

4. Firm's costs and revenues

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Definition of costs

➤ **accountable costs:**

all costs that the firm really pays – explicit costs, „visible“ in firm's accountancy

➤ **economic costs:**

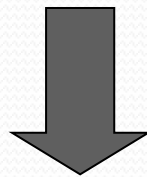
accountable (explicit) costs + opportunity (implicit) costs

Costs on labour and capital

- **labour costs** = wage rate (w) – costs per one working hour
- **capital costs** = rental (r) – costs per one machine hour – derived from the interest rate, which is the firm's opportunity cost
- **sunk costs** – costs with zero opportunity costs (i.e. costs on very special capital equipment with no alternate usage)

Short run costs – total values

- Short Total Costs, $STC = w.L + r.K_{fix}$
- $w.L$ = labour costs; variable costs (VC)...
- ...are changing with changing output (mostly costs on wages, materials, energy etc.)
- $r.K_{fix}$ = capital costs; fixed costs (FC)...
- ...remain constant with changing output (mostly amortization, rents, insurance etc.)



$$\mathbf{STC = w.L + r.K_{fix} = VC + FC}$$

Short run costs – average values

- Short Average Costs:

$$SAC = STC/Q = (FC+VC)/Q$$

- Average Fixed Costs:

$$AFC = FC/Q = r.K/Q = r.1/AP_K = r/AP_K$$

- Average Variable Costs:

$$AVC = VC/Q = w.L/Q = w.1/AP_L = w/AP_L$$

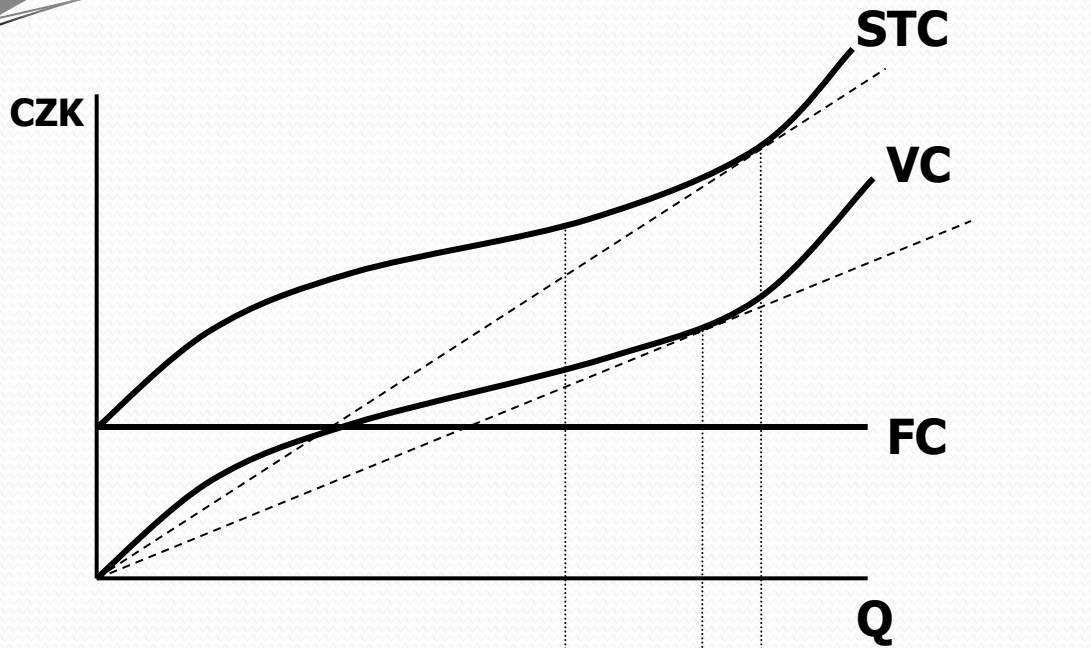
- ... and again... Short Average Costs:

$$SAC = AVC + AFC$$

Short run costs – marginal values

- Short Marginal Costs (SMC) = costs on additional output; change of total costs induced with the unity output increase
- $SMC = \partial STC / \partial Q = \partial VC / \partial Q$

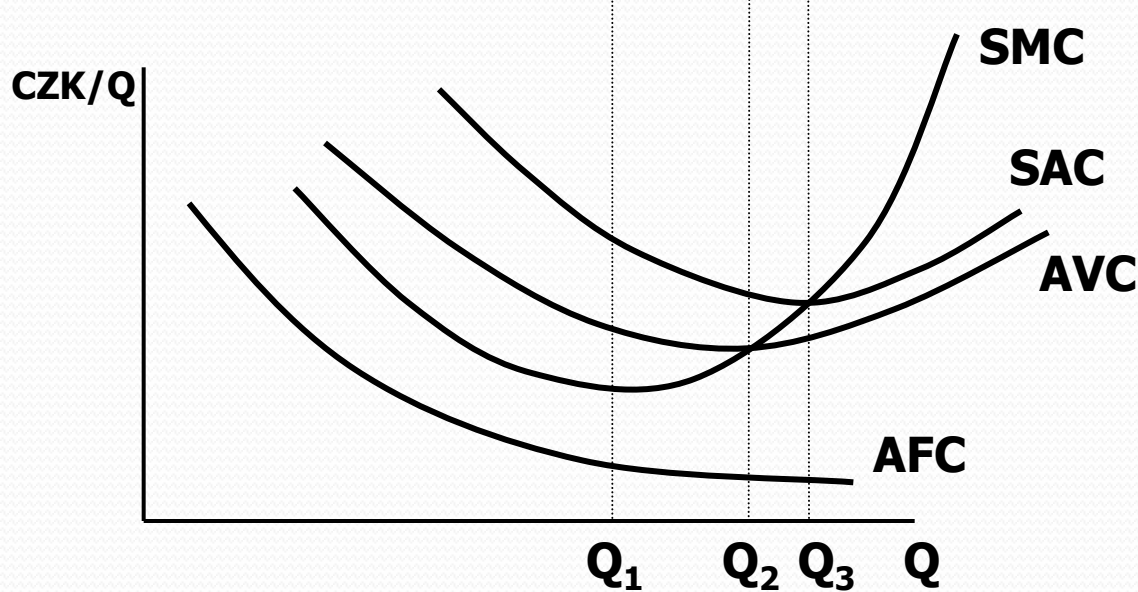
Relationship between total, average, marginal costs in short run



Q_1 – minimal SMC – increasing returns to labour change into diminishing returns to labour

Q_2 – minimal AVC

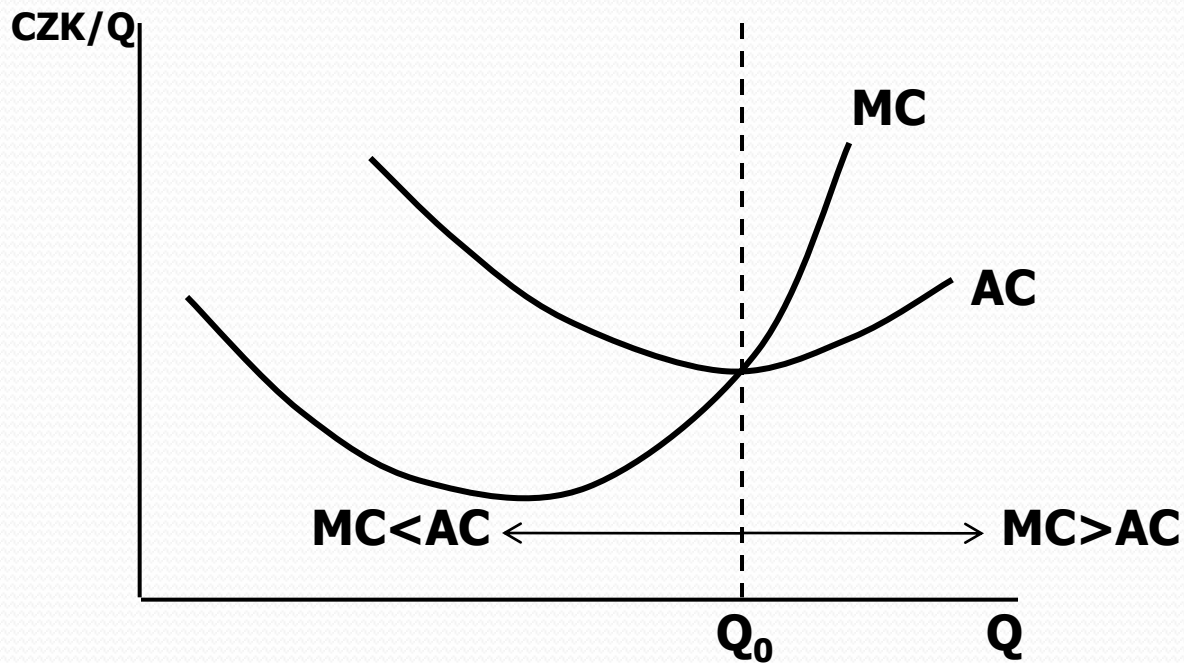
Q_3 – minimal SAC – to this spot the firm increases the effectiveness of capital



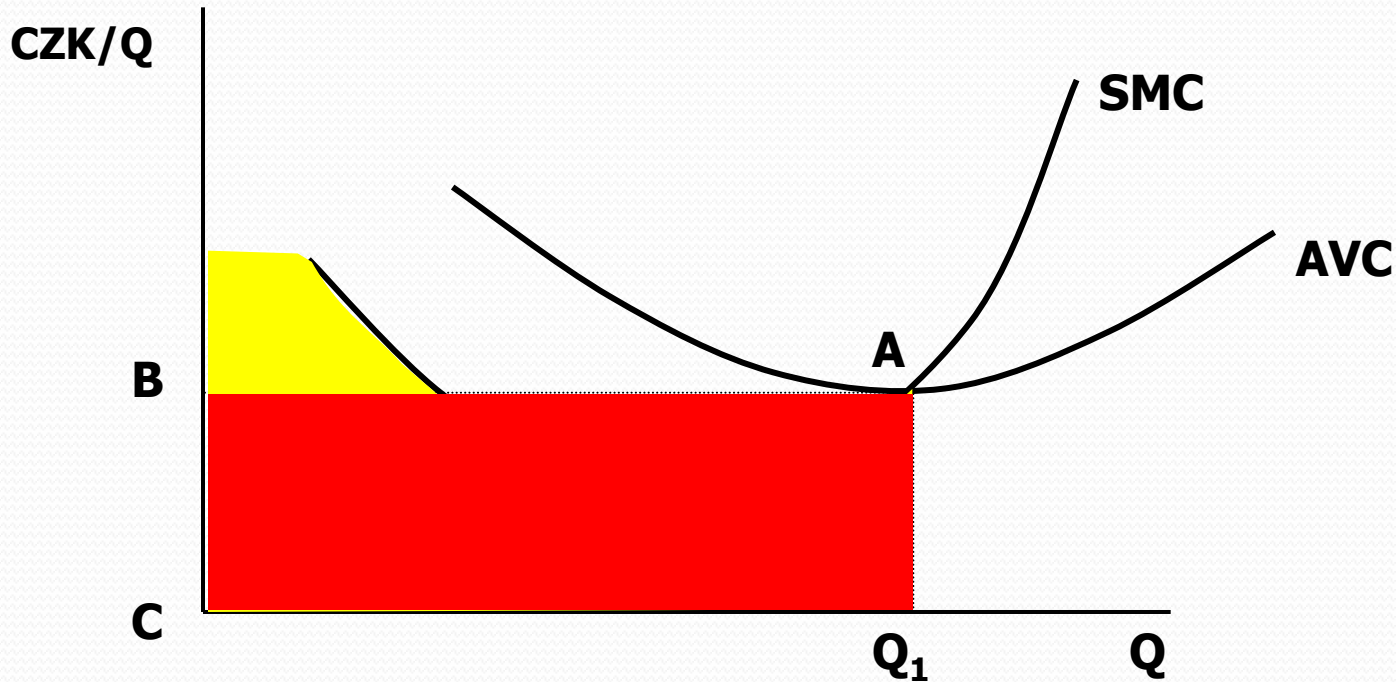
Relationship between marginal and average costs

- intersection of MC function and AC function lies in the minimum of AC functions
- ...general relationship of marginal and average values
- if $MC < AC$, then AC decrease
- if $MC > AC$, then AC increase
- development of MC depends on the character of returns to labour (in SR) or returns to scale (in LR)

Relationship between marginal and average costs



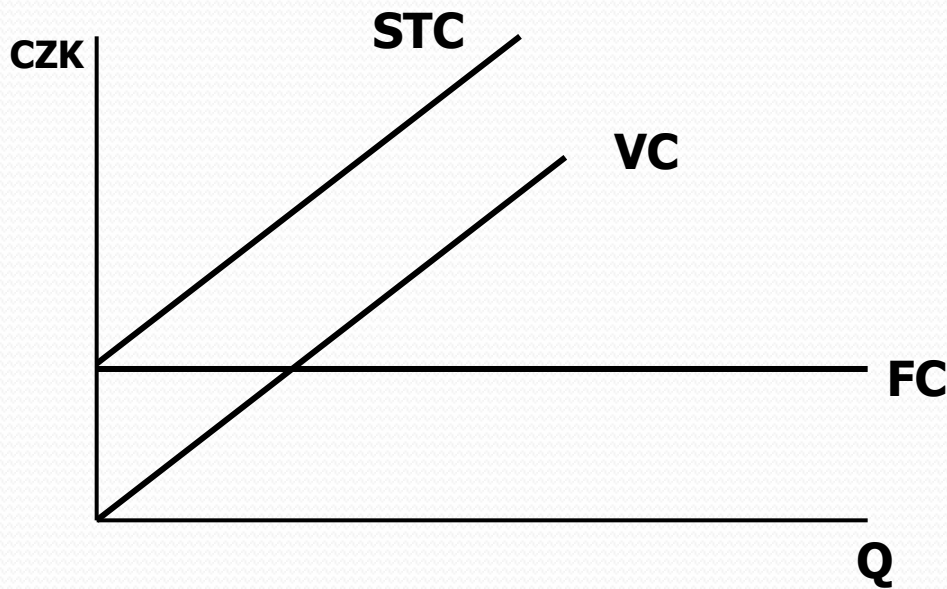
Average, marginal and total costs in SR



area under the SMC curve (to output Q_1) represents total variable costs on output Q_1 ...

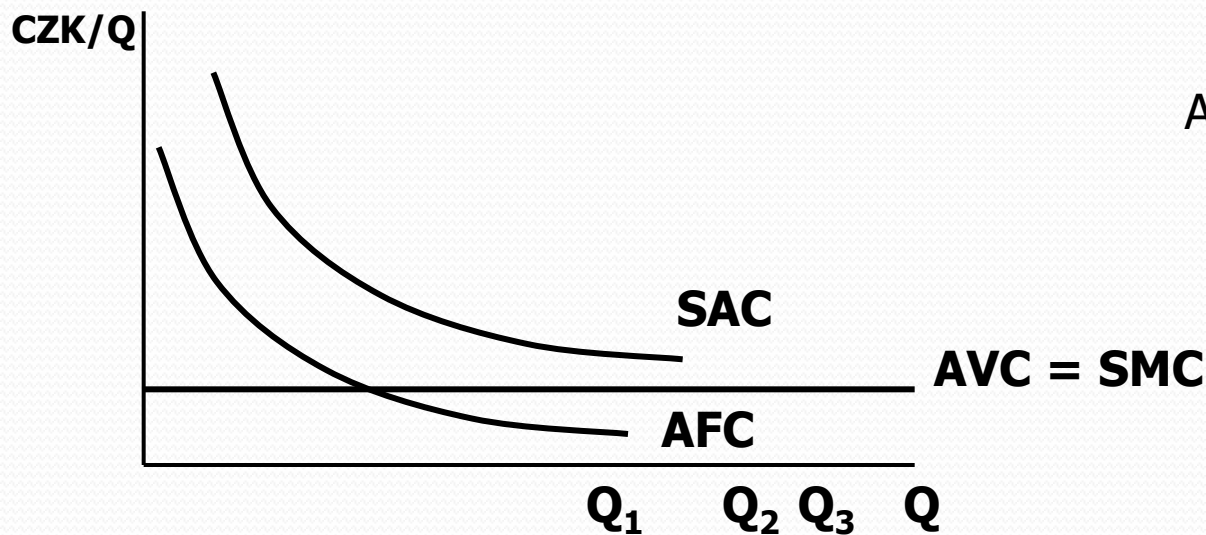
...as well as the surface of the rectangle Q_1ABC does

SR costs and constant returns to labour



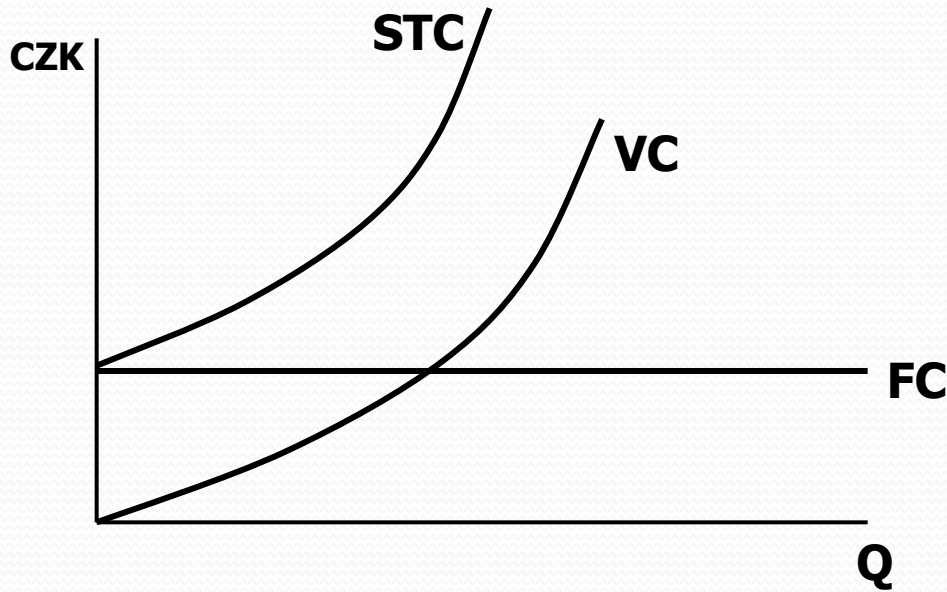
STC and VC grow by the constant rate

$$\begin{aligned} \text{STC} &= a + b \cdot Q \\ \text{AVC} &= b = \text{SMC} \end{aligned}$$



AVC and SMC are constant

SR costs and diminishing returns to labour



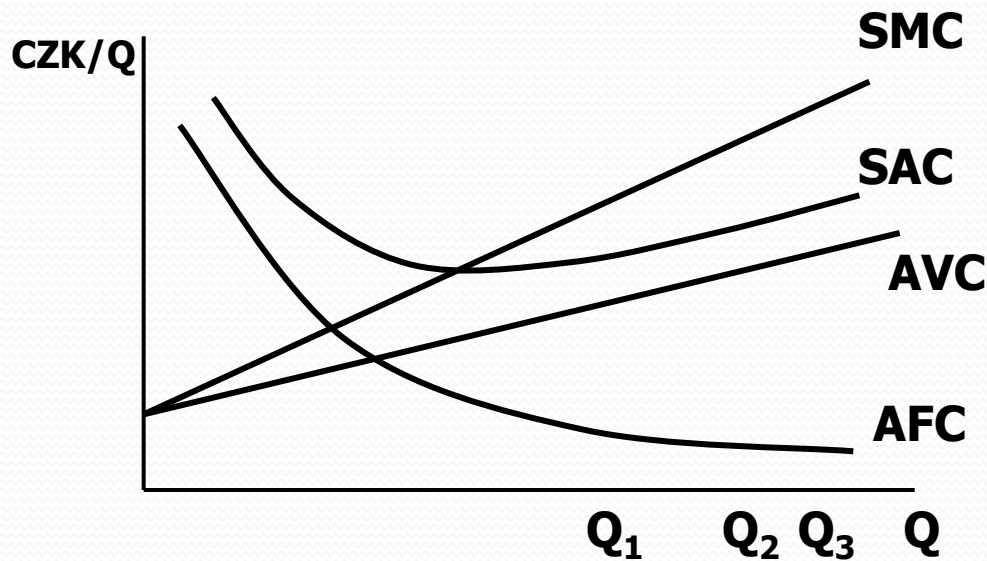
STC and VC grow by growing rate

$$STC = a + b \cdot Q + c \cdot Q^2$$

$$AVC = b + c \cdot Q$$

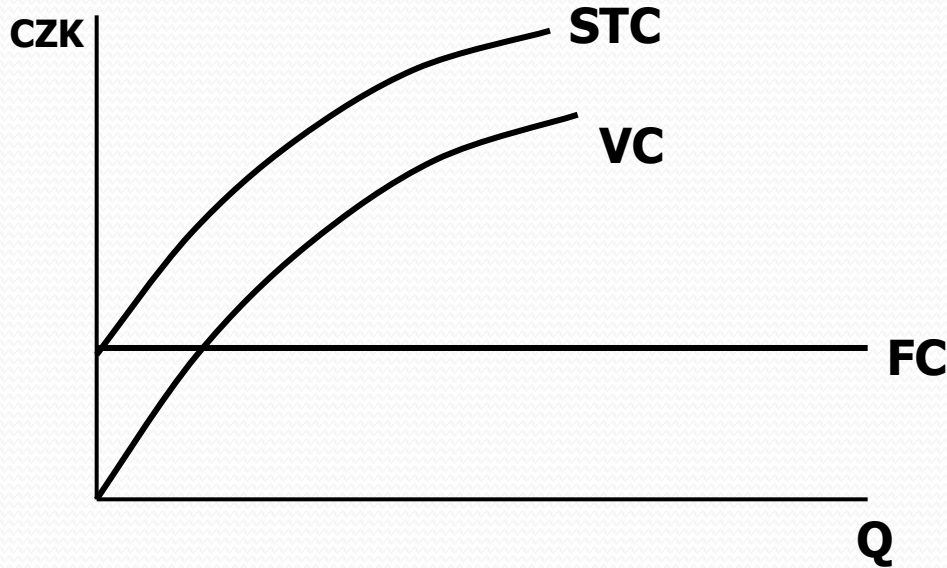
$$SMC = b + 2 \cdot c \cdot Q$$

→ SMC grow 2x faster than AVC



AVC and SMC grow with increasing output

SR costs and increasing returns to labour



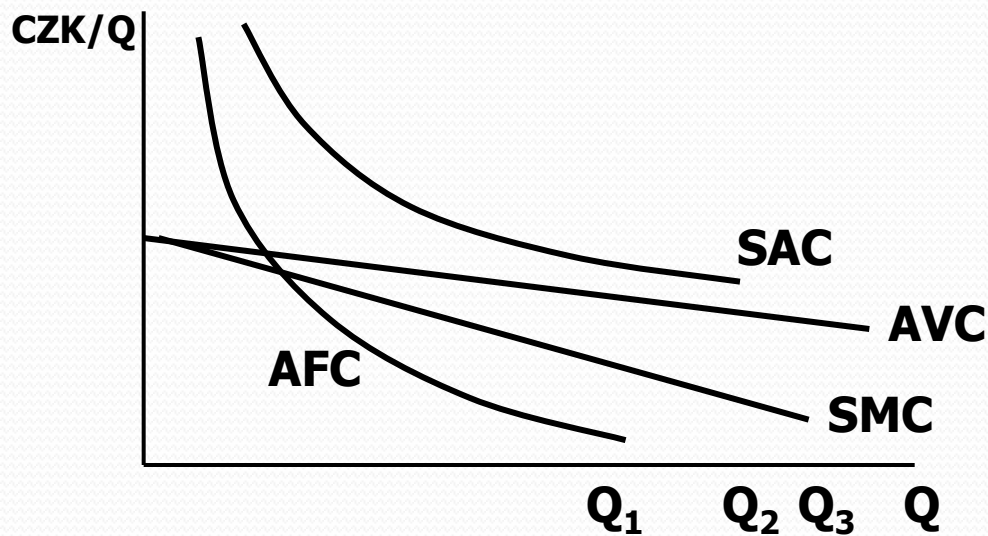
STC and VC grow by decreasing rate

$$STC = a + b \cdot Q - c \cdot Q^2$$

$$AVC = b - c \cdot Q$$

$$SMC = b - 2 \cdot c \cdot Q$$

→ **SMC decrease 2x faster than AVC**

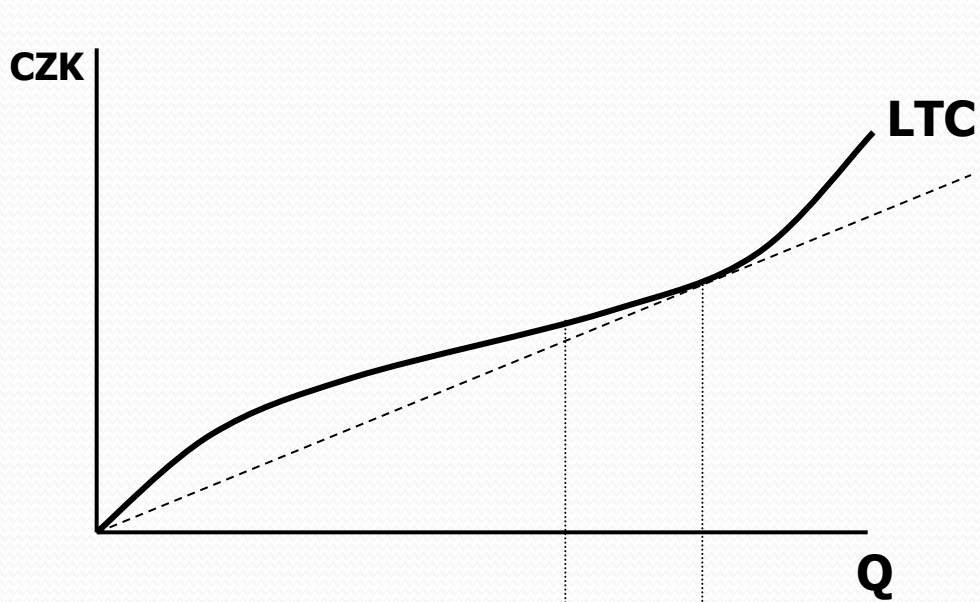


AVC and SMC decrease with increasing output

Long run costs

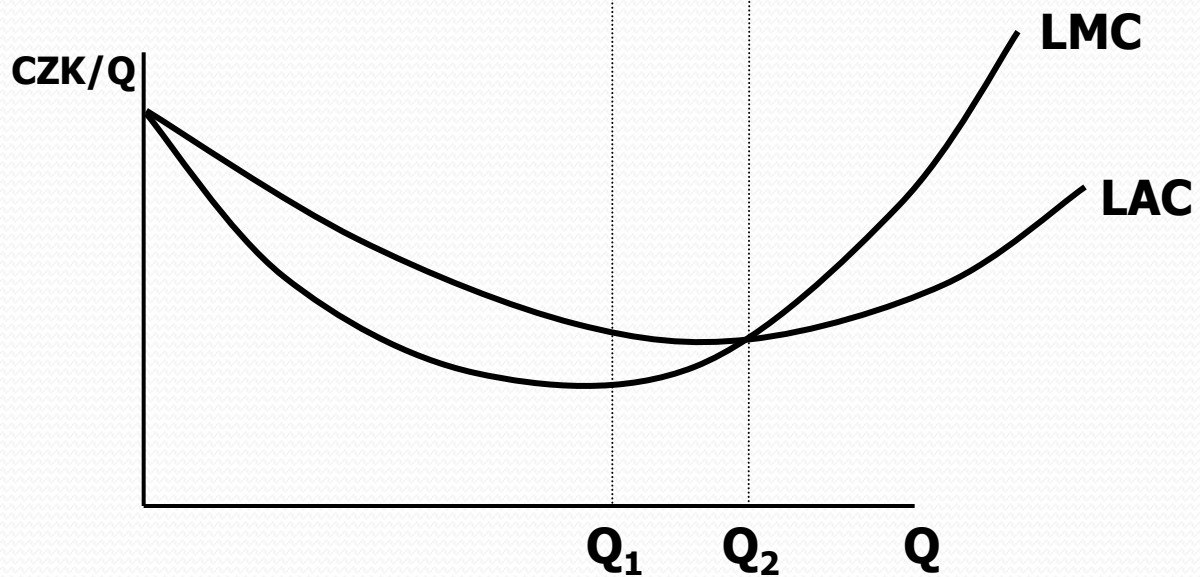
- in long run fixed costs do not exist – all costs are variable, firm is able to change the volume of all inputs... all costs
- Long Total Costs (LTC) = $w.L + r.K$
- Long Average Costs (LAC) = LTC/Q
- Long Marginal Costs (LMC) = $\partial LTC/\partial Q$
- development of LTC, LMC and LAC is determined with the type of returns to scale

Long run costs



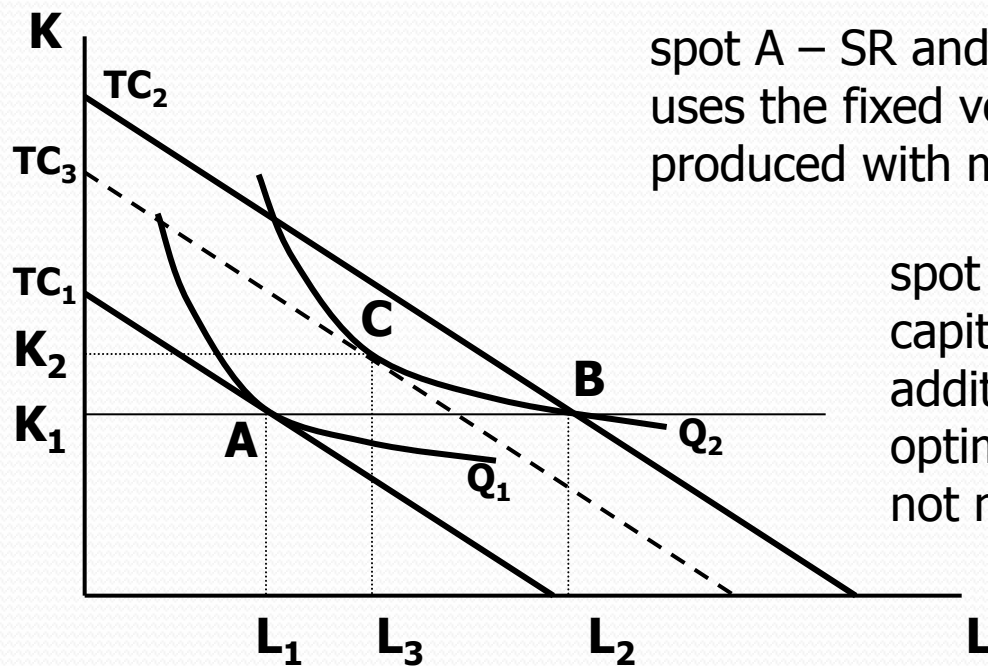
Q_1 – minimum LMC – increasing returns to scale switch into the diminishing ones

Q_2 – minimum LAC



Relationship between short run and long run costs

The existence of FC in short run may disallow the firm to minimize its total costs

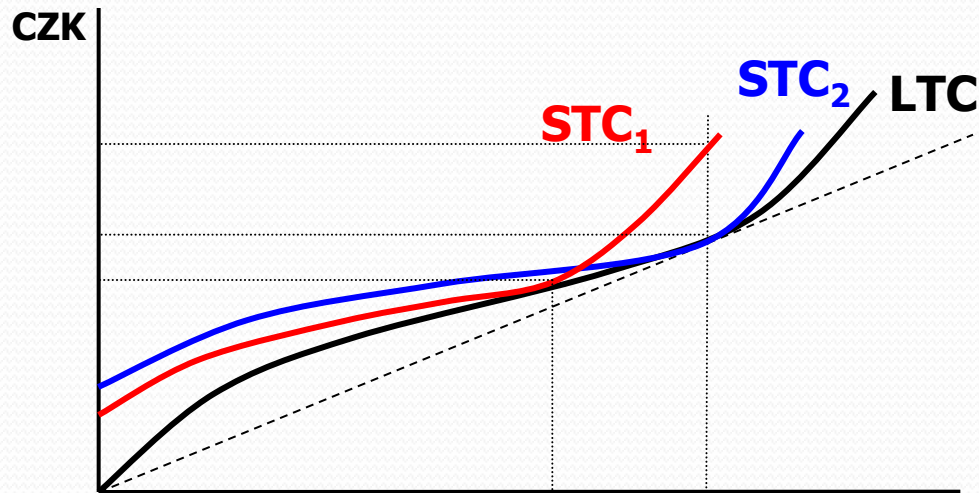


spot A – SR and LR firm's equilibrium – firm uses the fixed volume of capital K_1 , output Q_1 is produced with minimal total costs

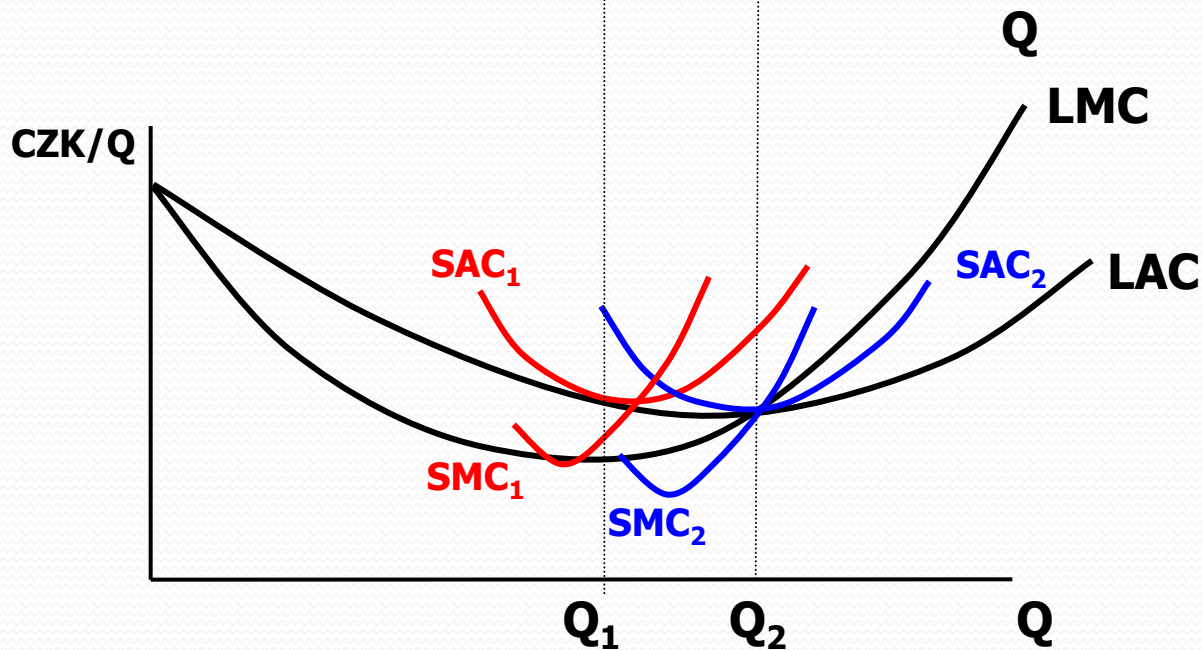
spot B – firm raises its output to Q_2 , but capital stock is fixed – the firm recruits additional labour force (L_2) – K_1, L_2 is optimal only in short run (total costs are not minimized)

spot C – LR firm's equilibrium – firm hires additional capital and dismisses some labour force – total costs on output Q_2 are minimized

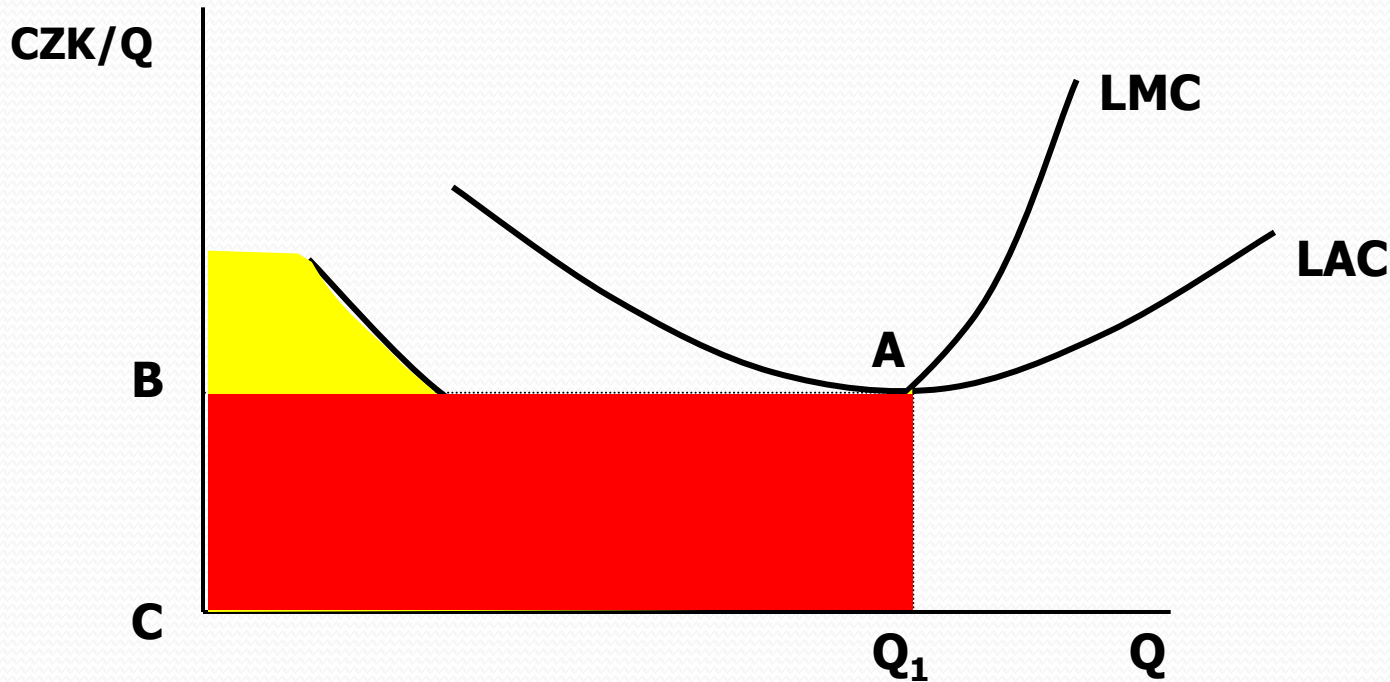
Relationship between short run and long run costs



LAC and LTC are envelope curves of short run cost curves – sets of spots for those stands: SAC=LAC and/or STC=LTC



Average, marginal and total costs in LR



area under LMC curve (to output Q_1) represents total long run costs on output Q_1 ...

...as well as the surface of rectangle Q_1ABC

Relationship between production and cost function

- development of cost functions is determined with:
- type of returns to labour (short run), returns to scale (long run)... in other words:
- development of MP_L (short run), multifactorial marginal productivity (long run)

Production and cost functions in SR

- if each additional unit of labour force produces **more** (MP_L grows), then each additional unit of output is less costly (SMC decline, STC grow by decreasing rate)
- if each additional unit of labour force produces **equally** (MP_L constant) then each additional unit of output is equally costly (SMC constant, STC grow by constant rate)
- if each additional unit of labour force produces **less** (MP_L decreases) then each additional unit of output is more costly (SMC grow, STC grow by growing rate)

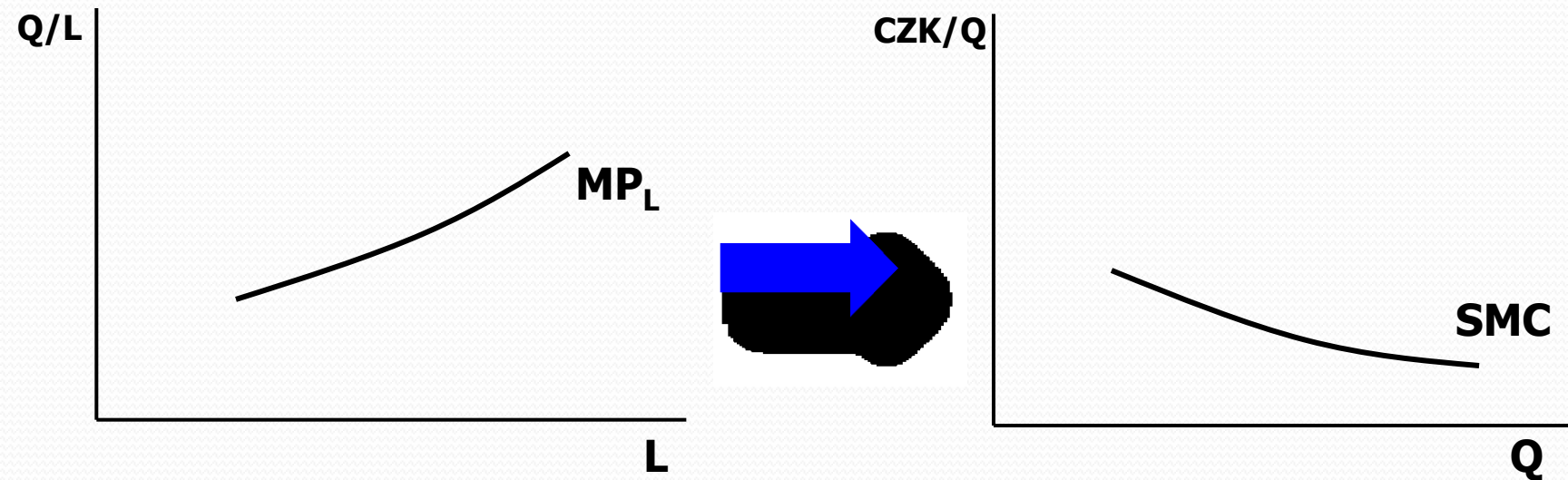
Production and cost functions in SR – increasing MP_L

Scrat (the prehistoric squirrel) feeds itself with nuts – the table shows its production function

L (hrs)	1	2	3	4
MP_L (p. of nuts)	1	1,5	3	4
Q (p. of nuts)	1	2,5	5,5	9,5
SMC (working hrs)	1	0,67	0,33	0,25

what would be the development of its SMC?

Production and cost functions in SR - increasing MP_L

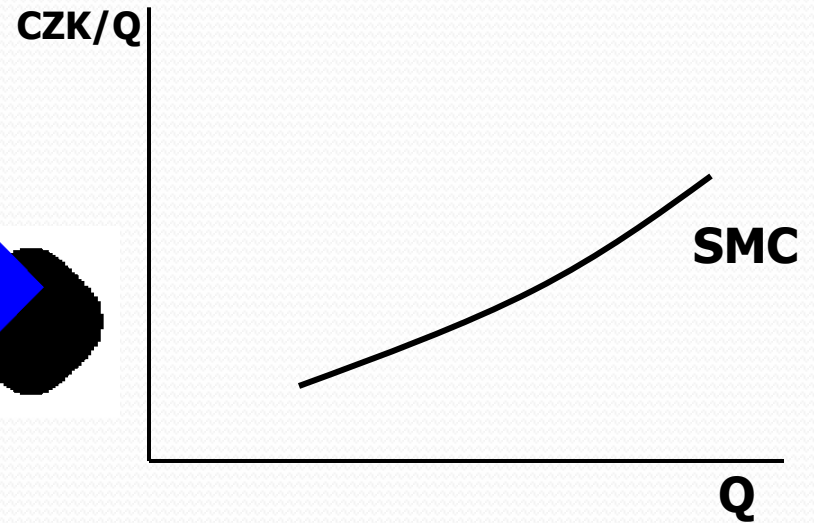
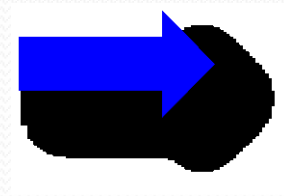
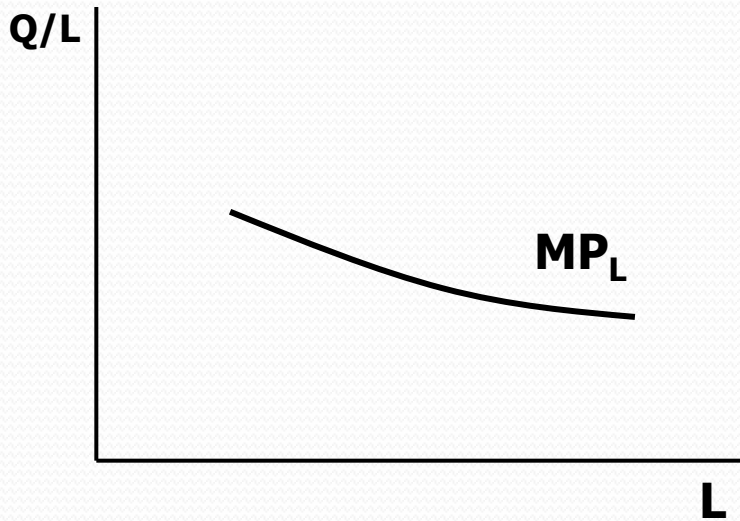


Production and cost functions in v SR – decreasing MP_L

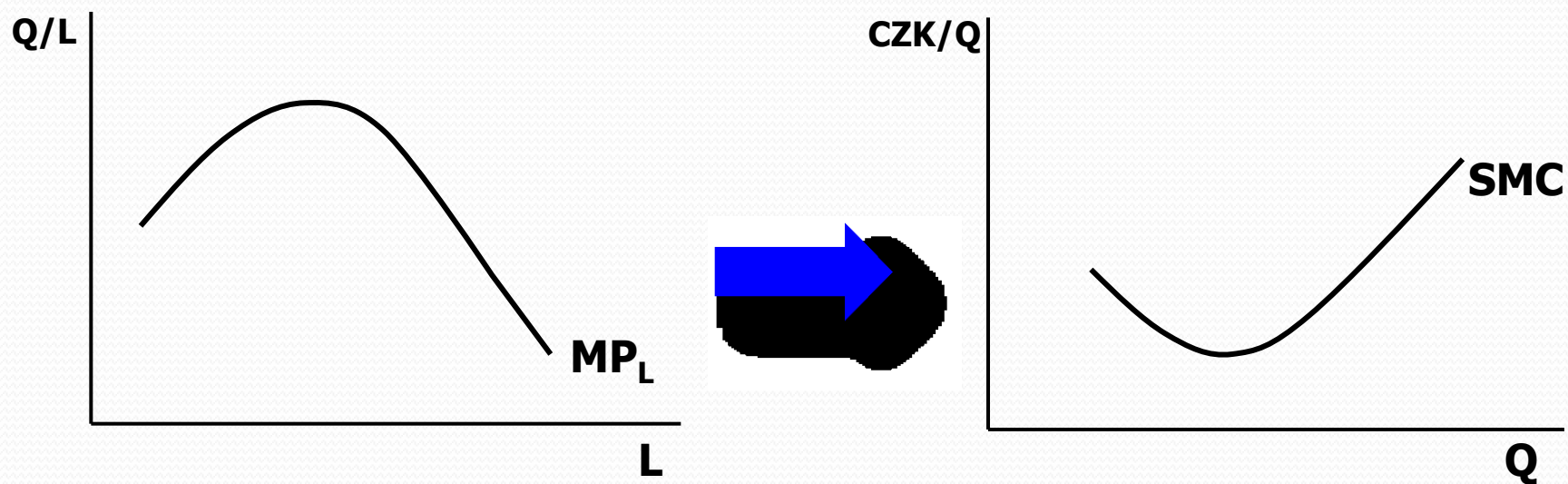
L (hrs)	1	2	3	4
MP_L (p.of nuts)	3	2	1,5	1
Q (p.of nuts)	3	5	6,5	7,5
MC (working hrs)	0,33	0,5	0,67	1

what would be the development of SMC?

Production and cost functions in SR – decreasing MP_L

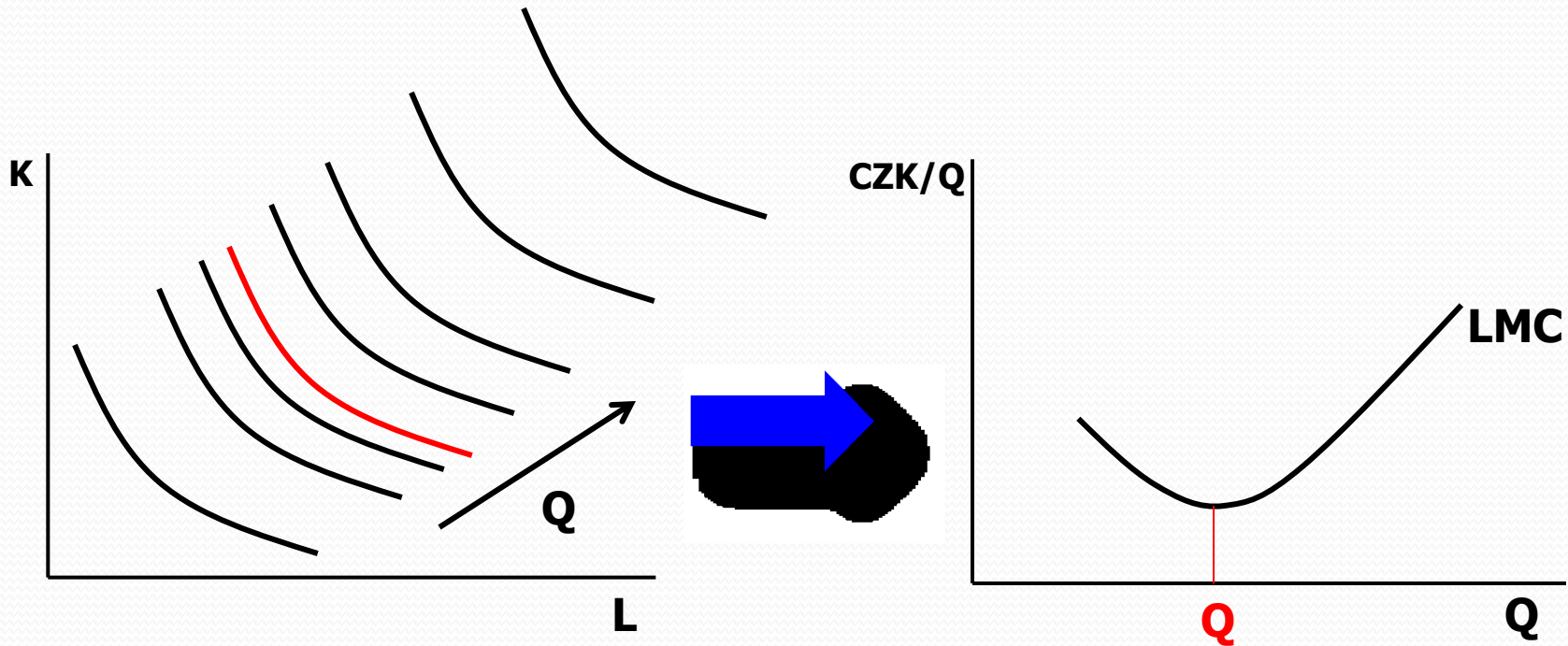


...if we join both cases



...we acquire the opposite developments of both functions

In long run it is very similar

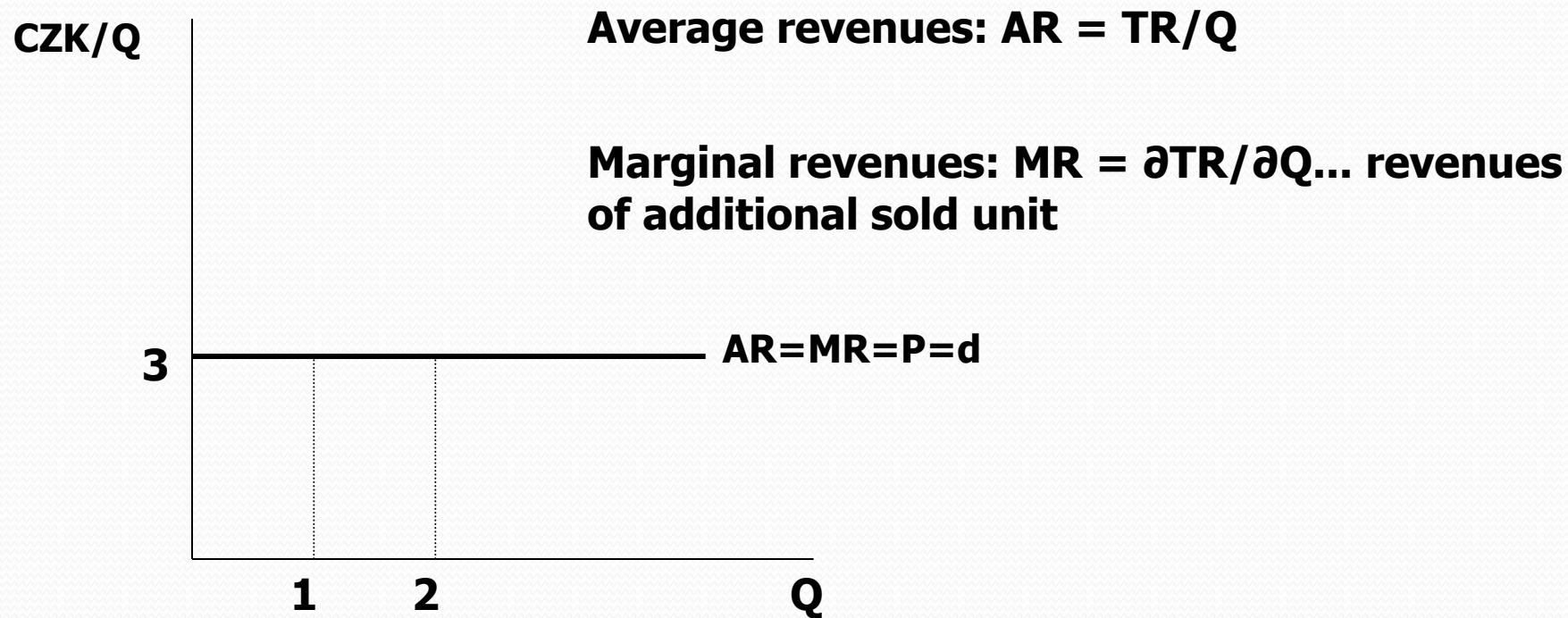


To **Q** the multifactorial productivity grows, then decreases – increasing returns to scale switch into the diminishing ones

Firm's revenues

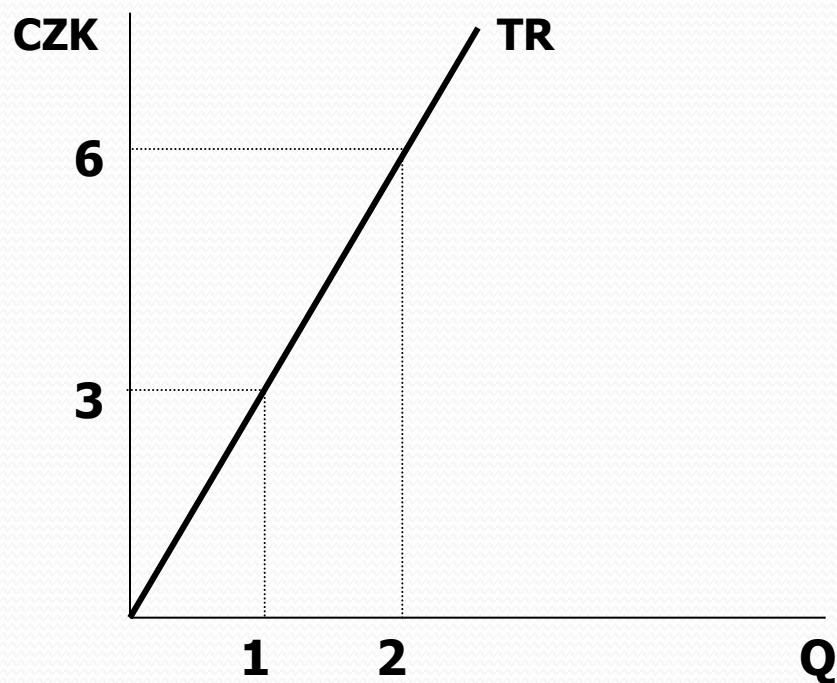
- = a sum of money gained by sales of its output
- revenues' development is determined by the type of final market competition, and by the price elasticity of firm's demand respectively

Average and marginal revenues upon the perfect competition market



The selling price in perfect competition market is objectively set with the market (intersection of D and S curves) – the firm is able to sell each additional unit for a constant price – AR and MR are constant on the level of market equilibrium price (AR, MR functions represent also the firm's demand function)

Total revenues upon the perfect competition market



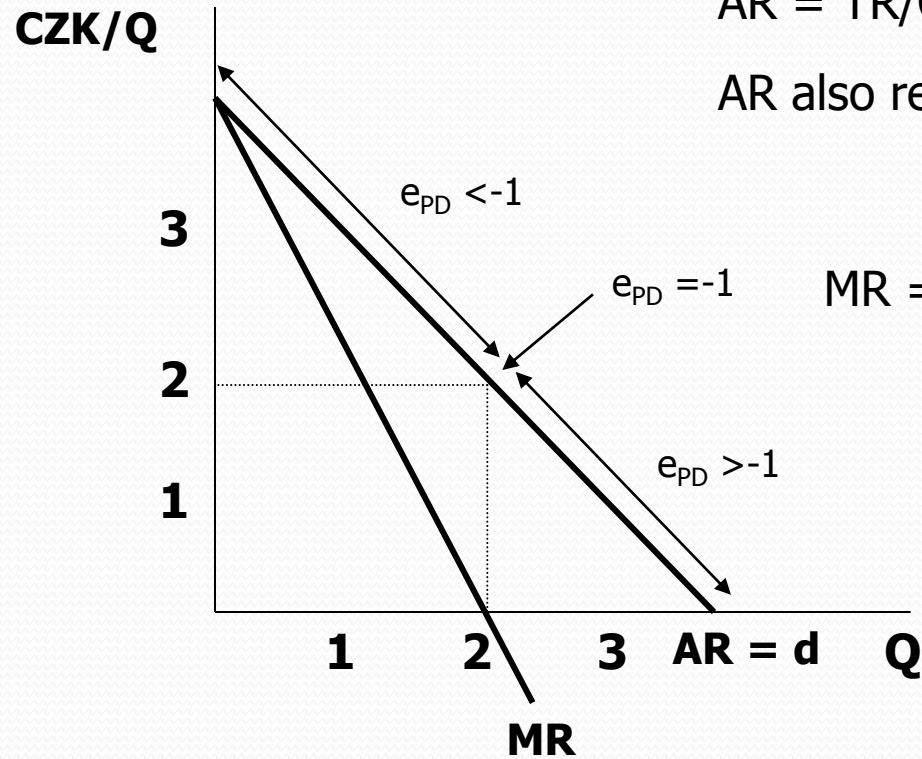
Total revenues grow by the constant rate (the slope of TR function equals to the market equilibrium price)

Average and marginal revenues upon imperfect competition market

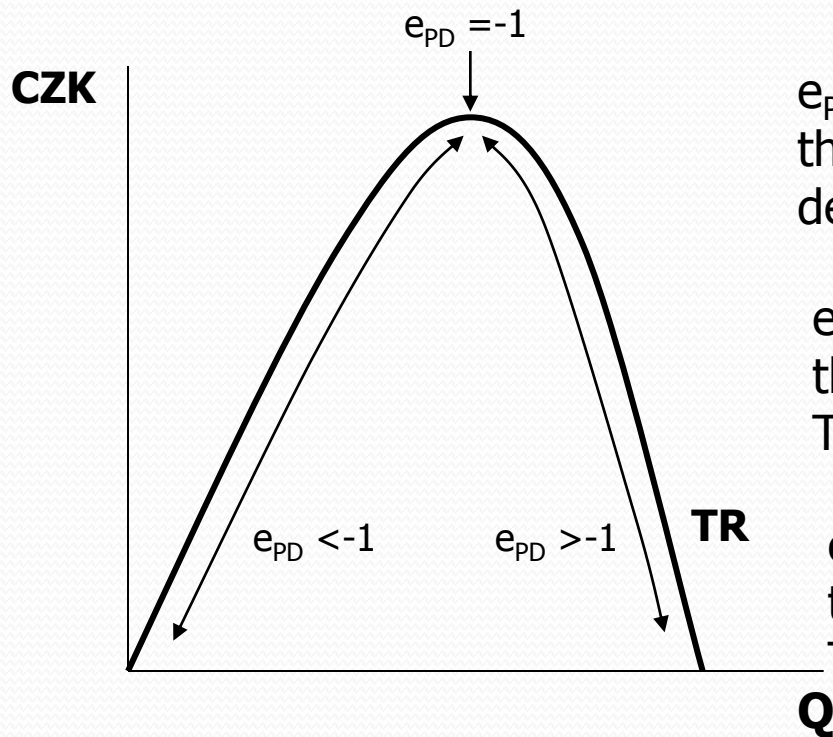
$$AR = TR/Q = (a-b \cdot Q) \cdot Q / Q = a - b \cdot Q$$

AR also represents the firm's demand function (d)

$$MR = \partial TR / \partial Q = \partial (a - b \cdot Q) \cdot Q / \partial Q = a - 2b \cdot Q$$



Total revenues upon imperfect competition market



$e_{PD} < -1$ relative drop of price is smaller than the relative increase of quantity demanded – TR grows

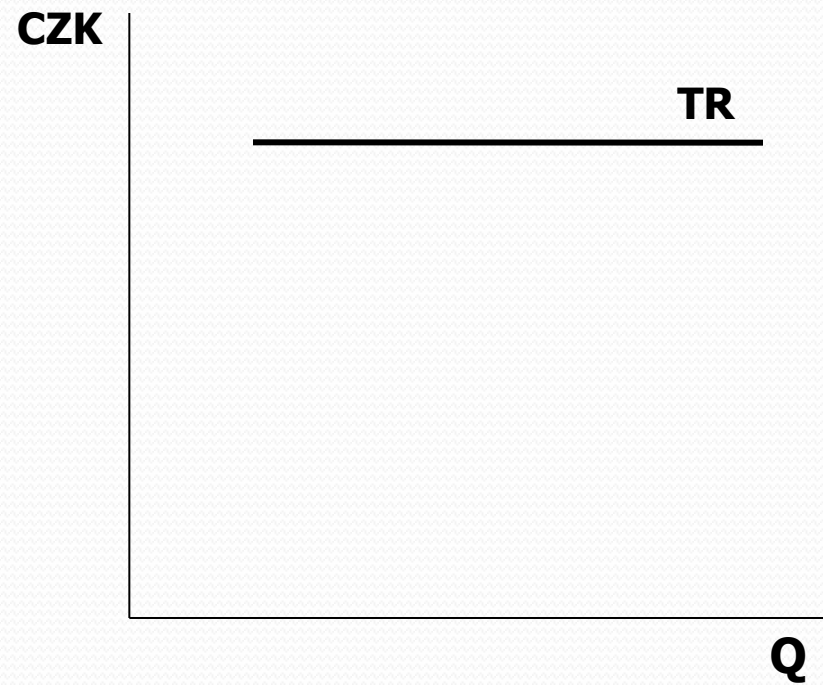
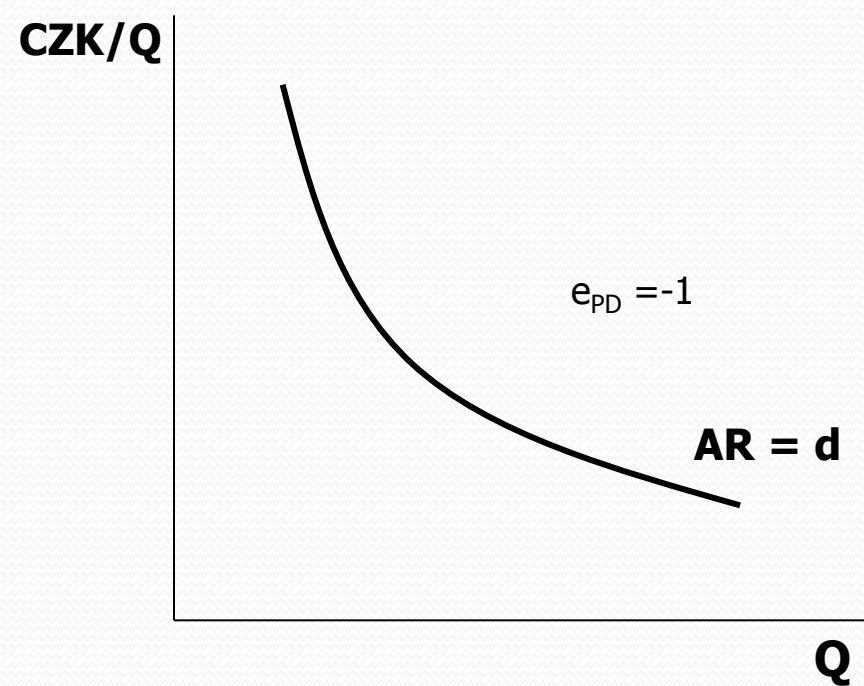
$e_{PD} = -1$ relative change of price equals to the relative change of quantity demanded – TR is constant (and maximal)

$e_{PD} > -1$ relative drop of price is bigger than the relative increase of quantity demanded – TR decreases

$$TR = P \cdot Q$$

$$TR = (a - b \cdot Q) \cdot Q$$

Firm's revenues upon unitary price elasticity of demand



TR – constant, MR = 0

Firm's revenues upon fluctuating price elasticity of demand

