Econometrics - Lecture 3

Regression Models: Interpretation and Comparison

Contents

- The Linear Model: Interpretation
- Selection of Regressors
- Specification of the Functional Form

Economic Models

Describe economic relationships (not only a set of observations), have an economic interpretation

Linear regression model:

$$y_i = \beta_1 + \beta_2 x_{i2} + \ldots + \beta_K x_{iK} + \varepsilon_i = x_i'\beta + \varepsilon_i$$

- Variables Y, X_2, …, X_K: observable
- Observations: y_i, x_{i2}, …, x_{iK}, i = 1, …, N
- Error term ε_i (disturbance term) contains all influences that are not included explicitly in the model; unobservable
- Assumption (A1), i.e., E{ε_i | X} = 0 or E{ε_i | x_i} = 0, gives

  $$E\{y_i \mid x_i\} = x_i'\beta$$

  the model describes the expected value of y_i given x_i

Example

Wage equation

$$wage_i = \beta_1 + \beta_2\, male_i + \beta_3\, school_i + \beta_4\, exper_i + \varepsilon_i$$

Answers questions like:

- Expected wage p.h. of a female with 12 years of education and 10 years of experience

Wage equation fitted to all 3294 observations

$$wage_i = -3.38 + 1.34{*}male_i + 0.64{*}school_i + 0.12{*}exper_i$$

- Expected wage p.h. of a female with 12 years of education and 10 years of experience: 5.50 USD

Regression Coefficients

Linear regression model:

$$y_i = \beta_1 + \beta_2 x_{i2} + \ldots + \beta_K x_{iK} + \varepsilon_i = x_i'\beta + \varepsilon_i$$

Coefficient β_k measures the change of Y if X_k changes by one unit

$$\frac{\Delta E\{y_i \mid x_i\}}{\Delta x_k} = \beta_k \quad \text{for } \Delta x_k = 1$$

- For continuous regressors

$$\frac{\partial E\{y_i | x_i\}}{\partial x_{ik}} = \beta_k$$

Marginal effect of changing X_k on Y

- Ceteris paribus condition: measuring the effect of a change of Y if X_k changes by one unit by β_k implies
  - knowledge which other X_i, $i \neq k$, are in the model
  - that all other X_i, $i \neq k$, remain unchanged

Example

Wage equation

$$wage_i = \beta_1 + \beta_2\ male_i + \beta_3\ school_i + \beta_4\ exper_i + \varepsilon_i$$

β_3 measures the impact of one additional year at school upon a person's wage, keeping gender and years of experience fixed

$$\frac{\partial E\{wage_i \mid male_i, school_i, exper_i\}}{\partial\, school_i} = \beta_3$$

Wage equation fitted to all 3294 observations

$$wage_i = -3.38 + 1.34*male_i + 0.64*school_i + 0.12*exper_i$$

- One extra year at school, e.g., at the university, results in an increase of 64 cents; a 4-year study results in an increase of 2.56 USD of the wage p.h.
- This is true for otherwise (gender, experience) identical people

Regression Coefficients, cont'd

- The marginal effect of a changing regressor may be depending on other variables

Example

- Wage equation: $wage_i = \beta_1 + \beta_2\, male_i + \beta_3\, age_i + \beta_4\, age_i^2 + \varepsilon_i$

  the impact of changing age depends on age:

$$\frac{\partial E\{y_i | x_i\}}{\partial age_i} = \beta_3 + 2\beta_4 age_i$$

- Wage equation may contain $\beta_3\, age_i + \beta_4\, age_i\, male_i$: marginal effect of age depends upon gender

$$\frac{\partial E\{y_i | x_i\}}{\partial age_i} = \beta_3 + \beta_4 male_i$$

Elasticities

Elasticity: measures the *relative* change in the dependent variable Y due to a *relative* change in X_k

- For a linear regression, the elasticity of Y with respect to X_k is

$$\frac{\partial E\{y_i|x_i\} / E\{y_i|x_i\}}{\partial x_{ik} / x_{ik}} = \frac{\partial E\{y_i|x_i\}}{\partial x_{ik}} \frac{x_{ik}}{E\{y_i|x_i\}} = \frac{x_{ik}}{x_i'\beta}\beta_k$$

- For a loglinear model

$$\log y_i = (\log x_i)' \beta + \varepsilon_i \quad \text{with } (\log x_i)' = (1, \log x_{i2}, \ldots, \log x_{ik})$$

elasticities are the coefficients β

$$\frac{\partial E\{y_i|x_i\} / E\{y_i|x_i\}}{\partial x_{ik} / x_{ik}} = \beta_k$$

Elasticities, cont'd

This follows from

$$\frac{\partial E\{\log y_i | x_i\}}{\partial x_{ik}} = \frac{\beta_k}{x_{ik}} = \frac{\partial E\{\log y_i | x_i\}}{\partial E\{y_i | x_i\}} \frac{\partial E\{y_i | x_i\}}{\partial x_{ik}}$$

$$\approx \frac{\partial \log E\{y_i | x_i\}}{\partial E\{y_i | x_i\}} \frac{\partial E\{y_i | x_i\}}{\partial x_{ik}} = \frac{1}{E\{y_i | x_i\}} \frac{\partial E\{y_i | x_i\}}{\partial x_{ik}}$$

and

$$\frac{\partial E\{y_i | x_i\}}{\partial x_{ik}} \frac{x_{ik}}{E\{y_i | x_i\}} = \frac{\partial E\{\log y_i | x_i\}}{\partial x_{ik}} \frac{x_{ik} E\{y_i | x_i\}}{E\{y_i | x_i\}}$$

$$= \frac{\beta_k}{x_{ik}} x_{ik} = \beta_k$$

Semi-Elasticities

Semi-elasticity: measures the *relative* change in the dependent variable *Y* due to a one-unit-change in X_k

- Linear regression for

  $\log y_i = x_i' \beta + \varepsilon_i$

  the elasticity of *Y* with respect to X_k is

  $$\frac{\partial E\{y_i | x_i\} / E\{y_i | x_i\}}{\partial x_{ik} / x_{ik}} = \beta_k x_{ik}$$

  β_k measures the relative change in *Y* due to a change in X_k by one unit

Example

Wage equation, fitted to all 3294 observations:

$$log(wage_i) = 1.09 + 0.20\ male_i + 0.19\ \log(exper_i)$$

- The coefficient of $male_i$ measures the semi-elasticity of wages with respect to gender: The wage differential between males ($male_i$ =1) and females is obtained from $w_f = \exp\{1.09 + 0.19 \log(exper_i)\}$ and $w_m = w_f \exp\{0.20\} = 1.22\ w_f$; the wage differential is 0.22 or 22%, i.e., approximately the coefficient 0.20[1)]
- The coefficient of $\log(exper_i)$ measures the elasticity of wages with respect to experience: 10% more time of experience results in a 1.9% higher wage

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[1)] For small x, $\exp\{x\} = \Sigma_k x^k/k! \approx 1+x$

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Selection of Regressors

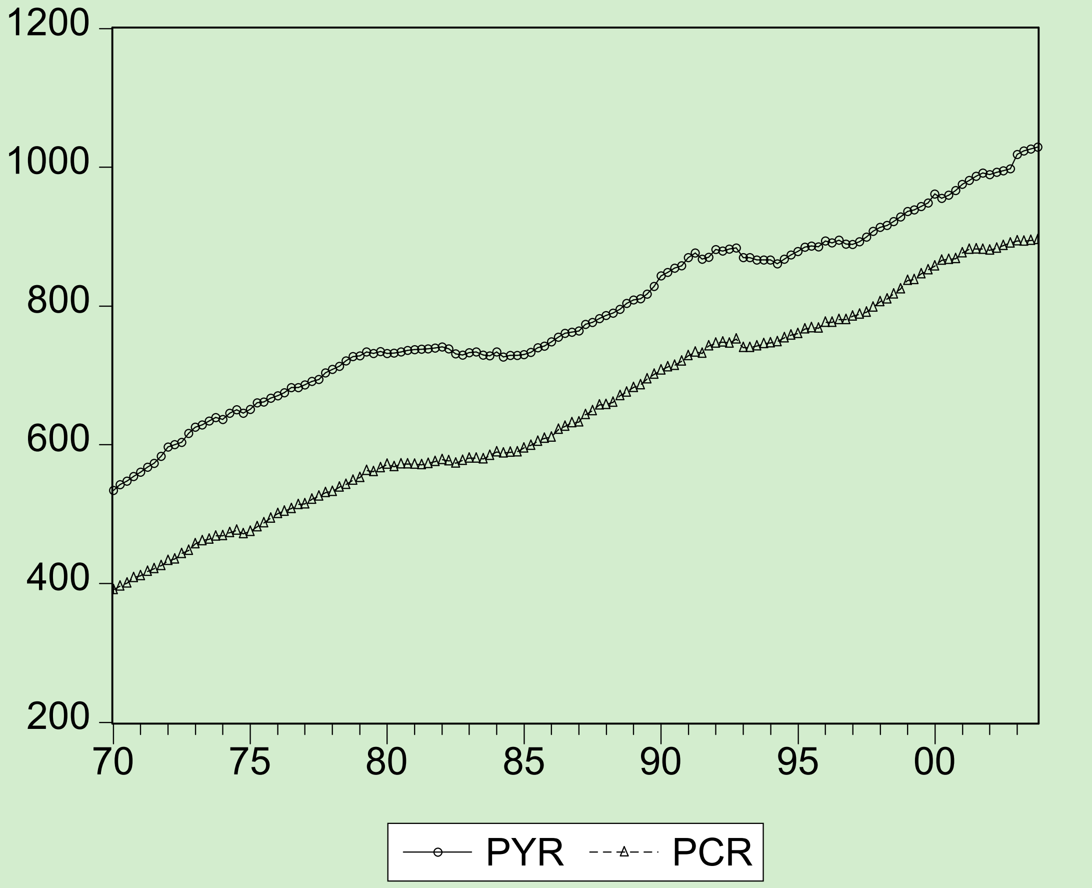
Specification errors:

- Omission of a relevant variable
- Inclusion of an irrelevant variable

Questions:

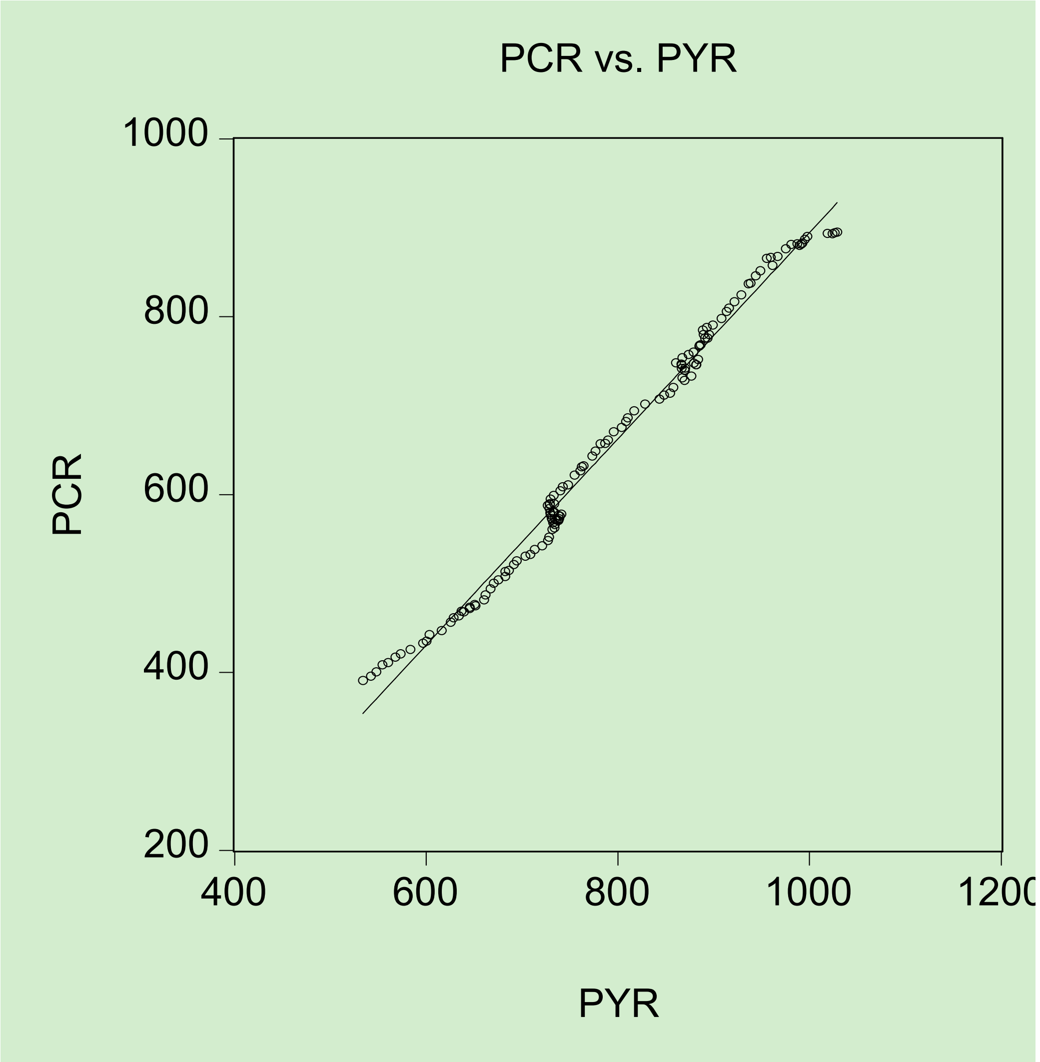
- What are the consequences?
- How to avoid specification errors?
- How to detect a committed specification error?

Example: Income and Consumption



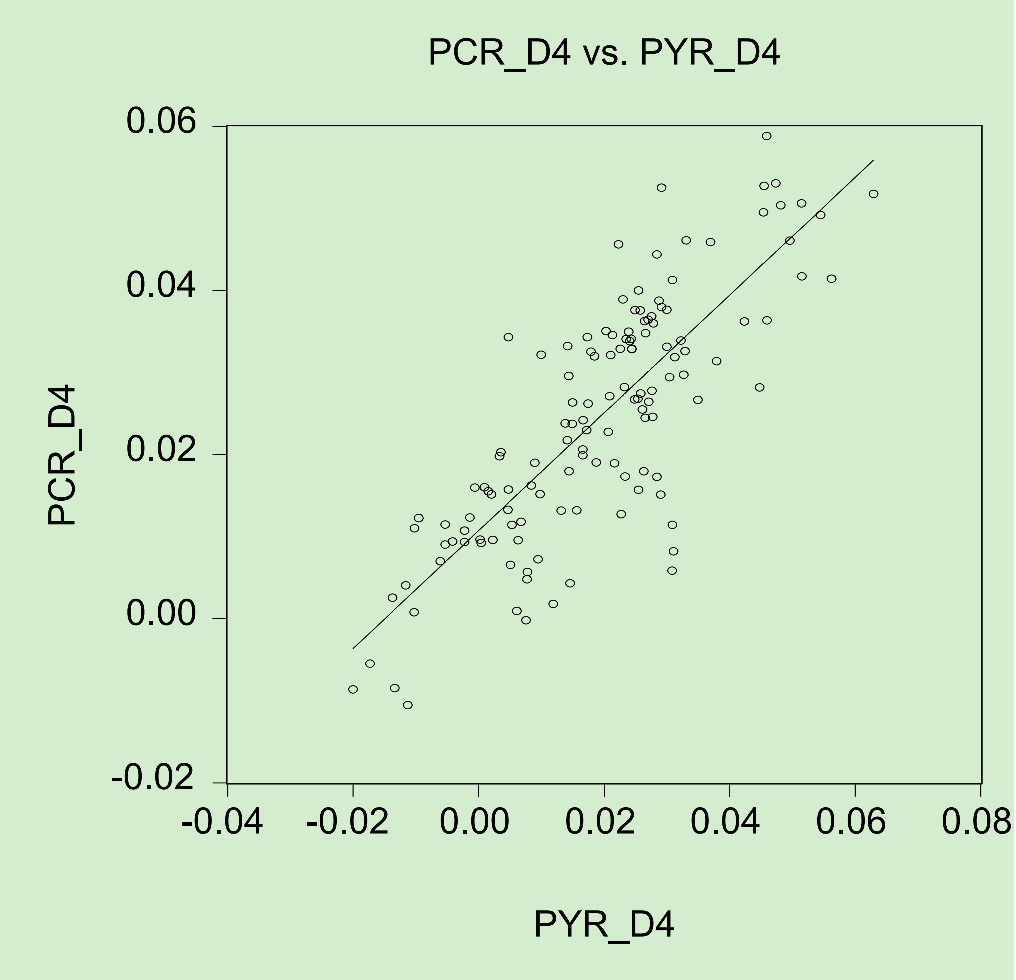
PCR: Private Consumption, real, in bn. EUROs
PYR: Household's Disposable Income, real, in bn. EUROs
1970:1-2003:4
Basis: 1995
Source: AWM-Database

Income and Consumption



PCR: Private Consumption, real, in bn. EUROs
PYR: Household's Disposable Income, real, in bn. EUROs
1970:1-2003:4
Basis: 1995
Source: AWM-Database

Income and Consumption: Growth Rates



PCR_D4: Private Consumption, real, growth rate
PYR_D4: Household's Disposable Income, real, growth rate
1970:1-2003:4
Basis: 1995
Source: AWM-Database

Consumption Function

C: Private Consumption, real, growth rate (PCR_D4)

Y: Household's Disposable Income, real, growth rate (PYR_D4)

T: Trend ($T_i = i/1000$)

$$\hat{C} = 0.011 + 0.761Y, \quad adj R^2 = 0.717$$

Consumption function with trend $T_i = i/1000$:

$$\hat{C} = 0.016 + 0.708Y - 0.068T, \quad adj R^2 = 0.741$$

Consumption Function, cont'd

OLS estimated consumption function: Output from GRETL

Dependent variable : PCR_D4

	coefficient	std. error	t-ratio	p-value
const	0,0162489	0,00187868	8,649	1,76e-014 ***
PYR_D4	0,707963	0,0424086	16,69	4,94e-034 ***
T	-0,0682847	0,0188182	-3,629	0,0004 ***

Mean dependent var	0,024911	S.D. dependent var	0,015222
Sum squared resid	0,007726	S.E. of regression	0,007739
R- squared	0,745445	Adjusted R-squared	0,741498
F(2, 129)	188,8830	P-value (F)	4,71e-39
Log-likelihood	455,9302	Akaike criterion	-905,8603
Schwarz criterion	-897,2119	Hannan-Quinn	-902,3460
rho	0,701126	Durbin-Watson	0,601668

Consequences

Consequences of specification errors:

- Omission of a relevant variable
- Inclusion of a irrelevant variable

Misspecification: Omitted Regressor

Two models, with *J*-vector z_i:

$$y_i = x_i'\beta + z_i'\gamma + \varepsilon_i \qquad \text{(A)}$$

$$y_i = x_i'\beta + v_i \qquad \text{(B)}$$

OLS estimates b_B of β from (B) can be written with y_i from (A):

$$b_B = \beta + \left(\sum_i x_i x_i'\right)^{-1} \sum_i x_i z_i'\gamma + \left(\sum_i x_i x_i'\right)^{-1} \sum_i x_i \varepsilon_i$$

If (A) is the true model but (B) is specified, i.e., relevant regressors z_i are omitted, b_B is biased by

$$E\left\{\left(\sum_i x_i x_i'\right)^{-1} \sum_i x_i z_i'\gamma\right\}$$

Omitted variable bias

No bias if (a) $\gamma = 0$ or if (b) variables in x_i and z_i are orthogonal

Misspecification: Irrelevant Regressor

Two models:

$$y_i = x_i'\beta + z_i'\gamma + \varepsilon_i \qquad \text{(A)}$$

$$\mathbf{y_i = x_i'\beta + v_i} \qquad \text{(B)}$$

If (B) is the true model but (A) is specified, i.e., the model contains irrelevant regressors z_i

The OLS estimates b_A

- are unbiased
- Have higher variances and standard errors than the OLS estimate b_B obtained from fitting model (B)

Specification Search

General-to-specific modeling:

1. List all potential regressors, based on, e.g.,
   - economic theory
   - empirical results
   - availability of data
2. Specify the most general model: include all potential regressors
3. Iteratively, test which variables have to be dropped, re-estimate
4. Stop if no more variable has to be dropped

The procedure is known as the LSE (London School of Economics) method

Alternatively, one can start with a small model and add variables as long as they contribute to explaining Y

Specification Search, cont'd

Alternative procedures

- Specific-to-general modeling: start with a small model and add variables as long as they contribute to explaining Y
- Stepwise regression

Specification search can be subsumed under *data mining*

Practice of Specification Search

Applied research

- Starts with a – in terms of economic theory – plausible specification
- Tests whether imposed restrictions are correct
  - Tests for omitted regressors
  - Tests for autocorrelation of residuals
  - Tests for heteroskedasticity
- Tests whether further restrictions need to be imposed
  - Tests for irrelevant regressors

Obstacles for good specification

- Complexity of economic theory
- Limited availability of data

Regressor Selection Criteria

Criteria for adding and deleting regressors

- *t*-statistic, *F*-statistic
- Adjusted R^2
- Information Criteria: penalty for increasing number of regressors
  - Akaike's Information Criterion

$$AIC = \log \tfrac{1}{N} \sum_i e_i^2 + \tfrac{2K}{N}$$

  - Schwarz's Bayesian Information Criterion

$$BIC = \log \tfrac{1}{N} \sum_i e_i^2 + \tfrac{K}{N} \log N$$

model with smaller BIC (or AIC) is preferred

The corresponding probabilities for type I and type II errors can hardly be assessed

Individual Wages

Are *school* and *exper* relevant regressors in

$$wage_i = \beta_1 + \beta_2\, male_i + \beta_3\, school_i + \beta_4\, exper_i + \varepsilon_i$$

or shall they be omitted?

- *t*-test: *p*-values are 4.62E-80 (*school*) and 1.59E-7 (*exper*)
- *F*-test: $F = [(0.1326-0.0317)/2]/[(1-0.1326)/(3294-4)] = 191.24$, with *p*-value 2.68E-79
- adj R^2: 0.1318 for the wider model, much higher than 0.0315
- AIC: the wider model (AIC = 16690.18) is preferable; for the smaller model: AIC = 17048.46
- BIC: the wider model (BIC = 16714.58) is preferable; for the smaller model: BIC = 17060.66

All criteria suggest the wider model

Individual Wages, cont'd

OLS estimated smaller wage equation (Table 2.1, Verbeek)

Dependent variable: *wage*		
Variable	Estimate	Standard error
constant	5.1469	0.0812
male	1.1661	0.1122
$s = 3.2174$	$R^2 = 0.0317$	$F = 107.93$

with AIC = 17048.46, BIC = 17060.66

Individual Wages, cont'd

OLS estimated wider wage equation (Table 2.2, Verbeek)

Table 2.2 OLS results wage equation

Dependent variable: *wage*

Variable	Estimate	Standard error	t-ratio
constant	−3.3800	0.4650	−7.2692
male	1.3444	0.1077	12.4853
school	0.6388	0.0328	19.4780
exper	0.1248	0.0238	5.2530

$s = 3.0462 \quad R^2 = 0.1326 \quad \overline{R}^2 = 0.1318 \quad F = 167.63$

with AIC = 16690.18, BIC = 16714.58

The AIC Criterion

Various versions in literature

- Verbeek, also Greene:

$$AIC = \log \tfrac{1}{N} \sum_i e_i^2 + \tfrac{2K}{N} = \log(s^2) + 2K / N$$

- Akaike‘s original formula is

$$AIC = -\frac{2\ell(b)}{N} + \frac{2K}{N}$$

with the log-likelihoodfunktion

$$\ell(b) = -\frac{N}{2}\left(1 + \log(2\pi) + \log s^2\right)$$

- GRETL:

$$AIC = N \log(s^2) + 2K + N\left(1 + \log(2\pi)\right)$$

Nested Models: Comparison

Model (B), p.20, is nested in model (A); (A) is extended by *J* additional regressors

Do the *J* added regressors contribute to explaining *Y*?

- *F*-test (*t*-test when $J = 1$) for testing H_0: coefficients of added regressors are zero

$$F = \frac{(R_A^2 - R_B^2)/J}{(1 - R_A^2)/(N-K)}$$

  R_B^2 and R_A^2 are the R^2 of the models without (B) and with (A) the *J* additional regressors, respectively

- Comparison of adjusted R^2: adj R_A^2 > adj R_B^2 equivalent to $F > 1$
- Information Criteria: choose the model with the smaller value of the information criterion

Comparison of Non-nested Models

Non-nested models: A: $y_i = x_i'\beta + \varepsilon_i$, B: $y_i = z_i'\gamma + v_i$ with components in z_i that are not in x_i

- Non-nested or encompassing *F*-test: compares by *F*-tests artificially nested models

  $y_i = x_i'\beta + z_{2i}'\delta_B + \varepsilon^*_i$ with z_{2i}: regressors from z_i not in x_i

  $y_i = z_i'\gamma + x_{2i}'\delta_A + v^*_i$ with x_{2i}: regressors from x_i not in z_i

  - Test validity of model A by testing H_0: $\delta_B = 0$
  - Analogously, test validity of model B by testing H_0: $\delta_A = 0$
  - Possible results: A or B is valid, both models are valid, none is valid
- Other procedures: *J*-test, PE-test

Individual Wages

Which of the models is adequate?

$$\log(wage_i) = 0.119 + 0.260\, male_i + 0.115\, school_i \qquad \text{(A)}$$

adj R^2 = 0.121, BIC = 5824.90,

$$\log(wage_i) = 0.119 + 0.064\, age_i \qquad \text{(B)}$$

adj R^2 = 0.069, BIC = 6004.60

- The artificially nested model is

$$-0.472 + 0.243\, male_i + 0.088\, school_i + 0.035\, age_i$$

- Test of model validity
  - model A: t-test for *age*, p-value 5.79E-15; model A is not adequate
  - model B: F-test for *male* and *school*: model B is not adequate

Comparison of Non-nested Models: *J*-Test

Non-nested models: A: $y_i = x_i'\beta + \varepsilon_i$, B: $y_i = z_i'\gamma + v_i$ with components of z_i that are not in x_i

- Combined model

  $$y_i = (1 - \delta)\, x_i'\beta + \delta\, z_i'\gamma + u_i$$

  δ indicates model adequacy

- Transformed model

  $$y_i = x_i'\beta^* + \delta z_i'c + u_i = x_i'\beta^* + \delta \hat{y}_{iB} + u^*_i$$

  with OLS-estimate c for γ and predicted values $\hat{y}_{iB}$ obtained from fitting model B; $\beta^* = (1-\delta)\beta$

- *J*-test for validity of model A by testing H_0: $\delta = 0$
- Less computational effort than the encompassing *F*-test

Individual Wages

Which of the models is adequate?

$$\log(wage_i) = 0.119 + 0.260\, male_i + 0.115\, school_i \quad \text{(A)}$$

adj R^2 = 0.121, BIC = 5824.90,

$$\log(wage_i) = 0.119 + 0.064\, age_i \quad \text{(B)}$$

adj R^2 = 0.069, BIC = 6004.60

Test of model validity by means of the *J*-test

- Extend the model B to

$$\log(wage_i) = -0.587 + 0.034\, age_i + 0.826\, \hat{y}_{iA}$$

  with values $\hat{y}_{iA}$ predicted for log($wage_i$) from model A
- Test of model validity: *t*-test for coefficient of $\hat{y}_{iA}$, t = 15.96, *p*-value 2.65E-55
- Model B is not a valid model

Linear vs. Loglinear Model

Choice between linear and loglinear functional form

$$y_i = x_i'\beta + \varepsilon_i \qquad \text{(A)}$$

$$\log y_i = (\log x_i)'\beta + v_i \qquad \text{(B)}$$

- On the basis of economic interpretation: are effects additive or multiplicative?
- Log-transformation stabilizes variance, particularly if the dependent variable has a skewed distribution (wages, income, production, firm size, sales,…)
- Loglinear models are easily interpretable in terms of elasticities

Linear vs. Loglinear Model: The PE-Test

Choice between linear and loglinear functional form

- Estimate both models

  $y_i = x_i'\beta + \varepsilon_i$ (A)

  $\log y_i = (\log x_i)'\beta + v_i$ (B)

  calculate the fitted values $\hat{y}$ (from model A) and $\log \ddot{y}$ (from B)
- Test $\delta_{LIN} = 0$ in

  $y_i = x_i'\beta + \delta_{LIN} (\log \hat{y}_i - \log \ddot{y}_i) + u_i$

  not rejecting $\delta_{LIN} = 0$ favors the model A
- Test $\delta_{LOG} = 0$ in

  $\log y_i = x_i'\beta + \delta_{LOG} (\hat{y}_i - \exp\{\log \ddot{y}_i\}) + u_i$

  not rejecting $\delta_{LOG} = 0$ favors the model B
- Both null hypotheses are rejected: find a more adequate model

Individual Wages

Test of validity of models by means of the PE-test

The fitted models are (with *l_x* for log(*x*))

$$wage_i = -2.046 + 1.406\ male_i + 0.608\ school_i \qquad \text{(A)}$$

$$l\_wage_i = 0.119 + 0.260\ male_i + 0.115\ l\_school_i \qquad \text{(B)}$$

- *x_f*: predicted value of *x*: *d_lg* = log(*wage_f*) – *l_wage_f*, *d_ln* = *wage_f* – exp(*l_wage_f*)
- Test of model validity, model A:

  $$wage_i = -1.708 + 1.379\ male_i + 0.637\ school_i - 4.731\ d\_lg_i$$

  with *p*-value 0.013 for *d_lg*; validity in doubt
- Test of model validity, model B:

  $$l\_wage_i = -1.132 + 0.240\ male_i + 1.008\ l\_school_i + 0.171\ d\_ln_i$$

  with *p*-value 0.076 for *d_ln*; model B to be preferred

The PE-Test

Choice between linear and loglinear functional form

- The auxiliary regressions are estimated for testing purposes
- If the linear model is not rejected: accept the linear model
- If the loglinear model is not rejected: accept the loglinear model
- If both are rejected, neither model is appropriate, a more general model should be considered
- In case of the Individual Wages example:
  - Linear model: *t*-statistic is – 4.731, *p*-value 0.013: the model is rejected
  - Loglinear model: *t*-statistic is 0.171, *p*-value 0.076 : the model is not rejected

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Non-linear Functional Forms

Model specification

$$y_i = g(x_i, \beta) + \varepsilon_i$$

instead of $y_i = x_i'\beta + \varepsilon_i$: violation of linearity

Non-linearity in regressors (but linear in parameters)

- Powers of regressors
- Interactions of regressors

OLS-technique still works; *t*-test, *F*-test for specification check

- Non-linearity in regression coefficients, e.g.,
  - $g(x_i, \beta) = \beta_1 x_{i1}^{\beta 2} x_{i2}^{\beta 3}$

    logarithmic transformation: $\log g(x_i, \beta) = \log \beta_1 + \beta_2 \log x_{i1} + \beta_3 \log x_{i2}$
  - $g(x_i, \beta) = \beta_1 + \beta_2 x_i^{\beta 3}$

    non-linear least squares estimation, numerical procedures

Various test procedures, e.g., RESET test, Chow test

Individual Wages: Effect of Gender

Effect of gender may be depending of education level

- Separate models for males and females
- Interaction terms between dummies for education level and male

Example: Belgian Household Panel, 1994 (N=1472)

- Five education levels
- Model with education dummies
- Model with interaction terms between education dummies and gender dummy
- F-statistic for interaction terms:

$$F(5, 1460) = \{(0.4032-0.3976)/5\}/\{(1-0.4032)/(1472-12)\}$$
$$= 2.74$$

with a p-value of 0.018

Wages: Education Dummies

Model with education dummies: Verbeek, Table 3.11

Table 3.11 OLS results specification 5

Dependent variable: log(*wage*)

Variable	Estimate	Standard error	t-ratio
constant	1.272	0.045	28.369
male	0.118	0.015	7.610
$educ = 2$	0.144	0.033	4.306
$educ = 3$	0.305	0.032	9.521
$educ = 4$	0.474	0.033	14.366
$educ = 5$	0.639	0.033	19.237
log(*exper*)	0.230	0.011	21.804

$s = 0.282$ $R^2 = 0.3976$ $\bar{R}^2 = 0.3951$ $F = 161.14$ $S = 116.47$

Wages: Interactions with Gender

Wage equation with interactions *educ*male*

Table 3.12 OLS results specification 6

Dependent variable: log(*wage*)

Variable	Estimate	Standard error	t-ratio
constant	1.216	0.078	15.653
male	0.154	0.095	1.615
$educ = 2$	0.224	0.068	3.316
$educ = 3$	0.433	0.063	6.851
$educ = 4$	0.602	0.063	9.585
$educ = 5$	0.755	0.065	11.673
$\log(exper)$	0.207	0.017	12.535
$educ = 2 \times male$	−0.097	0.078	−1.242
$educ = 3 \times male$	−0.167	0.073	−2.272
$educ = 4 \times male$	−0.172	0.074	−2.317
$educ = 5 \times male$	−0.146	0.076	−1.935
$\log(exper) \times male$	0.041	0.021	1.891

$s = 0.281$ $R^2 = 0.4032$ $\bar{R}^2 = 0.3988$ $F = 89.69$ $S = 115.37$

Wages: Effect of Gender

Wage equation with interaction *educ*male*

Table 3.12 OLS results specification 6

Dependent variable: log(*wage*)

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$educ = 4 \times male$	−0.172	0.074	−2.317
$educ = 5 \times male$	−0.146	0.076	−1.935
$\log(exper) \times male$	0.041	0.021	1.891

$s = 0.281 \quad R^2 = 0.4032 \quad \bar{R}^2 = 0.3988 \quad F = 89.69 \quad S = 115.37$

RESET Test

Test of the linear model $E\{y_i | x_i\} = x_i'\beta$ against misspecification of the functional form:

- Null hypothesis: linear model is correct functional form
- Test of H_0: RESET test (Regression Equation Specification Error Test)
- Test idea: non-linear functions of $\hat{y}_i$, the fitted values from the linear model, e.g., $\hat{y}_i^2$, $\hat{y}_i^3$, ... , do not improve model fit unter H_0
- Test procedure: linear model extended by adding $\hat{y}_i^2$, $\hat{y}_i^3$, ...
- *F*-test to decide whether powers of fitted values like $\hat{y}_i^2$, $\hat{y}_i^3$, ... contribute as additional regressors to explaining *Y*
- Power *Q* of fitted values: typical choice is $Q = 2$ or $Q = 3$

Individual Wages: RESET Test

The fitted models are (with *l_x* for log(*x*))

$$wage_i = -2.046 + 1.406\, male_i + 0.608\, school_i \qquad (A)$$

$$l\_wage_i = 0.119 + 0.260\, male_i + 0.115\, l\_school_i \qquad (B)$$

Test of specification of the functional form with $Q = 2$

- Model A: Test statistic: $F(2, 3288) = 10.23$, p-value = 3.723e-005
- Model B: Test statistic: $F(2, 3288) = 4.52$, p-value = 0.011

For both models the adequacy of the functional form is in doubt

Structural Break: Chow Test

In time-series context, coefficients of a model may change due to a major policy change, e.g., the oil price shock

- Modeling a process with structural break

  $$E\{y_i \mid x_i\} = x_i'\beta + g_i x_i'\gamma$$

  with dummy variable g_i=0 before the break, g_i=1 after the break
- Regressors x_i, coefficients β before, β+γ after the break
- Null hypothesis: no structural break, γ=0
- Test procedure: fitting the extended model, *F*- (or *t*-) test of γ=0

  $$f = \frac{S_r - S_u}{S_u} \frac{N - 2K}{K}$$

  with S_r (S_u): sum of squared residuals of the (un)restricted model
- Chow test for structural break or structural change

Chow Test: The Practice

Test procedure is performed in the following steps

- Fit the restricted model: S_r
- Fit the extended model: S_u
- Calculate f and the p-value from the F-distribution with K and $N-2K$ d.f.

Needs knowledge of break point

Your Homework

1. Show that the OLS estimator for β from $y_i = x_i'\beta + z_i'\gamma + \varepsilon_i$ can be written as (a) $b = (X'X)^{-1}X'(y-Zc)$ with estimator c for γ, or as (b) $b = (X'M_zX)^{-1}X'M_zy$ with residual generating matrix $M_z=I-Z(Z'Z)^{-1}Z'$.
2. Use the data set "wages" of Verbeek for the following analyses:
   a. Estimate the model where the log hourly wages are explained by *male*, *age* and *educ* with *age* = *school* + *exper* + *6*; interpret the results.
   b. Repeat the analysis after adding four dummy variables for the educational levels 2 through 5 instead of the variable *educ*; compare the model by using (a) the non-nested *F*-test and (b) the JE-test; interpret the results.
   c. Use the PE-test to decide whether the model in b. (where log hourly wages are explained) or the same model but with levels of hourly wages as explained variable is to be preferred; interpret the result.
   d. Repeat a. with the interaction *age*educ* as added regressor; interpret the result.