International Data on Labor Inputs and Wage Rates (cont.)

Country	*Primary Iron and Steel* 341		*Primary Non-ferrous Metals* 342		*Metal Products (Except Machinery)* 350		*Electrical Appliances and Equipment* 370	
	L/V	$\$w$	L/V	$\$w$	L/V	$\$w$	L/V	$\$w$
1. United States (1954)	0.1266	4387	0.1119	4540	0.1332	4314	0.1514	4119
2. Canada (1954)	0.1410	3769	0.0768	3953	0.1562	3507	0.1579	3536
3. New Zealand (1955/56)	0.2340[d]	2190[d]			0.2639	2190	0.3122	1856
4. Australia (1955/56)	0.2338	2306	0.2351	2194	0.3268	1966	0.3431	1898
5. Denmark (1954)	0.2173	1656	0.1775	1695	0.4006	1441	0.4272	1466
6. Norway (1954)	0.2918	1568	0.2684	1648	0.4142	1438	0.3266	1489
7. United Kingdom (1951)	0.4503	1224	0.3870	1208	0.5464	979	0.6226	1024
8. Ireland (1953)								
9. Puerto Rico (1952)					0.2800	1307	0.1370	1535
10. Colombia (1953)	0.4199	1168	0.4560	1526	0.5640	879	0.4370	972
11. Brazil (1949)							0.5350	572
12. Mexico (1951)	0.4728	594			0.5380	455	0.5537	643
13. Argentina (1950)	0.7920	591	0.5512	542	0.8747	463	0.6826	619
14. El Salvador (1951)					2.7322	243		
15. Southern Rhodesia (1952)								
16. Iraq (1954)	1.2866	210						
17. Ceylon (1952)								
18. Japan (1953)	0.7256	664	0.5432	621	1.2388	422	0.8054	517
19. India (1953)	0.9860	450	2.2000	262			1.7170	306

[d] Refers to primary iron and steel as well as primary non-ferrous metals.

B. Analyses of Costs

MARC NERLOVE

Returns to Scale in Electricity Supply

The study of returns to scale in public-utility enterprises has a long, if not always honorable, history. The question of whether there are increasing or decreasing returns to scale and over what range of output has, as we know, an important bearing on the institutional arrangements necessary to secure an optimal allocation of resources. If, as many writers in the field appear to believe, there are increasing returns to scale over the relevant range of outputs produced by utility undertakings, then these companies must either receive subsidies or resort to price discrimination in order to cover costs at socially optimal outputs.

In addition, as Chenery [2] has pointed out, the extent of returns to scale is a determinant of investment policies in growing industries. If there are increasing returns to scale and a growing demand, firms may find it profitable to add more capacity than they expect to use in the immediate future.

In studying the problem of returns to scale, the first question one must ask is "To what use are the results to be put?" It is inevitable that the

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If the efficiency of firms varies neutrally,[1] as indicated by the error term in (1), and the prices paid for factors vary from firm to firm, then the levels of input are not determined independently but are determined jointly by the firm's efficiency, level of output, and the factor prices it must pay. In short, a fitted relationship between inputs and output is a *confluent* relation that does not describe the production function at all but only the net effects of differences among firms. (For a more general discussion, see [13, 15].)

In such cases, however, it may be possible to fit the *reduced form* of a system of structural relations such as (1) and (3) and to derive estimates of the structural parameters from estimates of the reduced-form parameters. Not only does it turn out to be possible in this case, but an important reduced form turns out to be the cost function:

$$c = ky^{1/r}p_1^{a_1/r}p_2^{a_2/r}p_3^{a_3/r}v, \tag{4}$$

where

$$k = r(a_0 a_1^{a_1} a_2^{a_2} a_3^{a_3})^{-1/r},$$

$$v = u^{-1/r},$$

and

$$r = a_1 + a_2 + a_3.$$

The parameter r measures the degree of returns to scale. The fundamental duality between cost and production functions, demonstrated by Shephard [17], assures us that the relation between the cost function, obtained empirically, and the underlying production function is unique.[2] Under the cost minimization assumption, they are simply two different, but equivalent, ways of looking at the same thing.

Note that the cost function must include factor prices if the correspondence is to be unique. The problem of changing (over time) or differing (in a cross section) factor prices is an old one in statistical cost analysis; see [10], pp. 170–76. Most generally, it seems to have been handled by deflating cost figures by an index of factor prices, a procedure that Johnston [10] shows typically leads to bias in the estimation of the cost curve unless correct weights, which depend on (unknown) parameters of the production function, are used. It seems strange that no one has taken the obvious step of *including factor prices directly in the cost function.* If price data are available for the construction of an index and prices do not move proportionately, in which case no bias would result from deflation, why not use the extra information afforded?

What form of production function is appropriate for electric power? The generalized Cobb-Douglas function presented above is attractive for two reasons: First, it leads to a cost function that is linear in the logarithms of the variables

$$C = K + \frac{1}{r}Y + \frac{a_1}{r}P_1 + \frac{a_2}{r}P_2 + \frac{a_3}{r}P_3 + V, \tag{5}$$

where capital letters denote logarithms of the corresponding lower-case letters. The linearity of (5) makes it especially easy to estimate. Second, a single estimate of returns to scale is possible (it is the reciprocal of the coefficient of the logarithm of output), and returns to scale do not depend on output or factor prices. (The last-mentioned advantage turns out to be a defect as we shall see when we come to examine a few statistical results.) But does such a function accurately characterize the conditions of production in the electric power industry?

A casual examination of trade publications suggests that once a plant is built, fixed proportions are more nearly the rule. Support for this view is given by Komiya [11], who found that data on inputs and output for individual plants were better approximated by a fixed-proportions model that allowed differences in the proportions due to scale. A simplified version of Komiya's model is[3]

$$\begin{aligned} x_1 &= a_1 y^{b_1}, \\ x_2 &= a_2 y^{b_2}, \\ x_3 &= a_3 y^{b_3}. \end{aligned} \tag{6}$$

At the firm level, however, there are many possibilities for substitution that may go unnoticed at the plant level; for example, labor and fuel may be substituted for capital by using older, less efficient plants more intensively or by using a large number of small plants rather than a few large ones. Given persistent differences in the factor prices paid by different firms, cross-section data should reflect such possibilities of substitution. Certainly, as a provisional hypothesis, a generalized Cobb-Douglas function may be appropriate.

[1] A model incorporating non-neutral variations in efficiency of the form

$$y = (a_0 u_0) x_1^{a_1 u_1} x_2^{a_2 u_2} x_3^{a_3 u_2}$$

was discussed in my paper "On Measurement of Relative Economic Efficiency," abstract, *Econometrica*, 28 (July 1960), 695. It is interesting to note that despite the complex way in which the random elements u_0, u_1, and u_2 enter, there are circumstances under which it is possible to estimate the parameters in such a production function.

[2] I owe this point to Hirofumi Uzawa. It is true, of course, only if all firms have the same production function, except perhaps for differences in the constant term, so that aggregation difficulties may be neglected.

[3] Since y is exogenous, it would be appropriate to estimate the coefficients in (6) by least squares. An objection to this, however, is the fact that, if individual plants are considered, the output allocated to *each* is not exogenous; see Westfield ([19], pp. 15–81). Furthermore, Komiya does not use output but name-plate rated capacity and input levels adjusted to full capacity operation. It is even more doubtful whether the former can be considered as exogenous in a cross section. My objection here is closely related to the one raised by Hughes (see p. 169); however, while the endogenicity of output at the plant level is clear, its endogenicity at the firm level for a member of a power pool is conjectural.

It would, of course, be preferable to *test* whether significant substitution among factors occurs at the firm level. The use of the generalized Cobb-Douglas unfortunately does not permit us to do so except in a very general way, since its form implies that the elasticity of substitution between any pair of factors is one. A more general form, which has both the Cobb-Douglas and fixed coefficients as limiting cases, has recently been suggested by Arrow, Minhas, Chenery, and Solow [1]. Constant returns to scale are assumed, but the form can be easily generalized; in a more general form it is

$$y = [a_1 x_1^b + a_2 x_2^b + a_3 x_3^b]^{1/f}. \tag{7}$$

In this case returns to scale are given by the ratio b/f and the elasticity of substitution between any pair of factors can be shown to be $1/(1-b)$. In the special case in which $b = f$ it can be shown that the limiting form of (7) as the elasticity of substitution goes to zero is

$$y = \min\left\{\frac{x_1}{(a_1+a_2+a_3)^{1/b}-1}, \frac{x_2}{(a_1+a_2+a_3)^{1/b}-1}, \frac{x_3}{(a_1+a_2+a_3)^{1/b}-1}\right\}, \tag{8}$$

or fixed coefficients, and the limiting form as the elasticity of substitution goes to one is

$$y = (a_1+a_2+a_3)^{1/b} x_1^{a_1/(a_1+a_2+a_3)} x_2^{a_2/(a_1+a_2+a_3)} x_3^{a_3/(a_1+a_2+a_3)}, \tag{9}$$

or Cobb-Douglas. Although I have not formally demonstrated the fact, it is possible that the limiting form of the more general case (7) is something like the Komiya model as the elasticity of substitution tends to zero, and like the generalized Cobb-Douglas as it tends to one.

Unfortunately, in its generalized form (7) is quite difficult to estimate from the data available. Furthermore, although clearly superior to the generalized Cobb-Douglas form, (7) still implies that the elasticity of substitution between any pair of factors (e.g., labor capital and fuel capital) is the same, which hardly seems reasonable. Other generalizations are possible, but none that I have found thus far offers much hope of being amenable to a reasonable estimation procedure.

If the generalized Cobb-Douglas form is adopted, however, relatively simple estimation procedures can be devised for evaluating the parameters of the production function. The reduced form of (1) and (3) that incorporates all but one of the restrictions on the parameters in the derived demand equations (which are the more usual reduced form) is nothing but the cost function. The only restriction not incorporated in (4) or (5) is that the coefficients of the prices must add up to one. It is a simple matter to incorporate this restriction, however, by dividing costs and two of the prices by the remaining price (it doesn't matter either economically or statistically which price we choose). When fuel price is used as the divisor, the result is

$$C - P_3 = K + \frac{1}{r} Y + \frac{a_1}{r}(P_1 - P_3) + \frac{a_2}{r}(P_2 - P_3) + V, \tag{10}$$

which will be called Model A.

Model A assumes that we have relevant data on the "price" of capital and that this price varies significantly from firm to firm. If neither is the case, we are in trouble. Most of the results presented here are based on Model A, but the data used for this price of capital are clearly inadequate. (See Appendix B.) If one supposes, however, that the price of capital is the same for all firms, which is not implausible, one can do without data on capital price and use the restriction on the coefficients of output and prices to estimate the elasticity of output with respect to capital input. The assumption that capital price is the same for all firms implies

$$C = K' + \frac{1}{r} Y + \frac{a_1}{r} P_1 + \frac{a_3}{r} P_3 + V, \tag{11}$$

where $K' = K + (a_2/r)P_2$, since the exponents of the input levels in (1) are assumed to be the same for all firms. Equation (11) is called Model B.

2. Some Statistical Results and Their Interpretation

Estimation of Model A from a cross section of firms requires that we obtain data on production costs, total physical output, and the prices of labor, capital, and fuel for each firm; for Model B we do not need the price of capital, since it is assumed to be the same for all firms. Details of the construction of these data for a sample of 145 privately owned utilities in 1955 are given in Appendix B and are not discussed here at any length. Suffice it to say that these data are far from adequate for the purpose, and I now believe that a better job could have been done with other sources.

The results from the least-squares regression suggested by equation (10) are given in line I of Table 3; the interpretation of these results in terms of the parameters of the production function is given in line I of Table 4. The R^2 is 0.93, which is somewhat unusual for such a large number of observations; increasing returns to scale are indicated, and the elasticities of output with respect to labor and fuel have the right sign and are of plausible magnitude; however, the elasticity of output with respect to capital price has the wrong sign (fortunately, it is statistically insignificant).

The difficulties with capital may be due in part to the difficulty I encountered in measuring both capital costs and the price of capital. The former were measured as depreciation charges plus the proportion of interest on long-term debt attributable to the production plant; the figure for capital price was compounded of the yield on the firm's long-term debt and an index of construction costs. Depreciation figures reflect past prices and purchases of capital equipment, whereas the price of capital as I constructed

TABLE 3

Results from Regressions Based on Model A for 145 Firms in 1955

Regression No.	Coefficient: Y	$P_1 - P_3$	$P_2 - P_3$	x	R^2
I	0.721 (±0.175)	0.562 (±0.198)	−0.003 (±0.192)	—	0.931
II	0.696 (±0.173)	0.512 (±0.199)	0.033 (±0.185)	−0.046 (±0.022)	0.932
IIIA	0.398 (±0.079)	0.641 (±0.691)	−0.093 (±0.669)	—	0.512
IIIB	0.668 (±0.116)	0.105 (±0.275)	0.364 (±0.277)	—	0.635
IIIC	0.931 (±0.198)	0.408 (±0.199)	0.249 (±0.189)	—	0.571
IIID	0.915 (±0.108)	0.472 (±0.174)	0.133 (±0.157)	—	0.871
IIIE	1.045 (±0.065)	0.604 (±0.197)	−0.295 (±0.175)	—	0.920
IVA	0.394 (±0.055)			—	
IVB	0.651 (±0.189)			—	
IVC	0.877 (±0.376)	0.435 (±0.207)	0.100 (±0.196)	—	0.950
IVD	0.908 (±0.354)			—	
IVE	1.062 (±0.169)			—	

Figures in parentheses are the standard errors of the coefficients.
The dependent variable in all analyses was $C - P_3$.
The variables are defined as follows:

C = log costs Y = log output P_1 = log wage rate P_2 = log capital "price"

$$P_3 = \text{log fuel price} \quad x = \left|\frac{\text{output 1955} - \text{output 1954}}{\text{output 1954}}\right|.$$

it does not; it is perhaps not so surprising then that the price has little effect on costs. Model B is designed to evade this difficulty. Results based on Model B are presented in line V of Table 5 and the implications of this

TABLE 4

Returns to Scale and Elasticities of Output with Respect to Various Inputs Derived from Results Presented in Table 3 for 145 Firms in 1955

Regression No.	Returns to Scale	Elasticity of Output with Respect to: Labor	Capital	Fuel
I	1.39	0.78	−0.00	0.61
II	1.44	0.74	0.01	0.69
IIIA	2.52	1.61	−0.02	0.93
IIIB	1.50	0.16	0.53	0.81
IIIC	1.08	0.44	0.27	0.37
IIID	1.09	0.52	0.15	0.42
IIIE	0.96	0.58	0.29	0.67
IVA	2.52	1.10	0.25	1.17
IVB	1.53	0.65	0.15	0.73
IVC	1.14	0.50	0.11	0.53
IVD	1.10	0.48	0.11	0.51
IVE	0.94	0.41	0.09	0.44

regression for the parameters in the production function are given in line V of Table 6. It is apparent that the estimates of returns to scale and the elasticities of output with respect to labor and fuel are changed very little; the elasticity with respect to capital is of the right sign but still unreasonably low for an industry that is so capital-intensive.[4]

[4] K. Arrow has pointed out that considerations of plausibility implicitly involve an alternative method of estimating the coefficients in the production function: From the marginal productivity conditions (3), we find that for any pair of inputs i and j,

$$\frac{p_i x_i}{p_j x_j} = \frac{a_i}{a_j}.$$

Hence, by constructing some average of the ratios of expenditures on factors, we obtain estimates of the ratios of exponents in the production function. Had the data been arranged in such a manner as to facilitate computation of expenditures on individual factors, a comparison of the ratios a_i/a_j obtained in this way with those derived from the cost function would have been a useful supplement to the analysis. Arrow also pointed out that one could also verify the results by the fit of the production function derived from them. Unfortunately, it is not feasible to obtain good physical measures of the inputs, and such measures are required for this test.

TABLE 5

Results from Regressions Based on Model B for 145 Firms in 1955. Dependent Variable Was C — Log Costs

Regression No.	Coefficient Y	P_1	P_2	R^2
V	0.723 (±0.019)	0.483 (±0.303)	0.496 (±0.106)	0.914
VIA	0.361 (±0.086)	0.212 (±1.259)	0.655 (±0.350)	0.438
VIB	0.661 (±0.106)	−0.401 (±0.333)	0.490 (±0.134)	0.672
VIC	0.985 (±0.180)	−0.014 (±0.261)	0.330 (±0.138)	0.647
VID	0.927 (±0.106)	0.327 (±0.228)	0.426 (±0.064)	0.884
VIE	1.035 (±0.067)	0.704 (±0.272)	0.643 (±0.132)	0.934

Figures in parentheses are the standard errors of the coefficients.

A second difficulty with these regressions is not apparent from an examination of the coefficients and their standard errors. As part of these analyses, the residuals from the regressions were plotted against the logarithm of output. The result is schematically pictured in Figure 1. It is clear that neither regression relationship is truly linear in logarithms. To test this visual impression the observations were arranged in order of ascending output, and Durbin-Watson statistics were computed; the values of the statistics indicated highly significant positive serial correlation, which confirmed the visual evidence.

Aside from difficulties with the basic data, there appear to be at least two plausible and interesting hypotheses accounting for the result.

1. The first explanation of the result derives from dynamic considerations closely related to those underlying Friedman's Permanent-Income Hypothesis [7]. The important thing to note is that actual costs are underestimated by the regressions at both high and low outputs. Consider the situation pictured in Figure 2. Firms operate not on the long-run cost curve, but at points on the various short-run curves. If firms are evenly distributed about their optimal outputs (i.e., outputs at which long-run marginal cost equals short-run marginal cost), the effect will be to increase the estimate of the extent of increasing returns to scale if they are increasing, or diminish further

TABLE 6

Returns to Scale and Elasticities of Output with Respect to Various Inputs Derived from Results Presented in Table 5 for 145 Firms in 1955

Regression No.	*Returns to Scale*	*Elasticity of Output with Respect to* *Labor*	*Capital*	*Fuel*
V	1.38	0.67	0.03	0.69
VIA	2.77	0.59	1.39	0.74
VIB	1.51	−0.62	0.69	0.33
VIC	1.02	−0.01	0.27	0.46
VID	1.08	0.35	−0.34	0.62
VIE	0.97	0.68	0.03	0.68

the estimate of returns to scale if they are decreasing.[5] But elsewhere Friedman holds that a uniform distribution is not likely to occur; in fact he says, "The firms with the largest output are unlikely to be producing at an unusually low level: on the average they are likely to be producing at an unusually high level; and conversely for those that have the lowest output" ([14], p. 237).

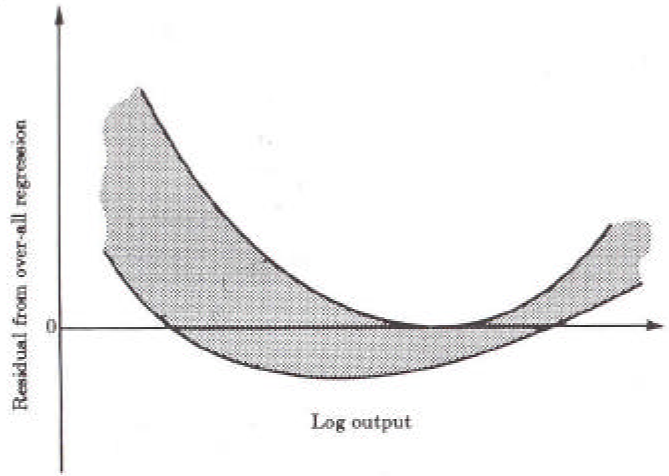

FIGURE 1

[5] This argument rests partly on the form of the function that constrains it to pass through the origin.

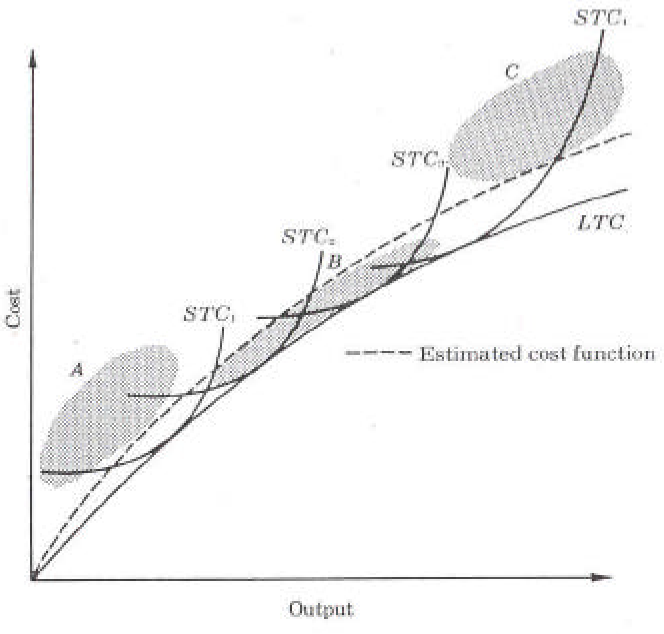

FIGURE 2

The situation described by Friedman is pictured in Figure 2 by the shaded areas A, B, and C, which refer, respectively, to observations on firms with unusually low, usual, and unusually high outputs. The Friedman explanation does produce a residual pattern similar to that observed. Regression II, Table 3, is designed to test this explanation for Model A. A corresponding test for Model B was not made. Since "usual" output cannot be directly observed, the hypothesis was modified slightly by identifying departure from the usual with large changes in output from the previous year, the assumption being that firms with stable output were likely to be near the optimal long-run output.[6] Thus, the absolute percentage changes in output should be positively related to total costs. Unfortunately, they are negatively related and significantly so.

Part of the explanation for this unexpected result is suggested by a more careful examination of the data. Almost all firms with large changes

[6] Capacity figures might have been used. However, those available appear to be somewhat unrealistic. These are based on generator name-plate ratings, which refer to the maximum output that can be produced without overheating. According to the Federal Power Commission, however, units of the same size, general design, and actual capability may show as much as a 20 per cent difference in rating ([5], p. xi). Furthermore, in a multiple-plant firm, total generator capacity is not the only factor to be considered. Such defects in the capacity figures also led to grouping firms by output rather than by capacity in the analyses of covariance presented below.

had positive changes and had been experiencing rapid growth for some time. It is well known, though unfortunately not taken into account in these analyses, that there is a steady rate of technological progress in generating equipment. Since expanding firms purchase new equipment in the process, the average age of a plant in those firms experiencing large changes in output is lower than that of firms with more stable outputs. Hence, the former tend to have lower costs because of the inadequacy of the capital-cost data to reflect obsolescence.[7] Thus, while one would not want to reject the Friedman hypothesis on the basis of this evidence, it clearly does not explain the residual pattern.

2. Fortunately, the observed result can be explained by a much simpler hypothesis, namely, that the degree of returns to scale is not independent of output, but varies inversely with it. Figure 3 illustrates this explanation: The solid line gives the traditional form of the total cost function, which shows increasing returns at low outputs and decreasing returns at high outputs. If we try to fit a function for which returns to scale are independent of the level of output, e.g., one linear in logarithms, a curve such as the dashed one will be obtained. The shaded areas A and B show the output ranges, high and low, for which total costs are underestimated.

If the true cost function is not linear in logarithms, we can either fit an over-all function that reflects this fact or attempt to approximate the actual function by a series of segments of functions linear in logarithms. Because of fitting difficulties and the problem of determining the form in which factor prices enter the cost function, I initially chose the latter course. Firms, arrayed in order of ascending output, were divided into 5 groups containing 29 observations each. A list of the firms used in the analysis appears in Appendix C. The results of fitting five separate regressions of the form indicated by Model A are given in lines IIIA through IIIE of Table 3 and the corresponding implications for the parameters in the production function in lines IIIA through IIIE of Table 4. Similar results for regressions of the form indicated by Model B are presented in lines VIA through VIE of Tables 5 and 6.

The results of these regressions with respect to returns to scale are appealing: Except for statistically insignificant reversal between groups C and D, returns to scale diminish steadily, falling from a high of better than 2.5 to a low of slightly less than 1, which indicates increasing returns at a diminishing rate for all except the largest firms in the sample. However, in the case of regressions III, the elasticity of output with respect to capital

[7] Treatment of capital costs is the source of one of the most serious shortcomings of the present study, as indeed capital measurement is in most studies of production. Solow's recent contribution to the study of the aggregate production function [18] offers considerable promise of an appropriate measure of capital used in the production of electric power. I hope, in future work, to make use of a model of production that involves fixed coefficients *ex post* at the plant level, but that permits substitution of inputs and that changes over time *ex ante*.

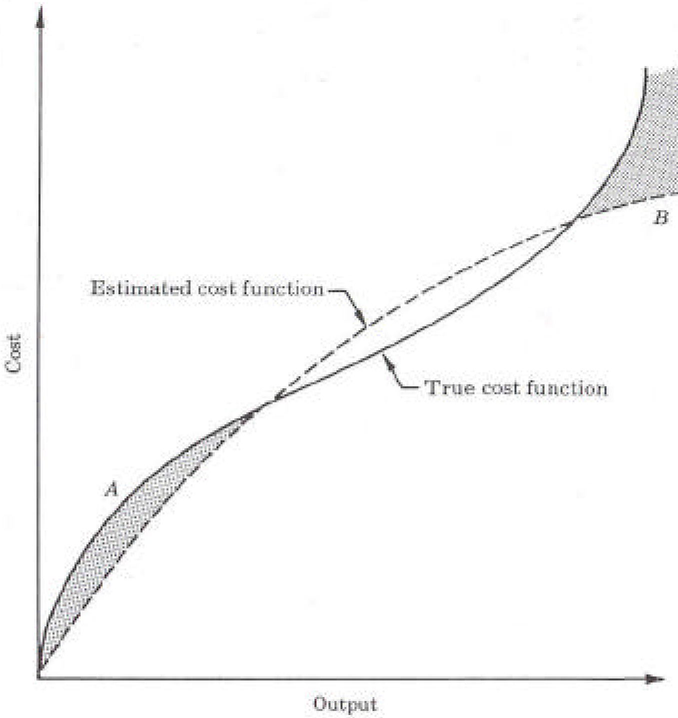

FIGURE 3

price behaves very erratically from group to group and has the wrong sign in groups A and E; in regressions VI the elasticity of output behaves erratically, both with respect to labor and with respect to capital, having the wrong sign in groups B and C for the former and in group D for the latter.

Analyses of covariance for regressions III and VI, compared with the over-all regressions I and V, respectively, gave F-ratios of 1.569 and 1.791 in that order. With 141 and 125 degrees of freedom, these ratios are significant at better than the 99 per cent level. Thus, breaking the sample into five groups significantly reduces the residual variance. However, because of the erratic behavior of the coefficients of independent variables other than output, it appears that we may have gone too far. Regressions III and VI are based on the assumption that *all* coefficients differ from group to group. Economically, this may be interpreted as the hypothesis of *non-neutral variations in returns to scale*; i.e., scale affects not only returns to scale but also marginal rates of substitution.

A halfway house between the hypothesis of no variation in returns to scale with output level and the hypothesis of non-neutral variations in scale

is the hypothesis of *neutral variations in returns to scale*. A general test of this hypothesis is equivalent to testing the hypothesis that the coefficients for the various prices in the individual group regressions are the same for all groups while allowing the constant terms and the coefficients of output to differ.[8] The hypothesis of neutral variations in returns to scale is tested in this way only in the context of Model A. The regression results are presented in lines IVA through IVE of Table 3 and their implications for the production function in Table 4. An analysis of covariance comparing regressions III and IV gives an F-ratio of 1.576. With 133 and 225 degrees of freedom, a ratio this high is significant at better than the 99 per cent level; hence, we cannot confidently reject the hypothesis of non-neutral variations in returns to scale on statistical grounds alone with this test. Examining the results derived from regressions IV, however, we find that the degree of returns to scale steadily declines with output until, for the group consisting of firms with the largest outputs, we find some evidence of diminishing returns to scale.[9] Furthermore, the elasticities of output with respect to the various input levels are all of the correct sign and of reasonable magnitude, although I still feel that the elasticity with respect to capital is implausibly low.[10] Thus, on economic grounds, one might tentatively accept the hypothesis of neutral variations in returns to scale.

If one accepts the hypothesis of neutral variations in returns to scale, a somewhat more refined analysis is possible, since we may then treat the degree of returns to scale as a continuous function of output. That is, instead of grouping the firms as we did previously, we estimate a cost function of the form

$$C = K + \frac{1}{r(Y)} Y + \frac{a_1}{r} P_1 + \frac{a_2}{r} P_2 + \frac{a_3}{r} P_3, \tag{12}$$

where $r(Y)$, the degree of returns to scale, is a function of the output level. Since neutral variations in returns to scale are assumed, the coefficients of the prices are unaffected. A preliminary graphical analysis indicated that returns to scale as a continuous function of output might be approximated by a function of the form

$$r(y) = \frac{1}{\alpha + \beta \log y}. \tag{13}$$

[8] For a generalized Cobb-Douglas the marginal rate of substitution between x_i and x_j is

$$\frac{\partial y/\partial x_i}{\partial y/\partial x_j} = \frac{a_i/a_j}{x_i/x_j}.$$

Hence, if the ratio of a_i to returns to scale, r, is restricted to be the same for each output group, the marginal rates of substitution will be invariant with respect to output level at each given factor ratio.

[9] Note, however, that the estimated value is insignificantly different from one, so that we cannot reject the hypothesis of constant returns to scale for this group of firms.

[10] See p. 179.

Thus, instead of regressions of the form suggested by (10) or (11), we fit

$$(14)\quad C - P_3 = K + \alpha Y + \beta Y^2 + \frac{a_1}{r}[P_1 - P_3] + \frac{a_2}{r}[P_2 - P_3] + V$$

(Model C)

and

$$(15)\quad C = K' + \alpha Y + \beta Y^2 + \frac{a_1}{r}P_1 + \frac{a_3}{r}P_3 + V$$

(Model D).

The results obtained for regressions based on Model C and Model D are reported in Table 7 for regressions VII and VIII, respectively. The implications of these results for the production function are given in Table 8. Note that returns to scale and the other parameters have been computed at five output levels only, so that the results in Table 8 may be readily compared with those in Tables 4 and 6.

Perhaps the most striking result of the assumption of continuously and neutrally variable returns to scale of the form suggested in (13) is the substantial increase in our estimate of the degree of returns to scale for firms in the three largest size groups. Whereas before, we found nearly constant returns to scale, it now appears that they are increasing.[11] In addition, all

TABLE 7

Results from Regressions Based on Models C and D for 145 Firms in 1955; Continuous Neutral Variations in Returns to Scale

Model C: Dependent Variable Was $C - P_3$

Regression No.	Coefficient: Y	Y^2	$P_1 - P_3$	$P_2 - P_3$	R^2
VII	0.151 (±0.062)	0.117 (±0.012)	0.498 (±0.161)	0.062 (±0.151)	0.958

Model D: Dependent Variable Was C

Regression No.	Coefficient: Y	Y^2	P_1	P_3	R^2
VIII	0.137 (±0.064)	0.118 (±0.013)	0.279 (±0.224)	0.255 (±0.054)	0.952

Figures in parentheses are the standard errors of the coefficients.

[11] Using the variance-covariance matrix for the coefficients in (14) or (15), one could easily compute, for a given y, a conditional standard error for $1/r$, which could then be used to test whether $1/r$ were significantly less than one (i.e., whether the finding of increasing returns was statistically significant). Unfortunately, the regression program used did not print out the inverse of the moment matrix, so this test could not be made. But there is little doubt, in view of the extremely small standard errors of the estimated α and β, that such a test would have shown the increasing returns found to be statistically significant.

TABLE 8

Returns to Scale and Elasticities of Output with Respect to Various Inputs Derived from Results Presented in Table 7 for 145 Firms in 1955

Regression VII (Model C)

Group	*Returns to Scale*[a]	*Elasticity of Output with Respect to*[a]: *Labor*	*Capital*	*Fuel*
A	2.92	1.45	0.18	1.29
B	2.24	1.12	0.14	0.98
C	1.97	0.98	0.12	0.87
D	1.84	0.92	0.11	0.81
E	1.69	0.84	0.10	0.75

Regression VIII (Model D)

Group	*Returns to Scale*[a]	*Elasticity of Output with Respect to*[a]: *Labor*	*Capital*	*Fuel*
A	3.03	0.85	1.41	0.77
B	2.30	0.64	1.07	0.59
C	2.01	0.56	0.94	0.51
D	1.88	0.52	0.88	0.48
E	1.72	0.48	0.80	0.44

[a] Evaluated at the median output for each group.

the coefficients in both analyses are of the right sign, and the results based on Model D yield results of plausible magnitude for the elasticity of output with respect to capital as compared with the elasticities with respect to labor and fuel. Analyses of covariance, comparing regressions VII and I with regressions VIII and V, yield F-ratios of 1.631 and 9.457, respectively; both are highly significant, with 141 and 140 degrees of freedom. A comparison of regression VII with regression III yields an F-ratio of 1.032, which, though not significant, does suggest that neutral variations in returns to scale of the form used are indistinguishable from non-neutral. Hence the hypothesis of neutral variations in returns to scale may be accepted both on economic grounds and on grounds of simplicity.

3. Conclusions and Prospects

The major substantive conclusions of this paper are that

1. There is evidence of a marked degree of increasing returns to scale at the firm level; but the degree of returns to scale varies inversely with output and is considerably less, especially for large firms, than that previously estimated for individual plants.

2. Variation in returns to scale may well be neutral in character; i.e., although the scale of operation affects the degree of returns to scale, it may not affect the marginal rates of substitution between different factors of production for given factor ratios.

These substantive conclusions derive from two conclusions of methodological interest:

1. The appropriate model at the firm level is a statistical cost function which includes factor prices and which is uniquely related to the underlying production function.

2. At the firm level it is appropriate to assume a production function that allows substitution among factors of production. When a statistical cost function based on a generalized Cobb-Douglas production function is fitted to cross-section data on individual firms, there is evidence of such substitution possibilities.

Inadequacies in the estimation of capital costs and prices and in the treatment of transmission suggest, however, that a less aggregative approach is called for. On a less aggregative level, it may be possible to produce more adequate measures of capital and to introduce transmission explicitly. A simple model of optimal behavior on the part of the firm may then allow us to combine this information in a way that will yield more meaningful results on returns to scale at the firm level.

APPENDIX A

A Relation between Returns to Scale at the Plant Level and at the Firm Level for an Electric Utility

Consider a firm that produces x, units in each of n identical plants. If plants and demand are uniformly distributed, all plants will produce identical outputs, so that the total output produced will be nx, where x is the common value. Under these circumstances, a general formula that has been developed by electrical engineers to express transmission losses [8] reduces to

$$y = bn^2x^2, \tag{A.1}$$

where y is the aggregate loss of power. That is, with uniformly distributed demand and identical plants, transmission losses are proportional to the square of total output.

If z is delivered power, we have

$$z = nx - y = nx - bn^2x^2. \tag{A.2}$$

Let $c(x)$ be the cost of producing x units in one plant. Production costs of the nx units are thus $nc(x)$. And let $t = T(n, x)$ be the cost of maintaining a network with n plants, each of which produces x units. We may expect that t increases with x, $\partial T/\partial x > 0$, since larger outputs require more and heavier wires and more and larger transformers. However, t may or may not increase with n. It is likely to decrease with n if the expense of operating and maintaining long transmission lines is large relative to the cost of a number of short lines, and likely to increase if the converse is true.

The total cost of delivering an amount z of power $T(z)$ is the sum of production costs of a larger amount of power and transmission costs:

$$T(z) = nc(x) + T(n, x). \tag{A.3}$$

Suppose that the firm chooses the number and size of its plants in order to minimize $T(z)$ for any given z. The values of n and x that minimize $T(z)$ subject to (A.2) are given by solving

$$c(x) + \frac{\partial T}{\partial n} - x\lambda\mu = 0, \tag{A.4}$$

$$nc'(x) + \frac{\partial T}{\partial x} - n\lambda\mu = 0, \tag{A.5}$$

$$z - (nx - bn^2x^2) = 0, \tag{A.6}$$

where

$$\mu = 1 - 2bnx$$

$$= \frac{z - y}{nx}. \tag{A.7}$$

The degree of returns to scale at the plant level, $p(x)$, may be defined as the reciprocal of the elasticity of production costs with respect to output:

$$p(x) = \frac{c(x)}{xc'(x)}. \tag{A.8}$$

It follows from (A.4), (A.5), and (A.8) that

$$p(x) = 1 + \frac{t}{(nx)c'(x)}(e_x - e_n), \tag{A.9}$$

where

$$e_x = \frac{x}{t}\frac{\partial T}{\partial x}, \qquad e_n = \frac{n}{t}\frac{\partial T}{\partial n}.$$

Since nx, t and $c'(x)$ are positive, it follows that returns to scale are greater or less than one, according to whether the elasticity of transmission cost with respect to output exceeds or falls short of the elasticity with respect to number of plants. If transmission costs decrease with a larger number of plants, then under the particular assumptions made here, the firm will operate plants in the region of increasing returns to scale. It may nonetheless operate as a whole in the region of decreasing returns to scale.

Let $P(z)$ be the degree of returns to scale for the firm as a whole when it delivers a supply of z units to its customers:

$$P(z) = \frac{\Gamma(z)}{z\Gamma'(z)}. \tag{A.10}$$

It is well known that the Lagrangian multiplier λ is equal to marginal cost; hence, from (A.5),

$$\Gamma'(z) = \lambda = \frac{1}{n\mu}\left[nc'(x) + \frac{\partial T}{\partial x}\right]. \tag{A.11}$$

Substituting for $\Gamma'(z)$ from (A.11), μ from (A.7), and $\Gamma(z)$ from (A.3), we obtain the following expression for $P(z)$:

$$P(z) = \frac{\Gamma(z)}{z}\cdot\frac{n(z-y)}{nx[nc'(x) + \partial T/\partial x]} = \left(1 - \frac{y}{z}\right)\frac{nc(x) + t}{n[xc'(x)] + x(\partial T/\partial x)}. \tag{A.12}$$

By definition,

$$p(x) = \frac{c(x)}{xc'(x)},$$

hence

$$P(z) = p(x)\left(1 - \frac{y}{z}\right)\frac{nc(x) + t}{nc(x) + [p(x)e_x]t}. \tag{A.13}$$

Neglecting the last term in the product on the right-hand side of (A.13) for the moment, we see that returns to scale at the firm level will typically be less than at the plant level, solely because of transmission losses; how much less depends on the ratio of losses to the quantity of power actually delivered. The final term in the product is a more complicated matter: If there are increasing returns to scale and if the costs of transmission increase rapidly with the average load (i.e., $e_x > 1$), then it is clear that the tendency toward diminishing returns at the level of the individual firm will be reinforced. It is perfectly possible under these circumstances that firms will operate individual plants in the range of increasing returns to scale and yet, considered as a unit, be well within the range of decreasing returns to scale.

Although this argument rests on a number of extreme simplifying assumptions, it nonetheless may provide an explanation for the divergent views and findings concerning the nature of returns to scale in electricity supply. Davidson [3] and Houthakker [9], for example, hold that there are diminishing returns to scale, while much of the empirical evidence and many other writers support the contrary view. The existing empirical evidence, however, refers to individual plants, not firms, and many writers in the public-utility field may have plants rather than firms in mind.

APPENDIX B

The Data Used in the Statistical Analyses

Estimation of Equation (7) from cross-section data on individual firms in the electric power industry requires that we obtain data on production costs, total physical output, and the prices paid for fuel, capital, and labor. Data on various categories of cost are relatively easy to come by, although there are difficulties in deriving an appropriate measure of capital costs. Price data are more difficult to come by, in general, and conceptual as well as practical difficulties are involved in formulating an appropriate measure of the "price" of capital. Such problems are, in fact, the *raisons d'être* for Model B, which permits us to ignore capital prices altogether.

A cross section of 145 firms in 44 states in the year 1955 was used in the analyses. The firms used in the analysis are listed in Appendix C. Selection of firms was made primarily on the basis of data availability. The various series used in the analyses were derived as follows.

B.1. Production Costs

Data on expenditures for labor and fuel used in steam plants for electric power generation are available by firm in [6], but the capital costs of production had to be estimated. This was done by taking interest and depreciation charges on the firm's entire production plant and multiplying by the ratio of the value of steam plant to total plant as carried on the firm's books. Among the shortcomings of this approach, three are worthy of special note:

(a) For many well-known reasons, depreciation and interest charges do not reflect capital costs as defined in some economically meaningful way. Furthermore, depreciation practices vary from firm to firm (there are about four basic methods in use by electric utilities), and such variation introduces a non-comparability of unknown extent.

(b) The method of allocation used to derive our series assumes that steam and hydraulic plants depreciate at the same rate, which is clearly not the case.

(c) Because of their dependence on past prices of utility plant, the use of depreciation and interest charges raises serious questions about the relevant measure of the price of capital. The use of a current figure is clearly inappropriate, but unless we are prepared to introduce the same magnitude on both sides of the equation, it is difficult to see how else the problem can be handled.

B.2. Output

Total output produced by steam plant in kilowatt hours during the entire year 1955 may be obtained from [6]. This was the series used, despite the fact that the peak load aspect of output is thereby neglected. Since the distribution of output among residential, commercial, and industrial users varies from firm to firm, characteristics of the peak will also vary and this in turn will affect our estimate of returns to scale if correlated with the level of output.

B.3. Wage Rates

At the time this study was undertaken, I was unaware of the existence of data on payroll and employment by plant contained in [5]; hence, inferior information was used to obtain this series. Average hourly earnings of utility workers (including gas and transportation) were available for 19 states from Bureau of Labor Statistics files. A mail survey was made of the State Unemployment Compensation Commissions in the remaining 29 states. All replied, but only ten were able to supply data. A regression of the average hourly earnings of utility workers on those for all manufacturing was used to estimate the former for states for which it was unavailable. The resulting state figures were then associated with utilities having the bulk of their operations in each state. In only one case, Northern States Power, were operations so evenly divided among several states that the procedure could not be applied. In this case an average of the Minnesota and Wisconsin rates was employed.

B.4. Price of Capital

As indicated, many practical and conceptual difficulties were associated with this series. Be that as it may, what was done was as follows: First, an estimate of the current long-term rate at which the firm could borrow was obtained by taking the current yield on the firm's most recently issued long-term bonds (obtained from Moody's Investment Manual). These were mainly 30-year obligations, and in all cases had 20 or more years to maturity. This rate was in turn multiplied by the Handy-Whitman Index of Electric Utility Construction Costs for the region in which the firm had the bulk of its operations ([4], p. 69). Two shortcomings worth special mention are:

(a) The neglect of the possibility of equity financing by the method.

(b) The fact that the Handy-Whitman Index includes the construction costs of hydraulic installations.

B.5. Price of Fuel

Since coal, oil, or natural gas may be burned to produce the steam required for steam electric generation, and since many plants are set up to use more than one type of fuel, prices were taken on a per-Btu basis. These were available by state from ([4], p. 49), and the state figures were assigned to individual utilities in the same manner as wage rates.

APPENDIX C

Names of Firms and Corresponding Costs, Output, Wage Rate, Fuel Price, and Capital Price in 1955

Firms used in the analysis are listed here in order of ascending output (measured in billions of kilowatt-hours). They are divided into 5 groups containing 29 observations each. These appear on pp. 434–438 following.

(*References appear on p. 439, following this Appendix.*)

Group A	Production Costs (million $)	Output (billion kwh)	Wage Rate ($/hr)	Fuel Price (¢/million Btu)	Capital Price (index)
1. California Pacific Utilities Co.	0.082	002	2.09	17.9	183
2. Brockton Edison Co.	0.661	003	2.05	35.1	174
3. Essex County Elec. Co.	0.990	004	2.05	35.1	171
4. The Montana Power Co.	0.315	004	1.83	32.2	166
5. Upper Peninsula Power Co.	0.197	005	2.12	28.6	233
6. Alpena Power Co.	0.098	009	2.12	28.6	195
7. Blackstone Valley Gas and Elec. Co.	0.949	011	1.98	35.5	206
8. Lawrence Electric Co.	0.675	013	2.05	35.1	150
9. Wisconsin-Michigan Power Co.	0.525	013	2.19	29.1	155
10. Cheyenne Light, Fuel and Power Co.	0.501	022	1.72	15.0	188
11. Portland General Elec. Co.	1.194	025	2.09	17.9	170
12. New Hampshire Elec. Co.	0.670	025	1.68	39.7	167
13. Southern Utah Power Co.	0.349	035	1.81	22.6	213
14. Mt. Carmel Public Utility Co.	0.423	039	2.30	23.6	164
15. Bangor Hydro-elec. Co.	0.501	043	1.75	42.8	170
16. Community Public Service Co.	0.550	063	1.76	10.3	161
17. New Port Elec. Corp.	0.795	068	1.98	35.5	210
18. Mississippi Valley Public Service Co.	0.664	081	2.29	28.5	158
19. Superior Water Light and Power Co.	0.705	084	2.19	29.1	156
20. Maine Public Service Co.	0.903	073	1.75	42.8	176
21. Housatonic Public Service Co.	1.504	099	2.20	36.2	170
22. Northwestern Public Service Co.	1.615	101	1.66	33.4	192
23. Missouri Utilities Co.	1.127	119	1.92	22.5	164
24. The Western Colorado Power Co.	0.718	120	1.77	21.3	175
25. Pacific Power and Light Co.	2.414	122	2.09	17.9	180
26. Green Mountain Power Co.	1.130	130	1.82	38.9	176
27. The Central Kansas Power Co.	0.992	138	1.80	20.2	202
28. Arkansas Missouri Power Co.	1.554	149	1.92	22.5	227
29. Northern Virginia Power Co.	1.225	196	1.92	29.1	186

Group B	Production Costs (million $)	Output (billion kwh)	Wage Rate ($/hr)	Fuel Price (¢/million Btu)	Capital Price (index)
1. Lake Superior District Power Co.	1.565	197	2.19	29.1	183
2. Missouri Public Service Co.	1.936	209	1.92	22.5	169
3. Montana-Dakota Utilities Co.	3.154	214	1.52	27.5	168
4. Missouri Power and Light Co.	2.599	220	1.92	22.5	164
5. The Connecticut Power Co.	2.298	234	2.20	36.2	164
6. The United Gas Improvements Co.	2.441	235	2.11	24.4	170
7. St. Joseph Light and Power Co.	2.031	253	1.92	22.5	158
8. Worcester County Elec. Co.	4.666	279	2.05	35.1	177
9. Black Hills Power and Light Co.	1.834	290	1.66	33.4	195
10. Western Light and Telephone Co. Inc.	2.072	290	1.80	20.2	176
11. Southern Colorado Power Co.	2.039	295	1.77	21.3	188
12. Iowa Southern Utilities Co.	3.398	299	1.70	26.9	187
13. Cambridge Elec. Light Co.	3.083	324	2.05	35.1	152
14. Northern States Power Co.	2.344	333	2.19	29.1	157
15. The Tucson Gas, Elec., Light and Power Co.	2.382	338	1.85	24.6	163
16. Madison Gas and Elec. Co.	2.657	353	2.19	29.1	143
17. Central Louisiana Elec. Co.	1.705	353	2.13	10.7	167
18. Savannah Elec. and Power Co.	3.230	416	1.54	26.2	217
19. Otter Tail Power Co.	5.049	420	1.52	27.5	144
20. The Eastern Shore Public Service Co. of Maryland	3.814	456	2.09	30.0	178
21. Central Maine Power Co.	4.580	484	1.75	42.8	176
22. Central Illinois Elec. and Gas Co.	4.358	516	2.30	23.6	167
23. New Bedford Gas and Edison Light Co.	4.714	550	2.05	35.1	158
24. New Jersey Power and Light Co.	4.357	563	2.32	31.9	162
25. Rockland Light and Power Co.	3.919	566	2.31	33.5	198
26. The Empire District Elec. Co.	3.442	592	1.92	22.5	164
27. Western Massachusetts Elec. Co.	4.898	671	2.05	35.1	164
28. El Paso Elec. Co.	3.584	696	1.76	10.3	161
29. Interstate Power Co.	5.535	719	1.70	26.9	174

Group C	Production Costs (million $)	Output (billion kwh)	Wage Rate ($/hr)	Fuel Price (¢/million Btu)	Capital Price (index)
1. Southern Indiana Gas and Elec. Co.	4.406	742	2.04	20.7	157
2. California Elec. Power Co.	4.289	795	2.24	26.5	185
3. Iowa Public Service Co.	6.731	800	1.70	26.9	157
4. Public Service Co. of New Hampshire	6.895	808	1.68	39.7	203
5. Minnesota Power and Light Co.	5.112	811	2.29	28.5	178
6. Gulf Power Co.	5.141	855	2.00	34.3	183
7. Central Hudson Gas and Elec. Corp.	5.720	860	2.31	33.5	168
8. Mississippi Power Co.	4.691	909	1.45	17.6	196
9. Iowa-Illinois Gas and Elec. Co.	6.832	913	1.70	26.9	166
10. West Texas Utilities Co.	4.813	924	1.76	10.3	172
11. Iowa Elec. Light and Power Co.	6.754	984	1.70	26.9	158
12. The Potomac Edison Co.	5.127	991	2.09	30.0	174
13. South Carolina Elec. and Gas Co.	6.388	1000	1.35	28.2	225
14. Pennsylvania Power Co.	4.509	1098	2.11	24.4	168
15. Montana Elec. Co.	7.185	1109	2.05	35.1	177
16. Central Illinois Light Co.	6.800	1118	2.30	23.6	161
17. Wisconsin Public Service Corp.	7.743	1122	2.19	29.1	162
18. Northern Indiana Public Service Co.	7.968	1137	2.04	20.7	158
19. Rochester Gas and Elec. Corp.	8.858	1156	2.31	33.5	176
20. Iowa Power and Light Co.	8.588	1166	1.70	26.9	183
21. New England Power Co.	6.449	1170	2.05	35.1	166
22. Wisconsin Power and Light Co.	8.488	1215	2.19	29.1	164
23. Tampa Elec. Co.	8.877	1279	2.00	34.3	207
24. Atlantic City Elec. Co.	10.274	1291	2.32	31.9	175
25. South Carolina Generating Co.	6.024	1290	1.55	28.2	225
26. Deleware Power and Light Co.	8.258	1331	2.13	30.0	178
27. The United Illuminating Co.	13.376	1373	2.20	36.2	157
28. The Hartford Elec. Light Co.	10.690	1420	2.20	36.2	138
29. Arizona Public Service Co.	8.308	1474	1.85	24.6	163

Group D	Production Costs (million $)	Output (billion kwh)	Wage Rate ($/hr)	Fuel Price (¢/million Btu)	Capital Price (index)
1. Southwestern Gas and Elec. Co.	6.082	1497	1.76	10.3	168
2. The Kansas Power and Light Co.	9.284	1545	1.80	20.2	158
3. Jersey Central Power and Light Co.	10.879	1649	2.32	31.9	177
4. Kansas Gas and Elec. Co.	8.477	1668	1.80	20.2	170
5. Louisiana Power and Light Co.	6.877	1782	2.13	10.7	183
6. The Narragansett Elec. Co.	15.106	1831	1.98	35.5	162
7. Central Power and Light Co.	8.031	1833	1.76	10.3	177
8. Mississippi Power and Light Co.	8.082	1838	1.45	17.6	196
9. San Diego Gas and Elec. Co.	10.866	1787	2.24	26.5	164
10. Public Service Co. of Oklahoma	8.596	1918	1.69	12.9	158
11. Utah Power and Light Co.	8.673	1930	1.81	22.6	157
12. Metropolitan Edison Co.	15.437	2028	2.11	24.4	163
13. Dallas Power and Light Co.	8.211	2057	1.76	10.3	161
14. Public Service Co. of Colorado	11.982	2084	1.77	21.3	156
15. Florida Power Corp.	16.674	2226	2.00	34.3	217
16. Central Illinois Public Service Co.	12.620	2304	2.30	23.6	161
17. Indianapolis Power and Light Co.	12.905	2341	2.04	20.7	183
18. Oklahoma Gas and Elec. Co.	11.615	2353	1.69	12.9	167
19. Texas Power and Light Co.	9.321	2367	1.76	10.3	161
20. Chicago District Elec. Generating Corp.	12.962	2451	2.04	20.7	163
21. The Connecticut Light and Power Co.	16.932	2457	2.20	36.2	170
22. Gulf States Utilities Co.	9.648	2507	1.76	10.3	174
23. New York State Elec. and Gas Corp.	18.350	2530	2.31	33.5	197
24. Kansas City Power and Light Co.	17.333	2576	1.92	22.5	162
25. Southwestern Public Service Co.	12.015	2607	1.76	10.3	155
26. Texas Elec. Service Co.	11.320	2870	1.76	10.3	167
27. Long Island Lighting Co.	22.337	2993	2.31	33.5	176
28. Illinois Power Co.	19.035	3202	2.30	23.6	170
29. Monongahela Power Co.	12.205	3286	1.61	17.8	183

Group E	Production Costs (million $)	Output (billion kwh)	Wage Rate ($/hr)	Fuel Price (¢/million Btu)	Capital Price (index)
1. Carolina Power and Light Co.	17.078	3312	1.68	28.8	190
2. Baltimore Gas and Elec. Co.	25.528	3498	2.09	30.0	170
3. Potomac Elec. Power Co.	24.021	3538	2.09	30.0	176
4. Boston Edison Co.	32.197	3794	2.05	35.1	159
5. Northern State Power Co.	26.652	3841	2.29	28.5	157
6. West Pennsylvania Power Co.	20.164	4014	2.11	24.4	161
7. Arkansas Power and Light Co.	14.132	4217	1.53	18.1	173
8. Pennsylvania Elec. Co.	21.410	4305	2.11	24.4	203
9. Public Service Co. of Indiana Inc.	23.244	4494	2.04	20.7	167
10. Wisconsin Elec. Power Co.	29.845	4764	2.19	29.1	195
11. Virginia Elec. Power Co.	32.318	5277	1.92	29.1	161
12. Indiana and Michigan Elec. Co.	21.988	5283	2.04	20.7	159
13. Pennsylvania Power and Light Co.	35.229	5668	2.11	24.4	177
14. Houston Lighting and Power Co.	17.467	5681	1.76	10.3	157
15. Alabama Power Co.	22.828	5819	1.79	18.5	196
16. Duquesne Light Co.	33.154	6000	2.11	24.4	183
17. Georgia Power Co.	32.228	6119	1.54	26.2	189
18. Union Elec. Co. of Missouri	34.168	6136	1.92	22.5	160
19. Consumers Power Co.	40.594	7193	2.12	28.6	162
20. Appalachian Elec. Power Co.	33.354	7886	1.61	17.8	178
21. Public Service Elec. and Gas Co.	64.542	8419	2.32	31.9	199
22. Southern California Edison Co.	41.238	8642	2.24	26.5	182
23. Niagara Mohawk Power Corp.	47.993	8787	2.31	33.5	190
24. Philadelphia Elec. Co.	69.878	9484	2.11	24.4	165
25. Duke Power Co.	44.894	9956	1.68	23.8	203
26. Pacific Gas and Elec. Co.	67.120	11477	2.24	26.5	151
27. The Detroit Edison Co. and Subsidiaries	73.050	11796	2.12	28.6	148
28. Consolidated Edison Co. of N.Y. Inc.	193.422	14359	2.31	33.5	212
29. Commonwealth Edison Co.	119.939	16719	2.30	23.6	162

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