# The Labor Market in Real Business Cycle Theory*

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The basic objective of the real business cycle research program is to use the neoclassical growth model to interpret observed patterns of fluctuations in overall economic activity. If we take a simple version of the model, calibrate it to be consistent with long-run growth facts, and subject it to random technology shocks calibrated to observed Solow residuals, the model displays short-run cyclical behavior that is qualitatively and quantitatively similar to that displayed by actual economies along many important dimensions. For example, the model predicts that consumption will be less than half as volatile as output, that investment will be about three times as volatile as output, and that consumption, investment, and employment will be strongly positively correlated with output, just as in the postwar U.S. time series.[1] In this sense, the real business cycle approach can be thought of as providing a benchmark for the study of aggregate fluctuations.

In this paper, we analyze the implications of real business cycle theory for the labor market. In particular, we focus on two facts about U.S. time series: the fact that hours worked fluctuate considerably more than productivity and the fact that the correlation between hours worked and productivity is close to zero.[2] These facts and the failure of simple real business cycle models to account for them have received considerable attention in the literature. [See, for example, the extended discussion by Christiano and Eichenbaum (1992) and the references they provide.] Here we first document the facts. We then present a baseline real business cycle model (essentially, the divisible labor model in Hansen 1985) and compare its predictions with the facts. We then consider four extensions of the baseline model that are meant to capture features of the world from which this model abstracts. Each of these extensions has been discussed in the literature. However, we analyze them in a unified framework with common functional forms, parameter values, and so on, so that they can be more easily compared and evaluated in terms of how they affect the model's ability to explain the facts.

The standard real business cycle model relies exclusively on a single technology shock to generate fluctuations, so the fact that hours worked vary more than productivity implies that the short-run labor supply elasticity must be large. The first extension of the model we consider is to recognize that utility may depend not only on leisure today but also on past leisure; this possibility leads us to introduce *nonseparable pref-*

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[1]These properties are also observed in other countries and time periods. See Kydland and Prescott 1990 for an extended discussion of the postwar U.S. data, and see Blackburn and Ravn 1991 or Backus and Kehoe, forthcoming, for descriptions of other countries and time periods.

[2]Although we concentrate mainly on these cyclical facts, we also mention an important long-run growth fact that is relevant for much of our discussion: total hours worked per capita do not display trend growth despite large secular increases in average productivity and real wages.

*erences,* as in Kydland and Prescott 1982.[3] This extension of the baseline model has the effect of increasing the relevant elasticity, by making households more willing to substitute leisure in one period for leisure in another period in response to short-run productivity changes. At the same time, with these preferences, households do not increase their work hours in response to permanent productivity growth. Thus, the nonseparable leisure model generates an increased standard deviation of hours worked relative to productivity without violating the long-run growth fact that hours worked per capita have not increased over long periods despite large increases in productivity.

The second extension of the baseline real business cycle model we consider is to assume that labor is *indivisible,* so that workers can work either a fixed number of hours or not at all, as in Hansen 1985. In this version of the model, all variation in the labor input must come about by changes in the number of employed workers, which is the opposite of the standard model, where all variation comes about by changes in hours per worker. Although the data display variation along both margins, the indivisible labor model is perhaps a better abstraction, since the majority of the variance in the labor input in the United States can be attributed to changes in the number of employed workers. In the equilibrium of the indivisible labor model, individual workers are allocated to jobs randomly, and this turns out to imply that the aggregate economy displays a large labor supply elasticity even though individual hours do not respond at all to productivity or wage changes for continuously employed workers. The large aggregate labor supply elasticity leads to an increased standard deviation of hours relative to productivity, as compared to the baseline model.

Neither nonseparable utility nor indivisible labor changes the result that the real business cycle model implies a large positive correlation between hours and productivity while the data display a near-zero correlation. This result arises because the model is driven by a single shock to the aggregate production function, which can be interpreted as shifting the labor demand curve along a stable labor supply curve and inducing a very tight positive relationship between hours and productivity. Hence, the next extension we consider is to introduce *government spending* shocks, as in Christiano and Eichenbaum 1992. If public consumption is an imperfect substitute for private consumption, then an increase in government spending has a negative wealth effect on individuals, which induces them to work more if leisure is a normal good. Therefore, government spending shocks can be interpreted as shifting the labor supply curve along the labor demand curve. Depending on the size of and the response to the two shocks, with this extension the model can generate a pattern of hours versus productivity closer to that found in the data.

The final extension we consider is to introduce *household production* as in Benhabib, Rogerson, and Wright 1991. The basic idea is to recognize that agents derive utility from home-produced as well as market-produced consumption goods and derive disutility from working in the home as well as in the market. In this version of the model, individuals, by working less at home, can increase hours of market work without reducing leisure as much. Therefore, the addition of household production increases the short-run labor supply elasticity and the standard deviation of hours relative to productivity. Furthermore, to the extent that shocks to household production are less than perfectly correlated with shocks to market production, individuals will have an incentive to substitute between home and market activity at a point in time. This is in addition to the standard incentive to substitute between market activity at different dates. Therefore, home production shocks, like government spending shocks, shift the labor supply curve and can generate a pattern of hours versus productivity closer to that found in the data.

Our basic finding is that each of these four extensions to the baseline real business cycle model improves its performance quantitatively, even though the extensions work through very different economic channels. As will be seen, some of the resulting models seem to do better than others along certain dimensions, and some depend more sensitively than others on parameter values. Our goal here is not to suggest that one of these models is best for all purposes; which is best for any particular application will depend on the context. Rather, we simply want to illustrate here how incorporating certain natural features into the standard real business cycle model affects its ability to capture some key aspects of labor market behavior.

## The Facts

In this section, we document the relevant business cycle facts. We consider several measures of hours worked and productivity and two sample periods (since some of the measures are available only for a shorter period). As in Prescott 1986, we define the *business cycle* as fluctuations around some slowly moving trend. For any given data series, we first take logarithms and then use the Hodrick-Prescott filter (as described in Prescott 1986) to remove the trend.

Table 1 contains some summary statistics for quarterly U.S. data that are computed from deviations constructed in this manner. The sample period is from 1955:3 to 1988:2. The variables are $y$ = output, $c$ = consumption (nondurables plus services), $i$ = fixed investment, $h$ = hours worked, and $w$

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[3]Note that these preferences are nonseparable between leisure in different periods; they may or may not be separable between leisure and consumption in a given period.

Tables 1 and 2

## Cyclical Properties of U.S. Time Series

### Table 1 1955:3–1988:2

| Data Series* | Variable $j$ | % S.D. $\sigma_j$ | Variable vs. Output $\sigma_j/\sigma_y$ | Variable vs. Output cor($j,y$) | Hours vs. Productivity $\sigma_h/\sigma_w$ | Hours vs. Productivity cor($h,w$) |
|---|---|---|---|---|---|---|
| Output | $y$ | 1.74 | 1.00 | 1.00 | — | — |
| Consumption | $c$ | .84 | .48 | .75 | — | — |
| Investment | $i$ | 5.48 | 3.16 | .90 | — | — |
| *Labor Market:* | | | | | | |
| 1. Household Survey (All Industries) | | | | | | |
| Hours Worked | $h$ | 1.42 | .82 | .87 | 1.64 | .10 |
| Productivity | $w$ | .87 | .50 | .58 | | |
| 2. Establishment Survey (Nonag. Industries) | | | | | | |
| Hours Worked | $h$ | 1.63 | .94 | .88 | 1.95 | –.13 |
| Productivity | $w$ | .84 | .48 | .36 | | |
| 3. Nonag. Industries From Household Survey | | | | | | |
| Hours Worked | $h$ | 1.75 | 1.01 | .76 | 1.44 | –.35 |
| Productivity | $w$ | 1.21 | .70 | .34 | | |
| 4. Efficiency Units From Hansen 1991 | | | | | | |
| Hours Worked | $h$ | 1.66 | .96 | .74 | 1.37 | –.30 |
| Productivity | $w$ | 1.22 | .70 | .41 | | |

### Table 2 1947:1–1991:3

| Data Series* | Variable $j$ | % S.D. $\sigma_j$ | Variable vs. Output $\sigma_j/\sigma_y$ | Variable vs. Output cor($j,y$) | Hours vs. Productivity $\sigma_h/\sigma_w$ | Hours vs. Productivity cor($h,w$) |
|---|---|---|---|---|---|---|
| Output | $y$ | 1.92 | 1.00 | 1.00 | — | — |
| Consumption | $c$ | .86 | .45 | .71 | — | — |
| Investment | $i$ | 5.33 | 2.78 | .73 | — | — |
| *Labor Market:* | | | | | | |
| 1. Household Survey (All Industries) | | | | | | |
| Hours Worked | $h$ | 1.50 | .78 | .82 | 1.37 | .07 |
| Productivity | $w$ | 1.10 | .57 | .63 | | |
| 2. Establishment Survey (Nonag. Industries) | | | | | | |
| Hours Worked | $h$ | 1.84 | .96 | .90 | 2.15 | –.14 |
| Productivity | $w$ | .86 | .45 | .31 | | |

*All series are quarterly, are in 1982 dollars, and have been logged and detrended with the Hodrick-Prescott filter. The output series, $y$, is the gross national product; $c$ is consumption of nondurables and services; and $i$ is fixed investment. Productivity is $w = y/h$.

Sources: Citicorp's Citibase data bank and Hansen 1991

= average productivity (output divided by hours worked).[4] For each variable $j$, we report the following statistics: the (percent) standard deviation $\sigma_j$, the standard deviation relative to that of output $\sigma_j/\sigma_y$, and the correlation with output cor($j$,$y$). We also report the relative standard deviation of hours to that of productivity $\sigma_h/\sigma_w$ and the correlation between hours and productivity cor($h$,$w$).

We present statistics for four measures of $h$ and $w$. Hours series 1 is total hours worked as recorded in the household survey and covers all industries. Hours series 2 is total hours worked as recorded in the establishment survey and covers only nonagricultural industries. These two hours series could differ for two reasons: they are from different sources, and they cover different industries.[5] To facilitate comparison, we also report, in hours series 3, hours worked as recorded in the household survey but only for nonagricultural industries. Finally, hours series 4 is a measure of hours worked in efficiency units.[6]

The reason for the choice of 1955:3–1988:2 as the sample period is that hours series 3 and 4 are only available for this period. However, the other series are available for 1947:1–1991:3, and Table 2 reports statistics from this longer period for the available variables.

Both Table 1 and Table 2 display the standard business cycle facts. All variables are positively correlated with output. Output is more variable than consumption and less variable than investment. Hours are slightly less variable than or about as variable as output, with $\sigma_h/\sigma_y$ ranging between 0.78 and 1.01, depending on the hours series and the period. Overall, all variables are more volatile in the longer period, but the relative volatilities of the variables are about the same in the two periods. (An exception is investment, which looks somewhat less volatile relative to output in the longer period.)

We want to emphasize two things. First, hours fluctuate more than productivity, with the magnitude of $\sigma_h/\sigma_w$ ranging between 1.37 and 2.15, depending on the series and the period. Second, the correlation between hours and productivity is near zero or slightly negative, with cor($h$,$w$) ranging between –0.35 and 0.10, depending on the series and the period. Chart 1 shows the scatter plot of $h$ versus $w$ from hours series 1 for the longer sample period. (Plots from the other hours series look similar.)

## The Standard Model

In this section, we present a standard real business cycle model and investigate its implications for the facts just described.

The model has a large number of homogeneous households. The representative household has preferences defined over stochastic sequences of consumption $c_t$ and leisure $l_t$, described by the utility function

$$(1) \qquad U = E\sum_{t=0}^{\infty}\beta^t u(c_t,l_t)$$

where $E$ denotes the expectation and $\beta$ the discount factor, with $\beta \in (0,1)$. The household has one unit of time each period to divide between leisure and hours of work:

$$(2) \qquad l_t + h_t = 1.$$

The model has a representative firm with a constant returns-to-scale Cobb-Douglas production function that uses capital $k_t$ and labor hours $h_t$ to produce output $y_t$:

$$(3) \qquad y_t = f(\bar{z}_t,k_t,h_t) = \exp(\bar{z}_t)k_t^{\theta}h_t^{1-\theta}$$

where $\theta$ is the capital share parameter and $\bar{z}_t$ is a stochastic term representing random technological progress. In general, we would assume that $\bar{z}_t = z_t + \bar{z}t$, where $\bar{z}$ is a constant yielding exogenous deterministic growth and $z_t$ evolves according to the process

$$(4) \qquad z_{t+1} = \rho z_t + \varepsilon_t$$

where $\rho \in (0,1)$ and $\varepsilon_t$ is independent and normally distributed with mean zero and standard deviation $\sigma_\varepsilon$. However, in this paper, we abstract from exogenous growth by setting $\bar{z} = 0$.[7] Capital evolves according to the law of motion

$$(5) \qquad k_{t+1} = (1-\delta)k_t + i_t$$

where $\delta$ is the depreciation rate and $i_t$ investment. Finally, the economy must satisfy the resource constraint

$$(6) \qquad c_t + i_t = y_t.$$

We are interested in the competitive equilibrium of this economy. Since externalities or other distortions are not part of this model (or the other models that we consider), the com-

[4]We use the letter $w$ because average productivity is proportional to marginal productivity (given our functional forms), which equals the real *wage* rate in our models.

[5]The establishment series is derived from payroll data and measures hours paid for, while the household series is taken from a survey of workers that attempts to measure hours actually worked. These two measures could differ, for example, because some workers may be on sick leave or vacation but still get paid. The household series is a better measure of the labor input, in principle, but because it is based on a survey of workers rather than payroll records, it is probably less accurate.

[6]Efficiency units are constructed from hours series 3 by disaggregating individuals into age and sex groups and weighting the hours of each group by its relative hourly earnings; see Hansen 1991 for details.

[7]Adding exogenous growth does not affect any of the statistics we report (as long as the parameters are recalibrated appropriately) given the way we filter the data; therefore, we set $\bar{z} = 0$ in order to simplify the presentation. See Hansen 1989.

petitive equilibrium is efficient. Hence, we can determine the equilibrium allocation by solving the social planner's problem of maximizing the representative agent's expected utility subject to feasibility constraints. That problem in this case is to maximize $U$ subject to equations (2)–(6) and some initial conditions $(k_0, z_0)$. The solution can be represented as a pair of stationary decision rules for hours and investment, $h_t = h^*(k_t, z_t)$ and $i_t = i^*(k_t, z_t)$, that determine these two variables as functions of the current capital stock and technology shock. The other variables, such as consumption and output, can be determined from the decision rules using the constraints, while prices can be determined from the relevant marginal conditions.

Standard numerical techniques are used to analyze the model. We choose functional forms and parameter values and substitute the constraint $c_t + i_t = f(z_t, k_t, h_t)$ into the instantaneous return function $u$ to reduce the problem to one of maximizing an objective function subject to linear constraints. Then we approximate the return function with a quadratic return function by taking a Taylor's series expansion around the deterministic steady state. The resulting linear-quadratic problem can be easily solved for optimal linear decision rules, $h_t = h(k_t, z_t)$ and $i_t = i(k_t, z_t)$; see Hansen and Prescott 1991 for details. Using these decision rules, we simulate the model, take logarithms of the artificially generated data, apply the Hodrick-Prescott filter, and compute statistics on the deviations (exactly as we did to the actual time series). We run 100 simulations of 179 periods (the number of quarters in our longer data set) and report the means of the statistics across these simulations.

Preferences are specified so that the model is able to capture the long-run growth fact that per-capita hours worked display no trend despite large increases in productivity and real wages. When preferences are time separable, capturing this fact requires that the instantaneous utility function satisfy

$$(7) \qquad u(c,l) = \log(c) + v(l)$$

or

$$(8) \qquad u(c,l) = c^{\sigma} v(l)/\sigma$$

where $\sigma$ is a nonzero parameter and $v(l)$ is an increasing and concave function. (See King, Plosser, and Rebelo 1987, for example.) Intuitively, the growth facts imply that the wealth and substitution effects of long-run changes in productivity cancel, so the net effect is that hours worked do not change.[8] We consider only preferences that satisfy (7) or (8); in fact, for convenience, we assume that

$$(9) \qquad u(c,l) = \log(c) + A\log(l).$$

Parameter values are calibrated as follows. The discount factor is set to $\beta = 0.99$ so as to imply a reasonable steady-state real interest rate of 1 percent per period (where a period is one quarter). The capital share parameter is set to $\theta = 0.36$ to match the average fraction of total income going to capital in the U.S. economy. The depreciation rate is set to $\delta = 0.025$, which (given the above-mentioned values for $\beta$ and $\theta$) implies a realistic steady-state ratio of capital to output of about 10 and a ratio of investment to output of 0.26. The parameter $A$ in the utility function (9) is chosen so that the steady-state level of hours worked is exactly $h = 1/3$, which matches the fraction of discretionary time spent in market work found in time-use studies (for example, Juster and Stafford 1991). Finally, the parameter $\rho$ in (4) is set to $\rho = 0.95$, and the standard deviation of $\varepsilon$ is set to $\sigma_\varepsilon = 0.007$, which are approximately the values settled on by Prescott (1986).

We focus on the following statistics generated by our artificial economy: the standard deviation of output; the standard deviations of consumption, investment, and hours relative to the standard deviation of output; the ratio of the standard deviation of hours to the standard deviation of productivity; and the correlation between hours and productivity. The results are shown in Table 3, along with the values for the same statistics from our longer sample from the U.S. economy (from Table 2). We emphasize the following discrepancies between the simulated and actual data. First, the model has a predicted standard deviation of output which is considerably less than the same statistic for the U.S. economy in either period. Second, the model predicts that $\sigma_h/\sigma_w$ is less than one, while it is greater than one in the data. Third, the correlation between hours and productivity in the model is far too high.

The result that output is not as volatile in the model economy as in the actual economy is not too surprising, since the model relies exclusively on a single technology shock, while the actual economy is likely to be subject to other sources of uncertainty as well. The result that in the model hours worked do not fluctuate enough relative to productivity reflects the fact that agents in the model are simply not sufficiently willing to substitute leisure in one period for leisure in other periods. Finally, the result that hours and productivity are too

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[8] Other specifications can generate a greater short-run response of hours worked to productivity shocks; but while this is desirable from the point of view of explaining cyclical observations, it is inconsistent with the growth facts. For example, the utility function used in Greenwood, Hercowitz, and Huffman 1988, $u(c,l) = v(c+Al)$, has a zero wealth effect and hence a large labor supply elasticity, but implies that hours worked increase over time with productivity growth. This specification is consistent with balanced growth if we assume the parameter $A$ grows at the same average rate as technology. Although such an assumption may seem contrived, it can be justified as the reduced form of a model with home production in which the home and market technologies advance at the same rate on average, as shown in Greenwood, Rogerson, and Wright 1992.

Table 3

Cyclical Properties of U.S. and Model-Generated Time Series

| Type of Data or Model | % S.D. of Output $\sigma_y$ | Variable vs. Output: Consumption $\sigma_c/\sigma_y$ | Investment $\sigma_i/\sigma_y$ | Hours $\sigma_h/\sigma_y$ | Productivity $\sigma_w/\sigma_y$ | Hours vs. Productivity: $\sigma_h/\sigma_w$ | $\text{cor}(h,w)$ |
|---|---|---|---|---|---|---|---|
| U.S. Time Series* | | | | | | | |
| Output | 1.92 | .45 | 2.78 | — | — | — | — |
| *Hours Worked:* | | | | | | | |
| 1. Household Survey (All Industries) | — | — | — | .78 | .57 | 1.37 | .07 |
| 2. Establishment Survey (Nonag. Industries) | — | — | — | .96 | .45 | 2.15 | –.14 |
| Models** | | | | | | | |
| Standard | 1.30 | .31 | 3.15 | .49 | .53 | .94 | .93 |
| Nonseparable Leisure | 1.51 | .29 | 3.23 | .65 | .40 | 1.63 | .80 |
| Indivisible Labor | 1.73 | .29 | 3.25 | .76 | .29 | 2.63 | .76 |
| Government Spending | 1.24 | .54 | 3.08 | .55 | .61 | .90 | .49 |
| Home Production | 1.71 | .51 | 2.73 | .75 | .39 | 1.92 | .49 |

*U.S. data here are the same as those in Table 2; they are for the longer time period: 1947:1–1991:3.

**The standard deviations and correlations computed from the models' artificial data are the sample means of statistics computed for each of 100 simulations. Each simulation has 179 periods, the number of quarters in the U.S. data.

Source: Citicorp's Citibase data bank

highly correlated in the model reflects the fact that the only impulse driving the system is the aggregate technology shock.

Chart 2 depicts the scatter plot between $h$ and $w$ generated by the model. Heuristically, Chart 2 displays a stable labor supply curve traced out by a labor demand curve shifting over time in response to technology shocks. This picture obviously differs from that in Chart 1.

## Nonseparable Leisure

Following Kydland and Prescott (1982), we now attempt to incorporate the idea that instantaneous utility might depend not just on current leisure, but rather on a weighted average of current and past leisure. Hotz, Kydland, and Sedlacek (1988) find evidence in the panel data that this idea is empirically plausible. One interpretation they discuss concerns the fact that individuals need to spend time doing household chores, making repairs, and so on, but after doing so they can neglect these things for a while and spend more time working in the market until the results of their home work depreciate. The important impact of a nonseparable utility specification for our purposes is that, if leisure in one period is a relatively good substitute for leisure in nearby periods, then agents will be more willing to substitute intertemporally, and this increases the short-run labor supply elasticity.

Assume that the instantaneous utility function is $u(c_t,L_t) = \log(c_t) + A\log(L_t)$, where $L_t$ is given by

$$(10) \quad L_t = \sum_{i=0}^{\infty} a_i l_{t-i}$$

and impose the restriction that the coefficients $a_i$ sum to one. If we also impose the restriction that

$$(11) \quad a_{i+1} = (1-\eta)a_i$$

for $i = 1, 2, ...,$ so that the contribution of past leisure to $L_t$ decays geometrically at rate $\eta$, then the two parameters $a_0$ and $\eta$ determine all of the coefficients in (10). Since $L_t$, and not simply $l_t$, now provides utility, individuals are more willing to intertemporally substitute by working more in some periods and less in others. (At the same time, in a deterministic steady

Charts 1–5

## Hours Worked vs. Productivity in the Data and the Models

Percentage Deviations From Trend

Chart 1 The U.S. Data, 1947:1–1991:3
Based on the Household Survey

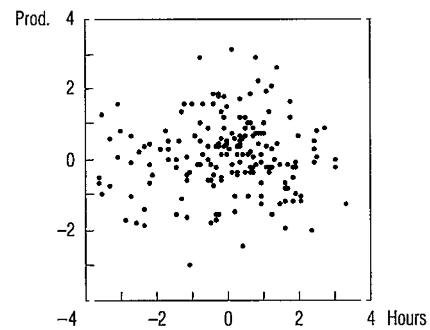



Chart 2 The Standard Model

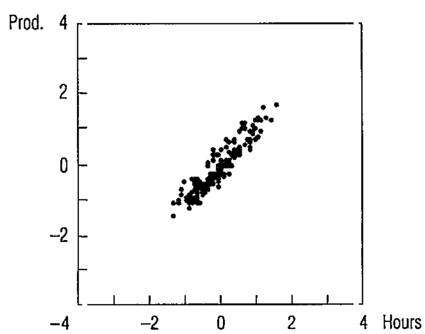



Chart 3 The Nonseparable Leisure Model

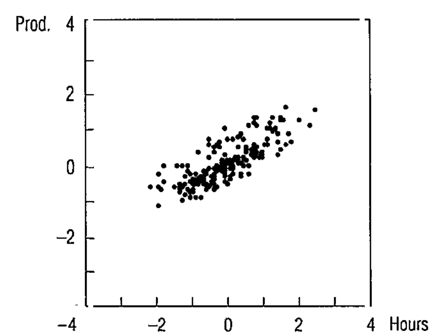



Chart 4 The Government Spending Model Without Technology Shocks . . .

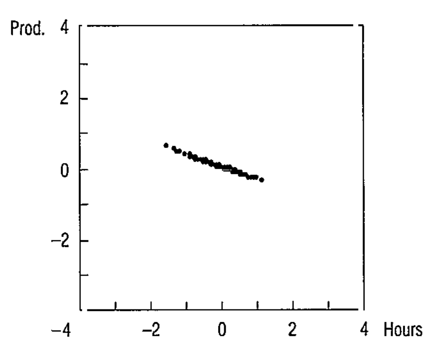
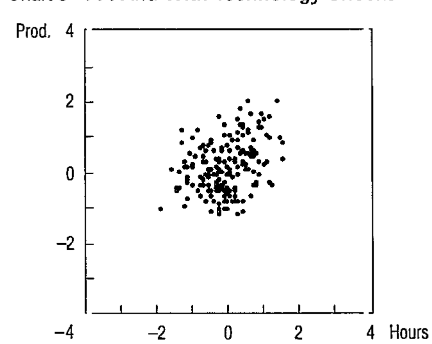



Chart 5 . . . And With Technology Shocks




Source of basic data: Citicorp's Citibase data bank state or along a deterministic balanced growth path, this model delivers the correct prediction concerning the effect of productivity growth on hours worked.)

The equilibrium can again be found as the solution to a social planner's problem, which in this case maximizes $U$ subject to (2)–(6), (10)–(11), and initial conditions.[9] The parameter values we use for the preference structure are $a_0 = 0.35$ and $\eta = 0.10$, which are the values implied by the estimates in Hotz, Kydland, and Sedlacek 1988; other parameter values are the same as in the preceding section.

The results are in Table 3. Notice that output is more volatile here than in the standard model, with $\sigma_y$ increasing from 1.30 to 1.51. Also, the standard deviation of hours worked relative to that of productivity has increased considerably, to $\sigma_h/\sigma_w = 1.63$, and the correlation between hours and productivity has decreased somewhat to 0.80. Chart 3 depicts the scatter plot of $h$ versus $w$ generated by this model. Although these points trace out a labor supply curve that is flatter than the one in Chart 2, the model still does not generate the cloud in Chart 1. We conclude that introducing nonseparable leisure improves things in terms of $\sigma_h/\sigma_w$, but does little for cor$(h,w)$.

## Indivisible Labor

We now take up the indivisible labor model of Hansen (1985), in which individuals are constrained to work either zero or $\hat{h}$ hours in each period, where $0 < \hat{h} < 1$. Adding this constraint is meant to capture the idea that the production process has important nonconvexities or fixed costs that may make varying the number of employed workers more efficient than varying hours per worker. As originally shown by Rogerson (1984, 1988), in the equilibrium of this model, individuals will be randomly assigned to employment or unemployment each period, with consumption insurance against the possibility of unemployment. Thus, this model generates fluctuations in the number of employed workers over the cycle. As we shall see, it also has the feature that the elasticity of total hours worked increases relative to the standard model.

Let $\pi_t$ be the probability that a given agent is employed in period $t$, so that $H_t = \pi_t\hat{h}$ is per-capita hours worked if we assume a large number of ex ante identical agents. Also, let $c_{0t}$ denote the consumption of an unemployed agent and $c_{1t}$ the consumption of an employed agent. As part of the dynamic social planning problem, $\pi_t$, $c_{0t}$, and $c_{1t}$ are chosen to maximize

$$(12) \qquad Eu(c_t,l_t) = \pi_t u(c_{1t},1-\hat{h}) + (1-\pi_t)u(c_{0t},1)$$

in each period, subject to the following constraint:

$$(13) \qquad \pi_t c_{1t} + (1-\pi_t)c_{0t} = c_t$$

where $c_t$ is total per-capita consumption. When $u(c,l) = \log(c) + A\log(l)$, the solution can be shown to imply that $c_{1t} = c_{0t} = c_t$.[10]

Therefore, in the case under consideration, expected utility can be written

$$(14) \qquad Eu(c_t,l_t) = \log(c_t) + \pi_t A\log(1-\hat{h}) = \log(c_t) - BH_t$$

where $B \equiv -A\log(1-\hat{h})/\hat{h} > 0$ and, as defined above, $H_t$ is hours worked per capita. Therefore, the indivisible labor model is equivalent to a divisible labor model with preferences described by

$$(15) \qquad \tilde{U} = E\sum_{t=0}^{\infty}\beta^t\tilde{u}(c_t,H_t)$$

where $\tilde{u}(c_t,H_t) = \log(c_t) - BH_t$. Based on this equivalence, we can solve the indivisible labor model as if it were a divisible labor model with a different instantaneous utility function, by maximizing $\tilde{U}$ subject to (2)–(6) and initial conditions.[11]

Two features of the indivisible labor economy bear mention. First, as discussed earlier, fluctuations in the labor input come about by fluctuations in employment rather than fluctuations in hours per employed worker. This is the opposite of the standard model and is perhaps preferable, since the majority of the variance in total hours worked in the U.S. data is accounted for by variance in the number of workers.[12] Second, the indivisible labor model generates a large intertemporal substitution effect for the representative agent because instantaneous utility, $\tilde{u}(c,H)$, is linear in $H$, and therefore the indifference curves between leisure in any two periods are linear. This is true despite the fact that hours worked are constant for a continuously employed worker.

Return to Table 3 for the results of our simulations of this model.[13] The indivisible labor model is considerably more volatile than the standard model, with $\sigma_y$ increasing from 1.30 to 1.73. Also, $\sigma_h/\sigma_w$ has increased from 0.94 to 2.63, actually

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[9]For the solution techniques that we use, this problem is expressed as a dynamic program. The stock of accumulated past leisure is defined to be $X_t$, and we write

$$L_t = a_0 l_t + \eta(1-a_0)X_t$$
$$X_{t+1} = (1-\eta)X_t + l_t.$$

These equations replace (10) and (11) in the recursive formulation.

[10]This implication follows from the fact that $u$ is separable in $c$ and $l$ and does not hold for general utility functions; see Rogerson and Wright 1988.

[11]Since the solution to the planner's problem in the indivisible labor model involves random employment, we need to use some type of lottery or sunspot equilibrium concept to support it as a decentralized equilibrium; see Shell and Wright, forthcoming.

[12]See Hansen 1985 for the U.S. data. Note, however, that European data display greater variance in hours per worker than in the number of workers; see Wright 1991, p. 17.

[13]The new parameter $B$ is calibrated so that steady-state hours are again equal to 1/3; the other parameters are the same as in the standard model.

somewhat high when compared to the U.S. data. Of course, this model is extreme in the sense that all fluctuations in the labor input result from changes in the number of employed workers, and models in which both the number of employed workers and the number of hours per worker vary fall somewhere between the standard divisible labor model and the indivisible labor model with respect to this statistic. (See Kydland and Prescott 1991 or Cho and Cooley 1989, for example.) Finally, the model implies that $\text{cor}(h,w) = 0.76$, slightly lower than the models discussed above but still too high. For the sake of brevity, the scatter plot between $h$ and $w$ is omitted; for the record, it looks similar to the one in Chart 3, although the indivisible labor model displays a little more variation in hours worked.

### Government Spending

We now introduce stochastic government spending, as in Christiano and Eichenbaum 1992. (That paper also provides motivation and references to related work.)

Assume that government spending, $g_t$, is governed by

$$(16) \quad \log(g_{t+1}) = (1-\lambda)\log(\bar{g}) + \lambda\log(g_t) + \mu_t$$

where $\lambda \in (0,1)$ and $\mu_t$ is independent and normally distributed with mean zero and standard deviation $\sigma_\mu$. Furthermore, as in Christiano and Eichenbaum 1992, assume that $\mu_t$ is independent of the technology shock. Also assume that government spending is financed by lump-sum taxation and that it enters neither the utility function nor the production function.[14] Then the equilibrium allocation for the model can be found by solving the planner's problem of maximizing $U$ subject to (16), (2)–(5), and, instead of (6), the new resource constraint

$$(17) \quad c_t + i_t + g_t = y_t.$$

An increase in $g_t$ is a pure drain on output here. Since leisure is a normal good, the negative wealth effect of an increase in $g_t$ induces households to work more. Intuitively, shocks to $g_t$ shift the labor supply curve along the demand curve at the same time that technology shocks shift the labor demand curve along the supply curve. This first effect produces a negative relationship between hours and productivity, while the second effect produces a positive relationship. The net effect on the correlation between hours and productivity in the model depends on the size of the $g_t$ shocks and on the implied wealth effect, which depends, among other things, on the parameter $\lambda$ in the law of motion for $g_t$ (because temporary shocks have a smaller wealth effect than permanent shocks). Hence, the calibration of this law of motion is critical. An ordinary least squares regression based on equation (16) yields estimates for $\lambda$ and $\sigma_\mu$ of 0.96 and 0.021, respectively. (In addition, the average of $g_t/y_t$ in our sample, which is 0.22, is used to calibrate $\bar{g}$.)

For the results, turn again to Table 3. The government spending model actually behaves very much like the standard model, except that the correlation between hours and productivity decreases to $\text{cor}(h,w) = 0.49$, which is better than the previous models although still somewhat larger than the U.S. data. Chart 4 displays the scatter plot generated by the model with only government spending shocks (that is, with the variance in the technology shock set to $\sigma_\varepsilon = 0$), and Chart 5 displays the scatter plot for the model with both shocks. These charts illustrate the intuition behind the results: technology shocks shift labor demand and trace out the labor supply curve, government shocks shift labor supply and trace out the labor demand curve, and both shocks together generate a combination of these two effects. The net results will be somewhat sensitive to the size of and the response to the two shocks; however, for the estimated parameter values, this model generates a scatter plot that is closer to the data than does the standard model.[15]

### Home Production

We now consider the household production model analyzed in Benhabib, Rogerson, and Wright 1991. (That paper also provides motivation and references to related work.)

Instantaneous utility is still written $u(c,l) = \log(c) + A\log(l)$, but now consumption and leisure have a different interpretation. We assume that

$$(18) \quad c_t = [ac_{Mt}^e + (1-a)c_{Ht}^e]^{1/e}$$

$$(19) \quad l_t = 1 - h_{Mt} - h_{Ht}$$

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[14]A generalization is to assume that instantaneous utility can be written $u(C,l)$, where $C = C(c,g)$ depends on private consumption and government spending. The special case where $C = c$ is the one we consider here, while the case where $C = c + g$ can be interpreted as the standard model, since then increases in $g$ can be exactly offset by reductions in $c$ and the other variables will not change. Therefore, the model with $C = c + g$ generates exactly the same values of all variables, except that $c + g$ replaces $c$. The assumption that $c$ and $g$ are perfect substitutes implies that they are perfectly negatively correlated, however. A potentially interesting generalization would be to assume that

$$C(c,g) = [\alpha c^\varphi + (1-\alpha)g^\varphi]^{1/\varphi}$$

where $1/(1-\varphi)$ is the elasticity of substitution.

[15]The size of the wealth effect depends on the extent to which public consumption and private consumption are substitutes. For example, if they were perfect substitutes, then a unit increase in $g$ would simply crowd out a unit of $c$ with no effect on hours worked or any of the other endogenous variables. We follow Christiano and Eichenbaum 1992 in considering the extreme case where $g$ does not enter utility at all. Also, the results depend on the (counterfactual) assumption that the shocks to government spending and technology are statistically independent. Finally, the results depend on the estimates of the parameters in the law of motion (16). The estimates in the text are from the period 1947:1–1991:3 and are close to the values used in Christiano and Eichenbaum 1992. Estimates from our shorter sample period, 1955:3–1988:2, imply a higher $\lambda$ of 0.98 and a lower $\sigma_\mu$ of 0.012, which in simulations yield $\text{cor}(h,w) = 0.65$.

where $c_{Mt}$ is consumption of a market-produced good, $c_{Ht}$ is consumption of a home-produced good, $h_{Mt}$ is hours worked in the market sector, and $h_{Ht}$ is hours worked in the home, all in period $t$. Notice that the two types of work are assumed to be perfect substitutes, while the two consumption goods are combined by an aggregator that implies a constant elasticity of substitution equal to $1/(1-e)$.

This model has two technologies, one for market production and one for home production:

$$(20) \quad f(z_{Mt},k_{Mt},h_{Mt}) = \exp(z_{Mt})k_{Mt}^{\theta}h_{Mt}^{1-\theta}$$

$$(21) \quad g(z_{Ht},k_{Ht},h_{Ht}) = \exp(z_{Ht})k_{Ht}^{\eta}h_{Ht}^{1-\eta}$$

where $\theta$ and $\eta$ are the capital share parameters. The two technology shocks follow the processes

$$(22) \quad z_{Mt+1} = \rho z_{Mt} + \varepsilon_{Mt}$$

$$(23) \quad z_{Ht+1} = \rho z_{Ht} + \varepsilon_{Ht}$$

where the two innovations are normally distributed with standard deviations $\sigma_M$ and $\sigma_H$, have a contemporaneous correlation $\gamma = \text{cor}(\varepsilon_{Mt},\varepsilon_{Ht})$, and are independent over time. In each period, a capital constraint holds: $k_{Mt} + k_{Ht} = k_t$, where total capital evolves according to $k_{t+1} = (1-\delta)k_t + i_t$. Finally, the constraints

$$(24) \quad c_{Mt} + i_t = f(z_{Mt},k_{Mt},h_{Mt})$$

$$(25) \quad c_{Ht} = g(z_{Ht},k_{Ht},h_{Ht})$$

imply that all new capital is produced in the market sector.

The parameters $\beta$, $\theta$, $\delta$, and $\rho$ are set to the values used in the previous sections. The two utility parameters $A$ and $a$ are set to deliver steady-state values of $h_M = 0.33$ and $h_H = 0.28$, as found in the time-use studies (Juster and Stafford 1991), and the capital share parameter in the household sector is set to $\eta = 0.08$, implying a steady-state ratio of $c_H/c_M$ of approximately 1/4.[16] The variances of the two shocks are assumed to be the same: $\sigma_H = \sigma_M = 0.007$. The parameter $e$, which determines the elasticity of substitution between $c_M$ and $c_H$, and $\gamma$, which is the correlation between $\varepsilon_M$ and $\varepsilon_H$, are set to the benchmark values used in Benhabib, Rogerson, and Wright 1991: $e = 0.8$ and $\gamma = 2/3$.

The results are at the bottom of Table 3. In the home production model, output is more volatile than in the standard model and about as volatile as in the indivisible labor model. The standard deviation of hours relative to productivity has increased considerably compared to the standard model, to $\sigma_h/\sigma_w = 1.92$. And cor($h$,$w$) has decreased to 0.49, the same as in the model with government spending.[17]

The intuition behind these results is that agents substitute in and out of market activity more in the home production model than in the standard model because they can use nonmarket activity as a buffer. The degree to which agents do this depends on their willingness to substitute $c_M$ for $c_H$, as measured by $e$, and on their incentive to move production between the two sectors, as measured by $\gamma$. (Lower values of $\gamma$ entail more frequent divergence between $z_M$ and $z_H$ and, hence, more frequent opportunities to specialize over time.) Note that some aspects of the results do not actually depend on home production being stochastic.[18] However, the correlation between productivity and market hours does depend critically on the size of the home technology shock, exactly as it depends on the size of the second shock in the government spending model. We omit the home production model's scatter plot between $h$ and $w$, but it looks similar to that of the model with government shocks.

## Conclusion

We have presented several extensions to the standard real business cycle model and analyzed the extent to which they help account for the U.S. business cycle facts, especially those facts concerning hours and productivity. Introducing nonseparable leisure, indivisible labor, or home production increases the elasticity of hours worked with respect to short-run productivity changes. Introducing a second shock, either to government spending or to the home production function, reduces the correlation between hours worked and productivity.[19]

Note that our goal has not been to convince you that any of these models is unequivocally to be preferred. Our goal has been simply to explain some commonly used real business cycle models and compare their implications for the basic labor market facts.

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[16]The two parameters $\theta$ and $\eta$ can be calibrated to match the observed average levels of market capital (producer durables and nonresidential structures) and home capital (consumer durables and residential structures) in the U.S. economy. This requires a lower value for $\theta$ and a higher value for $\eta$ than used here, as discussed in Greenwood, Rogerson, and Wright 1992.

[17]The exact results are somewhat sensitive to changes in the parameters $e$ and $\gamma$, for reasons discussed in the next paragraph.

[18]Even if the variance of the shock to the home technology is set to zero, shocks to the market technology will still induce relative productivity differentials across sectors. And even if the two shocks are perfectly correlated and of the same magnitude, agents will still have an incentive to switch between sectors over time because capital is produced exclusively in the market. It is these effects that are behind the increase in the labor supply elasticity.

[19]Other models can be constructed by combining the extensions considered here. Other extensions not considered here can also affect the implications of the model for the labor market facts, including distorting taxation as in Braun 1990 or McGrattan 1991 and nominal contracting as in Cho and Cooley 1990.

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