there should be equal protection under the law against the occurrence of crimes. The citizens argued that this equal protection should be interpreted as indicating that high-crime areas should have more police protection than low-crime areas. Therefore, police patrols and other methods for preventing crime (such as street lighting or cleaning up abandoned areas and buildings) should be used proportionately to crime occurrence.

In a fashion similar to Problem 10, the city has been broken down into 20 geographic areas, each containing 5,000 residences. The 1,000 sampled from each area showed the following incidence of crime during the past month:

AREA	NUMBER OF CRIMES	SAMPLE SIZE	CRIME RATE
1	14	1,000	0.014
2	3	1,000	0.003
3	19	1,000	0.019
4	18	1,000	0.018
5	14	1,000	0.014
6	28	1,000	0.028
7	10	1,000	0.010
8	18	1,000	0.018
9	12	1,000	0.012
10	3	1,000	0.003
11	20	1,000	0.020
12	15	1,000	0.015
13	12	1,000	0.012
14	14	1,000	0.014
15	10	1,000	0.010
16	30	1,000	0.030
17	4	1,000	0.004
18	20	1,000	0.020
19	6	1,000	0.006
20	30	1,000	0.030
	<u>300</u>		

Suggest a reallocation of crime protection effort, if indicated, based on a p chart analysis. To be reasonably certain in your recommendation, select a 95 percent confidence level (that is, $Z = 1.96$).

- 12. $\bar{X} = .499$
- UCL = .520
- LCL = .478
- $R = .037$
- UCL = .078
- LCL = .000
- Process is in control.

- 12 The following table contains the measurements of the key length dimension from a fuel injector. These samples of size five were taken at one-hour intervals.

SAMPLE NUMBER	OBSERVATIONS				
	1	2	3	4	5
1	.486	.499	.493	.511	.481
2	.499	.506	.516	.494	.529
3	.496	.500	.515	.488	.521
4	.495	.506	.483	.487	.489
5	.472	.502	.526	.469	.481
6	.473	.495	.507	.493	.506
7	.495	.512	.490	.471	.504

(continued)

SAMPLE NUMBER	OBSERVATIONS				
	1	2	3	4	5
8	.525	.501	.498	.474	.485
9	.497	.501	.517	.506	.516
10	.495	.505	.516	.511	.497
11	.495	.482	.468	.492	.492
12	.483	.459	.526	.506	.522
13	.521	.512	.493	.525	.510
14	.487	.521	.507	.501	.500
15	.493	.516	.499	.511	.513
16	.473	.506	.479	.480	.523
17	.477	.485	.513	.484	.496
18	.515	.493	.493	.485	.475
19	.511	.536	.486	.497	.491
20	.509	.490	.470	.504	.512

Construct a three-sigma \bar{X} chart and R chart (use Exhibit TN7.7) for the length of the fuel injector. What can you say about this process?

- 13 C-Spec, Inc., is attempting to determine whether an existing machine is capable of milling an engine part that has a key specification of $4 \pm .003$ inches. After a trial run on this machine, C-Spec has determined that the machine has a sample mean of 4.001 inches with a standard deviation of .002 inch.
- Calculate the C_{pk} for this machine.
 - Should C-Spec use this machine to produce this part? Why?

13. a. $C_{pk} = .333$
 b. No, the machine is not capable of producing this part.

14. a. $C_{pk} = \frac{110 - 100}{3 \times 4} = .8333$

b. $C_{pk} = \frac{92 - 90}{3 \times 4} = .1667$

- c. Many defects will be produced.

Left tail is a $z = -2/4 = -.5$, which corresponds to a probability of 0.1915. Right tail is at $z = 18/4 = 4.5$, which corresponds to a probability of approximately .5. The probability of being inside the limits is $.1915 + .5 = .6915$, and the probability of being outside is $1 - .6915 = .3085$. Approximately 31% of the parts will be defective.

ADVANCED PROBLEM

- 14 Design specifications require that a key dimension on a product measure 100 ± 10 units. A process being considered for producing this product has a standard deviation of four units.
- What can you say (quantitatively) regarding the process capability?
 - Suppose the process average shifts to 92. Calculate the new process capability.
 - What can you say about the process after the shift? Approximately what percentage of the items produced will be defective?

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FOOTNOTES

1 D. Birch, "The True Value of 6 Sigma," *Quality Progress*, April 1993, p. 6.

2 E. L. Grant and R. Leavenworth, *Statistical Quality Control* (New York: McGraw-Hill, 1964), p. 562.

3 There is some controversy surrounding AQLs. This is based on the argument that specifying some acceptable percentage of defectives is inconsistent with the philosophical goal of zero defects. In practice, even in the best QC companies, there is an acceptable quality level. The difference is that it may be stated in parts per million rather than in parts per hundred. This is the case in Motorola's six-sigma quality standard, which holds that no more than 3.4 defects per million parts are acceptable.

4 See, for example, H. F. Dodge and H. G. Romig, *Sampling Inspection Tables—Single and Double Sampling* (New York: John Wiley & Sons, 1959); and *Military Standard Sampling Procedures and Tables for Inspection by Attributes (MIL-STD-105D)* (Washington, DC: U.S. Government Printing Office, 1983).