4310 : Macroeconomics

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Administrative

1. Questions???

The envelope theorem

Consider an arbitrary maximization problem where the objective function f depends on some parameter a:

$$M(a) = \max_x f(x, a),$$

Let $x(a)$ be the value of x that solves the maximization problem in terms of the parameter a. Then $M(a) = f(x(a), a)$. The envelope theorem tells us how $M(a)$ changes as the parameter a changes, namely:

$$\frac{\mathrm{d}M(a)}{da} = \left.\frac{\partial f(x^*, a)}{\partial a}\right|_{x^*=x(a)}.$$

I.e. the derivative of M with respect to a is given by the partial derivative of $f(x, a)$ with respect to a, holding x fixed, and then evaluating at the optimal choice (x^*).

The Fundamental Theorems of Welfare Economics

- First Fundamental Welfare Theorem: every competitive equilibrium is Pareto-optimal.

- Second Fundamental Welfare Theorem: every Pareto-optimal allocation can be achieved as a competitive equilibrium after a suitable redistribution of initial endowments.

Solving the Social Planner's problem

When the first welfare theorem holds the solution to the social planner's problem is the competitive equilibrium allocation.

That is, there exists a set of prices such that the optimum solution can be decentralized as a competitive equilibrium with a price system that has an inner product representation.

The social planner's problem is much easier to solve since we get rid of the prices and the individuals' budget constraint.

Remember: 1WT does in general not hold in an overlapping generations economy.

Conditional expectation

In probability theory, a *conditional expectation* (also known as conditional expected value or conditional mean) is the expected value of a real random variable with respect to a *conditional probability distribution*.

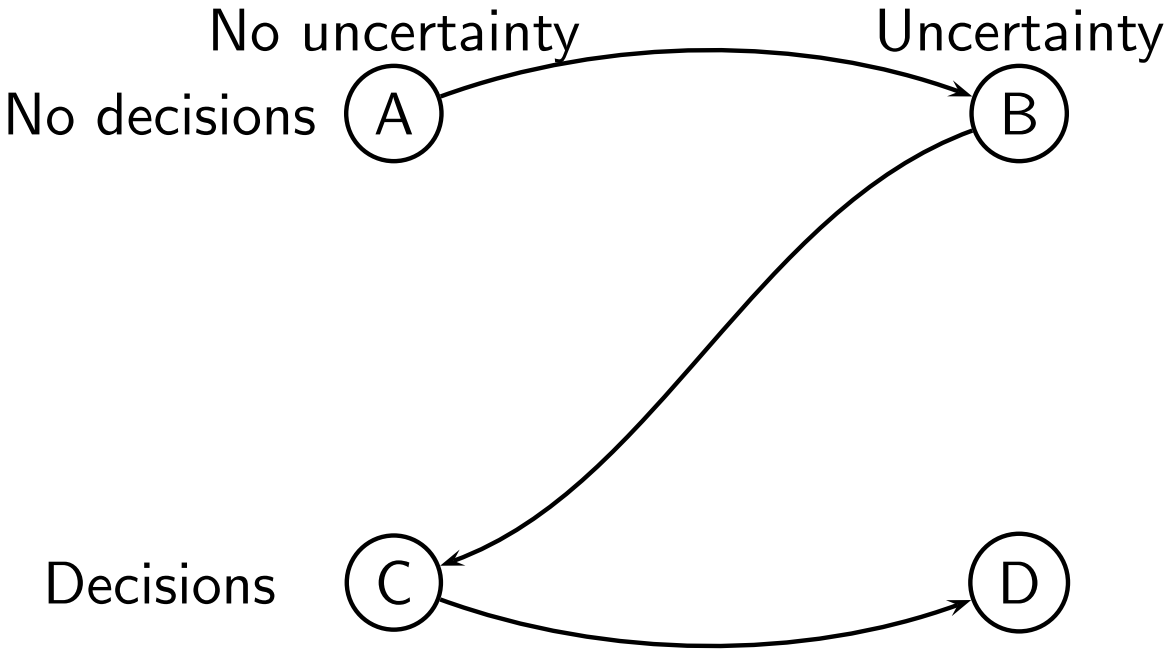
Usual notation either $\mathrm{E}\left(X_{t+1} = x \mid X_t = x_t\right)$, the conditional expectation of X_{t+1} given that X_t is equal to x_t, or simply $\mathrm{E}_t\left(X_{t+1}\right)$.

		$t+1$		
		R	O	S
	R	0.50	0.25	0.25
t	O	0.25	0.50	0.25
	S	0.50	0.50	0.00

Fixed-point Theorems and The Contraction Mapping Theorem

- Varian: Brouwer Fixed-Point Theorem
- Other fixed-point theorems like Kakutani.
- The Contraction Mapping Theorem.

Building the toolbox



A : Solow growth model
B : Value iteration on a Markov process
C : Ramsey growth model
D : Stochastic neoclassical growth model

The Euler Equation

$$\beta R_{t+1} \frac{u'(c_{t+1})}{u'(c_t)} = 1$$

where for the neoclassical growth model

$$R_{t+1} = 1 + f'(k_{t+1}) - \delta.$$

$$\underbrace{u'(c_t)}_{\text{marginal disutility of saving one more unit}} = \underbrace{\beta}_{\text{discount factor between two periods}} \text{ rate of ret}$$

The Euler Equation in a Stochastic Model Economy

$$\beta \mathrm{E}_t \left[R_{t+1} \frac{u'(c_{t+1})}{u'(c_t)} \right] = 1$$

where for the neoclassical growth model

$$R_{t+1} = 1 + z_{t+1} f'(k_{t+1}) - \delta.$$

$$c_t = \beta \mathrm{E}_t \left[R_{t+1} u'(c_{t+1}) \right]$$

Recursive formulation

Mathematical strategy:

- Seek the optimal savings/investment *function* directly and then to use this *function* to compute the optimal sequence of investments *from any initial capital stock*.
- This way of looking at the problem – decide on the immediate action to take as a function of the current situation – is called a *recursive formulation*.
- It exploits the observation that *a decision problem of the same structure recurs every period*.

Why bother?

- Ramsey completely solved this problem using variational methods – so why bother?
- Consider the stochastic variation of the problem:
  - In that case it makes no sense to choose a deterministic plan of investments for all future dates.
  - The best future choices will depend on how much output is available at the time – which again will depend on the current capital stock, the state of the technology, and the properties of the technology process.
- So in order solve stochastic dynamic decision problems we need recursive methods.

Contraction mapping

In general, $v(\cdot)$ is unknown, but the Bellman equation can be used to find it.
In most of the cases we will deal with, the Bellman equation satisfies **a contraction mapping theorem**, which implies that

1. There is a unique function $v(\cdot)$ which satisfies the Bellman equation.
2. If we begin with any initial function $v_0(k)$ and define

$$v_{i+1}(k) = \max_{c,k'}[u(c) + v_i(k')]$$

subject to

$$c + k' = f(k) + (1-d)k$$

for $i = 0, 1, 2, \ldots$ then $\lim_{i\to\infty} v_{i+1}(k) = v(k)$.

How to find the true value function

The above two implications give us two alternative means of uncovering the value function.

1. Implication 1 above, if we are fortunate enough to correctly guess the value function $v(\cdot)$; then we can simply plug $v(k_{t+1})$ into the right side and then verify that $v(k_t)$ solves the Bellman equation. This procedure only works in a few cases.

2. Implication 2 above is useful for doing numerical work. One approach is to find an approximation to the value function and iterate.

Approach

1. Pose a question
2. Use well-tested theory
3. Construct a model economy
4. Estimate the structural parameters of the model economy
5. Run the computational experiment and compare the performance of the model with what we have observed

What are business cycles?

Burns and Mitchell (1913, 1927, 1946): expansions occurring at the same time in many economic sectors, followed by similarly general recessions, contractions and revivals.

- fluctuations occurs in aggregate activity, not in particular sectors.
- cycles are recurrent, but not periodic.
- cycles have at least two different stages: expansions and contractions.
- once the economy enters into one of the stages, it stays there for some time.
- there are regular and predictable co-movements between variables over the cycles.

- Lucas (1981): deviations of real aggregate output from a trend, where there is co-movement of deviations from trend of various economic aggregates with those of output.
- NBER: a recession is a persistent period of decline in total output, income, employment, and trade, usually lasting from six months to one year, and marked by wide spread contracts in many sectors of the economy.

Business cycle facts

For each variable, study the following properties

- Procyclical, countercyclical or acyclical
- Leading or lagging
- More or less variable than the output (volatility)

Some definitions

- Procyclical: a variable that usually increases in booms, decreases in recession. For example, productivity is procyclical.
- Countercyclical: a variable that usually decreases in booms, increases in recession. For example, unemployment.
- Acyclical: a variable that shows no systematic relationship to the business cycle.

Hodrick-Prescott filter

The Hodrick-Prescott filter is a standard curve-fitting technique used to obtain a smoothed non-linear representation of a time series in order to filter out low-frequency components of the series.

$$\min \sum_{t=1}^{T} (y_t - \tau_t)^2 + \lambda \sum_{t=2}^{T-1} [(\tau_{t+1} - \tau_t) - (\tau_t - \tau_{t-1})]^2.$$

Facts of U.S. Business Cycles

- The magnitude of fluctuations in output and aggregate hours of work are nearly equal.
- Employment fluctuates almost as much as output.
- Productivity is slightly procyclical but varies less than output.
- Consumption of non-durables and services is much smoother than output.
- Investment in both producers' and consumers' durables fluctuations more than output
- Wages vary less than productivity and therefore output.

Co-movement over the Business Cycle

- Consumption, investment, inventories and imports are strongly procyclical but exports largely acyclical.
- Government expenditures are essentially uncorrelated with output.
- Average hourly compensation is uncorrelated with output.

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Question

To what extent can business cycles fluctuations be accounted for by a model economy populated by rational (not perfect foresight) agents responding optimally (not perfectly) to real shocks (random changes in technology, government purchase, taxes, taste, government regulation, terms of trade, energy prices, etc.) given their information set and then re-optimize when new information becomes available?

"Rational expectations"

> *"I should like to suggest that expectations, since they are informed predictions of future events, are essentially the same as the predictions of the relevant economic theory. At the risk of confusing this purely descriptive hypothesis with a pronouncement as to what firms ought to do, we call such expectations rational."*
>
> *John F. Muth. (1961). "Rational Expectations and the Theory of Price Movements"*

- *Not* normative
- Consistency. Ensure that the behavior of the model is consistent with individual decision makers' beliefs about the behavior of the model economy.