Welfare economic consequences of a mandatory reduction of the work-week

Extra problem set, ECON4310

1 Introduction

1.1 Background and question

You are a young employee at the Ministry of Finance working on quantitative economic analysis. One day a group of activists proposes that the government should enforce regulation in order to mandatory reduce the work-week from 36 hours per week to 30 hours per week. This proposal gets a lot of attention and creates a vivid debate in newspapers, radio and TV.

In order to answer the questions from members of the Storting and the press, the Minister asks you personally to answer the following question:

> What are the welfare consequences of a mandatory reduction of the work-week from 36hrs/week to 30hrs/week?

More specifically, she wants you to compute estimates of both the short-run effect and long-run consequences of such a reform. In order to make the analysis tractable and transparent she asks you to base your analysis on well established economic theory and consistency with how national accounts are measured. Because the evidence on possible effects on e.g. average hourly productivity and number of sick days so far is inconclusive, you are asked to leave such potentially relevant aspects of a reform for further, later analysis. As your identifying restriction you choose: In absence of work-time regulations the average individual would choose to work 36hrs/week.

2 Model economy

In order to answer the question you need a model of how individuals derive utility from leisure as well as consumption, how individuals make choices over of savings and labor supply over time, and how prices for capital and labor clear markets. The Minister asked you specifically to base your estimate on well-established theory. The natural choice is then to base your benchmark analysis on the detrended neoclassical growth model.

Specifically, individuals maximize the sum of discounted utility derived from consumption and leisure

$$\max_{\{c_t,l_t,i_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u(c_t, l_t).$$

The utility function is given the following parametric form

$$u(c_t, l_t) = \ln c_t + \psi \ln l_t, \qquad \forall t. \tag{1}$$

We assume the economy is closed and that consumption, c_t, is both private and government consumption. The expenditure approach to output in the model economy is

$$y_t = c_t + i_t, \qquad \forall t. \tag{2}$$

Markets are competitive and the firms make zero profit. Output is paid out to labor and capital. The income approach to output in the model economy is

$$y_t = r_t k_t + w_t h_t, \qquad \forall t. \tag{3}$$

Output is produced from capital and labor input with Cobb-Douglas production technology. The product approach to output in the model economy is

$$y_t = f(k_t, h_t) = k_t^{\alpha} h_t^{1-\alpha}, \qquad \forall t, \alpha \in (0,1). \tag{4}$$

The law of motion for capital accumulation is

$$k_{t+1} = (1-\delta) k_t + i_t \qquad \forall t, \delta \in [0,1]. \tag{5}$$

Total time available for an individual can be spent on leisure or supplied in the market. Without loss of generality we normalize total time available to 1

$$h_t + l_t = 1, \qquad \forall t. \tag{6}$$

Consumption, capital stock, hours supplied in the market and leisure are all strictly positive

$$c_t, k_t, h_t, l_t \geq 0, \qquad \forall t. \tag{7}$$

The first welfare theorem holds so we can solve this problem as a social planner's problem. As we covered in class, our interest in the social planner's problem is based on the fact that the solution to the social planner's problem is the competitive equilibrium allocation. That is, there exists a set of prices such that the optimum solution can be decentralized as a competitive equilibrium with a price system that has an inner product representation. The social planner's problem is much easier to solve since we get rid of the prices and the individuals' budget constraint (Eq. 3).

Since this problem obviously has a recursive structure, we reformulate the problem and write up the Bellman equation.

$$v\left(k_t\right) = \max_{k_{t+1}, h_t} \left\{u\left(c_t, l_t\right) + \beta v\left(k_{t+1}\right)\right\}$$

For this problem we have two control variables, k_{t+1} and h_t, and one (endogenous) state variable, k_t.

Problem 1. *Take the first order condition of the Bellman equation with respect to the two control variables and the envelope condition of the Bellman equation with respect to the endogenous state variable to derive and show that the intertemporal optimality condition is*

$$\beta R_{t+1} \frac{u_1\left(c_{t+1}, l_{t+1}\right)}{u_1\left(c_t, l_t\right)} = \beta\left(1 + f_1\left(k_{t+1}, h_{t+1}\right) - \delta\right) \frac{c_t}{c_{t+1}} = 1$$

and the intratemporal optimality condition is

$$w_t \frac{u_1\left(c_t, l_t\right)}{u_2\left(c_t, l_t\right)} = \frac{f_2\left(k_t, h_t\right)}{\psi} \frac{1 - h_t}{c_t} = 1.$$

2.1 Calibration and measurement

In order to answer the question about what will happen after the policy change, we want to calibrate the policy-invariant (preference and technology) parameters of the model so that the behavior of the model economy matches features of the measured data in as many dimensions as there are unknown parameters. We observe over time that certain ratios in actual economies are more or less constant.

Problem 2. *Show that along a detrended balanced growth path (steady state) the following ratios of endogenous variables can be expressed as functions of the structural parameters, i.e. technology and preference parameters:*

- *the investment/capital ratio*

$$\frac{i}{k} = \delta,$$

- *the factor prices*

$$\begin{aligned} w &= (1-\alpha)\, k^{\alpha} h^{-\alpha}, \\ r &= \alpha k^{\alpha-1} h^{1-\alpha}, \end{aligned}$$

- *capital's and labor's shares of national income*

$$\begin{aligned} \frac{wh}{y} &= (1-\alpha)\,, \\ \frac{rk}{y} &= \alpha, \end{aligned}$$

- *the capital/labor ratio*

$$\frac{k}{h} = \left(\frac{\frac{1}{\beta} - (1-\delta)}{\alpha} \right)^{\frac{1}{\alpha-1}},$$

- *the capital/output ratio*

$$\frac{k}{y} = \frac{\alpha}{\frac{1}{\beta} - (1-\delta)},$$

- *the investment/output ratio*

$$\frac{i}{y} = \frac{i}{k}\frac{k}{y} = \frac{\alpha\delta}{\frac{1}{\beta} - (1-\delta)},$$

- *and the consumption/output ratio*

$$\frac{c}{y} = 1 - \frac{\alpha\delta}{\frac{1}{\beta} - (1-\delta)}.$$

We must calibrate the following structural, time-invariant parameters from annual Norwegian national accounts:

- time-preference rate β,
- consumption-leisure trade-off ψ,
- capital's share of output α,
- depreciation rate δ.

Problem 3. *Download Norwegian National Accounts for the period 1970-2005, either accessed from Statistics Norway*

`http://www.ssb.no/english/subjects/09/01/nr_en/`

or Source OECD,

`http://new.sourceoecd.org/vl=6237706/cl=11/nw=1/rpsv/ij/oecdstats/16081188/v149n1/s1/p1`

to compute the following average ratios[1] *(see the Appendix for some comments on useful approximations given the SSB presentation of the data):*

- *labor's share of national income,*
- *investment to capital ratio, and*
- *capital to output ratio.*

Attach spreadsheets with the time series.

In addition is reasonable to assume that the average working-ago individual has 15×7 hours available per week net of sleep and personal care, i.e.

$$h = \frac{36}{105} \approx 0.343. \tag{8}$$

Problem 4. *Calibrate β, ψ, α and δ using steady-state ratios, the measured moments from Norwegian national accounts, and the fraction of available time at work. Without loss of generality we might normalize total steady-state output to* 1*.*

2.2 Welfare prior to a reform

Given the ratios along the detrended balanced growth path (steady state), the calibrated parameters, and the normalization of output to 1, we can now compute detrended balanced growth path values of the endogenous variables of the model prior to the policy reform.

Problem 5. *Compute the balanced-growth-path values for*

- *consumption c,*
- *leisure l,*
- *output y,*
- *investment i,*
- *capital stock k,*
- *labor income wh, and*
- *welfare, $u(c, l)$.*

[1] Remember to compute the average of the ratios and not the ratio of the averages....

3 Policy reform

In the model economy, a government mandatory reduction in the work week would amount to adding a constraint, $h \leq \bar{h} = \frac{30}{105} = \frac{2}{7}$, to the social planner's problem.

$$\max_{\{c_t, l_t, i_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u(c_t, l_t)$$

subject to

$$\begin{aligned}
c_t + i_t &\leq y_t, && \forall t \\
k_{t+1} &= (1-\delta) k_t + i_t && \forall t, \delta \in [0,1] \\
h_t + l_t &= 1, && \forall t \\
h_t &\leq \bar{h} = \frac{2}{7}, && \forall t \\
c_t, k_t, h_t, l_t &\geq 0, && \forall t \\
k_0 &> 0. && \text{given}
\end{aligned}$$

Functional forms

$$\begin{aligned}
u(c_t, l_t) &= \ln c_t + \psi \ln l_t, && \forall t, \sigma > 0 \\
y_t = f(k_t, h_t) &= k_t^{\alpha} h_t^{1-\alpha}, && \forall t, \alpha \in (0,1).
\end{aligned}$$

3.1 Comparison of long-run welfare

Problem 6. *Solve the constrained optimization problem. Denote all the new steady-state values with* ˜. *Show that the new steady-state values of the endogenous variables as functions of the policy-invariant preference and technology parameters will be*

$$\begin{aligned}
\tilde{l} &= 1 - \bar{h} \\
\tilde{k} &= \left(\frac{\frac{1}{\beta} - (1-\delta)}{\alpha} \right)^{\frac{1}{\alpha - 1}} \bar{h} \\
\tilde{y} &= \frac{\frac{1}{\beta} - (1-\delta)}{\alpha} \tilde{k} \\
\tilde{c} &= \left(1 - \frac{\alpha\delta}{\frac{1}{\beta} - (1-\delta)} \right) \tilde{y} \\
\tilde{\imath} &= \frac{\alpha\delta}{\frac{1}{\beta} - (1-\delta)} \tilde{y} \\
\tilde{w} &= (1-\alpha) \tilde{k}^{\alpha} \bar{h}^{-\alpha} \\
\tilde{r} &= \alpha \tilde{k}^{\alpha - 1} \bar{h}^{1-\alpha}.
\end{aligned}$$

Comment briefly on these results.

As economists we are not primarily interested in changes in real variables as such, but what effect is has for *welfare*.

Problem 7. *First compute the welfare after the policy reform*

$$u\left(\tilde{c},\tilde{l}\right)=\ln\tilde{c}+\psi\ln\tilde{l}.$$

Then compute the compensating variation between the two policies – how much you must compensate the representative individual under the new policy compared to the first policy, i.e. find λ *such that*

$$\begin{aligned} u\left(c,l\right) &= u\left(\lambda\tilde{c},\tilde{l}\right) \\ &\Updownarrow \\ \ln c+\psi\ln l. &= \ln\lambda\tilde{c}+\psi\ln\tilde{l}. \end{aligned}$$

Comment on your results and compare briefly with the results above

3.2 Dynamic welfare analysis

> *In the long run we are all dead.*
>
> John Maynard Keynes *A Tract on Monetary Reform*, ch. 3(1923).

The minister had also asked you to compute the short-run consequences of the reform. In order to answer that question, we must compute the transition between the two detrended balanced growth paths. In order to compute this transition, we need to solve for the individuals' decision rules.

In order to solve for the constrained equilibrium after a potential reform we can model $\bar{h}$ as an exogenous state variable

$$v\left(k_t,\bar{h}_t\right)=\max_{k_{t+1}}\left\{u\left(c_t,1-\bar{h}_t\right)+\beta v\left(k_{t+1},\bar{h}_{t+1}\right)\right\}.$$

The problem has now one control variable (k_{t+1}), one endogenous state variable (k_t) and one exogenous state variable ($\bar{h}_t$).

Problem 8. *Compute the time-consistent decision rules for the individuals as functions of the state variables*

$$\begin{aligned} k_{t+1} &= d\left(k_t, \bar{h}_t\right) \\ c_t &= f\left(k_t, \bar{h}_t\right) + (1-\delta) k_t - d\left(k_t, \bar{h}_t\right) \end{aligned}$$

using discrete value function iteration. Plot the decision rules. Attach your Matlab code.

Now that we have the time-consistent decision rules of the individuals we can also make predictions for the transition between the long-run growth paths.

Problem 9. *Let the first k be equal to the long-run growth path value* before *the reform and $\bar{h}$ be equal to value given by the proposed reform. Simulate 50 periods using the time-consistent decision rules you computed in the previous problem. Plot the transition for the following variables*

- *marginal product of labor,* $\{w_t\}_{t=1}^{50}$,
- *marginal product of capital,* $\{r_t\}_{t=1}^{50}$,
- *output,* $\{y_t\}_{t=1}^{50}$
- *consumption,* $\{c_t\}_{t=1}^{50}$,
- *investment,* $\{i_t\}_{t=1}^{50}$,
- *capital stock,* $\{k_t\}_{t=1}^{50}$,
- *labor income,* $\{w_t h_t\}_{t=1}^{50}$, *and*
- *welfare,* $\left\{u\left(c_t, 1-\bar{h}_t\right)\right\}_{t=1}^{50}$.

Comment on your findings. How do they modify your findings from the comparison of long-run growth paths?

4 Conclusion

Problem 10. *The last thing you need to do is to write a short and* well-written *"executive summary" which can be circulated to all the other ministries.*

A Measurement

A.1 Download time series for Statistics Norway

For some reason, SSB gives only time series for 1998-2006 in `html` format. In order to get the time series for 1970-2006 (*which you should!*) you have to download the `csv`-file. The `csv`-file can then be e.g. opened in Excel or imported into Stata.

`csv` (also known as comma-separated values, comma-separated list or comma-separated variables) file format is a file type that stores tabular data. The format dates back to the early days of business computing. For this reason, `csv`-files are common on all computer platforms.[2]

A.2 The factors' share of output

Ideally, taxes and subsidies should have been allocated to capital or labor in the presentation of the income approach to output. If this had been a serious research project, you should have done this allocation yourself. However, for this paper you might do the following approximation

$$\text{Labor's share of output} \approx \frac{\text{Compensation of employees}}{\text{Operating surplus} + \text{Compensation of employees}}.$$

The time series on the right hand side are available from "4 Gross domestic product by income components. Million kroner".

As you see, primarily due to the increase in revenue from the petroleum sector during the last 10 years or so, this time series does not seem stationary. Also "taking the model to the measurement", given the question we want to answer we are interested in the structural parameters for *the mainland economy*. In order to rather calculate this you might rather use the time series for mainland Norway from Tables 12 and 19.

A.3 Investment

As an approximation for investment, you might use "Gross fixed capital formation" at current prices from Table 26.

A.4 Capital stock

As an approximation for total capital stock, you might use "Fixed assets, total" at current prices from Table 35.

[2] Source: `http://en.wikipedia.org/wiki/Comma-separated_values`