Econometrics - Lecture 1

Introduction to Linear Regression

Contents

- Organizational Issues
- Some History of Econometrics
- An Introduction to Linear Regression
  - OLS as an algebraic tool
  - The Linear Regression Model
  - Small Sample Properties of OLS estimator
- Introduction to GRETL

Organizational Issues

Aims of the course

- Understanding of econometric concepts and principles
- Introduction to commonly used econometric tools and techniques
- Use of econometric tools for analyzing economic data: specification of adequate models, identification of appropriate econometric methods, interpretation of results
- Use of GRETL

Organizational Issues, cont'd

Literature

Course textbook

- Marno Verbeek, *A Guide to Modern Econometrics*, 3rd Ed., Wiley, 2008

Suggestions for further reading

- P. Kennedy, *A Guide to Econometrics*, 6th Ed., Blackwell, 2008
- W.H. Greene, *Econometric Analysis*. 6th Ed., Pearson International, 2008

Organizational Issues, cont'd

Prerequisites

- Linear algebra: linear equations, matrices, vectors (basic operations and properties)
- Descriptive statistics: measures of central tendency, measures of dispersion, measures of association, histogram, frequency tables, scatter plot, quantile
- Theory of probability: probability and its properties, random variables and distribution functions in one and several dimensions, moments, convergence of random variables, limit theorems, law of large numbers
- Mathematical statistics: point estimation, confidence intervals, hypothesis testing, *p*-value, significance level

Organizational Issues, cont'd

Teaching and learning method

- Course in six blocks
- Class discussion, written homework (computer exercises, GRETL) submitted by groups of (3-5) students, presentations of homework by participants
- Final exam

Assessment of student work

- For grading, the written homework, presentation of homework in class and a final written exam will be of relevance
- Weights: homework 70 %, final written exam 30 %
- Presentation of homework in class: students must be prepared to be called at random

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Empirical Economics Prior to 1930ies

The situation in the early 1930ies:

- Theoretical economics aims at “operationally meaningful theorems“; “operational” means purely logical mathematical deduction
- Economic theories or laws are seen as deterministic relations; no inference from data as part of economic analysis
- Ignorance of the stochastic nature of economic concepts
- Use of statistical methods for
  - measuring theoretical coefficients, e.g., demand elasticities,
  - representing business cycles
- Data: limited availability; time-series on agricultural commodities, foreign trade

Early Institutions

- Applied demand analysis: US Bureau of Agricultural Economics
- Statistical analysis of business cycles: H.L.Moore (Columbia University): Fourier periodogram ; W.M.Persons et al. (Harvard): business cycle forecasting; US National Bureau of Economic Research (NBER)
- Cowles Commission for Research in Economics
  - Founded 1932 by A.Cowles: determinants of stock market prices?
  - Formalization of econometrics, development of econometric methodology
  - R.Frisch, G.Tintner; European refugees
  - J.Marschak (head 1943-55) recruited people like T.C.Koopmans, T.M.Haavelmo, T.W.Anderson, L.R.Klein
  - Interests shifted to theoretical and mathematical economics after 1950

Early Actors

- R.Frisch (Oslo Institute of Economic Research): econometric project, 1930-35; T.Haavelmo, Reiersol
- J.Tinbergen (Dutch Central Bureau of Statistics, Netherlands Economic Institute; League of Nations, Genova): macro-econometric model of Dutch economy, ~1935; T.C.Koopmans, H.Theil
- Austrian Institute for Trade Cycle Research: O. Morgenstern (head), A.Wald, G.Tintner
- Econometric Society, founded 1930 by R.Frisch et al.
  - Facilitates exchange of scholars from Europe and US
  - Covers econometrics and mathematical statistics

First Steps

- R.Frisch, J.Tinbergen:
  - Macro-economic modeling based on time-series, ~ 1935
  - Aiming at measuring parameters, e.g., demand elasticities
  - Aware of problems due to quality of data
  - Nobel Memorial Prize in Economic Sciences jointly in 1969 (“for having developed and applied dynamic models for the analysis of economic processes”)
- T.Haavelmo
  - “The Probability Approach in Econometrics”: PhD thesis (1944)
  - Econometrics as a tool for testing economic theories
  - States assumptions needed for building and testing econometric models
  - Nobel Memorial Prize in Economic Sciences in 1989 ("for his clarification of the probability theory foundations of econometrics and his analyses of simultaneous economic structures”)

First Steps, Cont'd

- Cowles Commission
  - Methodology for macro-economic modeling based on Haavelmo's approach
  - Cowles Commission monographs by G.Tintner, T.C.Koopmans, et al.

The Haavelmo Revolution

- Introduction of probabilistic concepts in economics
  - Obvious deficiencies of traditional approach: Residuals, measurement errors, omitted variables; stochastic time-series data
  - Advances in probability theory in early 1930ies
  - Fisher‘s likelihood function approach
- Haavelmo‘s ideas
  - Critical view of Tinbergen‘s macro-econometric models
  - Thorough adoption of probability theory in econometrics
  - Conversion of deterministic economic models into stochastic structural equations
- Haavelmo‘s “The Probability Approach in Econometrics”
  - Why is the probability approach indispensible?
  - Modeling procedure based on ML and hypothesis testing

Haavelmo's Arguments for the Probabilistic Approach

- Economic variables in economic theory and econometric models
  - "Observational" vs. "theoretical" vs. "true" variables
  - Models have to take into account inaccurately measured data and passive observations
- Unrealistic assumption of permanence of economic laws
  - Ceteris paribus assumption
  - Economic time-series data
  - Simplifying economic theories
  - Selection of economic variables and relations out of the whole system of fundamental laws

Cowles Commission Methodology

Assumptions based to macro-econometric modeling and testing of economic theories

Time series model

$$Y_t = \alpha X_t + \beta W_t + u_{1t},$$
$$X_t = \gamma Y_t + \delta Z_t + u_{2t}$$

1. Specification of the model equation(s) includes the choice of variables; functional form is (approximately) linear
2. Time-invariant model equation(s): the model parameters α, …, δ are independent of time *t*
3. Parameters α, …, δ are structurally invariant, i.e., invariant wrt changes in the variables
4. Causal ordering (exogeneity, endogeneity) of variables is known
5. Statistical tests can falsify but not verify a model

Classical Econometrics and More

- "Golden age" of econometrics till ~1970
  - Multi-equation models for analyses and forecasting
  - Growing computing power
  - Development of econometric tools
- Scepticism
  - Poor forecasting performance
  - Dubious results due to
    - Wrong specifications
    - Imperfect estimation methods

Model	year	eq's
Tinbergen	1936	24
Klein	1950	6
Klein & Goldberger	1955	20
Brookings	1965	160
Brookings Mark II	1972	~200

- Time-series econometrics: non-stationarity of economic time-series
  - Consequences of non-stationarity: misleading *t*-, DW-statistics, R^2
  - Non-stationarity: needs new models (ARIMA, VAR, VEC); Box & Jenkins (1970: ARIMA-models), Granger & Newbold (1974, spurious regression), Dickey-Fuller (1979, unit-root tests)

Econometrics …

- … consists of the application of statistical data and techniques to mathematical formulations of economic theory. It serves to test the hypotheses of economic theory and to estimate the implied interrelationships. (Tinbergen, 1952)
- … is the interaction of economic theory, observed data and statistical methods. It is the interaction of these three that makes econometrics interesting, challenging, and, perhaps, difficult. (Verbeek, 2008)
- … is a methodological science with the elements
  - economic theory
  - mathematical language
  - statistical methods
  - software

The Course

1. Introduction to linear regression (Verbeek, Ch. 2): the linear regression model, OLS method, properties of OLS estimators
2. Introduction to linear regression (Verbeek, Ch. 2): goodness of fit, hypotheses testing, multicollinearity
3. Interpreting and comparing regression models (MV, Ch. 3): interpretation of the fitted model, selection of regressors, testing the functional form
4. Heteroskedascity and autocorrelation (Verbeek, Ch. 4): causes, consequences, testing, alternatives for inference
5. Endogeneity, instrumental variables and GMM (Verbeek, Ch. 5): the IV estimator, the generalized instrumental variables estimator, the generalized method of moments (GMM)
6. The practice of econometric modeling

The Next Course

- Univariate and multivariate time series models: ARMA-, ARCH-, GARCH-models, VAR-, VEC-models
- Models for panel data
- Models with limited dependent variables: binary choice, count data

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Linear Regression

Y: explained variable
X: explanatory or regressor variable
The linear regression model describes the data-generating process of *Y* under the condition *X*

simple linear regression model

$$Y = \quad +$$

β: coefficient of *X*
α: intercept

multiple linear regression model

$$Y = 1 + 2 \quad 2 + \cdots + \quad K$$

Example: Individual Wages

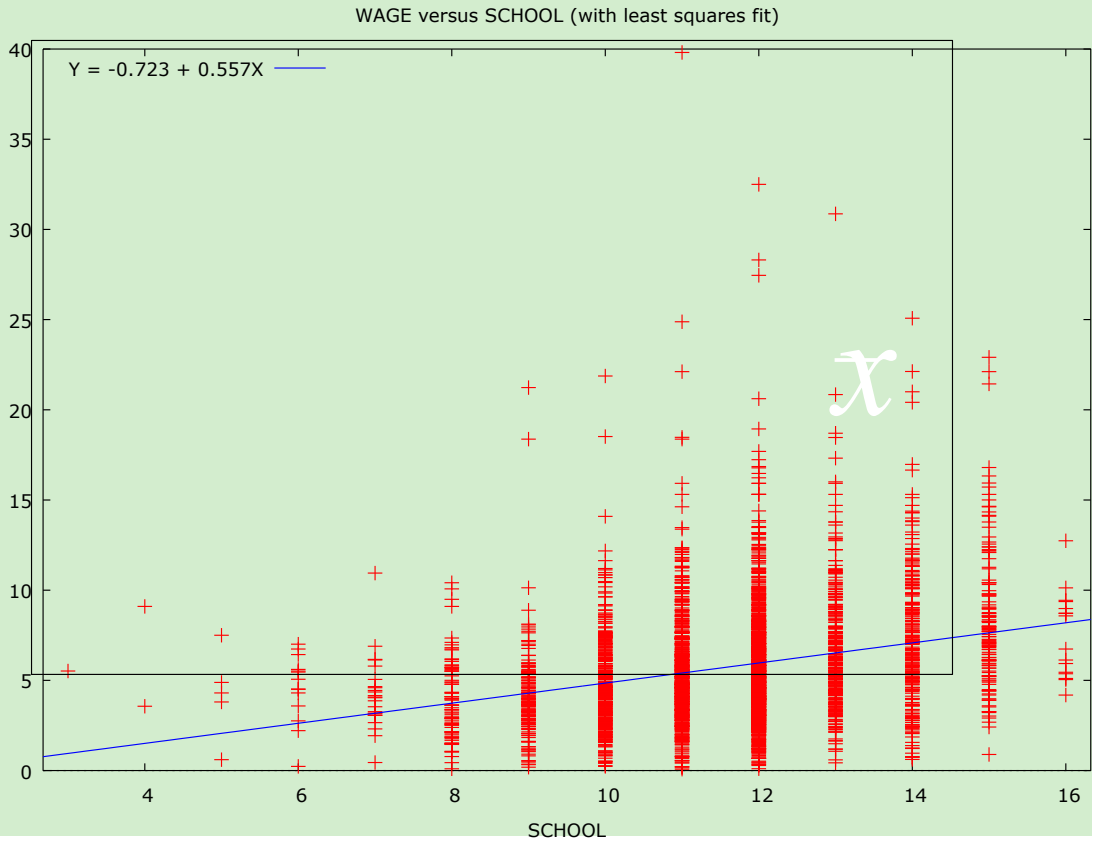
Sample (US National Longitudinal Survey, 1987)

- N = 3294 individuals (1569 females)
- Variable list
  - WAGE: wage (in 1980 $) per hour (p.h.)
  - MALE: gender (1 if male, 0 otherwise)
  - EXPER: experience in years
  - SCHOOL: years of schooling
  - AGE: age in years
- Possible questions
  - Effect of gender on wage p.h.: Average wage p.h.: 6,31$ for males, 5,15$ for females
  - Effects of education, of experience, of interactions, etc. on wage p.h.

x

Individual Wages, cont'd

Wage per hour vs. Years of schooling



Fitting a Model to Data

Choice of values b_1, b_2 for model parameters β_1, β_2 of $Y = \beta_1 + \beta_2 X$, given the observations (y_i, x_i), $i = 1,\ldots,N$

Principle of (Ordinary) Least Squares or OLS:

$$b_i = \arg\min_{\beta 1,\, \beta 2} S(\beta_1, \beta_2),\ i=1,2$$

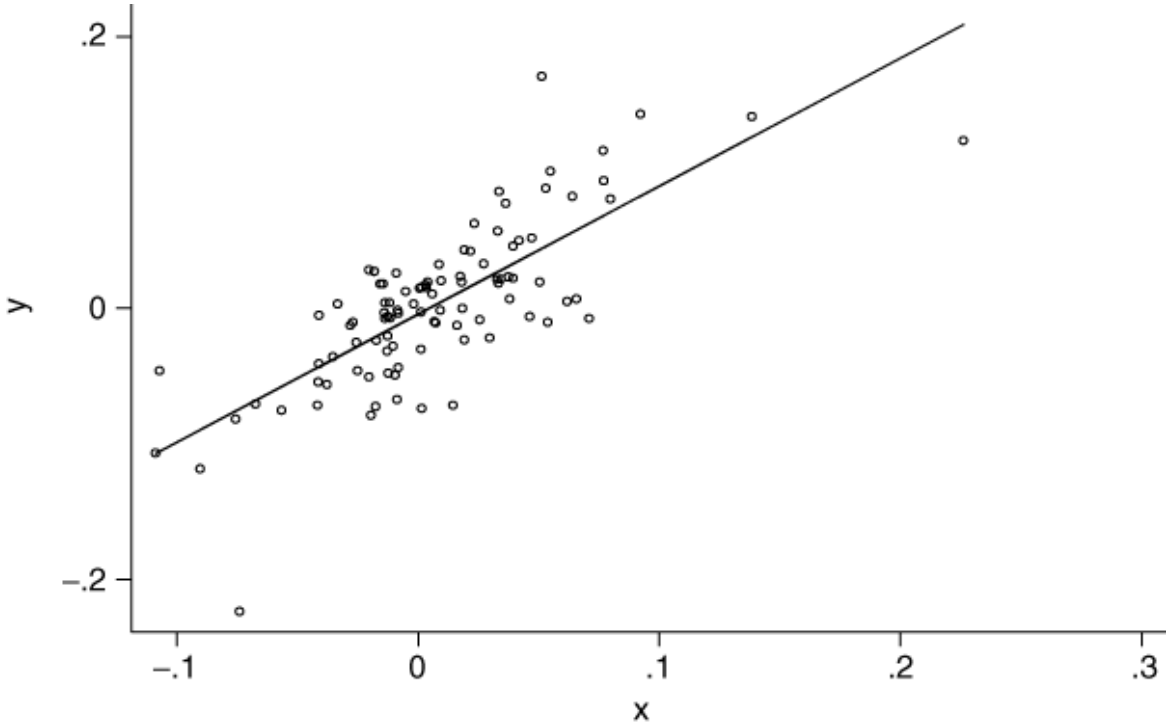
Objective function: sum of the squared deviations

$$S(\beta_1, \beta_2) = \Sigma_i [y_i - (\beta_1 + \beta_2 x_i)]^2 = \Sigma_i \varepsilon_i^2$$

Deviation between observation and fitted value: $\varepsilon_i = y_i - (\beta_1 + \beta_2 x_i)$

Observations and Fitted Regression Line

Simple linear regression: Fitted line and observation points (Verbeek, Figure 2.1)



OLS-Estimators

Equating the partial derivatives of $S(\beta_1, \beta_2)$ to zero: normal equations

$$b_1 + b_2 \sum_{i=1}^{N} x_i = \sum_{i=1}^{N} y_i$$

$$b_1 \sum_{i=1}^{N} x_i + b_2 \sum_{i=1}^{N} x_i^2 = \sum_{i=1}^{N} x_i y_i$$

OLS-estimators b_1 und b_2 result in

$$b_2 =$$

$$b_1 =$$

with mean values $\bar{x}$ and $\bar{y}$

and second moments

$$s_{xy} =$$

$$s_x^2 =$$

Individual Wages, cont'd

Sample (US National Longitudinal Survey, 1987): wage per hour, gender, experience, years of schooling; N = 3294 individuals (1569 females)

Average wage p.h.: 6,31$ for males, 5,15$ for females

Model:

$$wage_i = \beta_1 + \beta_2\, male_i + \varepsilon_i$$

$male_I$: male dummy, has value 1 if individual is male, otherwise value 0

OLS-estimation gives

$$wage_i = 5{,}15 + 1{,}17 * male_i$$

Compare with averages!

Individual Wages, cont'd

OLS estimated wage equation (Table 2.1, Verbeek)

Dependent variable: *wage*		
Variable	Estimate	Standard error
constant	5.1469	0.0812
male	1.1661	0.1122
$s = 3.2174$	$R^2 = 0.0317$	$F = 107.93$

$wage_i$ = 5,15 + 1,17*$male_i$

estimated wage p.h for males: 6,313

for females: 5,150

OLS-Estimators: General Case

Model for Y contains K-1 explanatory variables

$$Y = \beta_1 + \beta_2 X_2 + \ldots + \beta_K X_K = x'\beta$$

with $x = (1, X_2, \ldots, X_K)'$ and $\beta = (\beta_1, \beta_2, \ldots, \beta_K)'$

Observations: $(y_i, x_i') = (y_i, (1, x_{i2}, \ldots, x_{iK}))$, $i = 1, \ldots, N$

OLS-estimates $b = (b_1, b_2, \ldots, b_K)'$ are obtained by minimizing the objective function wrt the β_k's

$$S(\hat{\beta}) = \sum_{i=1}^{N} (y_i - x_i' \hat{\beta})^2$$

this results in

$$-2 \sum_{i=1}^{N} x_i (y_i - x_i b) = 0$$

OLS-Estimators: General Case,

cont'd

or

$$\sum_{i=1}^{N} x_i x_i' \, b = \sum_{i=1}^{N} x_i y_i$$

the so-called normal equations, a system of *K* linear equations for the components of *b*

Given that the symmetric *K*x*K*-matrix $\sum_{i=1}^{N} x_i x_i'$ has full rank *K* and is hence invertible, the OLS-estimators are

$$b = \left(\sum_{i=1}^{N} x_i x_i' \right)^{-1} \sum_{i=1}^{N} x_i y_i$$

Best Linear Approximation

Given the observations: $(y_i, x_i') = (y_i, (1, x_{i2}, \ldots, x_{iK}))$, $i = 1, \ldots, N$

For y_i, the linear combination or the fitted value

$$\hat{y}_i =$$

is the best linear combination for Y from $X_2, \ldots, X_K$ and a constant (the intercept)

Some Matrix Notation

N observations

$$(y_1,x_1), \ldots , (y_N,x_N)$$

Model: $y_i = \beta_1 + \beta_2 x_i + \varepsilon_i$, $i = 1, \ldots, N$, or

$$y = X\beta + \varepsilon$$

with

$$y = \begin{pmatrix} y_1 \\ \vdots \\ y_N \end{pmatrix}, X = \begin{pmatrix} 1 & x_1 \\ \vdots & \vdots \\ 1 & x_N \end{pmatrix}, \beta = \begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix}, \varepsilon = \begin{pmatrix} \varepsilon_1 \\ \vdots \\ \varepsilon_N \end{pmatrix}$$

OLS Estimators in Matrix Notation

Minimizing

$$S(\beta) = (y - X\beta)' (y - X\beta) = y'y - 2y'X\beta + \beta' X'X\beta$$

with respect to β gives the normal equations

$$\frac{\partial S'}{\partial \beta} = -2X'y + 2X'Xb = 0$$

resulting from differentiating $S(\beta)$ with respect to β and setting the first derivative to zero

The OLS-solution or OLS-estimators for β are

$$b = (X'X)^{-1}X'y$$

The best linear combinations or predicted values for Y given X or projections of y into the space of X are obtained as

$$\hat{y} = Xb = X(X'X)^{-1}X'y = P_x y$$

the NxN-matrix P_x is called the projection matrix or hat matrix

Residuals in Matrix Notation

The vector y can be written as $y = Xb + e = \hat{y} + e$ with residuals

$$e = y - Xb \text{ or } e_i = y_i - x_i'b,\ i = 1, \ldots, N$$

- From the normal equations follows

  $$-2(X'y - X'Xb) = -2\,X'e = 0$$

  i.e., each column of X is orthogonal to e

- With

  $$e = y - Xb = y - P_x y = (I - P_x)y = M_x y$$

  the residual generating matrix M_x is defined as

  $$M_x = I - X(X'X)^{-1}X' = I - P_x$$

  M_x projects y into the orthogonal complement of the space of X

- Properties of P_x and M_x: symmetry ($P'_x = P_x$, $M'_x = M_x$) idempotence ($P_xP_x = P_x$, $M_xM_x = M_x$), and orthogonality ($P_xM_x = 0$)

Properties of Residuals

Residuals: $e_i = y_i - x_i'b$, $i = 1, \ldots, N$

- Minimum value of objective function

  $$S(b) = e'e = \Sigma_i e_i^2$$

- From the orthogonality of $e = (e_1, \ldots, e_N)'$ to each $x_i = (x_{1i}, \ldots, x_{Ni})'$, i.e., $e'x_i = 0$, follows that

  $$\Sigma_i e_i = 0$$

  i.e., average residual is zero, if the model has an intercept

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Economic Models

Describe economic relationships (not only a set of observations), have an economic interpretation

Linear regression model:

$$y_i = \beta_1 + \beta_2 x_{i2} + \ldots + \beta_K x_{iK} + \varepsilon_i = x_i'\beta + \varepsilon_i$$

- Variables y_i, x_{i2}, …, x_{iK}: observable, sample (i = 1, …, N) from a well-defined population or universe
- Error term ε_i (disturbance term) contains all influences that are not included explicitly in the model; unobservable; assumption $E\{\varepsilon_i \mid x_i\} = 0$ gives

  $$E\{y_i \mid x_i\} = x_i'\beta$$

  the model describes the expected value of y given x
- Unknown coefficients β_1, …, β_K: population parameters

The Sampling Concept

The regression model $y_i = x_i'\beta + \varepsilon_i$, $i = 1, \ldots, N$; or $y = X\beta + \varepsilon$

describes one realization out of all possible samples of size N from the population

A) Sampling process with fixed, non-stochastic x_i's

- New sample: new error terms ε_i and new y_i's
- Random sampling of ε_i, $i = 1, \ldots, N$: joint distribution of ε_i's determines properties of b etc.

B) Sampling process with samples of (x_i, y_i) or (x_i, ε_i)

- New sample: new error terms ε_i and new x_i's
- Random sampling of (x_i, ε_i), $i = 1, \ldots, N$: joint distribution of (x_i, ε_i)'s determines properties of b etc.

The Sampling Concept, cont'd

- The sampling with fixed, non-stochastic x_i's is not realistic for economic data
- Sampling process with samples of (x_i, y_i) is appropriate for modeling cross-sectional data
  - Example: household surveys, e.g., EU-SILC
- Sampling process with samples of (x_i, y_i) from time-series data: sample is seen as one out of all possible realizations of the underlying data-generating process

The Ceteris Paribus Condition

- The linear regression model needs assumptions to allow interpretation
- Assumption for ε_i's: $E\{\varepsilon_i \mid x_i\} = 0$; exogeneity of variables X
- This implies

  $$E\{y_i \mid x_i\} = x_i'\beta$$

  i.e., the regression line describes the conditional expectation of y_i given x_i
- Coefficient β_k measures the change of the expected value of Y if X_k changes by one unit and all other X_j values, $j \neq k$, remain the same (ceteris paribus condition)
- Exogeneity can be restrictive

Regression Coefficients

Linear regression model:

$$y_i = \beta_1 + \beta_2 x_{i2} + \ldots + \beta_K x_{iK} + \varepsilon_i = x_i'\beta + \varepsilon_i$$

Coefficient β_k measures the change of the expected value of Y if X_k changes by one unit and all other X_j values, $j \neq k$, remain the same (ceteris paribus condition); marginal effect of changing X_k on Y

$$\frac{\partial E[y_i|x_i]}{\partial x_{ik}} = \beta_k$$

Example

- Wage equation: $wage_i = \beta_1 + \beta_2\, male_i + \beta_3\, school_i + \beta_4\, exper_i + \varepsilon_i$

  β_3 measures the impact of one additional year at school upon a person's wage, keeping gender and years of experience fixed

Estimation of β

Given a sample (x_i, y_i), $i = 1, \ldots, N$, the OLS-estimators for β

$$b = (X'X)^{-1}X'y$$

can be used as an approximation for β

- The vector b is a vector of numbers, the estimates
- The sampling concept and assumptions on ε_i's determine the quality, i.e., the statistical properties, of b

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Fitting Economic Models to Data

Observations allow

- to estimate parameters
- to assess how well the data-generating process is represented by the model, i.e., how well the model coincides with reality
- to improve the model if necessary

Fitting a linear regression model to data

- Parameter estimates $b = (b_1, \ldots, b_K)'$ for coefficients $\beta = (\beta_1, \ldots, \beta_K)'$
- Standard errors $se(b_k)$ of the estimates b_k, $k=1,\ldots,K$
- *t*-statistics, *F*-statistic, R^2, Durbin Watson test-statistic, etc.

OLS Estimator and OLS Estimates *b*

OLS estimates *b* are a realization of the OLS estimator

The OLS estimator is a random variable

- Observations are a random sample from the population of all possible samples
- Observations are generated by some random process

Distribution of the OLS estimator

- Actual distribution not known
- Theoretical distribution determined by assumptions on
  - model specification
  - the error term ε_i and regressor variables x_i

Quality criteria (bias, accuracy, efficiency) of OLS estimates are determined by the properties of the distribution

Gauss-Markov Assumptions

Observation y_i is a linear function

$$y_i = x_i'\beta + \varepsilon_i$$

of observations x_{ik}, k =1, …, K, of the regressor variables and the error term ε_i

for i = 1, …, N; $x_i' = (x_{i1}, \ldots, x_{iK})$; $X = (x_{ik})$

A1	$E\{\varepsilon_i\} = 0$ for all i
A2	all ε_i are independent of all x_i (exogeneous x_i)
A3	$V\{\varepsilon_i\} = \sigma^2$ for all i (homoskedasticity)
A4	$Cov\{\varepsilon_i, \varepsilon_j\} = 0$ for all i and j with $i \neq j$ (no autocorrelation)

In matrix notation: $E\{\varepsilon\} = 0$, $V\{\varepsilon\} = \sigma^2 I_N$

Systematic Part of the Model

The systematic part $E\{y_i \mid x_i\}$ of the model $y_i = x_i'\beta + \varepsilon_i$, given observations x_i, is derived under the Gauss-Markov assumptions as follows:

(A2) implies $E\{\varepsilon \mid X\} = E\{\varepsilon\} = 0$ and $V\{\varepsilon \mid X\} = V\{\varepsilon\} = \sigma^2 I_N$

- Observations x_i, $i = 1, \ldots, N$, do not affect the properties of ε
- The systematic part

  $$E\{y_i \mid x_i\} = x_i'\beta$$

  can be interpreted as the conditional expectation of y_i, given observations x_i

Is the OLS-estimator a Good Estimator?

- Under the Gauss-Markov assumptions, the OLS estimator has nice properties; see below
- Gauss-Markov assumptions are very strong and often not satisfied
- Relaxations of the Gauss-Markov assumptions and consequences of such relaxations are important topics

Properties of OLS Estimators

1. The OLS estimator *b* is unbiased: $E\{b \mid X\} = E\{b\} = \beta$

   Needs assumptions (A1) and (A2)

2. The variance of the OLS estimator *b* is given by

   $$V\{b \mid X\} = V\{b\} = \sigma^2(\Sigma_i x_i x_i')^{-1} = \sigma^2(X' X)^{-1}$$

   Needs assumptions (A1), (A2), (A3) and (A4)

3. Gauss-Markov theorem: The OLS estimator *b* is a BLUE (best linear unbiased estimator) for β

   Needs assumptions (A1), (A2), (A3), and (A4) and requires linearity in parameters

The Gauss-Markov Theorem

OLS estimator *b* is BLUE (best linear unbiased estimator) for β

- Linear estimator: $b^* = Ay$ with any full-rank KxN matrix A
- b^* is an unbiased estimator: $E\{b^*\} = E\{Ay\} = \beta$
- *b* is BLUE: $V\{b^*\} - V\{b\}$ is positive definite, i.e. the variance of any linear combination $d'b^*$ is not smaller than that of $d'b^*$

  $$V\{d'b^*\} \geq V\{d'b\}$$

  e.g., $V\{b_k^*\} \geq V\{b_k\}$ for any k
- The OLS-estimator is most accurate among the linear unbiased estimators

Standard Errors of OLS Estimators

- Variance of the OLS-estimators:

  $$V\{b\} = \sigma^2(X' X)^{-1} = \sigma^2(\Sigma_i\, x_i\, x_i')^{-1}$$

- Standard error of OLS estimate b_k: The square root of the k^{th} diagonal element of $V\{b\}$
- Estimator $V\{b\}$ is proportional to the variance σ^2 of the error terms
- Estimator for σ^2: sampling variance s^2 of the residuals e_i

  $$s^2 = (N - K)^{-1}\, \Sigma_i\, e_i^2$$

  Under assumptions (A1)-(A4), s^2 is unbiased for σ^2

  Attention: the estimator $(N - 1)^{-1}\, \Sigma_i\, e_i^2$ is biased
- Estimated variance (covariance matrix) of b:

  $$\tilde{V}\{b\} = s^2(X' X)^{-1} = s^2(\Sigma_i\, x_i\, x_i')^{-1}$$

Standard Errors of OLS Estimators, cont'd

- Variance of the OLS-estimators:

  $$V\{b\} = \sigma^2(X' X)^{-1} = \sigma^2(\Sigma_i x_i x_i')^{-1}$$

- Standard error of OLS estimate b_k: The square root of the k^{th} diagonal element of $V\{b\}$

  $$\sigma\sqrt{c_{kk}}$$

  with c_{kk} the k-th diagonal element of $(X' X)^{-1}$

- Estimated variance (covariance matrix) of b:

  $$\tilde{V}\{b\} = s^2(X' X)^{-1} = s^2(\Sigma_i x_i x_i')^{-1}$$

- Estimated standard error of b_k:

  $$se(b_k) = s\sqrt{c_{kk}}$$

Two Examples

Simple regression $Y_i = \alpha + \beta X_i + \varepsilon_t$

The variance for β is

$$V\{b\} = \frac{\sigma^2}{N s_x^2}$$

b is the more accurate, the larger N and s_x^2 and the smaller σ^2

Regression with two regressors:

$$Y_i = \beta_1 + \beta_2 X_{i2} + \beta_3 X_{i3} + \varepsilon_t$$

The variance for β_2 is

$$V\{b_2\} = \frac{1}{1 - r_{23}^2} \cdot \frac{\sigma^2}{N s_{x2}^2}$$

b_2 is most accurate if X_2 and X_3 are uncorrelated

Normality of Error Terms

For statistical inference purposes, a distributional assumption for the ε_i's is needed

A5	ε_i normally distributed for all *i*

Together with assumptions (A1), (A3), and (A4), (A5) implies

$\varepsilon_i \sim NID(0,\sigma^2)$ for all *i*

i.e., all ε_i are

- independent drawings
- from the *normal* distribution
- with mean 0
- and variance σ^2

Error terms are "normally and independently distributed"

Properties of OLS Estimators

1. The OLS estimator *b* is unbiased: $E\{b\} = \beta$
2. The variance of the OLS estimator is given by

$$V\{b\} = \sigma^2(X'X)^{-1}$$

3. The OLS estimator *b* is a BLUE (best linear unbiased estimator) for β

4. The OLS estimator *b* is normally distributed with mean β and covariance matrix $V\{b\} = \sigma^2(X'X)^{-1}$

$$b \sim N(\beta, \sigma^2(X'X)^{-1}) \; , \; b_k \sim N(\beta_k, \sigma^2 c_{kk})$$

Needs assumptions (A2) + (A5)

Example: Individual Wages

$wage_i = \beta_1 + \beta_2\, male_i + \varepsilon_i$

What do the assumptions mean?

(A1): $\beta_1 + \beta_2\, male_i$ contains the whole systematic part of the model; no regressors besides gender relevant?

(A2): x_i uncorrelated with ε_i for all *i*: knowledge of a person's gender provides no information about further variables which affect the person's wage; is that realistic?

(A3) $V\{\varepsilon_i\} = \sigma^2$ for all *i*: variance of error terms (and of wages) is the same for males and females; is that realistic?

(A4) $Cov\{\varepsilon_i,\varepsilon_j\} = 0$, $i \neq j$: implied by random sampling

(A5) Normality of ε_i : is that realistic? (Would allow, e.g., for negative wages)

Individual Wages, cont'd

OLS estimated wage equation (Table 2.1, Verbeek)

Dependent variable: *wage*		
Variable	Estimate	Standard error
constant	5.1469	0.0812
male	1.1661	0.1122
$s = 3.2174$	$R^2 = 0.0317$	$F = 107.93$

b_1 = 5,147, se(b_1) = 0,081: mean wage p.h. for females: 5,15$, with std.error of 0,08$

b_2 = 1,166, se(b_2) = 0,112

95% confidence interval for β_1: 4,988 $\leq \beta_1 \leq$ 5,306

Your Homework

1. Verbeek's data set "WAGES" contains for a sample of 3294 individuals the wage and other variables. Using GRETL, draw box plots (a) for all wages p.h. in the sample, (b) for wages p.h. of males and of females.
2. For Verbeek's data set "WAGES", calculate, using GRETL, the mean wage p.h. (a) of the whole sample, (b) of males and females, and (c) of persons with schooling between (i) 0 and 6 years, (ii) 7 and 12 years, and (iii) 13 and 16 years.
3. For the simple linear regression ($Y = \beta_1 + \beta_2 X + \varepsilon$): write the OLS-estimator $b = (X'X)^{-1}X'y$ in summation form; X: NxK.
4. Show that $V\{b\} = \sigma^2(X'X)^{-1}$.
5. Show that $P_x M_x = 0$ for the hat matrix P_x and the residual generating matrix M_x.