

Question 1. θ_t is governed by the following law of motion

$$\theta_{t+1} = \theta_t + \begin{cases} .1 & \text{if } i_t > .5 \\ -.1 & \text{if } i_t \leq .5 \end{cases}$$

where $\theta_1 = 0$ and i_t is independent and identically distributed (iid) and drawn from a uniform distribution on the interval $(0,1)$.

Generate a sequence of length 200. Plot this sequence

Algorithm 1 Answer to Question 1 (pseudo-code)

```
 $\theta_1 \leftarrow 0$   
for  $t \leftarrow 1$  to 199 do  
   $d \leftarrow$  random draw from a uniform distribution over  $(0,1)$   
  if  $d > 0.5$  then  
     $\theta_{t+1} = \theta_t + 0.1$   
  else  
     $\theta_{t+1} = \theta_t - 0.1$   
  end if  
end for  
  
plot  $\{\theta_t\}_{t=1}^{200}$ .
```

Algorithm 2 Answer to Question 1 (Matlab implementation)

```
clear all;  
  
theta(1) = 0;  
for t = 1 : 199  
  d = rand;  
  if d > 0.5  
    theta(t+1) = theta(t) + 0.1;  
  else  
    theta(t+1) = theta(t) - 0.1;  
  end  
end  
  
plot(theta)
```

Question 2. Suppose you are going to evaluate a set of 50 fund managers over a period of four years. You will observe their performance every month (48 observations).

Before you start your study you decide to simulate a simple model where no manager is better than the other. You know the market has an expected drift with a large associated variance.

After some statistical analysis of historical data you decide to simulate the following little model: Suppose all managers starts in the first month with the same, given amount of available funds, for simplicity normalized to 1. Monthly returns are independent and identically distributed (iid). For any given month, with 40% probability a manager will outperform the market and earn a return of 0.01098% (14% annually), with 20% probability she will earn the average market return of 0.00327% (4% annually), and with 40% probability she will lose money and earn a return of -0.00514% (-6% annually).

Algorithm 3 Answer to Question 2 (pseudo-code)

```
for  $f \leftarrow 1$  to 50 do {Let  $f$  be a counter for the fund managers}
   $\theta_{1,f} \leftarrow 1$ 
  for  $m \leftarrow 1$  to 47 do {Let  $m$  be a counter for the months}
     $d \leftarrow$  random draw from a uniform distribution over  $(0, 1)$ 
    if  $d > 0.4$  then
       $\theta_{m+1,f} = \theta_{m,f} \cdot (1 + 0.01098)$ 
    else if  $0.4 \leq d < 0.6$  then
       $\theta_{m+1,f} = \theta_{m,f} \cdot (1 + 0.00327)$ 
    else if  $d \geq 0.6$  then
       $\theta_{m+1,f} = \theta_{m,f} \cdot (1 - 0.00514)$ 
    end if
  end for
end for
```

Question 3. In a given quarter, state of the economy can be either high (“boom”) or low (“recession”) and it follows a Markov process. The Markov transition matrix is

$$\Pi = \begin{bmatrix} 0.6 & 0.4 \\ 0.4 & 0.6 \end{bmatrix}.$$

Total factor productivity of the economy is given by $A = e^{z_t}$ where the parameter z is governed by the following stochastic process: If the economy is in the high state, the z increases by 0.025 relative to trend, and if the economy is in the low state, the z decreases by 0.025 relative to trend, i.e. z is governed by the following law of motion

$$z_{t+1} = z_t + \begin{cases} 0.025 & \text{if the state is high} \\ -0.025 & \text{if the state is low} \end{cases}.$$

Simulate a sequence of 40 quarters.

Algorithm 4 Answer to Question 2 (pseudo-code)

```

Π =  $\begin{bmatrix} 0.6 & 0.4 \\ 0.4 & 0.6 \end{bmatrix}$ 
 $z_1 \leftarrow 1$ 
 $s \leftarrow 1$  {Let  $s$  be an indicator of the state of the economy}
for  $q \leftarrow 1$  to 39 do {Let  $q$  be a counter for the quarters}
   $d \leftarrow$  random draw from a uniform distribution over  $(0, 1)$ 
  if  $s = 1$  then
    if  $d < \Pi(s, 1)$  then
       $z_{q+1} \leftarrow z_q + 0.025$ 
       $s \leftarrow 1$ 
    else
       $z_{q+1} \leftarrow z_q - 0.025$ 
       $s \leftarrow 2$ 
    end if
  else if  $s = 2$  then
    if  $d < \Pi(s, 2)$  then
       $z_{q+1} \leftarrow z_q + 0.025$ 
       $s \leftarrow 2$ 
    else
       $z_{q+1} \leftarrow z_q - 0.025$ 
       $s \leftarrow 1$ 
    end if
  end if
end for

```
