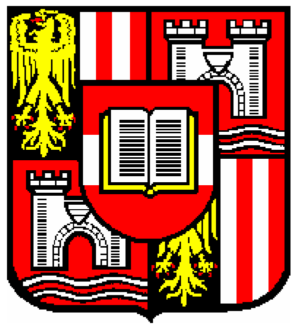
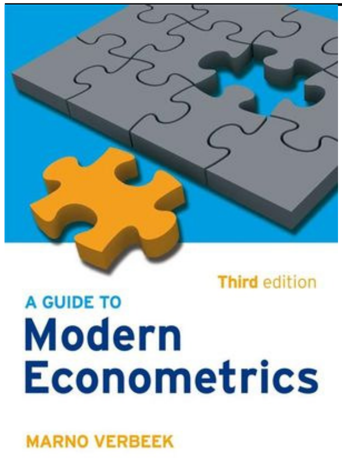
Econometrics

Werner G. Müller
Department of Applied Statistics
(Institut für Angewandte Statistik) .
Johannes-Kepler-Universität Linz

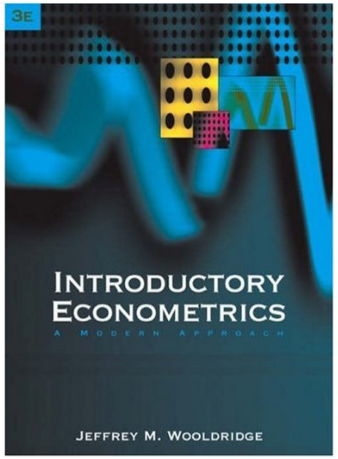


WS 2009/10

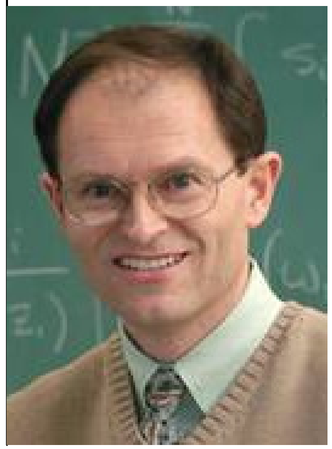
What is Econometrics?



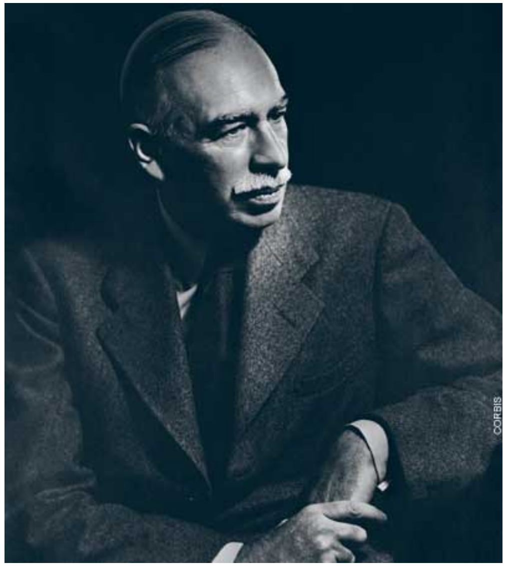
„[…], econometrics is the interaction of economic theory, observed data and statistical methods. It is the interaction of these three that makes econometrics interesting, challenging, and perhaps, difficult.“ Verbeek (2000, 2008)



„Econometrics is based upon the development of statistical methods for estimating economic relationships, testing economic theories, and evaluating and implementing government and business policy.“ Wooldridge (2006)



Criticism



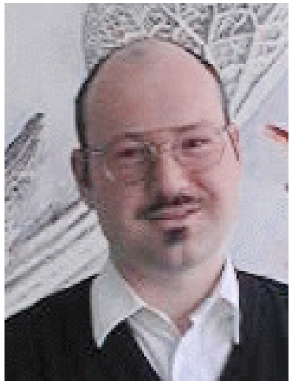
„That there is anyone I would trust with it at the present stage, or that this brand of statistical alchemy is ripe to become a branch of science, I am not yet persuaded. But Newton, Boyle, and Locke all played with alchemy. So let him [Tinbergen] continue.“ Keynes (1939)

„Die Ökonometriker sind im Wesentlichen Statistiker, die sich einbilden, die komplexe Welt der Wirtschaft in Zahlen abbilden zu können. Dagegen war selbst der Kommunismus subtil.“ Gansterer (2003)

Slides are based on

Manuscript for the planned textbook
„Ökonometrie Praxis!“
of W.G. Müller (JKU) and T.Url (Wifo),

in German.



Excerpts and additional material on the IFAS-Homepage: www.ifas.jku.at.

Complimentary GRETL course by Daniel Němec (Masarykovy Univerzity Brno).

Used Software

Examples in the manuscript are made in Eviews6.

Homeworks should be done in GRETL.

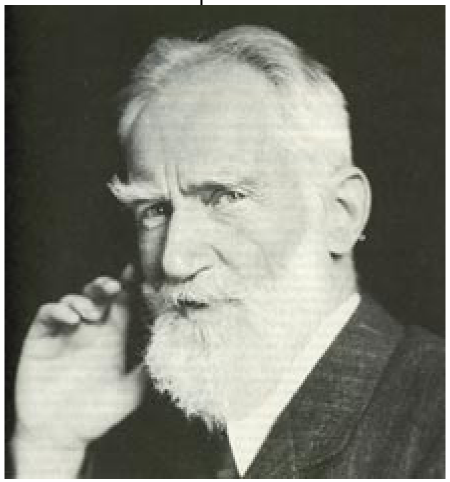
GRETL is for free, a cheap student version of Eviews4.1 is fully sufficient for replicating most of the examples in this course. Tipp!

Chapter 1 - Contents

1. Inflation

„If the governments devalue the currency in order to betray all creditors, you politely call this procedure *inflation*."

George Bernard Shaw (1856-1950)

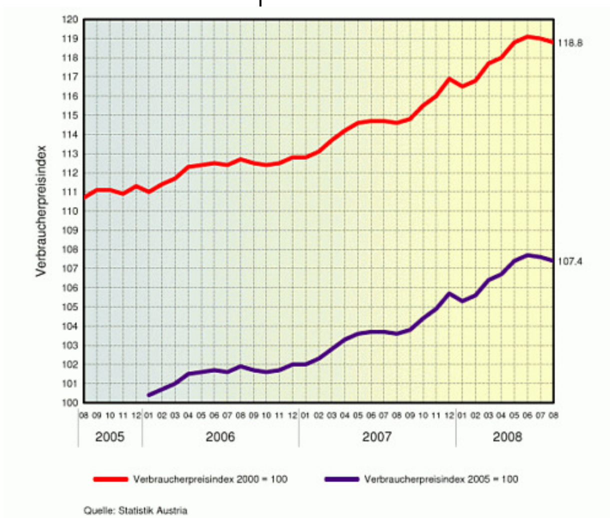


The Consumer Price Index

In a consumer price index (CPI) P_{Ct} the prices of the most important $1, \ldots, N$ consumed goods and services will be comprised as follows for a period t:

$$P_{Ct} = w_1\left(\frac{P_{1t}}{P_{1B}}\right) + w_2\left(\frac{P_{2t}}{P_{2B}}\right) + \ldots + w_N\left(\frac{P_{Nt}}{P_{NB}}\right) = \sum_{i=1}^{N} w_i\left(\frac{P_{it}}{P_{iB}}\right)$$

and compared to the price in a basis year B of the index. The importance of the i-th component of the basket of goods is reflected by the weight w_i. The more of a component is consumed, the higher is his weight in the index and the stronger will its price changes affect the value of P_{Ct}. In the basis year the value of the index is $P_{Ct}=1$, because $t=B$ holds and the prices are thus identical. The published price indices are usually multiplied by a factor 100, such that $P_{Ct}=100$ holds for the basis year.



The Inflationrate

The Inflationsrate $\pi_t \times 100\%$ measures the percent change of a price index between two consecutive observation periods, i.e:

$$\pi_t = (P_t - P_{t-1})/P_{t-1}.$$

An alternative definition is: $\pi_t = \log P_t - \log P_{t-1}$.

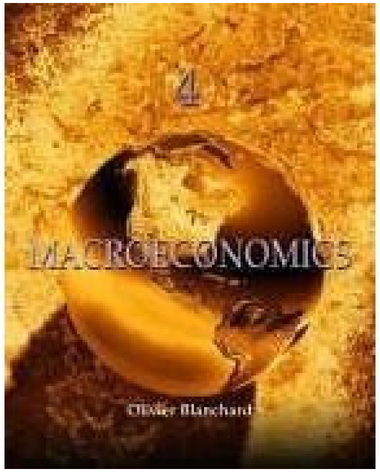
This assumes a continuous exponential growth process $P_t = P_{t-T} \cdot \exp(\pi_t \cdot T)$, which holds only approximately. For $T=1$ therefore holds

$P_t = P_{t-1} \cdot exp(\pi_t)$ and hence $\pi_t = log(P_t / P_{t-1})$.

The Economic Impact of Inflation

- Unemployement: Phillipscurve
- Growth: money demand

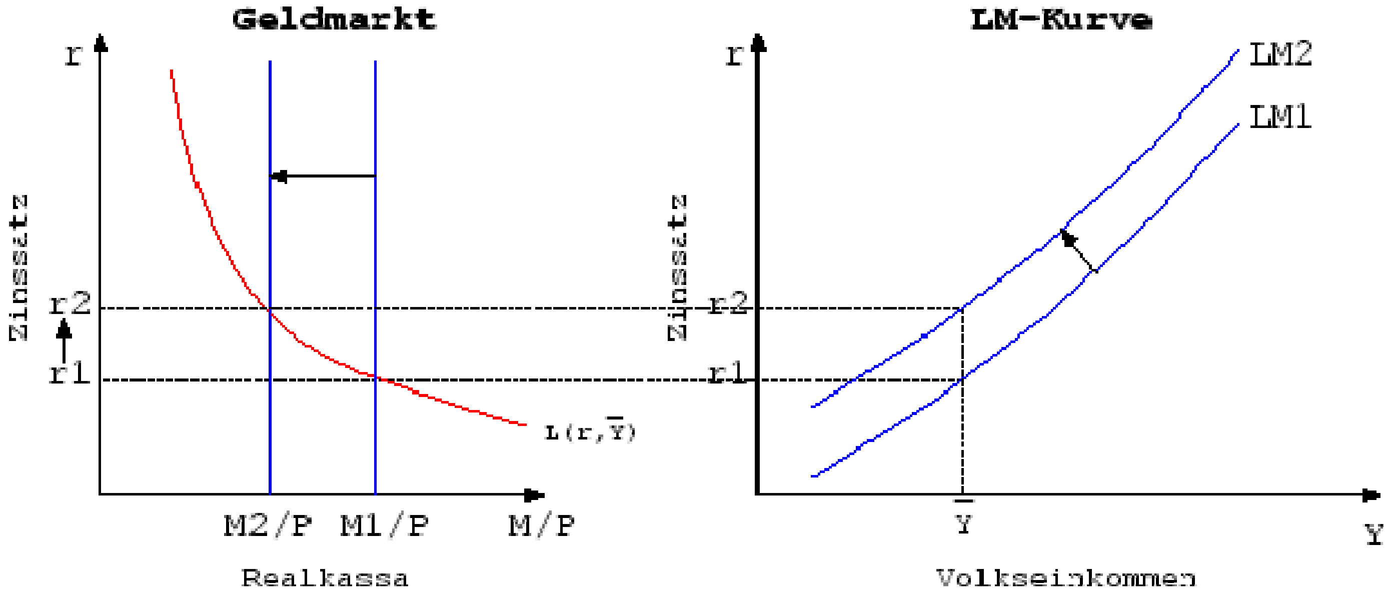
Look e.g. into the fourth edition of the famous textbook by Blanchard (2006)



Theory of Money Demand

Money market equilibrium:

$$M^d/P = Y\,L(r+\pi^e) = M/P.$$



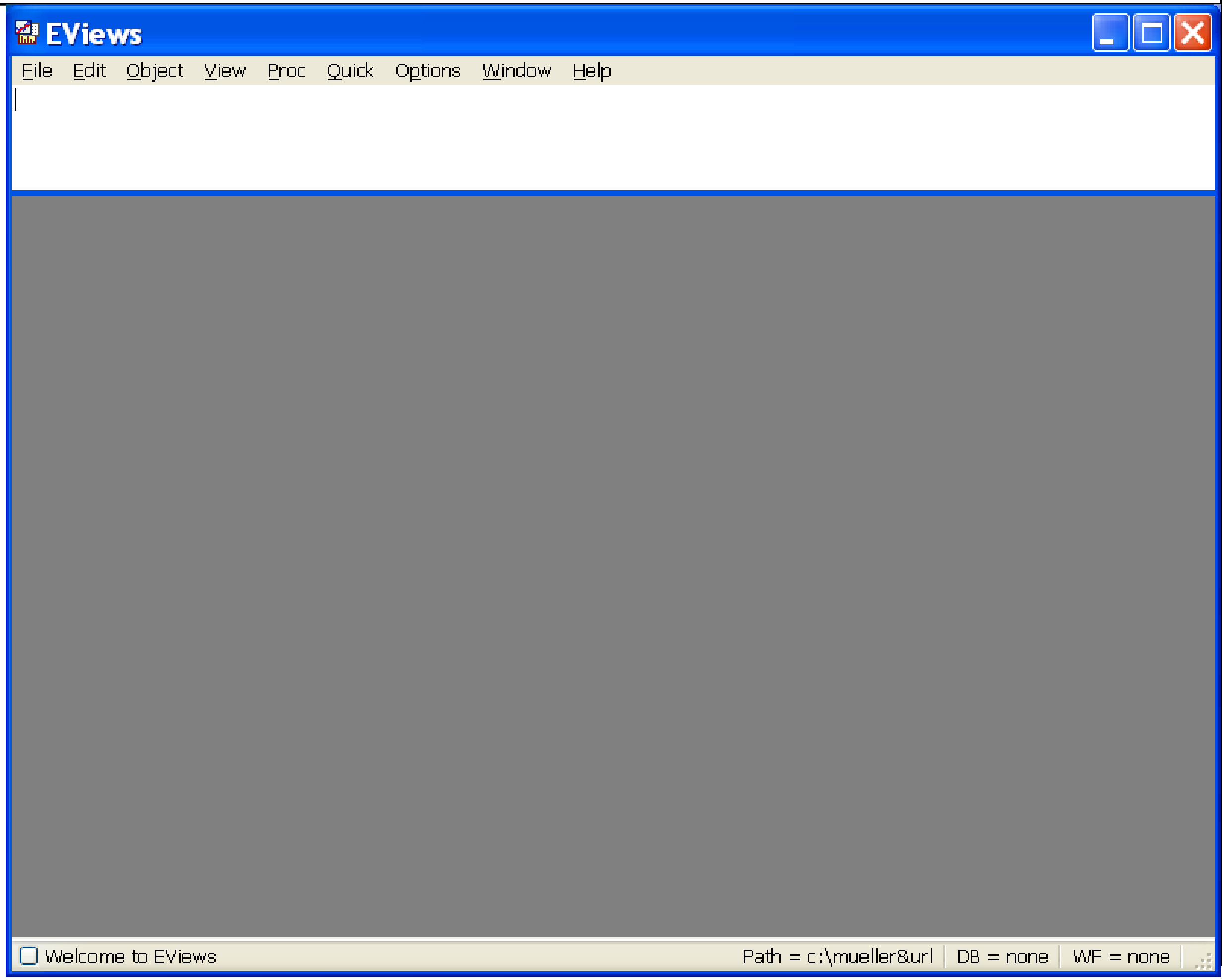
Graphics and Java-Applet from the thesis of Walter Ebner, WU 1999

1.1 Hyperinflation

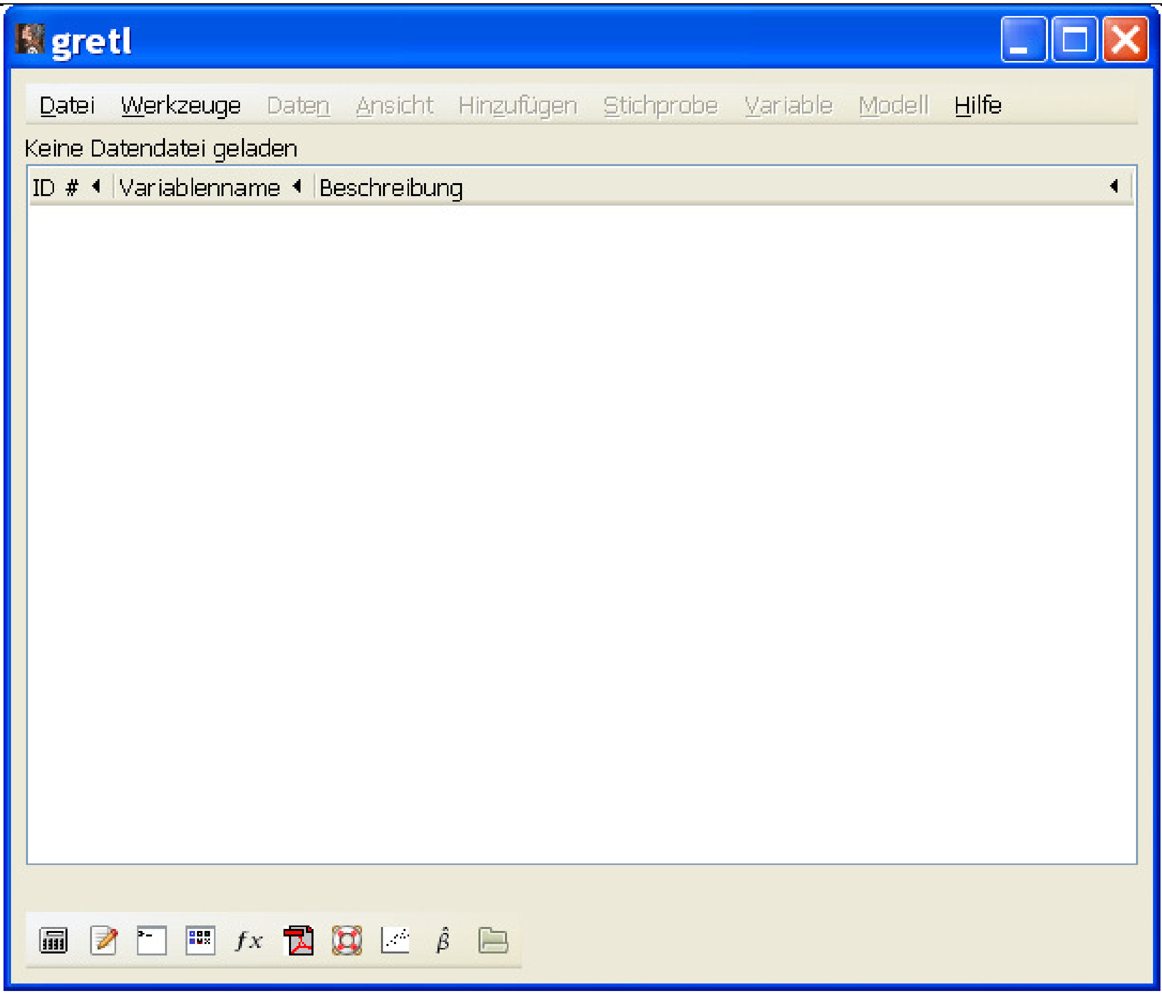
$$\frac{\Delta M}{P} = \frac{\Delta M}{M}\frac{M}{P} = \frac{\Delta M}{M}\left[\bar{Y}L\left(\bar{r}+\pi^{e}\right)\right]$$

Cagan, P., „The Monetary Dynamics of Hyperinflation“, in Friedman, M., „Studies in the Quantity Theory of Money“, University of Chicago Press, Chicago, 1956, S. 25-117.

1.2 Data Entry



1.2 Data Entry



Dataset:

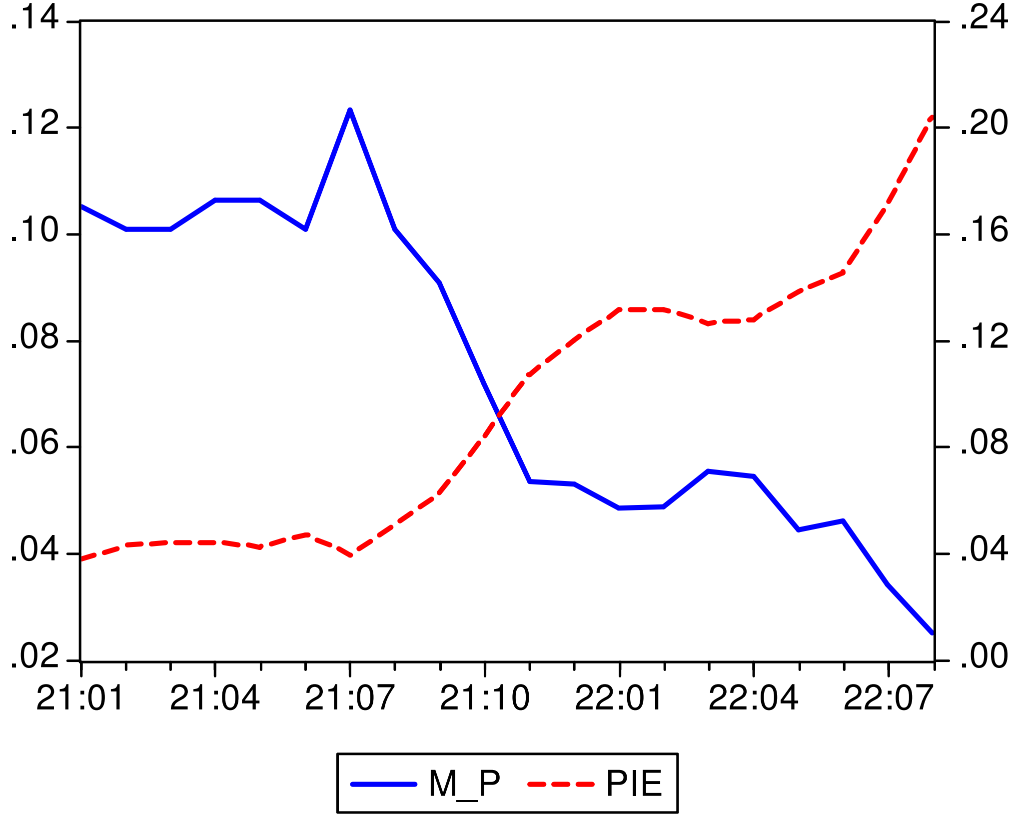
Monthly data,

from january 1921 until august 1922 from Austria.

M/P	π	π^e
0.1053	0.0601	0.0382
0.1010	0.1407	0.0428
0.1010	0.0682	0.0440
0.1064	0.0431	0.0440
0.1064	0.0058	0.0421
0.1010	0.1391	0.0470
0.1235	-0.1075	0.0394
0.1010	0.2747	0.0509
0.0909	0.2832	0.0622
0.0719	0.4960	0.0834
[illegible]	0.5782	0.1075
[illegible]	0.3753	0.1204
[illegible]	0.3569	0.1319
[illegible]	0.1283	0.1319
[illegible]	0.0269	0.1266
[illegible]	0.1492	0.1278
[illegible]	0.3435	0.1384
[illegible]	0.2871	0.1458
[illegible]	0.6544	0.1704
0.0252	0.8517	0.2038

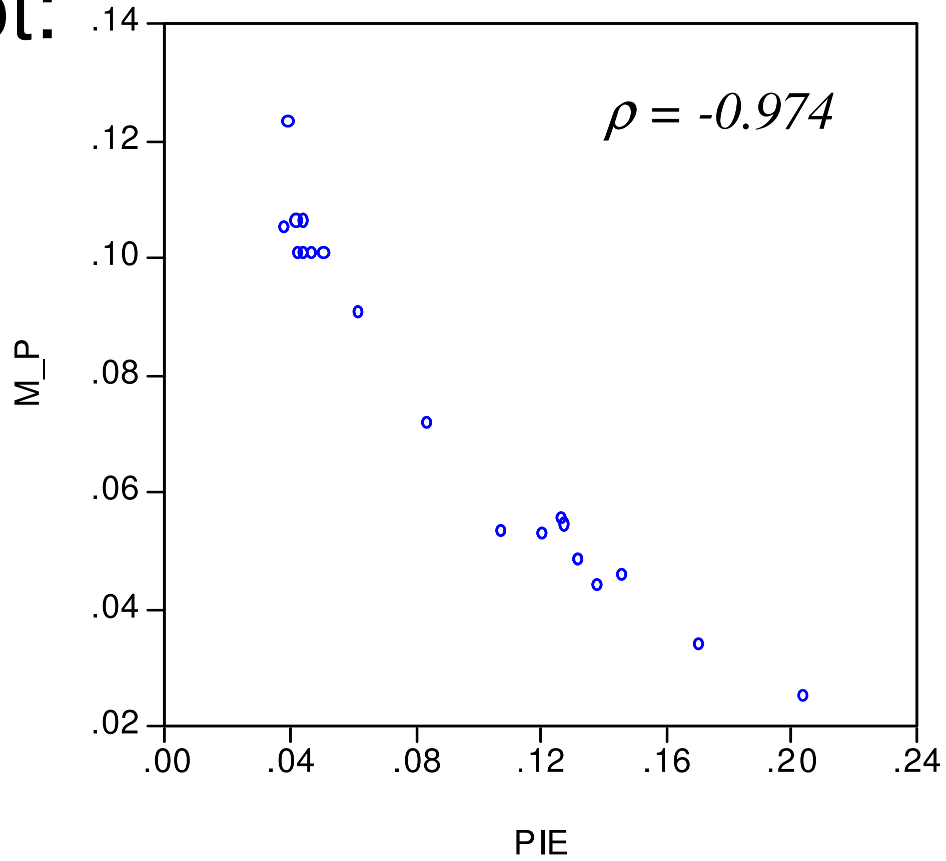
1.3 The Graphical Display of Data

The line-graph:



1.3 Graphical Display of Data

The Scatterplot:



Quantification by means of the coefficient of correlation (p429):

$$\rho = \frac{Kov(M/P, \pi^e)}{\sqrt{Var(M/P)Var(\pi^e)}}$$

1.4 Datatransformation

- Data-driven, e.g. Box-Cox transformation:

$$\tau(Y;\lambda) = \begin{cases} (Y^\lambda - 1)/\lambda & \text{if } \lambda \neq 0, \\ \ln(Y) & \text{if } \lambda = 0. \end{cases}$$

- Theory-driven, e.g. from a money demand function with proportional elasticity of inflation:

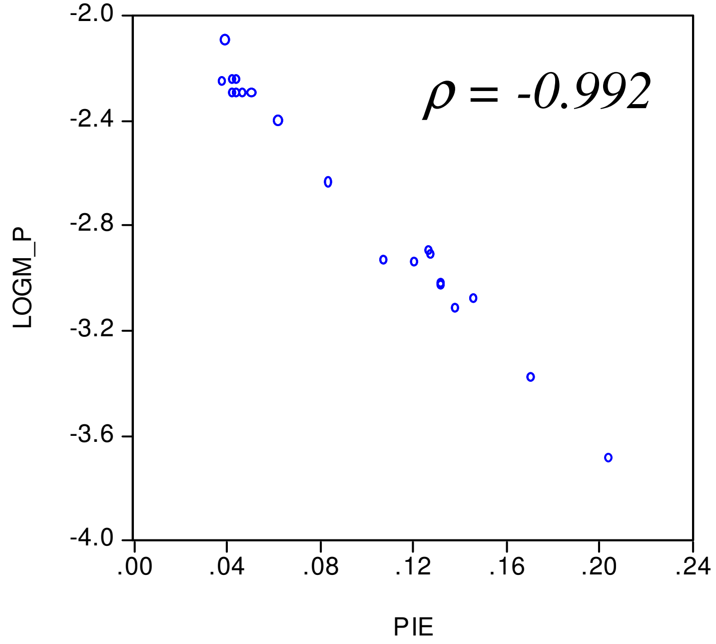
$$M/P = \exp\{-\alpha\pi^e - \gamma\}.$$

1.4 Datatransformation

- The non-linear relationship between money demand and expected inflation can be transformed to a linear relationship by taking logarithms on both sides:

$$\log M/P = -\alpha\pi^e - \gamma.$$

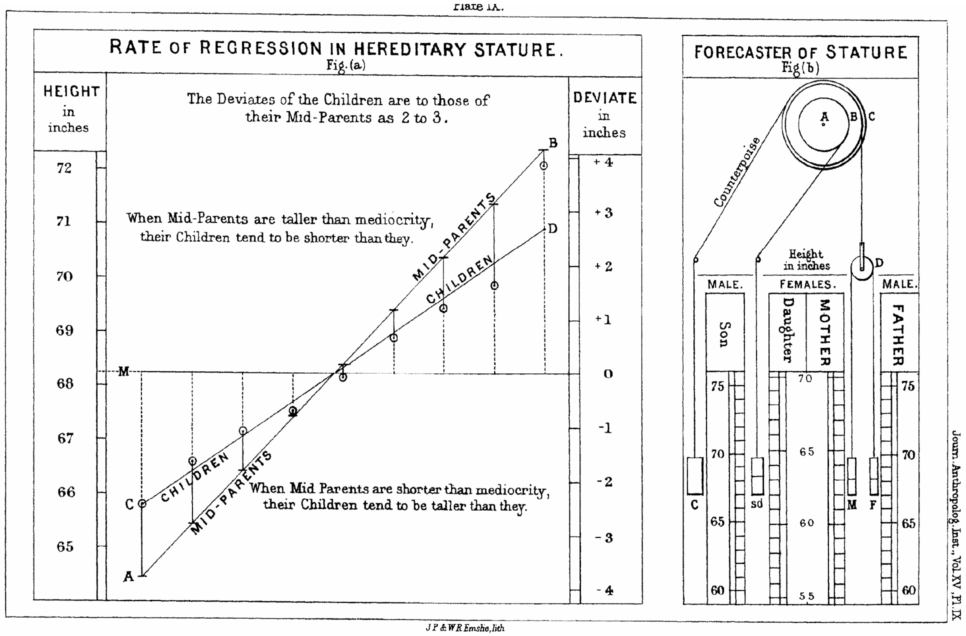
- For further analysis the variable m_p must thus be logarithmically transformed.

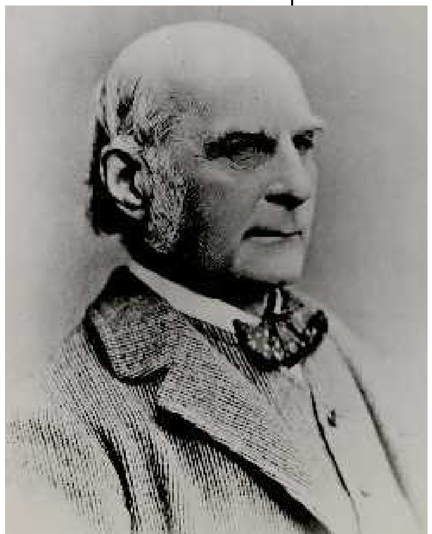


Parameter Estimation

Regression towards the mean

Sir Francis Galton (1886)

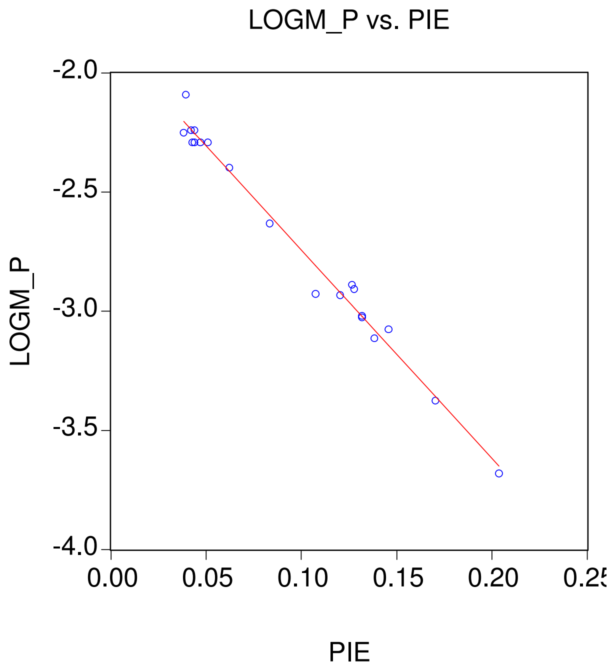




1.5 An Estimation Method

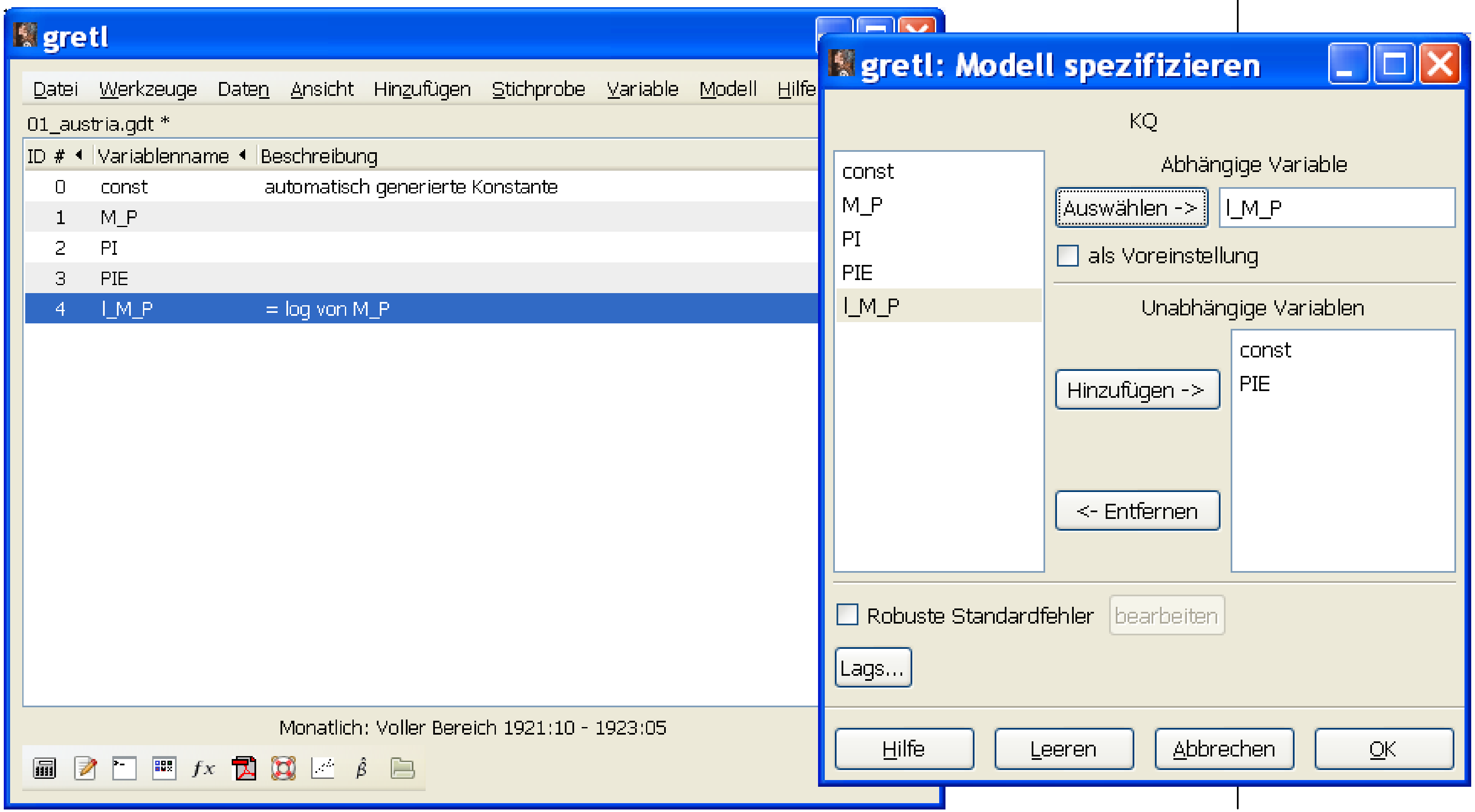
The most common method of regression analysis is the so-called **least-squares (LS** approach (8pp). Here, the sum of squared vertical deviations of the entries in the scatter-plot from the regression lines is minimized, thus in our example

$$\min_{\alpha,\gamma} \Sigma_t (\log M_t / P_t + \alpha \pi_t^e + \gamma)^2.$$

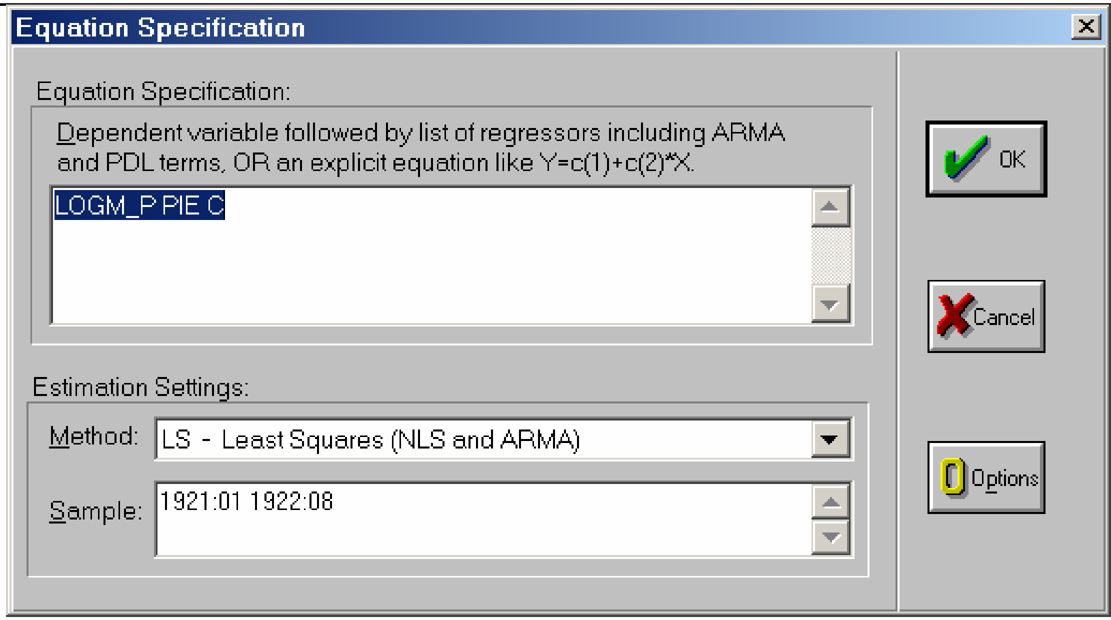


The solutions of this minimization task $\hat{\alpha}$ and $\hat{\gamma}$ are the (ordinary) least squares estimators of the parameters. They define the location (intercept) and slope of the regression line.

In GRETL


gretl
Datei Werkzeuge Daten Ansicht Hinzufügen Stichprobe Variable Modell Hilfe
01_austria.gdt *
ID # Variablenname Beschreibung
0 const automatisch generierte Konstante
1 M_P
2 PI
3 PIE
4 l_M_P = log von M_P
Monatlich: Voller Bereich 1921:10 - 1923:05
gretl: Modell spezifizieren
KQ
const
M_P
PI
PIE
l_M_P
Abhängige Variable
Auswählen ->
l_M_P
als Voreinstellung
Unabhängige Variablen
Hinzufügen ->
<- Entfernen
const
PIE
Robuste Standardfehler
bearbeiten
Lags...
Hilfe
Leeren
Abbrechen
OK

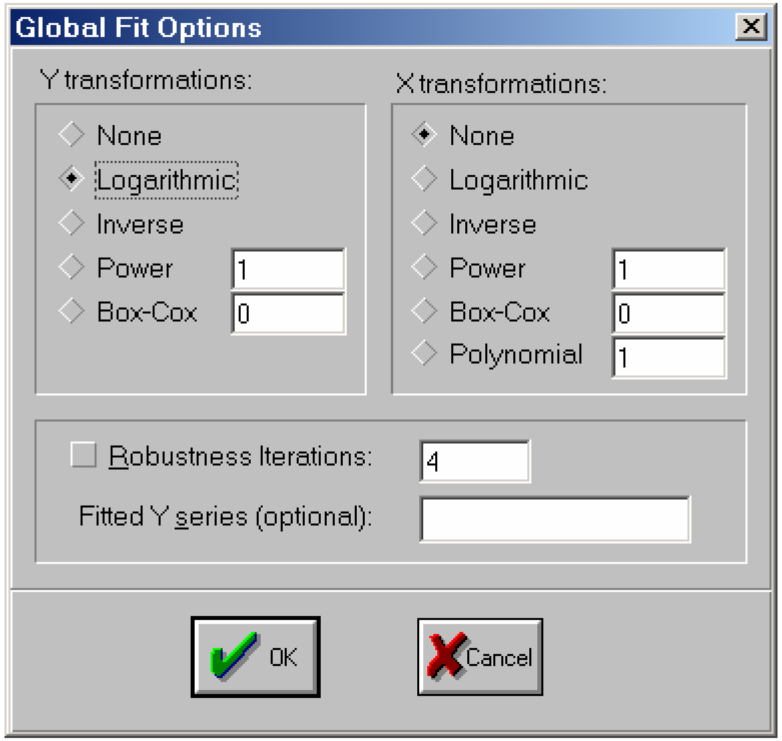
Concrete Estimates

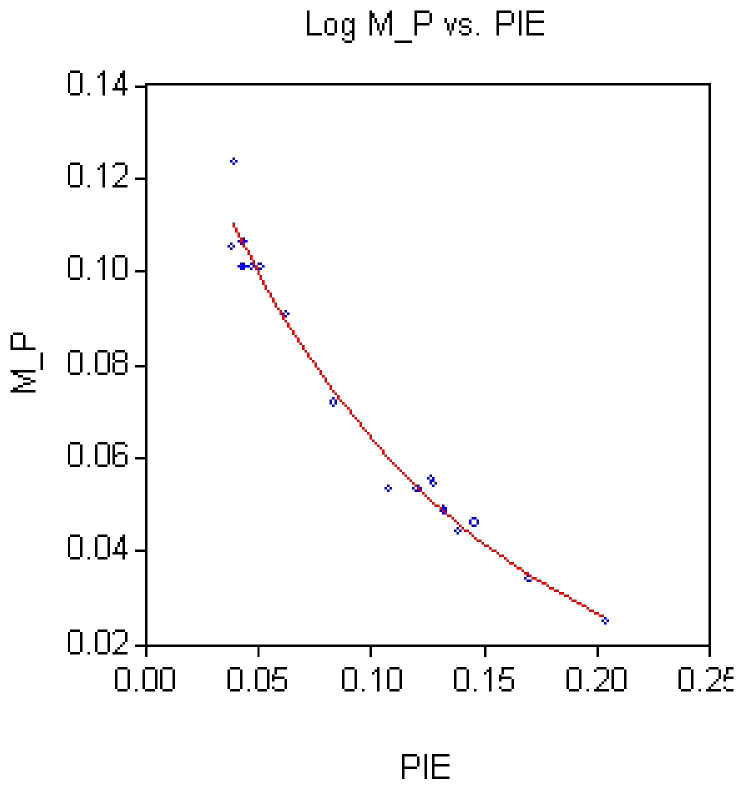


Entries under **Coefficient**, which denote the estimates $-\hat{\alpha} = -8.74$ and $-\hat{\gamma} = -1.87$

Backtransformation

via Global-Fit-Options:





Requirements (p15&16)

- The statistical model is linear in the parameters.
- The errors ε_t are random, they are independent of each other, have expectation 0 and a constant variance σ_ε^2.
- The regressors X are strictly exogeneous, i.e. independent from past, current and future errors and linearly independent.
- The number of observations exceeds the number of regressors.

Matrixnotation (p12)

The linear model can be written as

$$y = X\beta + \varepsilon$$

with $y = \begin{pmatrix} y_1 \\ \vdots \\ y_T \end{pmatrix}, X = \begin{pmatrix} 1 & x_1 \\ \vdots & \vdots \\ 1 & x_T \end{pmatrix}, \beta = \begin{pmatrix} \beta_0 \\ \beta_1 \end{pmatrix}, \varepsilon = \begin{pmatrix} \varepsilon_1 \\ \vdots \\ \varepsilon_T \end{pmatrix}$

Here is/are

$\beta=\{-\gamma,-\alpha\}'=\{\beta_0,\beta_1\}'$ the parameters,

$\varepsilon=\{\varepsilon_1,\dots,\varepsilon_T\}'$ the error terms,

$y=\{y_1,\dots,y_T\}'$ the regressand with $y_t = \log M_t/P_t$,

X the regressormatrix with $x=\{x_1,\dots,x_T\}'$ and $x_t = \pi_t^e$.

OLS-Estimator in Matrixnotation (p13)

The least squares estimator for β can now be compactly written as

$$\hat{\beta} = (X'X)^{-1}X'y\,, \qquad (2.19)$$

that is a special linear combination of the observations of the regressand.

Derivation (p12&13 and Appendix A)

Principle of least squares: $\hat{\beta} = \arg\min_\beta S(\beta)$

Partial differntiation of

$$S(\beta) = \varepsilon'\varepsilon = (y-X\beta)'(y-X\beta)$$
$$= y'y - 2y'X\beta + \beta'X'X\beta$$

leads to

$$\frac{\partial S}{\partial \beta} = -2X'y + 2X'X\beta$$

Setting equal zero yields the normal equations:

$$X'X\beta = X'y.$$

Invertibility

We require the matrix $X'X$ to be of full rank and thus also X (see requirements).

Caution! We do not have that, when

- $T < k$ (numbers of observations is smaller than numbers of parameters)
- there are linear relations between the vectors of regressors.

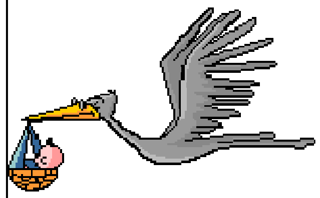
Residuals (p9)

Almost as a side-product during estimation we produce the so-called residuals. They correspond to the vertical distances of the observations form the regression line, i.e.

$$\hat{\varepsilon} = y - X\hat{\beta} = y - \hat{y}.$$

The residuals thus reflect the difference between the individual observed value of the explained variable y and the value $\hat{y}$, which is predicted (estimated) through the model.

1.6 Spurious Correlation (p323)



When two time series consist of growing or falling values, it often happens, that they exhibit high correlations, although they do not have any causal relationship.

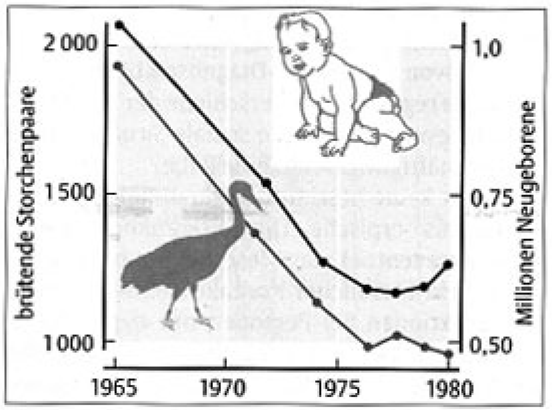
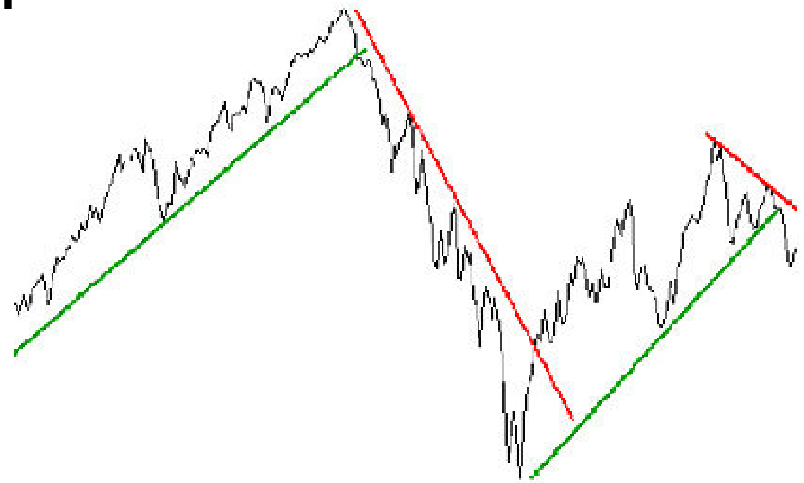


Abb. 2.3 Korrelation zwischen der Abnahme brütender Storchenpaare und dem Geburtenrückgang in der Bundesrepublik Deutschland zwischen 1965–1980 (nach Sies [30])

Solution: „detrending“

Trend

Longterm development of a variable



Linear detrending:

$$\hat{\varepsilon}_{yt} = \log M_t/P_t - \hat{\beta}_{y0} - \hat{\beta}_{y1}\, t$$

$$\hat{\varepsilon}_{xt} = \pi_t^e - \hat{\beta}_{x0} - \hat{\beta}_{x1}\, t$$

Corrected Result

Dependent Variable: RESIDLOGM_P

Method: Least Squares

Sample: 1921:01 1922:08

Included observations: 20

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	1.99E-16	0.009838	2.02E-14	1.0000
RESIDPIE	-10.84415	0.637443	-17.01195	0.0000

R-squared	0.941446	Mean dependent var	2.08E-16
Adjusted R-squared	0.938193	S.D. dependent var	0.176964
S.E. of regression	0.043995	Akaike info criterion	-3.314839
Sum squared resid	0.034840	Schwarz criterion	-3.215266
Log likelihood	35.14839	F-statistic	289.4065
Durbin-Watson stat	1.973688	Prob(F-statistic)	0.000000

The (multiple)2-Regressor Model (p19)

Again the linear model is written as

$$y = X\beta + \varepsilon$$

with $y = \begin{pmatrix} y_1 \\ \vdots \\ y_T \end{pmatrix}, X = \begin{pmatrix} 1 & x_{11} & x_{21} \\ \vdots & \vdots & \vdots \\ 1 & x_{1T} & x_{2T} \end{pmatrix}, \beta = \begin{pmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \end{pmatrix}, \varepsilon = \begin{pmatrix} \varepsilon_1 \\ \vdots \\ \varepsilon_T \end{pmatrix}$

Here is/are

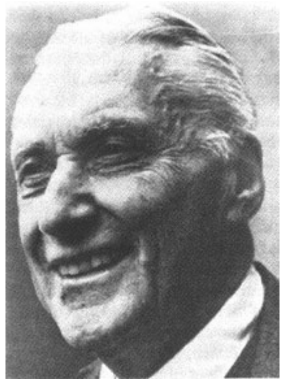
$\beta=\{-\gamma,-\alpha,\delta\}'=\{\beta_0,\beta_1,\beta_2\}'$ the parameters,

$\varepsilon=\{\varepsilon_1,\ldots,\varepsilon_T\}'$ the error terms,

$y=\{y_1,\ldots,y_T\}'$ the regressand with $y_t = \log M_t/P_t$,

X the regressormatrix with $x_1=\{\pi_1^e,\ldots, \pi_T^e\}'$ and $x_2=\{1,\ldots,t,\ldots,T\}'$.

Frisch-Waugh(-Lovell) Theorem



The coefficients from the linear detrended regressions correspond to the ones from multiple regressions with the trend as an additional regressor.

A more general version later….

1st Homework: Inflation during the southern Sung-Dynasty

Lui, F.T., “Cagan's Hypothesis and the First Nationwide Inflation of Paper Money in World History”, *Journal of Political Economy*, 91(6), 1983, S. 1067-1074 (also in “Major Inflations in History”, herausgegeben von F.H. Capie, Cheltenham, U.K., Edward Elgar Publishing Ltd., 1991).

Periode	M_t	P_t
1161-1170	100	100.0
1171-1180	204	86.7
1181-1190	224	107.3
1191-1200	827	183.9
1201-1210	1429	279.8
1211-1220	2347	280.2
1221-1230	2755	335.5
1240	4949	4032.2

2.3 The coefficient of determination (p21)

is a measure of Goodness of fit, determined from the variance of the residuals as follows

$$R^2 = 1 - \frac{\sum_t \hat{\varepsilon}_t^2}{\sum_t (y_t - \bar{y})^2} \qquad (2.42)$$

In the regression model with intercept, it holds that

$$0 \leq R^2 \leq 1$$

Exhibit 1

For each of the five questions below circle the most correct response. In answering these questions use the following definitions and diagram: Regression line: the line that minimizes the sum of squared errors; coefficient of determination: R^2, calculated as one minus the ratio of the sum of squared errors to the total sum of squares; total sum of squares: the sum of squared deviations of Y around its mean.

1. For the four sample points, which line best represents the average of the *Y* data? (Note that the point (0, 0) is one of the observations.)
A. *Ya* B. *Yb* C. *Yc* D. *Yd* E. *Ye*

2. For the four samples points, which line best represents the regression line?
A. *Ya* B. *Yb* C. *Yc* D. *Yd* E. *Ye*

3. The coefficient of determination for the regression line in Question 2 is:
A. Less than 0 B. 0 C. Between .0 and .5 D. .5 E. Greater than .5

4. If the regression line is forced to pass through the origin, which line best represents the regression line incorporating this zero intercept constraint:
A. *Ya* B. *Yb* C. *Yc* D. *Yd* E. *Ye*

5. The coefficient of determination for the constrained regression in Question 4 is:
A. Less than 0 B. 0 C. Between .0 and .5 D. .5 E. Greater than .5

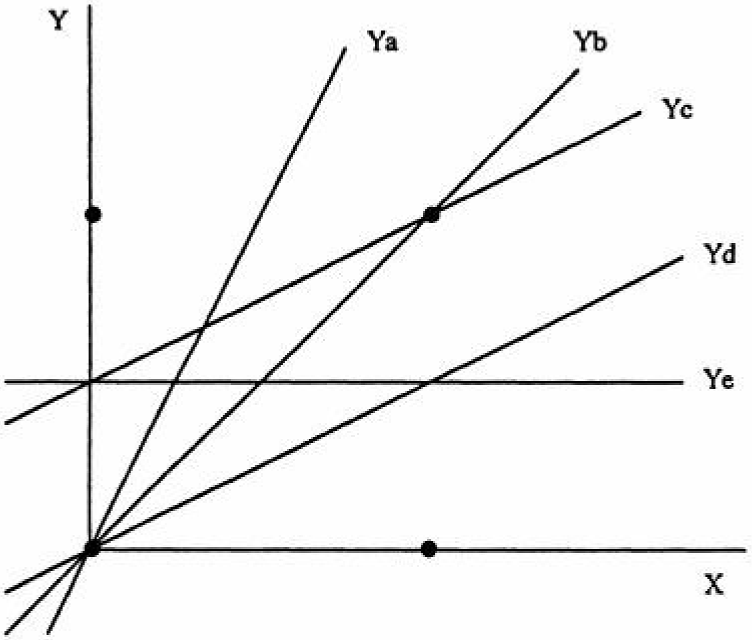


Figure 1.

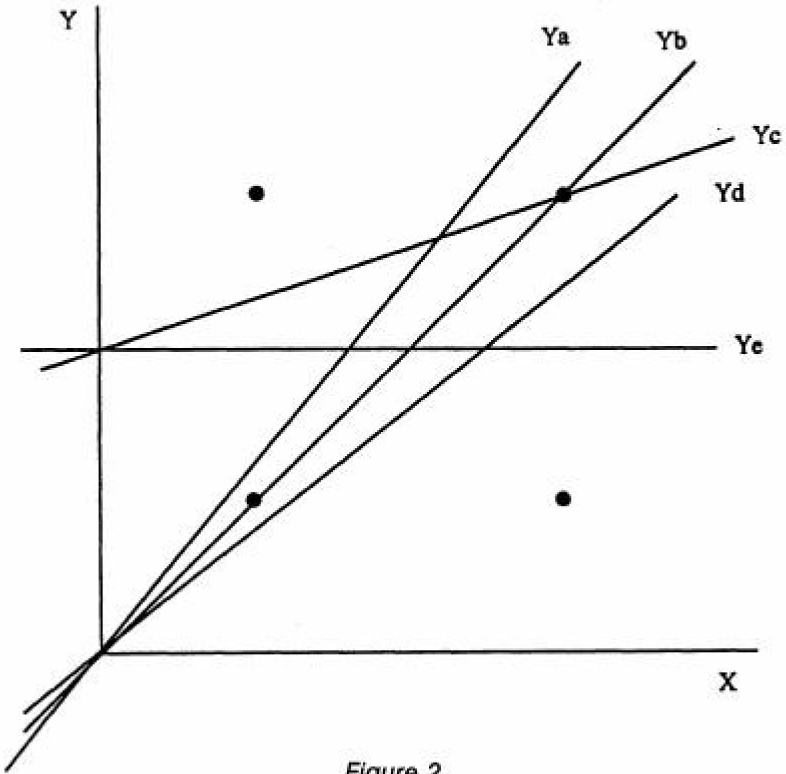


Figure 2.

Econometrics on TV

Series 11, Season 1 of NUMB3RS