The Standard RBC Theory

Kaiji Chen
University of Oslo

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Road map

- Stylized facts about hours fluctuations over the business cycles

- The baseline real business cycle model
  - introduce labor
  - introduce technology shocks
  - calibration

- quantitative implications of the model versus data.

1 The Standard RBC Theory

1.1 Endogenizing Labor Supply

One period example

- No initial wealth

$$\max_{c,h} \log c + \psi \frac{(1-h)^{1-\theta} - 1}{1-\theta}$$
$$s.t$$
$$c \leq wh$$

where $h \in [0,1]$ is the total number of hours the household works.

Normalize total time household has available in a period to 1

- ψ determines how painful it is for the representative household to work.

- Lagrangian

$$L = \log c + \psi \frac{(1-h)^{1-\theta} - 1}{1-\theta} - \lambda (c - wh)$$

- Focs

$$\frac{1}{c} = \lambda \tag{1}$$

$$\psi (1-h)^{-\theta} = \lambda w \tag{2}$$

- $$c = wh = \frac{1}{\lambda}$$

  where first equality comes from the budget constraint and the second from (1). Substitute result in (2) so that

  $$\psi\left(1-h\right)^{-\theta} = \frac{1}{wh}w = \frac{1}{h}$$

  This means that labor leisure choice is not affected by the level of wage.

WHY?

- (Intratemporal) substitution effect: higher w makes labor more productive, thus effectively leisure more expensive (because its opportunity costs increases). Hence, household substitutes consumption for leisure and work more.

- Income effect: higher w makes economy generate the more output with the same input. Hence, it is optimal to increase both consumption and leisure.

- An increase in wage rate leads to positive income and substitution effect on consumption. For leisure, negative substitution effect and positive income effect.

- Log utility on consumption implies that income and substitution effects for leisure (or labor) cancel each other.

Intertemporal substitution of labor supply

- With log utility for consumption, labor supply does not respond to a permanent increase in wage rate.

- How agents respond to temporarily high wage rate?

- Will show that it is optimal to intertemporally substitute labor supply: work harder when they are more productive. The magnitude depends on the utility function (labor supply elasticity)

A Two-Period Extension

- Life time budget constraint is now

$$c_1 + \frac{c_2}{1+r} = w_1 h_1 + \frac{w_2 h_2}{1+r}$$

- Lifetime utility is

$$\max_{c_1,h_1,c_2,h_2} \log c_1 + \psi \frac{(1-h_1)^{1-\theta} - 1}{1-\theta} + \beta \left(\log c_2 + \psi \frac{(1-h_2)^{1-\theta} - 1}{1-\theta} \right)$$

- Calculating FOCs using Lagrangian method and rearranging terms

$$
\begin{aligned}
\frac{1}{c_1} &= \beta(1+r)\frac{1}{c_2} \\
\frac{1-h_2}{1-h_1} &= \left[\beta(1+r)\frac{w_1}{w_2}\right]^{\frac{1}{\theta}}
\end{aligned}
\tag{3}
$$

Equation (3) shows that

- The relative labor supply in the two periods responds to relative wage. If w_1 is larger than w_2 (household expects that future productivity will be lower than the current productivity), household supplies more labor today than in the next period. In other words, household has a stronger incentive to substitute consumption for leisure in the first period due to higher w_1.

- The relative labor supply in the two periods responds to the interest rate. A higher interest rate induces household to increase her labor supply today as the return to saving is higher.

- The sensitivity of the responses decreases with θ (or increase with the intertemporal elasticity of substitution, defined as $\frac{1}{\theta}$).

1.2 Introduction of Technology Shocks

- Production function

$$Y_t = e^{z_t} k_t^{\alpha} h_t^{1-\alpha}$$

- Basic idea: make z_t (total factor productivity) vary over time.

- If current TFP is high, output will be high even if inputs stay the same. In addition, household will respond to temporarily higher TFP by working harder, increasing output even further.

- Thus the economy starts to display fluctuations that looks like business cycle driven by fluctuation in z_t, or TFP, and further amplified by endogenous response of labor supply.

Effects of a favorable technology shock on labor supply-through increase in w and r

- Temporary wage increase leads to increase in labor supply due to intertemporal substitution effect on labor supply.
  - the more transitory the shock is, the stronger is the intertemporal substitution effect on labor supply. Hence, labor supply responds more strongly to an increase in w, vise versa.
  - In other words, the more transitory is the shock, the smaller the increase in C, the smaller is the income effect. Hence, labor supply responds more strongly to an increase in w.
- Increase in r makes savings more attractive. Hence labor supply increases in response to a positive technology shock.

1.3 The Baseline Business Cycle Model

- The economy consists of a representative price taking firm and a representative price taking, infinitely-lived household.

- The representative household owns the capital stock k_t and maximizes her expected lifetime utility.

Timing within time t is

1. The beginning-of-period stock of capital is k_t, and the shock is realized.

2. Spot market opens. Firm demands capital services and labor. The household supplies these factors. Market clears for both factors at price r_t and w_t

3. The representative household allocates her total income optimally between consumption and investment.

The Representative Household's Problem

$$\max_{\{c_t,h_t,k_{t+1}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \left[\log c_t + \psi \log\left(1 - h_t\right)\right]$$

subject to

$$\begin{aligned} k_{t+1} + c_t &= w_t h_t + \left(1 + r_t\right) k_t, \forall t \\ c_t &\geq 0, h_t \in [0,1], k_0 \text{ given} \\ z_t &= \rho z_{t-1} + \varepsilon_t \end{aligned}$$

- Exploit 1st welfare theorem: this economy is Pareto optimal, implying that we can reformulate the problem as a social planner problem

- The resource constraint of the planner is given by

$$k_{t+1} + c_t = (1 - \delta)\, k_t + e^{z_t} k_t^{\alpha} h_t^{1-\alpha}$$

- Note resource constraint has to hold for every possible realization of the TFP shock.

Social Planner's Problem

$$\max_{\{c_t,h_t,k_{t+1}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \left[\log c_t + \psi \log\left(1 - h_t\right)\right]$$

subject to

$$\begin{aligned} k_{t+1} + c_t &= (1-\delta)\, k_t + e^{z_t} k_t^{\alpha} h_t^{1-\alpha} && (4) \\ c_t &\geq 0, h_t \in [0,1] \text{ and } k_0 \text{ given} && (5) \\ z_t &= \rho z_{t-1} + \varepsilon_t && (6) \end{aligned}$$

Optimality Conditions

- Intratemporal Optimality Conditions

$$\frac{\psi}{1-h_t} = \frac{(1-\alpha_t)\, e^{z_t} k_t^{\alpha} h_t^{-\alpha}}{c_t}$$

state that social planner equates costs and benefits of an extra hour of work. utility cost of a marginal increase in labor is $\frac{\psi}{1-h_t}$.Benefit is to increase production and thus consumption by marginal product of labor, and thus utility from consumption by $\frac{(1-\alpha_t) e^{z_t} k_t^{\alpha} h_t^{-\alpha}}{c_t}$.

- Intertemporal Optimality Condition

$$\frac{1}{c_t} = \beta E_t \left[\frac{1}{c_{t+1}} \left(1 + \alpha e^{z_{t+1}} k_{t+1}^{\alpha-1} h_{t+1}^{1-\alpha} - \delta \right) \right]$$

Back to Competitive Equilibrium

- In a decentralized competitive equilibrium, factor prices are equal to value of their respective marginal product, thus

$$\begin{aligned} r_t &= \alpha e^{z_t} k_t^{\alpha-1} h_t^{1-\alpha} - \delta \\ w_t &= (1-\alpha_t)\, e^{z_t} k_t^{\alpha} h_t^{-\alpha} \end{aligned}$$

Calibration

- Parameters of the model to be calibrated
  - technology parameters: $(\alpha, \delta, \rho, \sigma_{\varepsilon})$
  - Preference parameters: (ψ, β)
- Parameters that have a time dimension: δ, β

- Process of choosing these parameters is often called calibration.

- Idea: we want to make the model's predictions to match certain observations from the data.

- Since we are interested in business cycle properties of the model and do not want to cheat by choosing parameter values that help the model deliver good business cycle implications, we choose parameters to match long run observations from the data

- Calibration is country specific. We do it for the US.

- Need to specify period length, because some parameter values have time dimension.

- Since most business cycle research is done with quarterly data, we use a quarter as the length of a period in the model.

- For those parameters that have a time dimension, we compute the annual values and convert them to the quarterly values.

- Alternatively, we can directly compute the quarterly values, with the flow variables, say, I, Y, C, adjusted to their quarterly values when calibrating.

- In particular, ψ is calibrated to match the fact that on average people work a third of their non-sleeping time to market activity

- The social planner's intratemporal optimality condition implies

$$\frac{1-h}{h} = \frac{\psi}{1-\alpha}\frac{c}{y}$$

- the $\frac{c}{y}$is estimated to be 0.75, and $h = 0.31$. This gives $\psi = 1.78$.

- Solve the decision rules numerically using value function iteration or other methods.

Comparing Business Cycle Statistics of the Model with the Data

- Basic Idea
  - Start with initial condition for capital stock k_0 (normally equal to the steady state value) and for $z_0 = 0$.
  - Draw a sequence of technology shocks $\{\varepsilon_t\}_{t=0}^{T}$, and construct $\{z_t\}_{t=0}^{T}$ from the equation

$$z_t = \rho z_{t-1} + \varepsilon_t$$

  - Simulate a large number of realizations of the model economy 100 times, each simulation being 150 period long (which is the length of the periods in data)

– decompose both the artificial and real data into trend and cyclical components with H-P filter or other methods.

– With cyclical components of artificial data from the model can compute exactly the same statistics as we did from the real data.

x	$\sigma_x\,(m)$	$\sigma_x\,(d)$	$\sigma_x/\sigma_y\,(m)$	$\sigma_x/\sigma_y\,(d)$	$\rho_{x,y}\,(m)$	$\rho_{x,y}\,(d)$
Output	1.351	1.72	1	1	1	1
Consumption	0.329	1.27	0.244	0.738	0.84	0.83
Investment	5.954	8.24	4.407	4.791	0.99	0.91
Hours	0.769	1.65	0.569	0.930	0.99	0.86

- Output fluctuates less in the model that in the U.S. economy (the volatility predicted by the model is 70-75% of that in the data)

- Labor input fluctuates only about half as much as in the U.S. economy.

- Investment fluctuates much more than output and consumption much less than output. Relative to GDP, consumption fluctuate significantly more in the data than in the model.

- The correlation of all variables with output is very high, and higher than that in the data.

- The model predicts a very strong correlation between the labor input and labor productivity.

Underlying reasons for model weakness

- The technology shock by assumption is very persistent (to ensure the persistence of output). Thus wage adjusts very smoothly, implying small intertemporal substitution in labor supply.

- The movement in labor supply are instead mainly associated with changes in interest rate, Hence the volatility of hours is small compared to the volatility of output.

- In this frictionless economy, it is too easy to smooth consumption over time with saving alone.

- The high correlation of all variables with output is due to the fact that there is only one shock in this economy.

Problems and Potential Solutions

- Main failure: too little hours fluctuation.
- Potential solutions:
  - abandon the logarithm specification and allow the intertemporal elasticity of labor supply to take value larger than one.
  - recognize explicitly that most (about two thirds) of the fluctuation in labor supply comes from changes along the extensive margin (movement into and out of employment) rather than along the *intensive margin* (changes in hours when employed).

We need a volatile labor input fluctuation over the business cycle

- period utility in general form

$$\log(c_t) + \psi \frac{(1-h_t)^{1-\theta} - 1}{1-\theta}$$

- Frisch elasticity of leisure (labor supply): $\frac{1}{\theta}$.
  - the elasticity of the leisure (labor supply) with respect to wage leaving constant the marginal utility of consumption.

- More volatile fluctuation of labor input requires a higher Frisch elasticity.

- Maximum Frisch elasticity obtained when $\theta = 0$. The period utility

$$\log\left(c_t\right) - \psi h_t$$

  - Marginal disutility is labor is linear, implying that labor supply reacts strongly to changes in wages

- How can we justify that marginal disutility of labor is constant at ψ?