3. PRODUCTION ANALYSIS OF THE FIRM

CONTENTS

- ☞ basic background of firm´s analysis
- ☞ short run production function
- ☞ firm´s production in long run, firm´s equlibrium
- ☞ firm´s equilibrium upon different levels of total costs, and prices of inputs
- ☞ returns to scale
- ☞ examples of long run production functions

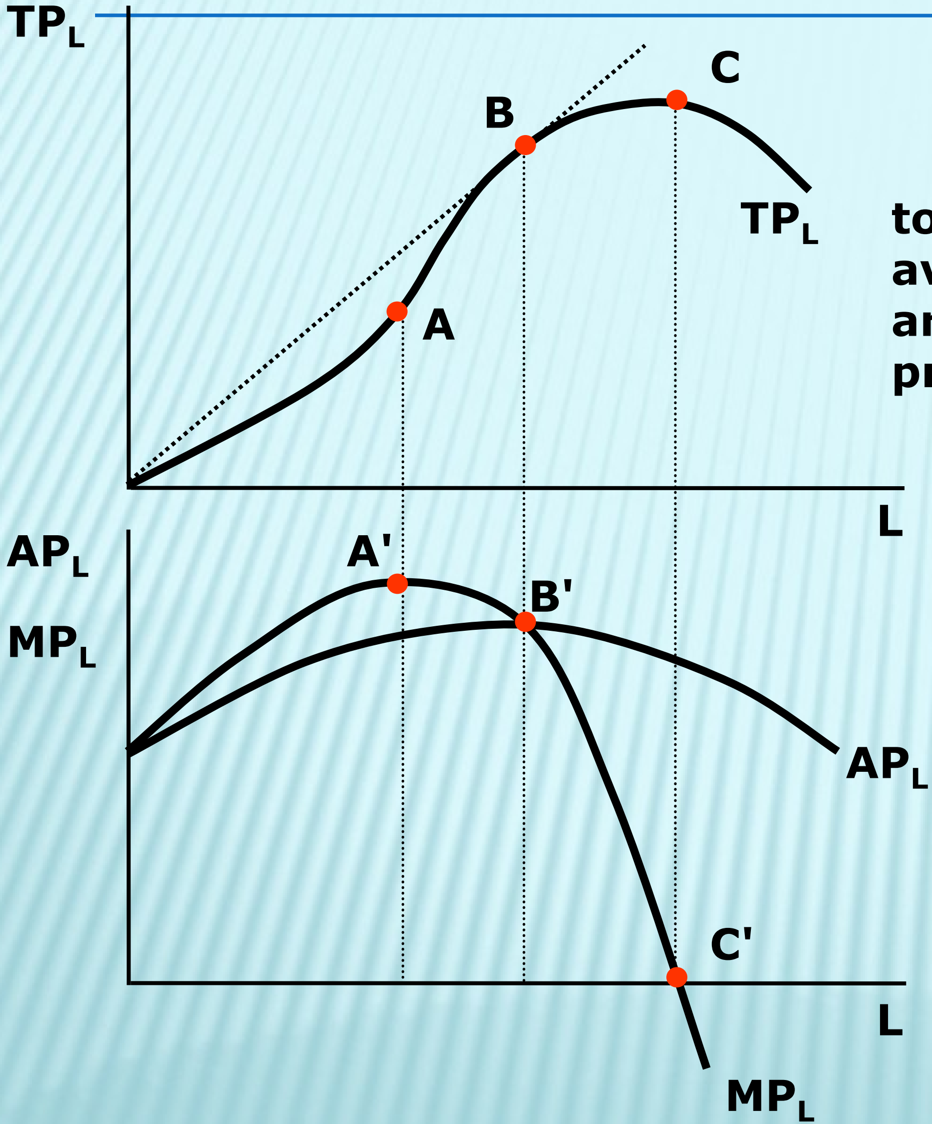
BASIC BACKGROUND OF FIRM´S ANALYSIS

- firm = subject producing goods and/or services... transformig inputs into outputs
- firm: recruits the inputs
  organizes their transformation into outputs
  sells its outputs
- firm´s goal is to maximize its profit
- economic vs. accountable profit
- ekonomic profit = accountable profit minus implicit costs

BASIC BACKGROUND OF FIRM´S ANALYSIS

- production limits – technological and financial
- production function – relationship between the volume of inputs and volume of outputs
- conventional inputs: labour (L), capital (K)
- other inputs: land (P), technological level (τ)
- production function: $Q = f(K,L)$
- short run – volume of capital is fixed
- long run – all inputs are variable

SHORT RUN PRODUCTION FUNCTION



to A – increasing returns to labour (MP_L increasing)

to B – 1st stage of production – average product of labour and capital are increasing; motivation to increase production, fixed input not fully utilized

between B and C – 2nd stage of production – average product of labour decreasing, but AP of capital still increasing

behind C – 3rd stage of production – both APs decreasing, total product decreasing

firm endeavours the 2nd stage of production

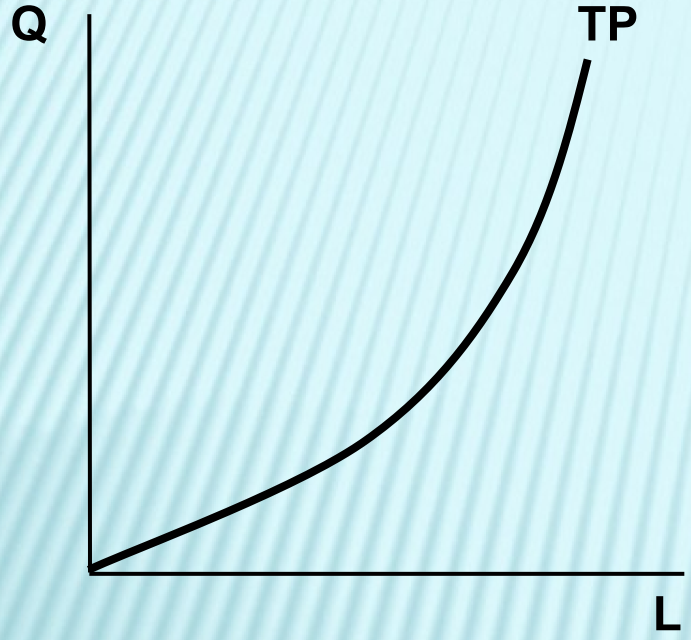
WHAT INFLUENCES THE SHAPE OF MP_L?

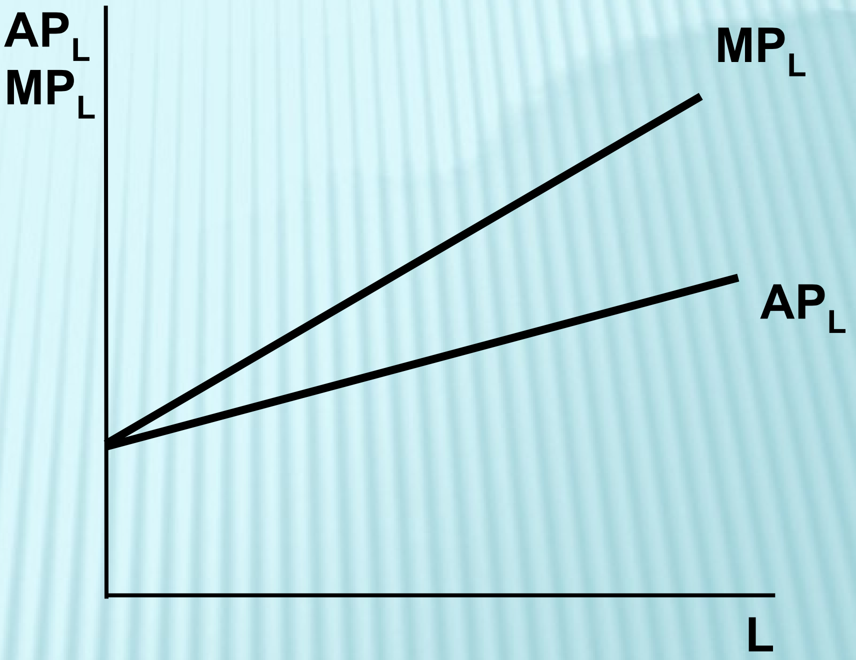
- MP_L = product of additional unit of labour
- we add: a) additional working hours... or b) additional workers?
- a): MP_L influenced with human´s nature
- b): MP_L influenced with the nature of production

SR PRODUCTION – SOME IDENTITIES

- $Q = f(K_{fix}, L)$
- $AP_L = Q/L$ $AP_K = Q/K$
- $MP_L = \partial Q/\partial L$ $MP_K = \partial Q/\partial K$

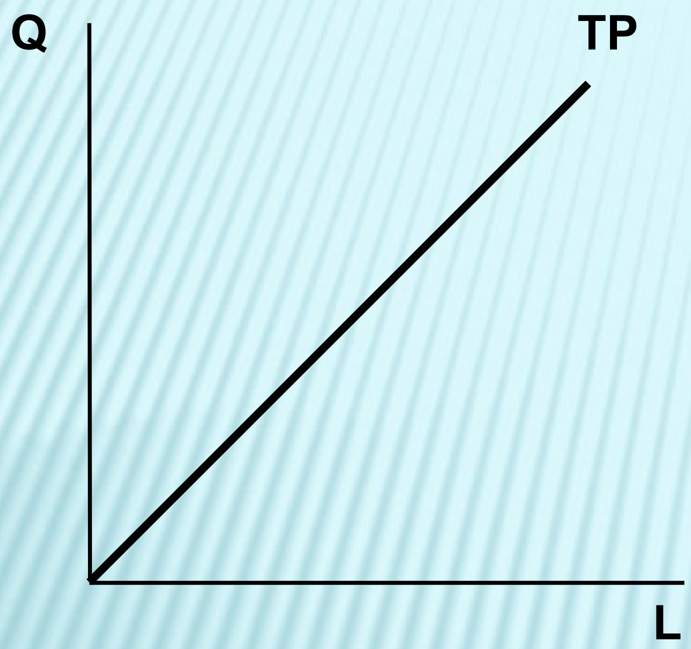
SR PRODUCTION – INCREASING RETURNS TO LABOUR

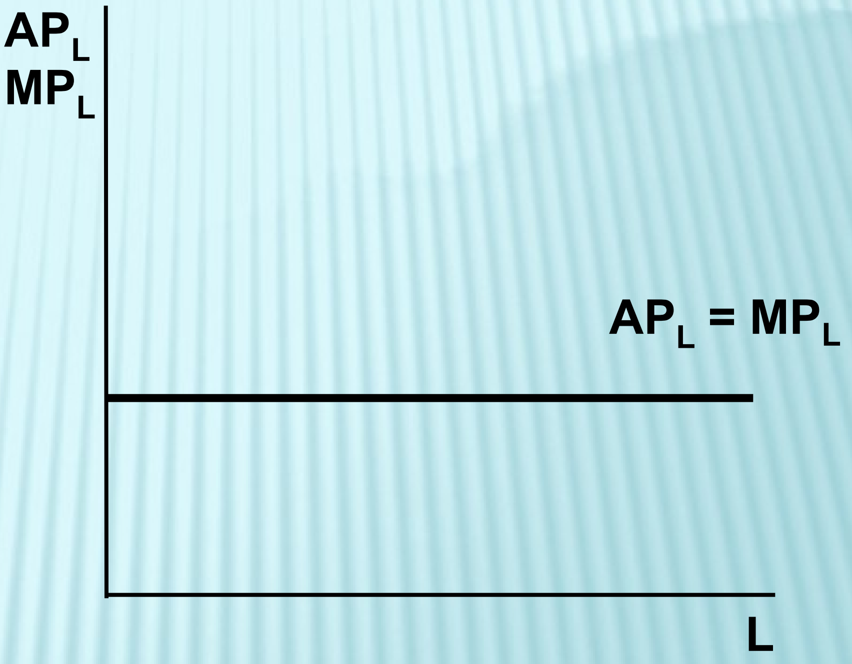




Total product increases with increasing rate – TP grows faster than the volume of labour recruited

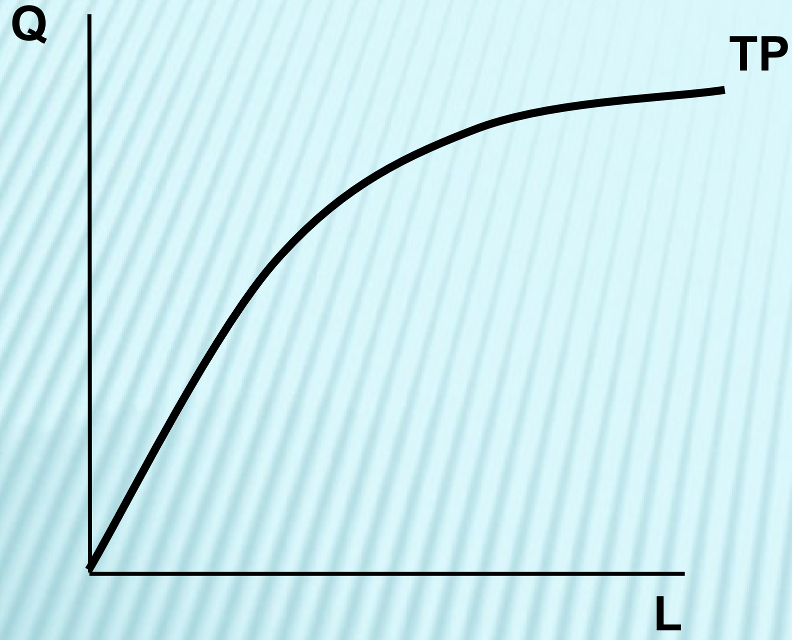
SR PRODUCTION – CONSTANT RETURNS TO LABOUR

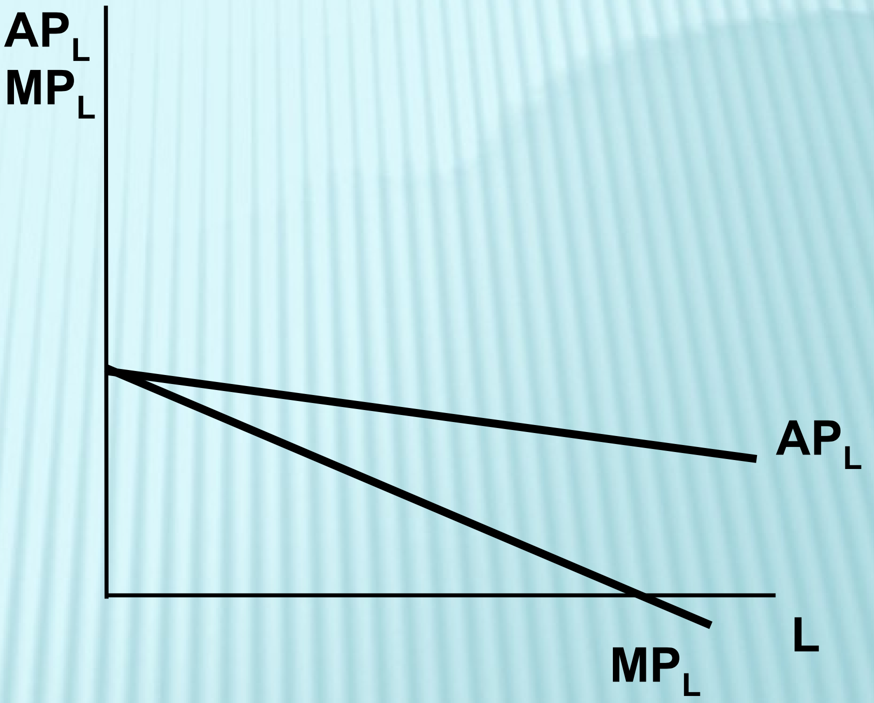




TP increases with constant rate – as fast as volume of labour recruited

SR PRODUCTION – DIMINISHING RETURNS TO LABOUR



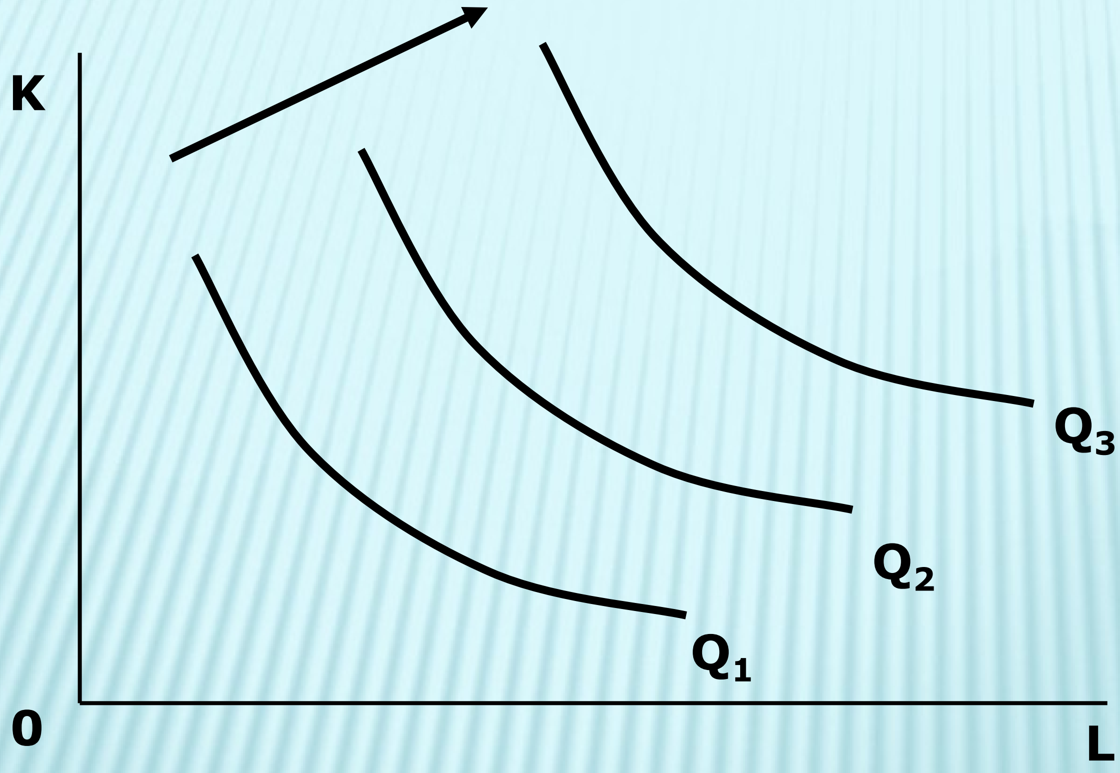


TP increases with decreasing rate – TP grows slower than the volume of labour recruitet

PRODUCTION IN LONG RUN (LR)

- both inputs, labour and capital are variable
- $Q = f(K,L)$
- LR production function as a „map“ of isoquants
- isoquant = a curve that represents the set of different combination of inputs leading to the constant volume of total product (output) – analogy to consumer´s IC

LR PRODUCTION FUNCTION – MAP OF ISOQUANTS



If both inputs are normal, total product grows with both inputs increasing

ISOQUANTS CHARACTERISTICS

- analogy to ICs
- ... are aligned from the cardinalistic point of view (total product is measureable)
- ... do not cross each other
- ... are generally convex to the origin (a firm usually needs both inputs)

MARGINAL RATE OF TECHNICAL SUBSTITUTION

- *MRTS – ratio expressing the firm´s possibility to substitute inputs with each other, total product remaining constant (compare with consumer´s MRS_C)*
- $MRTS = -\Delta K/\Delta L$
- $-\Delta K.MP_K = \Delta L.MP_L \rightarrow -\Delta K/\Delta L = MP_L/MP_K \rightarrow MRTS = MP_L/MP_K$

ELASTICITY OF SUBSTITUTION

- relative change of ratio K/L to relative change of MRTS
- implies the shape of isoquants
- $\sigma = \dfrac{d(K/L)/K/L}{dMRTS/MRTS}$
- σ = ∞ for perfectly substituteable inputs
- σ = 0 for perfect complementary inputs

OPTIMAL COMBINATION OF INPUTS

- again analogy of consumer´s equilibrium
- firm is limited with its budget
- budget constraint depends on total firm´s expenditures (total costs – TC), and prices of inputs
- firm´s budget line (isocost):

  TC = w.L + r.K, where

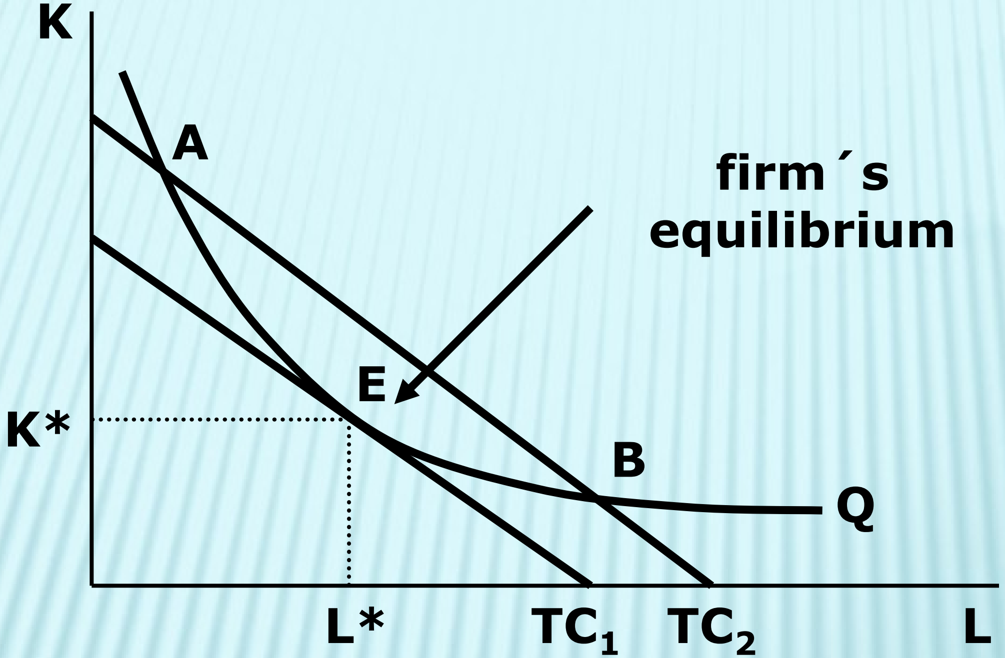
  w…wage rate

  r…rental (derived from interest rate)

OPTIMAL COMBINATION OF INPUTS – INNER SOLUTION

- if isoquant tangents the isocost:
- if the slope of isoquant equals the slope of isocost...
- ...if stands: MRTS = w/r , so:
- $MP_L/MP_K = w/r$
- only in the above case the firm produces the specific output with minimal total costs, or:
- ... firm produces maximal output with specific level of total costs

OPTIMAL COMBINATION OF INPUTS – INNER SOLUTION



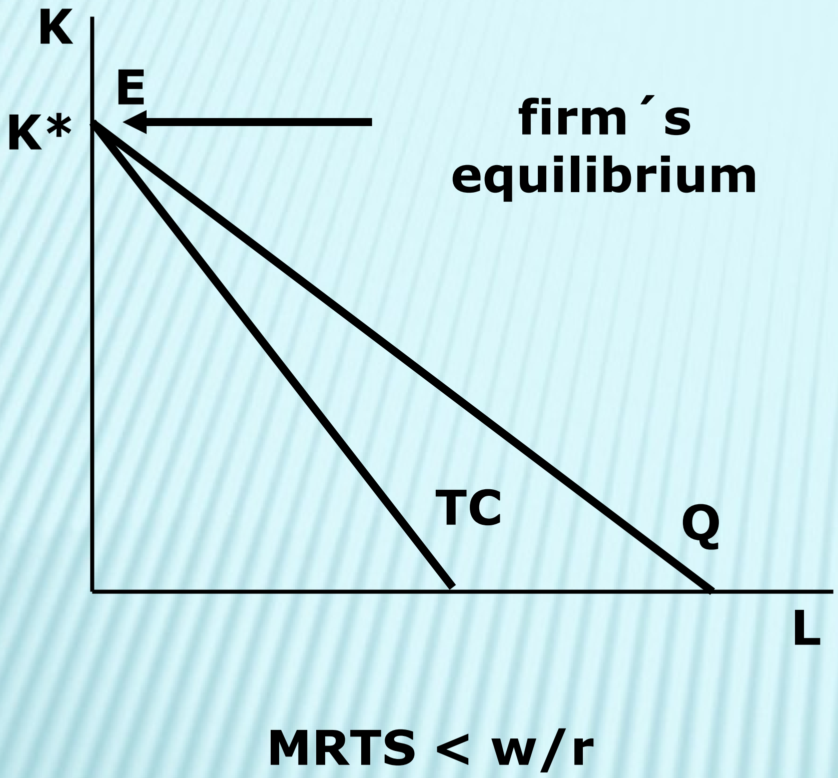
In A and B, the firm is not minimizing its total costs

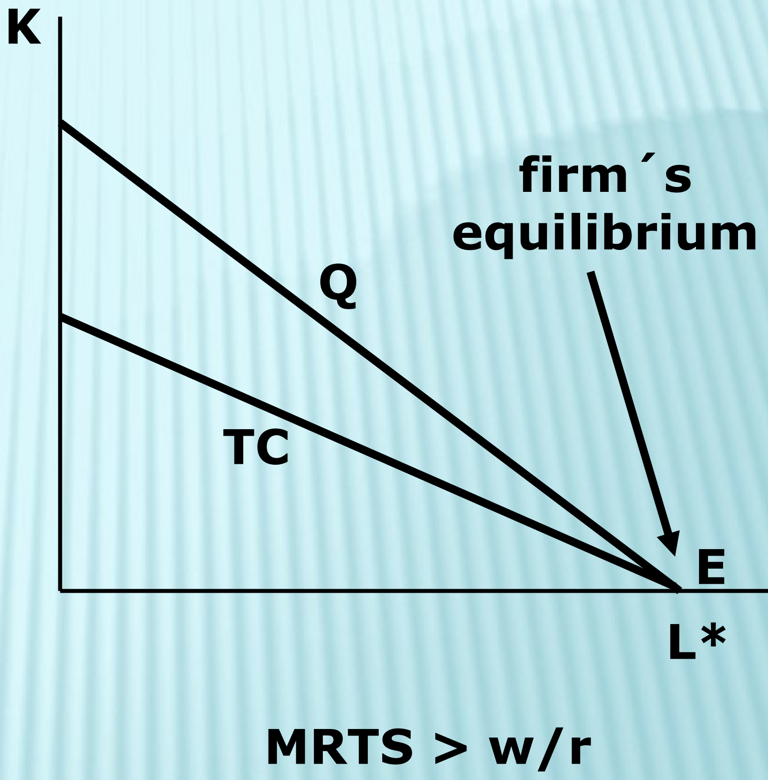
In A and B, the firm is not maximizing its total product

OPTIMAL COMBINATION OF INPUTS – CORNER SOLUTION

- usually in the case of perfectly substituteable inputs... then...
- ...firm´s equilibrium as an intersection of isoquant and isocost
- MRTS ≠ w/r

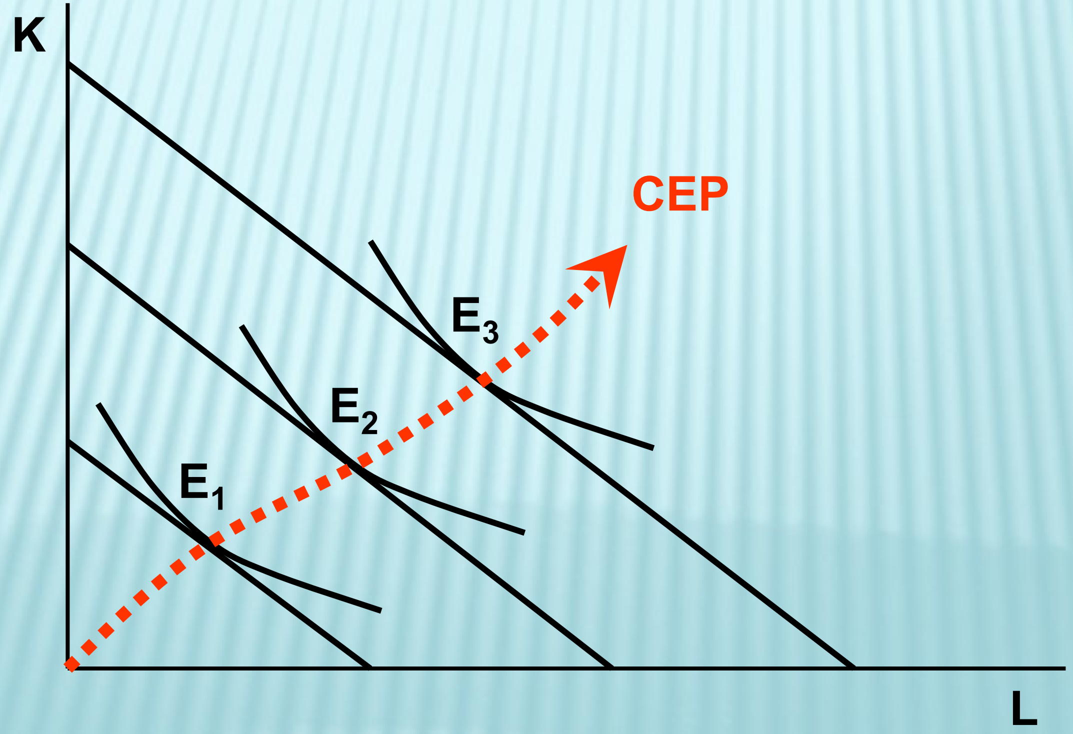
OPTIMAL COMBINATION OF INPUTS – CORNER SOLUTION





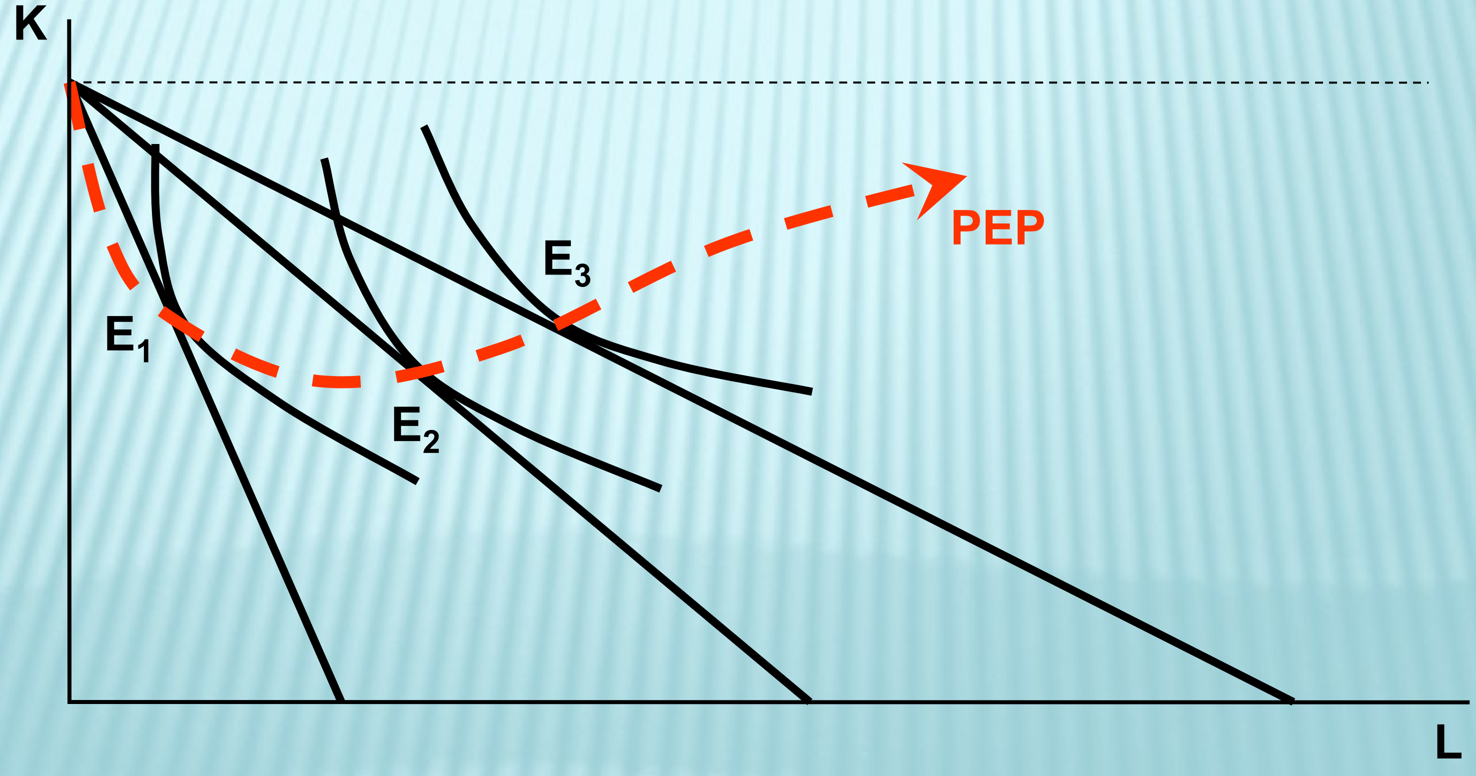
COST EXPANSION PATH (CEP)

- set of firm´s equilibria upon different levels of total costs (budgets)
- analogy to consumer´s ICC



PRICE EXPANSION PATH

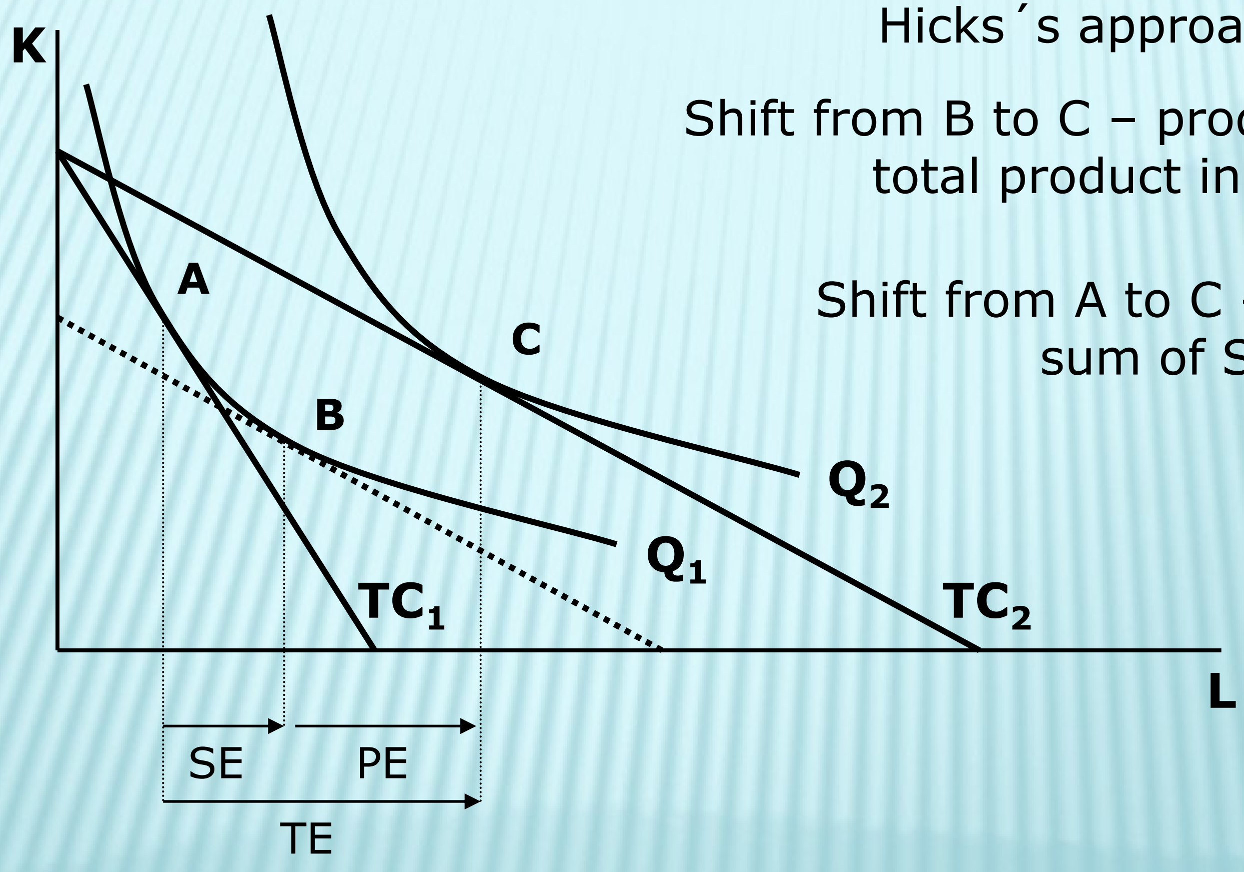
- set of firm´s equilibria upon different levels of price of one of the inputs
- analogy to consumer´s PCC



IMPACT OF THE CHANGE OF PRICE OF INPUT ON THE QUANTITY IN PRODUCTION PROCESS – SUBSTITUTION AND PRODUCTION EFFECT

- ***substitution effect (SE)*** – the firm substitutes relatively more costly input with the relatively cheaper one
- ***production effect (PE)*** – analogy to consumer´s IE – change of price of input leads to the change of real budget

SUBSTITUTION AND PRODUCTION EFFECT OF WAGE RATE DECREASE



Shift from A to B – substitution effect – total product remains constant (we use Hicks´s approach

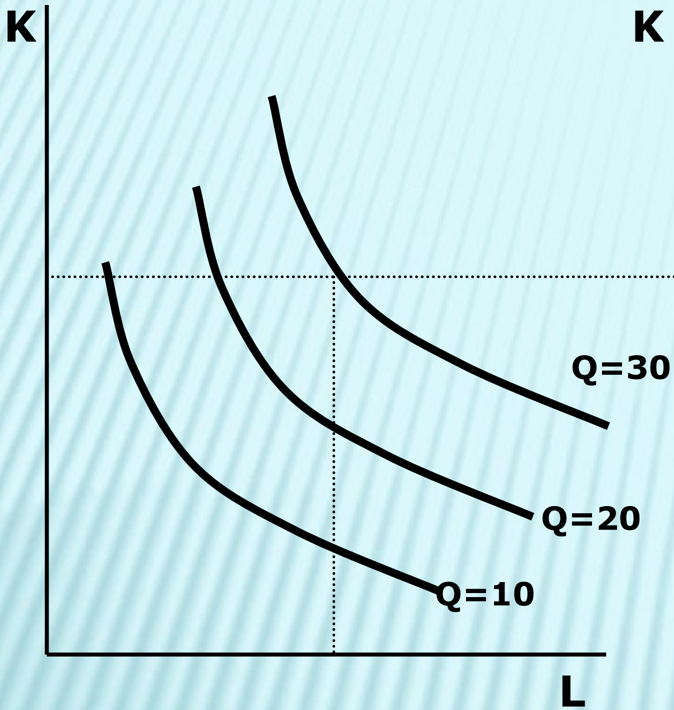
Shift from B to C – production effect – total product increases

Shift from A to C – total effect – the sum of SE and PE

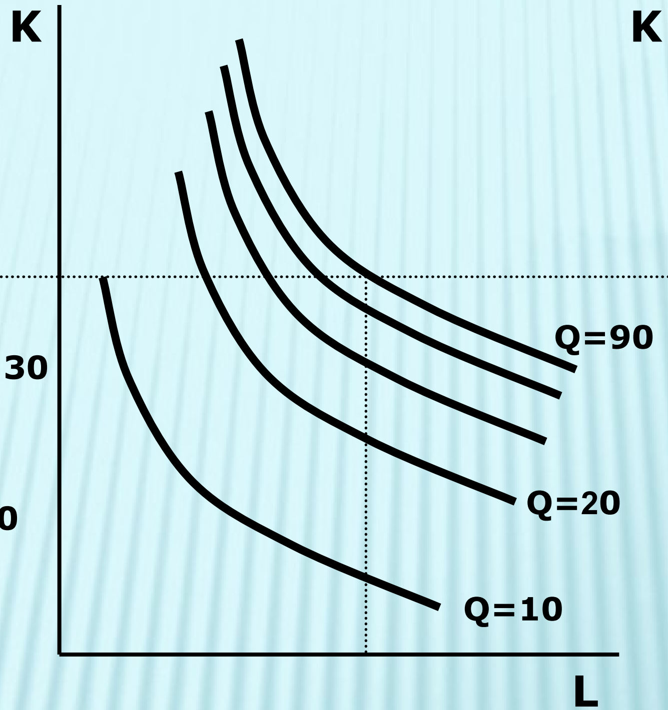
RETURNS TO SCALE

- we compare the relative change of total product and relative change of inputs volume
- diminishing, constant or increasing
- ***diminishing***: total product grows slower than the volume of inputs recruited
- ***constant***: total product and the volume of inputs grow by the same rate
- ***increasing***: total product grows faster than the volume of inputs recruited

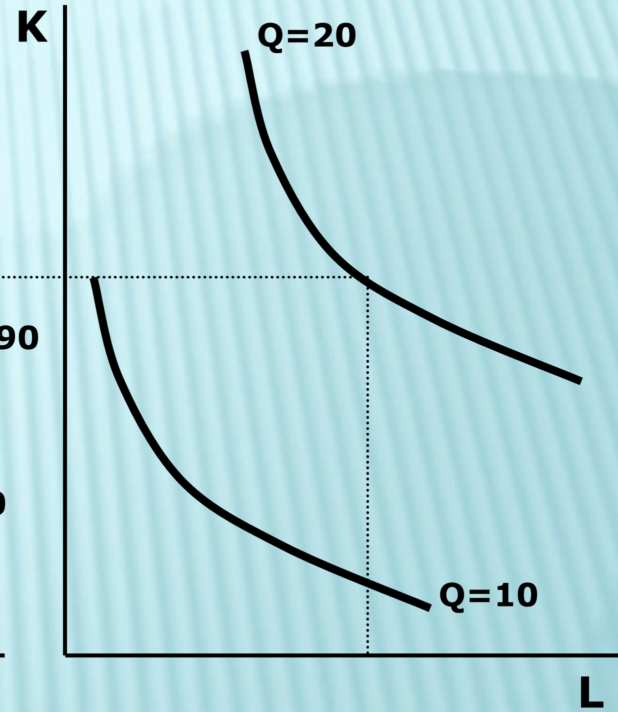
LR PRODUCTION FUNCTIONS UPON DIFFERENT TYPES OF RETURNS TO SCALE



constant – isoquants keep the same distance



increasing – isoquants get closer



diminsing – isoquants draw apart from each other

EXAMPLES OF PRODUCTION FUNCTIONS

1. Linear production function

   $Q = f(K,L) = a.K + b.L$

- ☞ contents constant returns to scale:

  $f(t.K,t.L) = a.t.K + b.t.L = t(a.K+b.L) = t.f(K,L)$

- ☞ elasticity of inputs substitution:

  $\sigma = \infty \rightarrow$ labour and capital are perfect substitutes

EXAMPLES OF PRODUCTION FUNCTIONS

2. With fixed inputs proportion:

$Q = \min(a.K, b.L)$

„min“ says that total product is limited with smaller value of one of the inputs – i.e. 1 lory needs 1 driver – if we add second driver, we do not raise the total volume of transported load

- ☞ contents constant returns to scale:

$f(t.K, t.L) = \min(a.t.K, b.t.L) = t.\min(a.K, b.L) = t.f(K, L)$

- ☞ elasticity of inputs substitution:

$\sigma = 0 \rightarrow$ labour and capital are perfect complements

EXAMPLES OF PRODUCTION FUNCTION

3. Cobb-Douglas production function:

$Q = f(K,L) = A.K^a.L^b$

☞ returns to scale:

$f(t.K,t.L) = A.(t.K)^a(t.L)^b = A.t^{a+b}.K^a.L^b = t^{a+b}.f(K,L)$

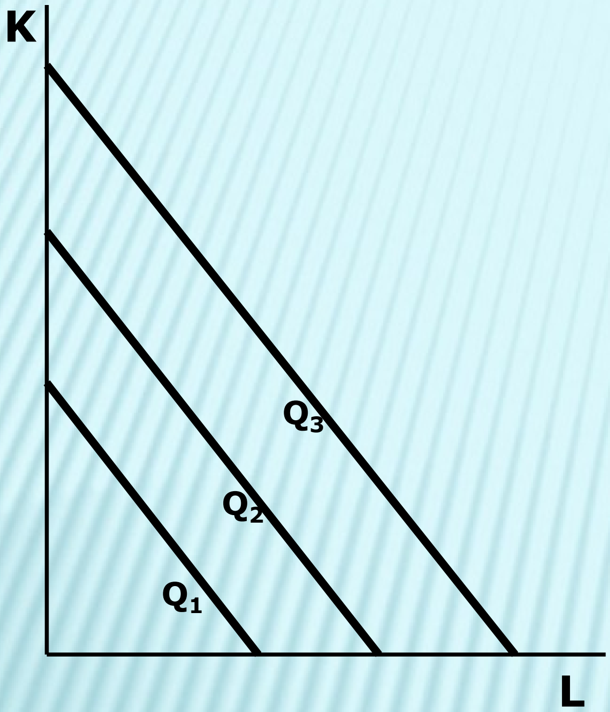
depend on the value of “a“ and “b“, if:

$a+b=1 \rightarrow$ constant

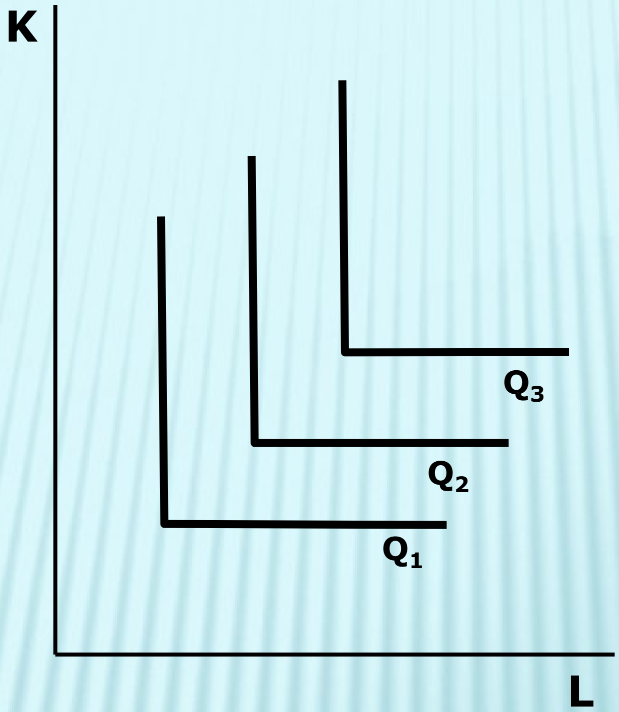
$a+b>1 \rightarrow$ increasing

$a+b<1 \rightarrow$ diminishing

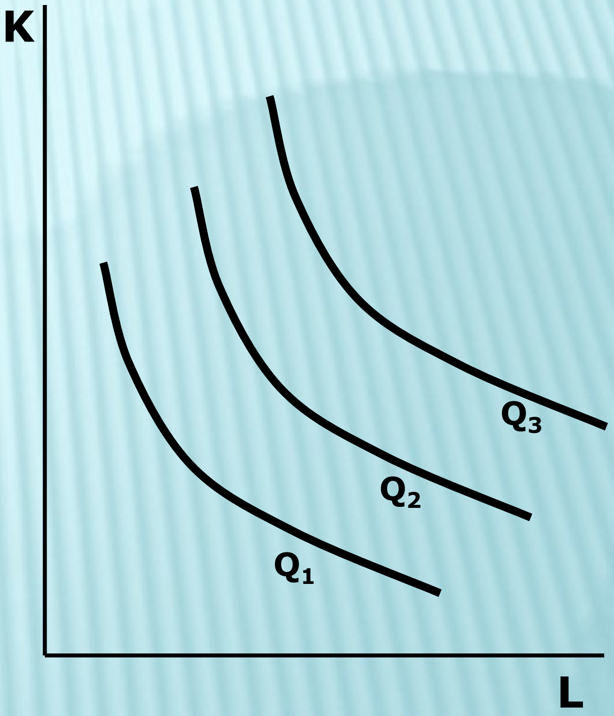
EXAMPLES OF PRODUCTION FUNCTIONS



Linear production funcstion



Production function with fixed proportion of inputs



Cobb-Douglas production function