RBC Model with Indivisible Labor

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Last Class

- Data on labor supply-fluctuations mostly driven by employment fluctuations

- Effect of wage rate on labor supply

  - substitution effect: magnitude depend on Frisch elasticity

  - income effect: magnitude depending on how persistent is the wage change.

- Effect of interest rate on labor supply

- Performance of standard business cycle models

Road map

- Model of Indivisible Labor

- RBC Model with indivisible labor (and full employment insurance)

- Aggregate and individual elasticity of labor supply

1 RBC Model with Indivisible Labor

Indivisible labor

- There is a continuum of ex-ante identical agents.

- Suppose the period utility function is given by
$$u(c_t, h_t) = \log(c_t) + v(1 - h_t)$$
where the function v satisfies $v' > 0$, $v'' < 0$.

- Assume that household can either work full time, $h_t = \widehat{h}$, $0 < \widehat{h} < 1$,or not at all $h_t = 0$. That is, they either have a job requiring fixed number of working hours, or they don't.

- Rationale for this assumption: in the U.S. economy, about two thirds of variations in hours worked comes from individuals moving into and out of unemployment, with only one third from variations in hours when employed.

- However, European data displays greater variance in hours worked per worker than in the number of workers.

Labor lottery

- Now, assume each individual can pick a probability π_t that he is employed and works $\widehat{h}$ hours in period $t, \pi_t \in [0,1]$. (This will make individuals happier than in the case $\pi_t = \{0,1\}$.)
  - Since all agents are *ex-ante* the same, they will choose the same probability π_t.
- Hence, π_t is also the fraction of agents that are employed each period.
- A lottery determines who will be actually be unemployed at each period.
- Individuals can insure each other against the contingency of unemployment.

The social planning problem

- Alternatively, we can let the social planner choose π_t, the fraction of population to work at each period.

- Assume that all people get picked to work $\widehat{h}$ hours with the same probability, so π_t is also the probability that a particular agent get picked.

- Denote $H_t = \pi_t \widehat{h}$ as the hours worked per capita.

- Assume that the social planner provides full insurance against being unemployed.

- As part of the dynamic social planning problem, the social planner solves

$$\max_{c_{1t},c_{0t}} \pi_t \left(\log c_{1t} + v\left(1 - \widehat{h}\right)\right) + (1 - \pi_t)\left(\log c_{0t} + v(1)\right),$$

  subject to

$$\pi_t c_{1t} + (1 - \pi_t)\, c_{0t} = c_t$$

  where c_t is total per capita consumption, which is given at this stage, c_{1t} (c_{0t}) is consumption of an employed (unemployed) agent.

- The solution to the above problem is $c_{1t} = c_{0t} = c_t$.

- The current period expected utility becomes

$$\begin{aligned} Eu(c_t, h_t) &= \pi_t \left(\log c_t + v\left(1 - \widehat{h}\right)\right) + (1 - \pi_t)(\log c_t + v(1)) \\ &= \log c_t - \pi_t \left(v(1) - v\left(1 - \widehat{h}\right)\right) + v(1) \end{aligned}$$

- Ignoring constants added to the utility function, we can rewrite the effective utility function as

$$Eu(c_t, h_t) = \log c_t - \psi H_t$$

- where $\psi = \left(v(1) - v\left(1 - \widehat{h}\right)\right) / \widehat{h} > 0$.

- Therefore, the decision variables for the social planner are the same as for a divisible labor model

$$\max_{\{c_t, H_t, k_{t+1}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \left(\log c_t - \psi H_t\right)$$

subject to

$$\begin{aligned} k_{t+1} + c_t &= (1-\delta)\, k_t + e^{z_t} k_t^{\alpha} H_t^{1-\alpha} && (1) \\ c_t &\geq 0, H_t \in [0,1] \text{ and } k_0 \text{ given} && (2) \\ z_t &= \rho z_{t-1} + \varepsilon_t && (3) \end{aligned}$$

Key assumptions and implications

- Key Assumptions
  - Full unemployment insurance.
  - All agents are ex-ante homogeneous.
- Fluctuations in labor input comes from fluctuations in employment rather than fluctuations in hours per employed worker.
- At the aggregate level, the Frisch elasticity of labor supply is infinity, though for a continuously employed worker, hours worked are constant (implying a value of 0 for Frisch elasticity of labor supply at the micro level).

A simple example

- Agents live for two periods, don't discount the future and only value consumption in the second period.
  - Abstract from capital accumulation, but let households store output between the first and the second period.

- Let A_1 and A_2 denote labor productivity in the first and second period (which is also the wage rate in this case).

- Social planner's problem

$$\begin{aligned} & \max \log c_2 - \psi h_1 - \psi h_2 \\ s.t.\ c_2 &= A_1 h_1 + A_2 h_2 \end{aligned}$$

- Optimal Choice: if $A_1 > A_2$, then the agent should worked only in period 1. *Vise Versa.*

- Extra unit of work brings about an extra disutility of work ψ, no matter when it is done. Thus should always work that extra unit in the period when labor is more productive.

- In this case, $h_2 = 0$, $h_1 = \frac{1}{\psi}$ and $c_2 = \frac{A_1}{\psi}$.

- For linear disutility of labor, effects of changes in productivity on labor supply are very strong.

- Suppose the utility is

$$\log c_2 + \psi \log (1 - h_1) + \psi \log (1 - h_2)$$

- Optimality condition

$$\frac{A_2}{A_1} = \frac{1 - h_1}{1 - h_2}$$

- Labor supply does not respond as drastically to difference in A_1 and A_2. As long as $\frac{A_2}{A_1}$ not too big, work in both period. Small changes in $\frac{A_2}{A_1}$ do not lead to drastic labor supply responses.

- Data: labor input varies substantially over cycle, real wages only moderately. Linear specification more successful.

Calibration of the RBC model with indivisible labor

- pick ψ so that the model economy reproduce amount of work equal to long run average in the data (about one third of their non-sleeping time).
  - Compute the stead state hours worked.
  - Back out ψ such that $H = \frac{1}{3}$.

- The calibration of other parameters are the same as before.

1.1 Quantitative Properties of the model

Impulse response functions

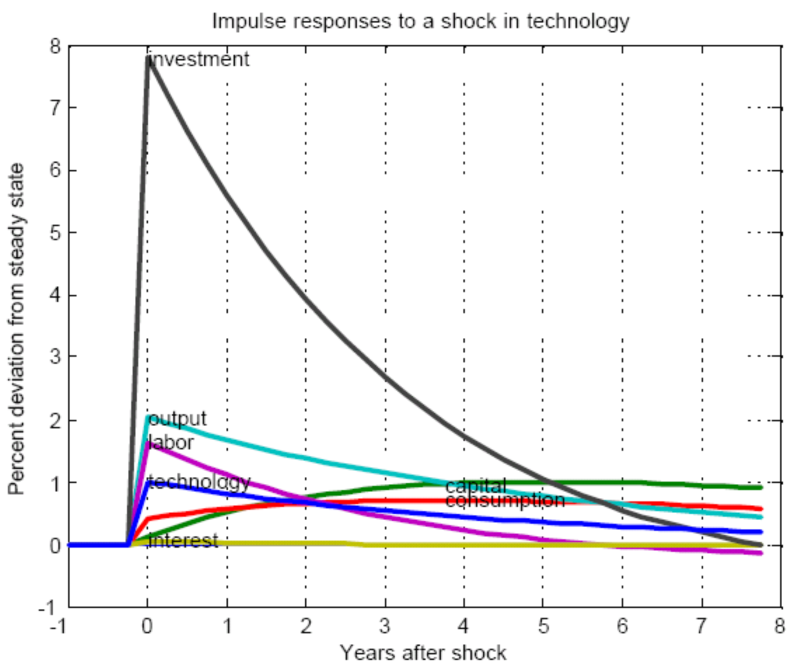
- Assume that before period 0, the economy is in the steady state.

- At period 0, there is a positive technology shock, in the size of one standard deviation, that is $z_0 = \sigma_z$.

- After this shock, the technology by assumption is not hit by further shocks and thus the technology follows

$$z_t = \rho z_{t-1}$$

- Impulse response functions trace how endogenous variables respond to the shock.



Impulse responses to a shock in technology
Percent deviation from steady state
Years after shock
investment
output
labor
technology
capital
consumption
interest
8
7
6
5
4
3
2
1
0
-1
-1
0
1
2
3
4
5
6
7
8

Main observations

- Labor supply responds positively to the increase in productivity. Consequently, output increase by more than the technology shock, due to intertemporal substitution of labor supply (amplification of business cycles).

- Consumption is hump shaped, because it is optimal to devote a lot of output to investment initially due to an increase interest rate.

- Eventually, technology goes back to the steady state, so do output, consumption labor and capital stock.

- Propagation of business cycles: persistent effect on output in this model is due to

- persistent technology process

- increase in capital stock

Compare business cycle statistics of the model with the data

Table 3
Cyclical Properties of U.S. and Model-Generated Time Series

Type of Data or Model	% S.D. of Output σ_y	Variable vs. Output: Consumption σ_c/σ_y	Investment σ_i/σ_y	Hours σ_h/σ_y	Productivity σ_w/σ_y	Hours vs. Productivity: σ_h/σ_w	cor(h,w)
U.S. Time Series*							
Output	1.92	.45	2.78	—	—	—	—
Hours Worked:							
1. Household Survey (All Industries)	—	—	—	.78	.57	1.37	.07
2. Establishment Survey (Nonag. Industries)	—	—	—	.96	.45	2.15	−.14
Models**							
Standard	1.30	.31	3.15	.49	.53	.94	.93
Nonseparable Leisure	1.51	.29	3.23	.65	.40	1.63	.80
Indivisible Labor	1.73	.29	3.25	.76	.29	2.63	.76
Government Spending	1.24	.54	3.08	.55	.61	.90	.49
Home Production	1.71	.51	2.73	.75	.39	1.92	.49

*U.S. data here are the same as those in Table 2; they are for the longer time period: 1947:1–1991:3.

**The standard deviations and correlations computed from the models' artificial data are the sample means of statistics computed for each of 100 simulations. Each simulation has 179 periods, the number of quarters in the U.S. data.

Source: Citicorp's Citibase data bank

Figure 1: Source: Hansen and Wright (1992)

- Compared with standard RBC model, models with indivisible labor supply generates hours volatility much close to the data.

- As a result, the volatility of output in this model is closer to the data.

- Also, model with indivisible labor supply somehow lower the correlation between labor input and labor productivity, but still much higher than the data.

- The volativity of consumption relative to output in this model is roughly the same as that in the standard model.

Other Extensions of RBC Models to deal with labor market shortcomings

- Home Production
  - Motivation: empirically, women's elasticity of labor supply along the extensive margin is much higher than men.
  - By introducing home production, increase the intertemporal elasticity of labor supply.
- Government spending shock.
  - Labor supply responds to government spending shock in addition to wages.

  - Somehow lower the correlation between labor supply and labor productivity.

- Labor market search frictions

- Learning by working

2 More on Labor Supply Elasticity

- The above model relies on the assumption that agent is ex-ante homogeneous, and there exists full insurance of employment risk.

- As a result, at equilibrium wage rate, the aggregate elasticity of labor supply is infinite.

- Problem: both assumptions are far from reality.

- Suppose workers differ in both preference ψ and asset holdings a and that the insurance of employment risks is not available.

- Static problem of choosing work (supplying $\overline{h}$ units of hours) or not.

- With indivisible labor, the agent i works if

$$\log\left(w\overline{h} + ra_i\right) - \psi_i \frac{\overline{h}^{1-\theta}}{1-\theta} \geq \log\left(ra_i\right)$$

- The reservation wage is

$$\widetilde{w} = \frac{ra_i}{\overline{h}}\left(\exp\left(\psi_i \Delta\right) - 1\right)$$

where $\Delta = \frac{\overline{h}^{1-\theta}}{1-\theta}$ is a constant, independent of individual characteristics.

- Workers with high ψ_i and larger a_i demand a higher reservation wage.

Aggregate labor supply and the reservation wage distribution-an example

- Suppose equal numbers of two types of workers exits in the economy, with reservation wage $10 and $20.

- Suppose labor is indivisible.

- At a wage rate of $10 and $20, the aggregate labor supply elasticity is infinite. otherwise, it is zero.

- Intuition: whenever a mass in the reservation wage distribution exists, the aggregate labor supply elasticity can take a large value.

Aggregate labor supply and the reservation wage distribution-in general

- Suppose many types of workers exit and that a worker works $\overline{h}$ hours if the market wage, w, exceeds the reservation wage.

$$h(w) = \begin{cases} \overline{h} \text{ if } w \geq \widetilde{w} \\ 0 \text{ otherwise} \end{cases}$$

- The reservation wage follows a distribution, where $\phi(\widetilde{w})$ is the probability density function of the reservation wage.

- The aggregate labor supply function, $H(w)$, is

$$H(w) = \int_0^w \overline{h}\phi(\widetilde{w})\, d\widetilde{w} = \Phi(w)\,\overline{h}$$

- The aggregate labor supply elasticity $\Gamma(w) = \frac{H'(w)w}{H(w)}$ is

$$\Gamma(w) = \frac{\Phi'(w)\,w}{\Phi(w)}$$

- The aggregate elasticity depends on the concentration of workers: the marginal density, $\Phi'(w)$, relative to the cumulative density, $\Phi(w)$.

- In the two-type workforce example, the aggregate elasticity is infinite where there is a mass of workers and zero elsewhere.

- In the lottery economy, the reservation wage distribution is degenerate (as the individuals are identical) at the equilibrium wage rate $(\Phi'(w) = \infty)$ and the aggregate elasticity becomes infinity.