2. Unemployment

„Auf die Arbeit schimpft man nur so lange, bis man keine mehr hat.“

Harry Sinclair Lewis
(1856-1950)

Chapter 2 - Contents

2. Unemployment (Okun's Law - *simple lineare regression inferential*)

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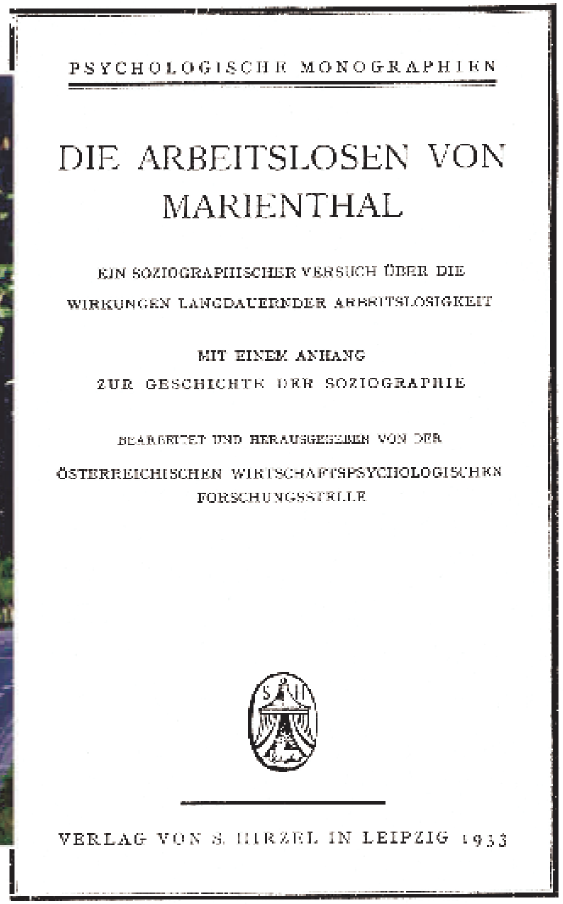
2.7 Homework (*Okun's Law in alternative forms*)

An eternal hot topic



PSYCHOLOGISCHE MONOGRAPHIEN

DIE ARBEITSLOSEN VON MARIENTHAL

EIN SOZIOGRAPHISCHER VERSUCH ÜBER DIE WIRKUNGEN LANGDAUERNDER ARBEITSLOSIGKEIT

MIT EINEM ANHANG ZUR GESCHICHTE DER SOZIOGRAPHIE

BEARBEITET UND HERAUSGEGEBEN VON DER ÖSTERREICHISCHEN WIRTSCHAFTSPSYCHOLOGISCHEN FORSCHUNGSSTELLE

VERLAG VON S. HIRZEL IN LEIPZIG 1933

Okun´s Law:

Negative but underproportional relationship between economic growth and increase of the rate of unemployment!

Arthur M. Okun

(1928-1980)

Okun, A.M., "Potential GNP: Its measurement and significance", Cowles Foundation Paper 190, 1962.

Okun's equation in first differences:

$$u_t - u_{t-1} = 0.30 - 0.30\, g_{yt}.$$

For the reduction of the unemployment rate of *1* percentage point we require a growth of the economy of $1\% / 0.3 = 3.3\%$ (the Okun coefficient).

Constantine Manos *USA. Unemployed, 1957*

The Potential Output:

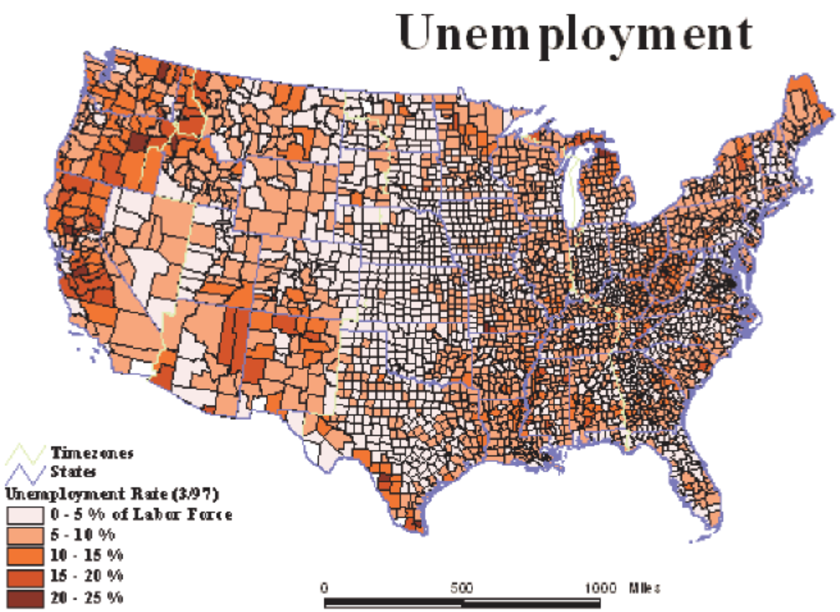
Level of output on which all resources are fully exhausted,

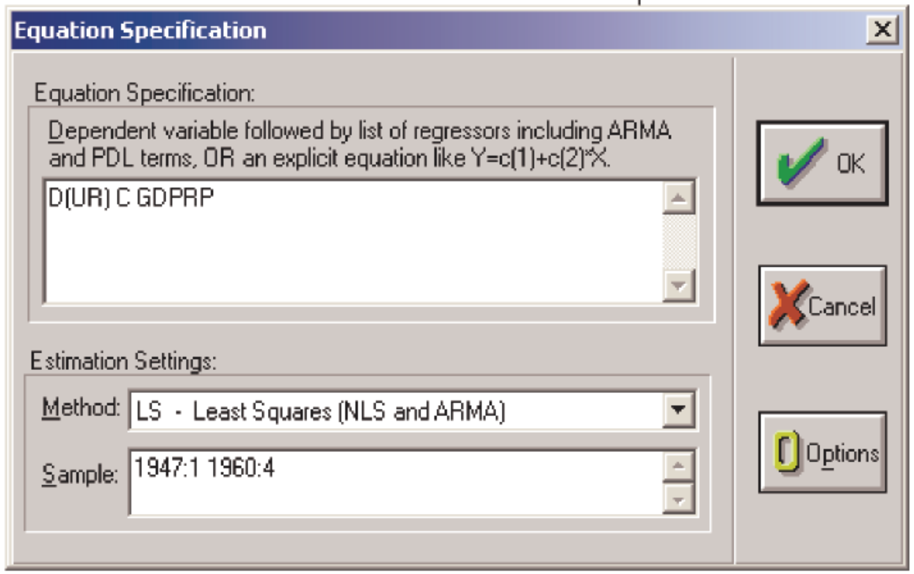
here $p_t = Y_t[1 + 0.033(u_t - 4)]$.

Mchael Lenson
Full Production and Full Employment under Our Democratic System of Private Enterprise, ca. 1944

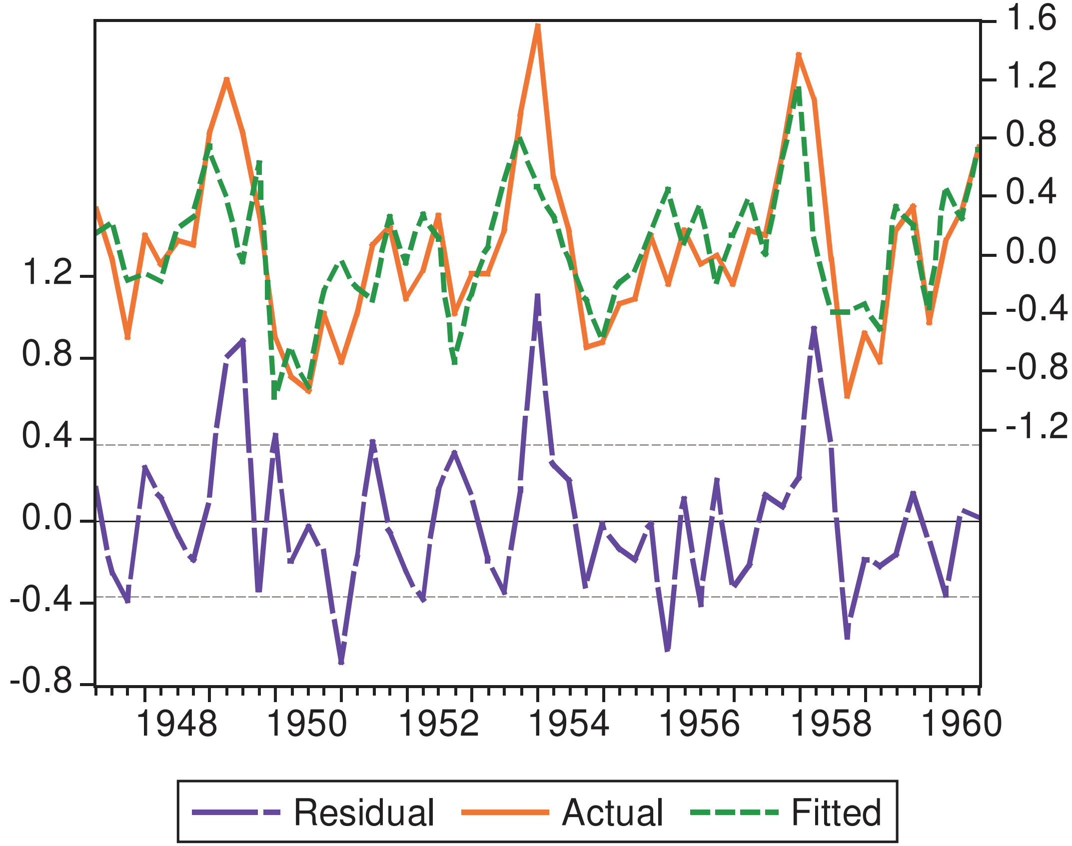
Okun´s regression

Data: USA from second quarter 1947 until fourth quarter 1960

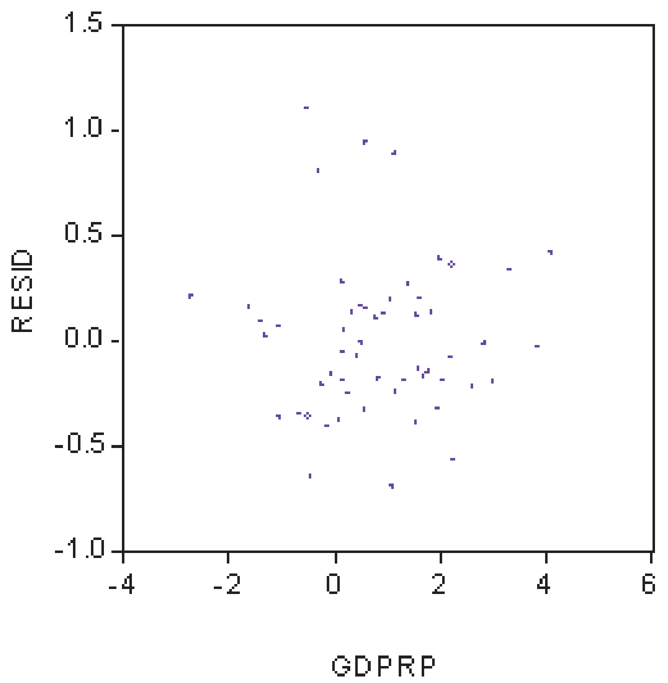


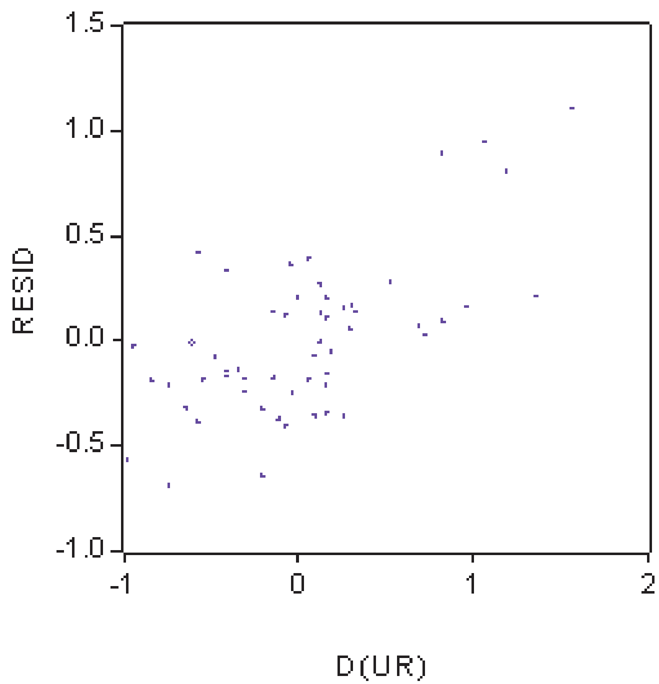


2.2 Residual Plots



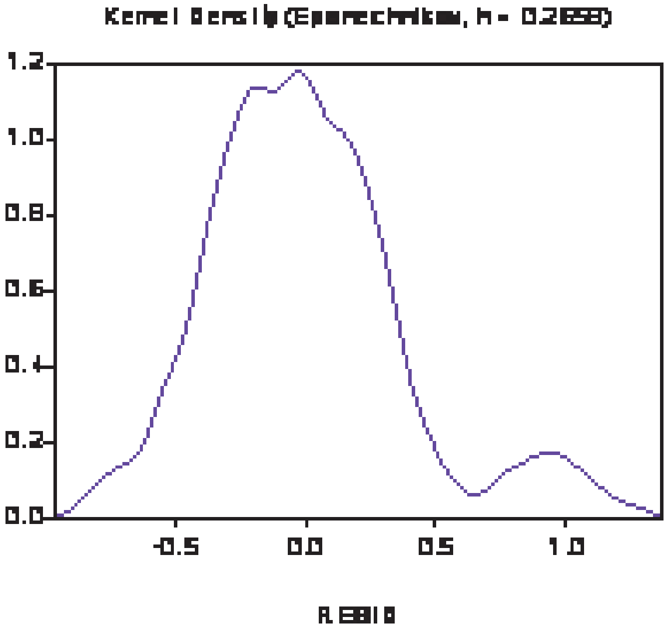
Residual Scatterplots

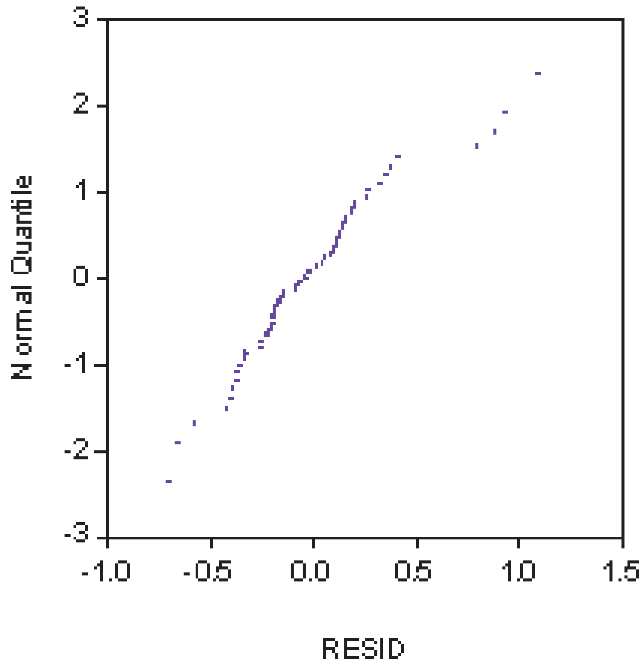




against regressor and regressand.

Distribution Plots





Kernel density and Q-Q Plot

2.3 The Coefficient of Determination (p21ff)

is motivated from the decomposition of sum of squares in the linear regression:
SST = SSR + SSE

$$\sum_t (y_t - \bar{y})^2 = \sum_t (\hat{y}_t - \bar{y})^2 + \sum_t \hat{\varepsilon}_t^2,$$

$$y'(I - \iota\iota'/T)y = \hat{y}'(I - \iota\iota'/T)\hat{y} + \hat{\varepsilon}'\hat{\varepsilon}$$

The fraction of the explained from the total variation is called the coefficient of determination or

$$R^2 = 1 - \frac{\sum_t \hat{\varepsilon}_t^2}{\sum_t (y_t - \bar{y})^2} = 1 - \frac{\hat{\varepsilon}'\hat{\varepsilon}}{y'(I - \iota\iota'/T)y}$$

R^2 in the output:

$$s_{\hat{\varepsilon}}^2 = \frac{\sum_t \hat{\varepsilon}_t^2}{T-k-1}$$

$$R^2 = 1 - \frac{53 \times 0.371312^2}{54 \times 0.569696^2} = 0.583059$$

$$s_y^2 = \frac{1}{T-1} \sum_t (y_t - \bar{y})^2$$

Dependent Variable: D(UR)
Method: Least Squares
Sample(adjusted): 1947:2 1960:4
Included observations: 55 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.311155	0.058945	5.278767	0.0000
GDPRP	-0.313616	0.036429	-8.609075	0.0000

R-squared	0.583059	Mean dependent var	0.043345
Adjusted R-squared	0.575192	S.D. dependent var	0.569696
S.E. of regression	0.371312	Akaike info criterion	0.892140
Sum squared resid	7.307262	Schwarz criterion	0.965134
Log likelihood	-22.53384	F-statistic	74.11617
Durbin-Watson stat	1.646644	Prob(F-statistic)	0.000000

Correspondence with the Correlation Coefficient

In the simple linear regression holds:

$$R^2 = Korr(x,y)^2.$$

Since $\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$ we have $\hat{y}_i - \bar{y} = \hat{\beta}_1 (x_i - \bar{x})$

and $\sum (\hat{y}_i - \bar{y})^2 = \hat{\beta}_1^2 \sum (x_i - \bar{x})^2$

since $\hat{\beta}_1 = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sum (x_i - \bar{x})^2}$ then holds

$$R^2 = \frac{\sum (\hat{y}_i - \bar{y})^2}{\sum (y_i - \bar{y})^2} = \frac{\left[\sum (x_i - \bar{x})(y_i - \bar{y})\right]^2}{\sum (x_i - \bar{x})^2 \sum (y_i - \bar{y})^2}$$

Correspondence with the Correlation Coefficient (p23)

Always holds (in linear eegression):

$$R^2 = Korr(\hat{y}, y)^2.$$

Define $M^0 = I - \iota\iota'/T$

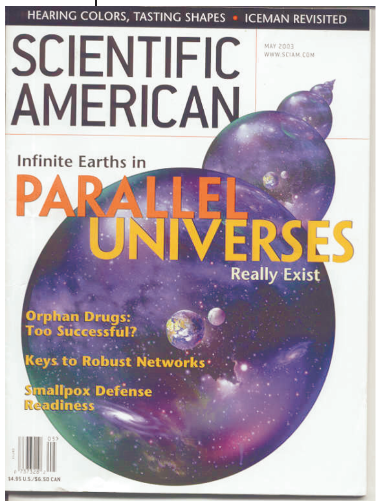
then we have $\hat{y}'M^0\hat{y} = \hat{y}'M^0 y$ and thus

$$R^2 = \frac{\hat{y}'M^0\hat{y}}{y'M^0 y} = \frac{\hat{y}'M^0\hat{y}}{y'M^0 y} \cdot \frac{\hat{y}'M^0 y}{\hat{y}'M^0\hat{y}}$$

2.4 *t* -test and *p* -values (p23ff)

Basic Assumption/Fiction:

one considers the existing data (pairs) as a random sample from a universe of possible configurations, that are generated from an economy in a constant state.



The Distribution of Estimators: (p17)

$$E[\hat{\beta}\,|X] = E[(X'X)^{-1}X'y\,|X]$$
$$= E[(X'X)^{-1}X'(X\beta+\varepsilon)\,|X]$$
$$= E[\beta + (X'X)^{-1}X'\varepsilon\,|X]$$
$$= \beta + (X'X)^{-1}X'E[\varepsilon\,|X] = \beta$$

$$Var[\hat{\beta}\,|X] = Var[\beta + (X'X)^{-1}X'\varepsilon\,|X]$$
$$= (X'X)^{-1}X'Var[\varepsilon\,|X]X(X'X)^{-1}$$
$$= \sigma_\varepsilon^2\,(X'X)^{-1}X'X(X'X)^{-1} = \sigma_\varepsilon^2\,(X'X)^{-1}.$$

$$\hat{\beta} = \beta + (X'X)^{-1}X'\varepsilon : \; N\left(\beta, \sigma_\varepsilon^2 (X'X)^{-1}\right)$$

The Gauß-Markov Theorem (1821,1912) p18

Amongst all linear, unbiased estimators is the OLS-estimator the one with the smallest variance!

(Johann) Carl Friedrich Gauß (1777-1855)

Андрей Андреевич Марков (1856-1922)

Derivation:

Be $\beta^* = Cy$ an unbiased estimator, then:

$$E[\beta^*] = E[CX\beta + C\varepsilon]\} = CX\beta = \beta,$$

which yields: $CX=I$; so one gets

$$\mathrm{Var}[\beta^*] = \mathrm{Var}[CX\beta + C\varepsilon] = C\sigma^2 IC' = CC'\sigma^2$$

Be $C = (X'X)^{-1}X'+D$; since $CX=I$ we have $DX=0$; one yields $CC' = (X'X)^{-1}+DD'$

and $CC' - (X'X)^{-1} \geq 0$

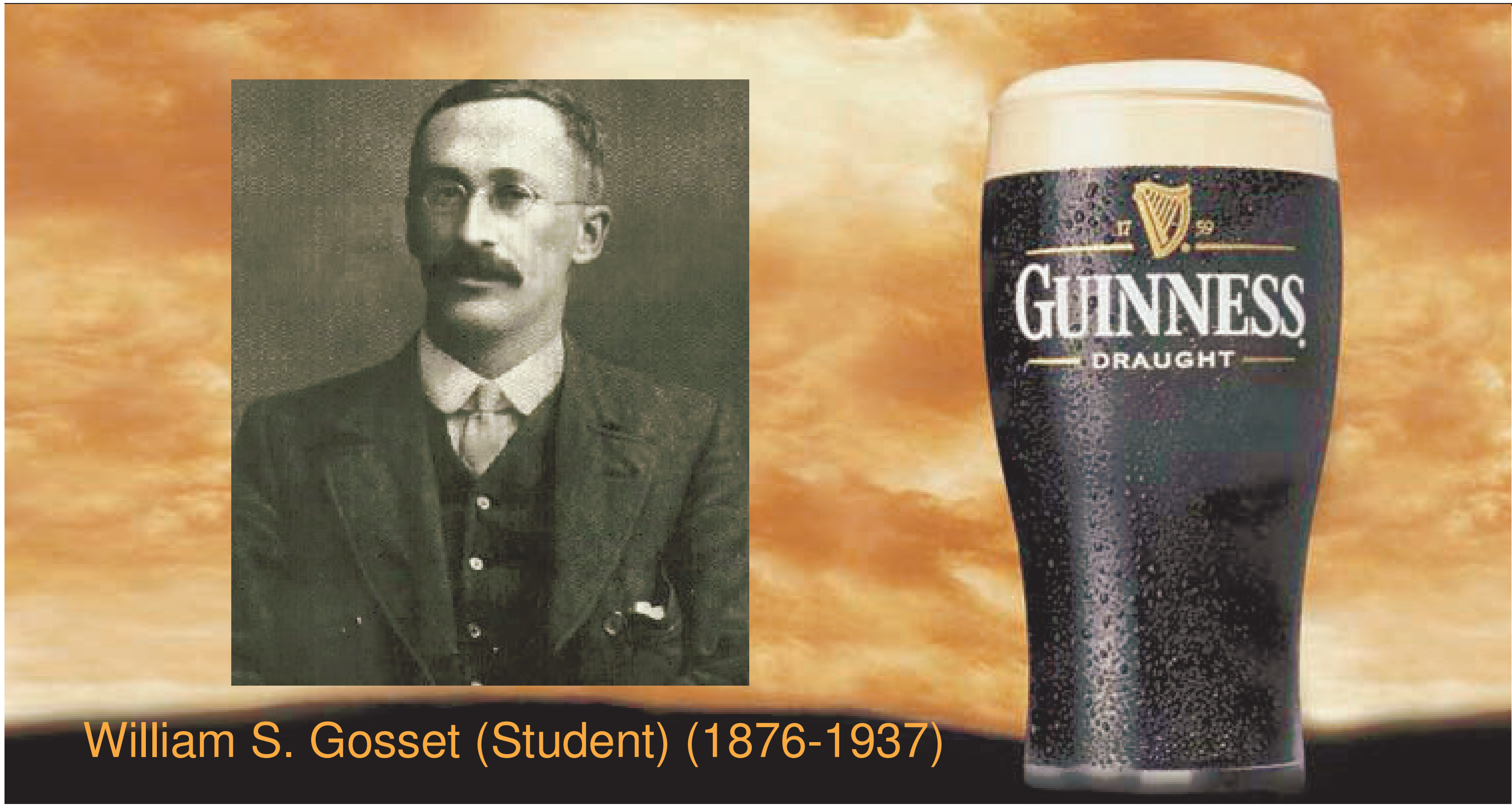
Thus $\hat{\beta}$ is of minimal variance.

Statistical Tests

Are used for checking hypotheses, e.g. of the form

$H_0: \beta_i = 0.$

What has that to do with Econometrics?



William S. Gosset (Student) (1876-1937)

$\hat{\beta}_i / \hat{\sigma}_\varepsilon \sqrt{(X'X)^{-1}_{ii}}$ ~ t-distribution, where $\hat{\sigma}_\varepsilon^2 = \sum_t \hat{\varepsilon}_t^2/(T-k-1)$

Output

$$t-\text{value} = \frac{\hat{\beta}}{\hat{\sigma}_\varepsilon \sqrt{(X'X)^{-1}_{ii}}}$$

$$p-\text{value} = 1 - F_t\left(\left|t-\text{value}\right|\right)$$

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Log likelihood	-22.53384	F-statistic	74.11617
Durbin-Watson stat	1.646644	Prob(F-statistic)	0.000000

Errors in Testing (p31f)

		Decision	
		H_0 accept	H_0 reject
Status of the world	H_0	correct decision	error of 1st kind (α)
	$\bar{H}_0$	error of 2nd kind (β)	correct decision

www.stauff.de/matgesch/dateien/fehlerersterzweiterart.htm

2.5 Studentized Residuals and …

Under the given assumptions residuals are normally distributed with expectation 0 and variance $\sigma_\varepsilon^2(1-H_{tt})$.

The matrix $H=X(X'X)^{-1}X'$ is called hat matrix, since it transforms the original values into the predicted values $\hat{y} = Hy$

Vectormanipulation in Eviews: external and internal studentizing

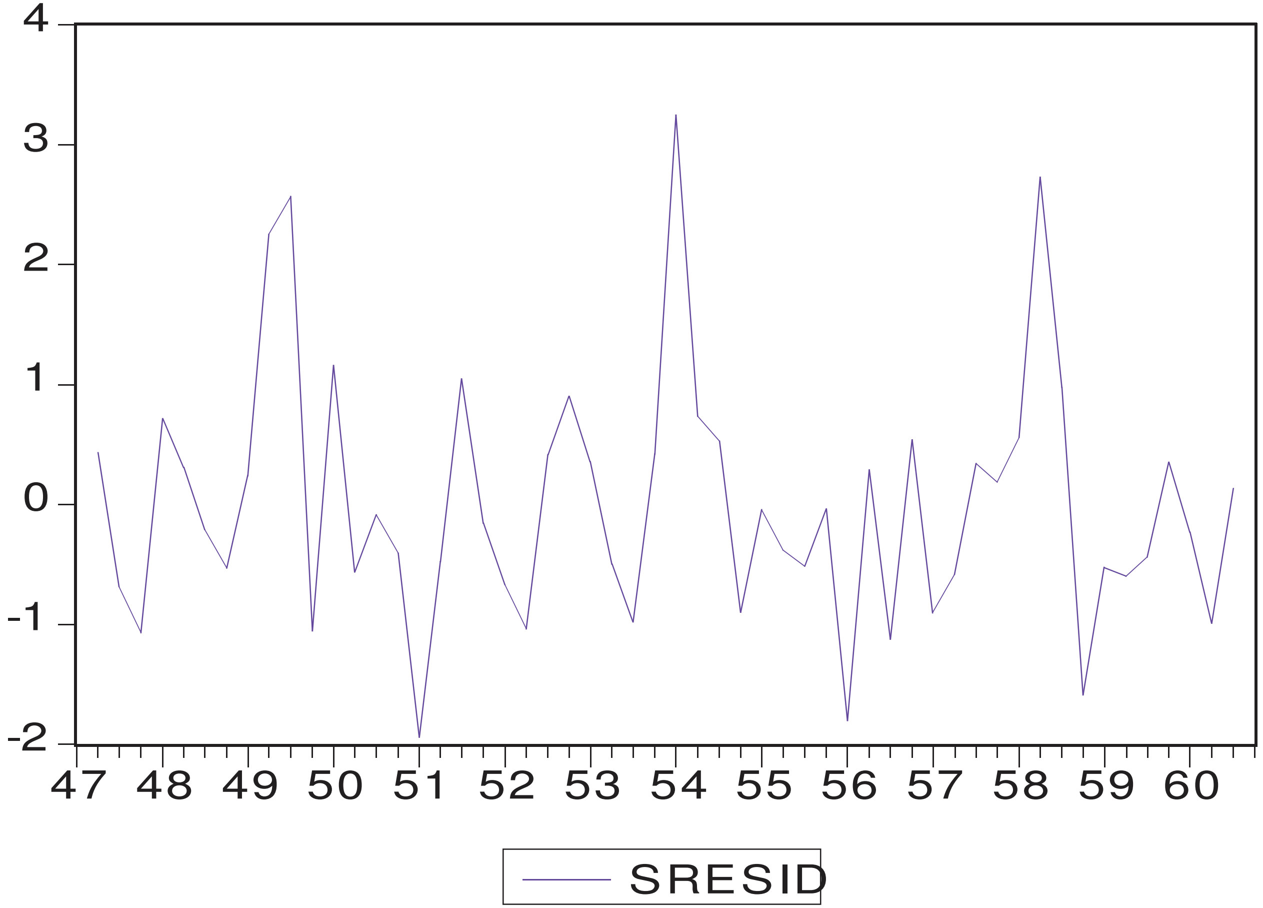
```
stom(groupx,x)

vector htt=@getmaindiagonal(x*@inverse(@inner(x))*  @transpose(x))

mtos(htt, hhtt)

series rresid=resid/@sqrt(1-hhtt)/okun.@se
```

$$r_t = \frac{\hat{\varepsilon}_t}{\hat{\sigma}_\varepsilon \sqrt{1 - H_{tt}}}$$

```
series sresid=rresid*@sqrt((okun.@regobs-2)/ (okun.@regobs-1-rresid^2))
```

$$s_t = r_t \sqrt{\frac{T - k - 1}{T - k - r_t^2}}$$

externally studentized residuals (t-distributed) for Okun regression



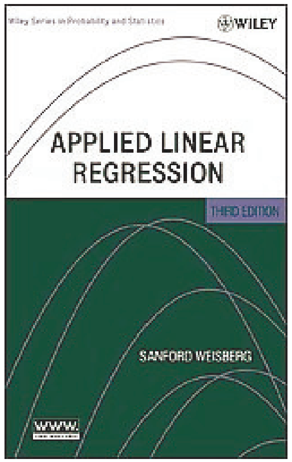
Multiple Testing

Based on the Bonferroni inequality:

$$P(A \cap B) \geq P(A) + P(B) - 1$$

Carlo Emilio Bonferroni (1892-1960)

With many! tests benchmark 3.5, see

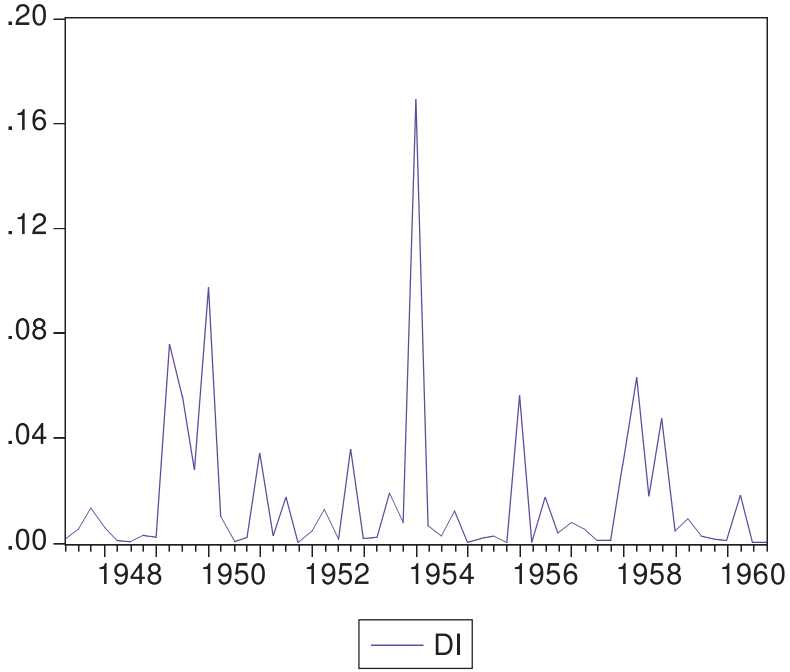


Sanford Weisberg

Cook´s distances

Distances in the parameter space, that arise when single observations get deleted:

$$D_t = \frac{1}{k+1} r_t^2 \left(\frac{H_{tt}}{1-H_{tt}} \right)$$



R. Dennis Cook

„Robustified“ Estimation

Dependent Variable: D(UR)
Method: Least Squares
Sample: 1947:2 1953:4 1954:2 1960:4
Included observations: 54

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.277743	0.055130	5.037979	0.0000
GDPRP	-0.298737	0.033789	-8.841121	0.0000

R-squared	0.600508	Mean dependent var	0.015130
Adjusted R-squared	0.592826	S.D. dependent var	0.534849
S.E. of regression	0.341288	Akaike info criterion	0.724154
Sum squared resid	6.056835	Schwarz criterion	0.797821
Log likelihood	-17.55217	F-statistic	78.16543
Durbin-Watson stat	1.680899	Prob(F-statistic)	0.000000

$$u_t - u_{t-1} = 0.278 - 0.299\, g_{yt} = -\,0.30\,(g_{yt} - 0.93).$$

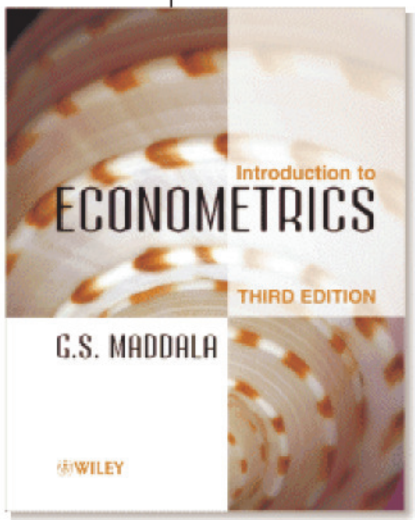
2.6 Reverse Regression

Exchange regressor and regressand!

Instead of $y_t = \beta_0 + \beta_1 x_t + \varepsilon_t$ use $x_t = \lambda_0 + \lambda_1 y_t + \xi_t$.

The estimator is analogously given as $\hat{\lambda} = (Y'Y)^{-1}Y'x$ with $Y = \{\iota, y\}$ and thus $\hat{\lambda}_1 = R^2/\hat{\beta}_1$.

Generalization: error-in-variables models

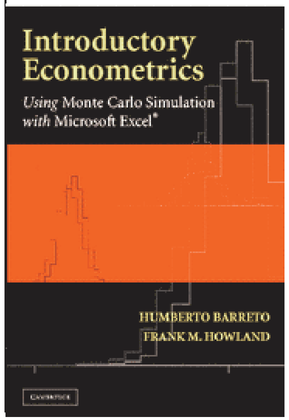


Reverse Regression

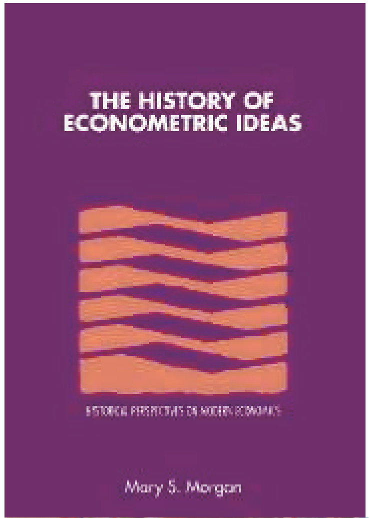
Dependent Variable: GDPRP
Method: Least Squares
Sample: 1947:2 1960:4
Included observations: 55

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.934528	0.122262	7.643630	0.0000
DUR	-1.859149	0.215952	-8.609075	0.0000

R-squared	0.583059	Mean dependent var	0.853942
Adjusted R-squared	0.575192	S.D. dependent var	1.387078
S.E. of regression	0.904060	Akaike info criterion	2.671845
Sum squared resid	43.31823	Schwarz criterion	2.744839
Log likelihood	-71.47573	F-statistic	74.11617
Durbin-Watson stat	2.070022	Prob(F-statistic)	0.000000



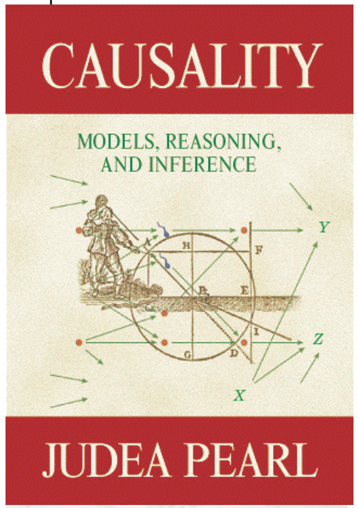
Causality

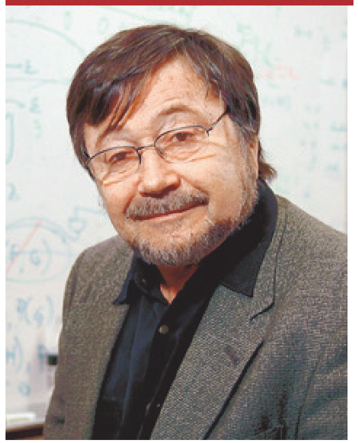


Controversial, fundamental Problem:

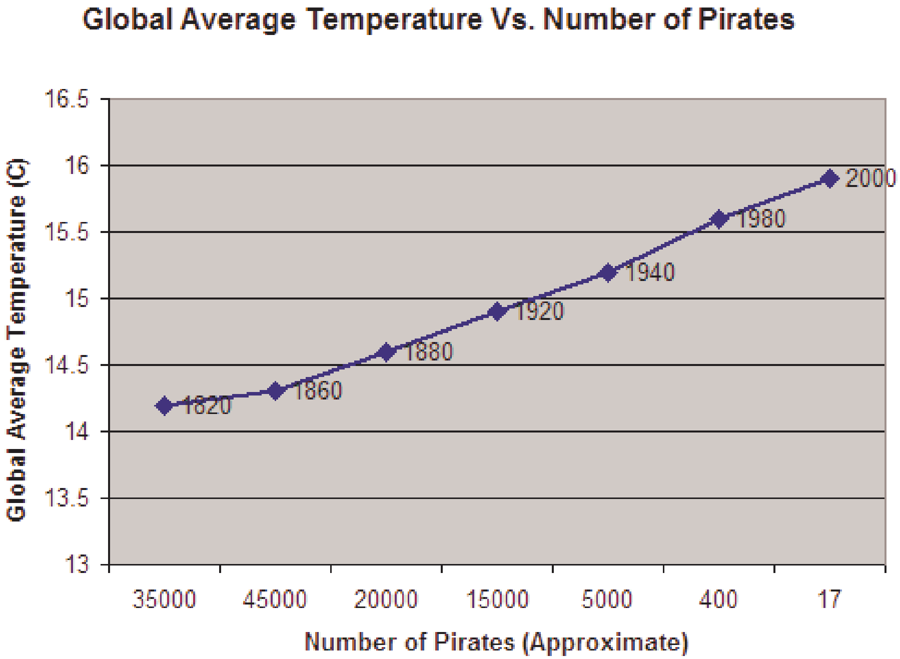
mostly - which variable shall be predicted by which, thus the dominant direction of prediction.

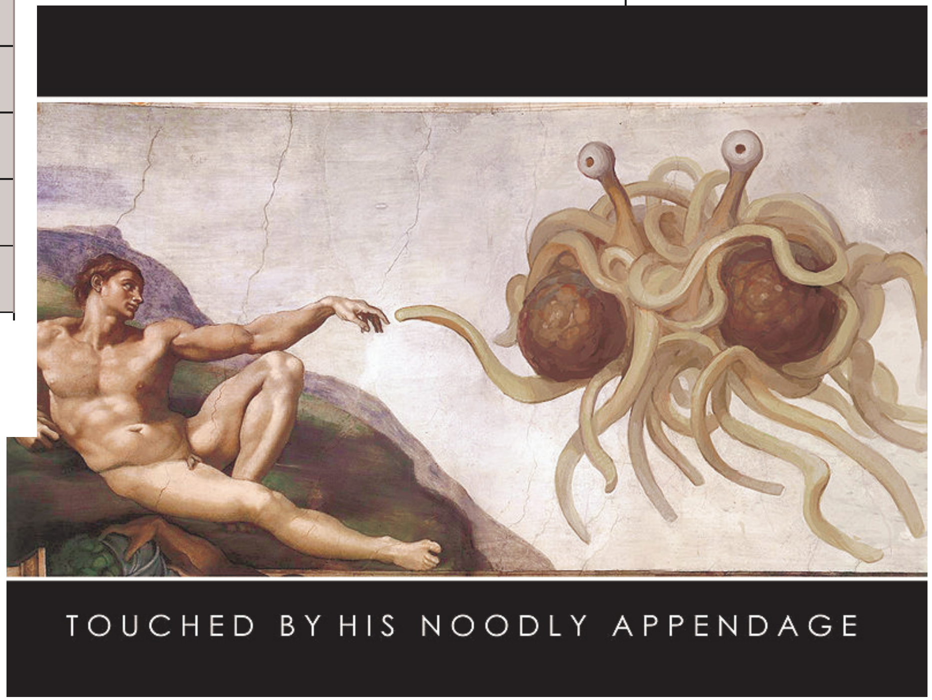
here rather $E[u_t - u_{t-1} \mid g_{yt}]$





Example: Pastafarianismus





Homework 2

- Blanchard (2006) employs in equation 9.2 not *3.5* % for the average growth rate of the potential output, but *3.0* %.
- Okun suggests in his article two further specifications. The first is

$$u_t = \beta_0 + \beta_1\, gap_t,$$

with $gap_t = yp_t / y_t - 1$.

- The second suggested equation is

$$\log\,(100 - u_t) = \beta_0 + \beta_1 \log y_t + \beta_2 t.$$