

ECON 4350: Growth and Investment

Lecture note 5

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10 Endogenous growth: An overview

Required reading: Romer (1990), sec. 1-2., BSiM: 4.1-4.2

10.1 What do we mean by endogenous growth?

- In the neo-classical growth models we ended up with the following conclusion:
 1. Factor-accumulation can not give rise to growth in the long run, i.e. it can not explain perpetual growth.
 2. The only source of long-run growth in the model is if we postulate a process of changes in technology, i.e. if we introduce exogenous technological progress.
- We will now turn to models falling under the heading “endogenous growth”. This terminology is somewhat misleading, since we will be focusing on models that departure from the neo-classical models with respect to either of the two conclusions above.
- Our models will follow either of four paths:
 1. Relaxing the upper Inada-condition
 2. Externalities and/or public goods (Abandoning CRS)
 3. Two-sector models, giving 'effective' increasing returns to the accumulated factors.

4. Models of research and technological progress (Endogenizing T)

- The first three gives rise to perpetual (long-run) growth from factor-accumulation. Only the last one explicitly endogenizes something that was exogenous in the neo-classical model (i.e. technological progress).
- It is thus not a clear distinction in terms of exogeneity/endogeneity between these models and the neo-classical models.
- The term endogenous growth theory is, however, fairly well established as a caption for the full array of models.

10.2 Inessential non-reproducible factors

- We consider the case with only two inputs to production, characterized by the production function

$$Y = F(K, L)$$

- Note that in the models we have looked at so far there is the following important distinction between the two inputs:
 1. Capital, K , is reproducible, i.e. it can be made from Y .
 2. Labor, L , is non-reproducible, i.e. it can not be produced/taken from Y .
- In our previous models also human capital has been reproducible, and more specifically been produced by the same technology as Y and K .
- Remember that

$$\dot{k}/k = sf(k)/k - (n + \delta)$$

and we have the graph in (\dot{k}, k) space

In order to have perpetual growth in k (and hence in y) we must have

$$\lim_{k \rightarrow \infty} [f(k)/k] > (n + \delta)/s \quad (1)$$

For this to hold we must have that the average productivity of capital, $f(k)/k$, approaches a positive number when k approaches infinity. (Note that since $f(k)/k = Y/K$ this also corresponds to $\lim_{K \rightarrow \infty} [Y/K] > (n + \delta)/s$ for any given L).

- Since $\lim_{k \rightarrow \infty} f(k) = \infty$ (Show this!) we must use l'Hopitals rule on the limit in expression (1), that is

$$\lim_{k \rightarrow \infty} [f(k)/k] = \lim_{k \rightarrow \infty} f'(k)$$

We hence see that when the upper Inada-condition holds, i.e.

$$\lim_{k \rightarrow \infty} f'(k) = 0$$

condition (1) cannot hold and we cannot have perpetual growth.

- Hence the upper Inada condition for the reproducible input, is the property of the neo-classical production function that is fundamental in securing convergence to a steady state, and thereby preventing long-run growth.
- It is also illustrating to see this in the familiar graph in (\dot{k}, k) space

- That is: When we relax the upper Inada condition we can get perpetual growth by. This is a form of endogenous growth as explained above.
- Remember from Lecture 2 that the upper Inada-conditions ensures that capital and labor are essential for production, i.e. $F(0, L) = F(K, 0) = 0$ for all K, L .
- Thus we can conclude that
 - *When the non-reproducible factors of production are inessential to production we can get perpetual growth.*

10.2.1 Two examples

- Let us therefore investigate some cases where $\lim_{K \rightarrow \infty} F_K = \mu > 0$.
- These are cases where only the reproduced factor of production (K) is essential to production, meaning $F(K, 0) > 0$ for all $K > 0$, while $F(0, L) = 0$ for all L .
- Consider a Constant Elasticity of Substitution (CES) production function:

$$Y = F(K, L) = A \{a(bK)^\psi + (1-a)[(1-b)L]^\psi\}^{1/\psi} \quad (2)$$

where $0 < a < 1$, $0 < b < 1$ and $\psi < 1$. It is easy to show that the production function exhibits constant returns to scale (do this!). When $\psi \rightarrow -\infty$ the production function approaches $Y = \min[bK, (1-b)L]$ i.e. a limitation law as in the Harrod-Domar case, while it approaches a Cobb-Douglas, $Y = CK^aL^{1-a}$ when $\psi \rightarrow 0$. For $\psi = 1$ the function is linear in L and K .

- For simplicity we let A be a constant. Dividing by L we then get output per capita:

$$y = f(k) = A \{a(bk)^\psi + (1-a)(1-b)^\psi\}^{1/\psi} \quad (3)$$

and hence

$$f'(k) = Aab^\psi \{ab^\psi + (1-a)(1-b)^\psi k^{-\psi}\}^{(1-\psi)/\psi} \quad (4)$$

and

$$f(k)/k = A \{ab^\psi + (1-a)(1-b)^\psi k^{-\psi}\}^{1/\psi} \quad (5)$$

- Consider the case with a high degree of substitutability between labor and capital, that is, $0 < \psi < 1$. Then the limits of the expressions above are

$$\lim_{k \rightarrow \infty} [f'(k)] = \lim_{k \rightarrow \infty} [f(k)/k] = Aba^{1/\psi} > 0$$

and

$$\lim_{k \rightarrow 0} [f'(k)] = \lim_{k \rightarrow 0} [f(k)/k] = \infty$$

- What happens if $\psi < 0$? We will get convergence, but can have quite problematic solutions. Consider what happens with $sAba^{1/\psi}$ as an exercise. This resembles what we have seen for the Harrod-Domar model.

- Hence the CES-production function gives an example of perpetual growth even if we have constant returns to scale.
- As another example consider the production function

$$Y = F(K, L) = AK + BK^\alpha L^{1-\alpha}$$

or in per capita terms

$$y = f(k) = Ak + Bk^\alpha$$

Note that this is a mixture of a AK -production function, and a Cobb-Douglas. The function exhibits constant returns to scale. In the limit the latter part becomes relatively unimportant.

- We now have

$$f(k)/k = A + Bk^{-(1-\alpha)}$$

which approaches A as k tends to infinity. We are thus approaching AK -like behavior and have perpetual growth.

10.3 Externalities and public provision

- Consider an economy characterized by a continuum of representative agents/workers. (For simplicity we keep their number fixed, i.e. there is no population growth). Each agent's output is given by a constant returns to scale production function

$$y = F(k, El)$$

where for each agent, k is the stock of available capital and l is the input of labor. E is the efficiency of each worker and is common to all workers.

- Assume that the total amount of capital in the economy has a positive impact on the productivity of each worker, i.e.

$$E = A(K) \tag{6}$$

where $A(K)$ is increasing in K .

- We will later look more closely at examples where we justify this assumption based on externalities in the use of capital (learning by doing/social knowledge) or by public provision of services (the level depending on K) that improve productivity and are provided as public goods.

- The relationship (6) adds to

$$Y = F(K, A(K)L)$$

and the growth rate is then

$$\dot{Y}/Y = (F_K + F_L A'(K)L) \dot{K}/Y$$

- With a constant savings rate $s = \dot{K}/Y$, we get

$$\dot{Y}/Y = sF_K + sF_L A'(K)L$$

- Due to the usual upper Inada-condition the term involving F_K vanishes when K approaches infinity, however \dot{Y}/Y might still be positive if $sF_L A'(K)L$ is bounded away from zero.
- Since we have CRS, the partial derivative $F_L(K, A(K)L)$ is homogenous of degree zero, so we have

$$F_L(K, A(K)L)A'(K)L = F_L(1, A(K)L/K)A'(K)L$$

- Assume that

$$\lim_{K \rightarrow \infty} A'(K)L = b \tag{7}$$

then (since $\lim_{K \rightarrow \infty} A'(K) = c \Rightarrow \lim_{K \rightarrow \infty} A(K)/K = c$) we also have that

$$\lim_{K \rightarrow \infty} \frac{A(K)}{K} \cdot L = b$$

and hence

$$\lim_{K \rightarrow \infty} F_L(1, \frac{A(K)}{K}L)A'(K)L = F_L(1, b)b$$

giving

$$\lim_{K \rightarrow \infty} \frac{\dot{Y}}{Y} = sF_L(1, b)b > 0$$

and perpetual growth.

- Hence given the framework described by (6) condition (7) is a sufficient condition for growth in the long run.
- We will be considering conditions (6) and (7) in detail in the more specific models to follow under topic 11 and 12.

10.4 Two-sector approaches

- We now consider another approach which takes a minimum departure from the neo-classical model. We show that

– *long-run growth is possible even with a neo-classical production function if at least one of the accumulated factors are produced only by use of reproducible factors.*

- To see an example of this consider an economy with the following two-sector structure.
- Output C of the consumption good is produced by a Cobb-Douglas production function

$$C = K_C^\alpha L^{1-\alpha}$$

where K_C is the amount of capital used in this production sector.

- The sector producing capital, the investment sector, uses only capital (K_I) but exhibits constant returns to scale

$$\dot{K} = aK_I$$

and where $K = K_C + K_I$.

- Assume that a constant fraction of capital goes to the investment sector, $K_I = \phi K$ (and hence $K_C = (1 - \phi)K$). Note that this replaces the assumption of a fixed savings rate.
- For simplicity we keep population constant. Then we see that

$$\frac{\dot{C}}{C} = \alpha \frac{\dot{K}_C}{K_C} = \alpha \frac{\dot{K}}{K} = \alpha a \phi$$

So the growth rate of consumption is constant and positive. We will see long-run growth even if we have not introduced any departures from constant returns to scale.

- The most interesting example of this model is one with a separate sector for production of human capital. We will return to this in more detail as topic 13.

10.5 Why is explaining technological progress so hard?

- The final approach we will follow is to try to explain the growth process of T where T is interpreted as a stock of knowledge.
- A typical trait of these models is that previous inventions (increases in the level of T) fuels back on the cost of future investment.
- A central feature of these models is a production function for knowledge (i.e. characterizing the R&D-sector)

$$\dot{T} = G(T, K, L)$$

- A serious problem for the models is that it is very hard to have a good idea about the nature of this production function (in particular the role of T as an input). This is serious since results tend to be strongly dependent upon parameters of the production function.
- Why is it so difficult to explain such research and development activities? At this point we should note two things:
 1. There are several reasons to expect the ideas/knowledge embodied in T to have the character of a public good. It is non-rival in its use (as reflected by the production function $Y = F(K, TL)$) and to a certain extent non-excludable. (Cf. the excellent discussion in Romer (1990), sec. 1-2.
 2. Due to constant returns to scale in the rival inputs K and L we know that if we have a perfectly competitive market structure we have

$$RK + wL = Y$$

Hence, there is no income left to remunerate production of T .

- Hence, under perfect competition private agents/firms will have no incentives to do research, and we will not get growth in T .
- Therefore we must seek models that introduces markets with imperfect competition.
- In topic 14 and 15, we will pursue two such paths
 1. Monopolistic competition in a market with a variety of products.
 2. Quality ladders with temporary monopoly power for the use of the newest technology.

10.6 Endogenous growth: Does it matter?

- An important lesson of our study of the neo-classical growth model is that there is basically no room for policy in promoting growth. This is so for two reasons
 1. The decentralized solution is pareto-optimal
 2. Policy can not affect the growth in the long run
- We now briefly sketch why this in general will change in models with endogenous growth.
- Consider the AK-model where

$$Y = AK$$

- The AK-model is in many ways the reduced form of several of the models we will encounter. The parameter A then summarizes various parameters (e.g. those related to policy), and should be thought of more broadly as characterizing the level of technology.
- An important rationale for the AK-model is that it can be seen as an alternative representation of a model with

$$Y = F(K, H)$$

and CRS in K and H , i.e. CRS in broad capital. See BSiM 4.2 for the details.

- Using our usual setup for household preferences, household behavior in this technological environment is characterized by the Euler-equation

$$\frac{\dot{c}}{c} = (1/\theta)(r - \rho) \tag{8}$$

- But we now have that the rental price must be a constant $R = A$. Hence, the interest rate is also constant

$$r = A - \delta \tag{9}$$

- So we have

$$\frac{\dot{c}}{c} = (1/\theta)(A - \delta - \rho) \tag{10}$$

- The growth rate of consumption is therefore constant (and we assume $A - \delta > \rho$, so that it is positive).
- Capital is evolving according to

$$\dot{k} = (sA - \delta - n)k - c \quad (11)$$

- It is easy to show (cf. BSiM p. 207) that in the steady state c/k must be fixed, so k must grow at the same (constant) rate as c . Further, if we investigate the transversality condition more carefully (see BSiM 4.1.4) we see that (not surprisingly) there is no transitional dynamics in this model, and

$$\frac{\dot{c}}{c} = \frac{\dot{k}}{k} = (1/\theta)(A - \delta - \rho) = \text{Constant} \quad (12)$$

at all times.

- We can also see this in the phase diagram.

- Notice that in the AK-model changes in parameters such as A , δ and ρ will affect both levels **and** growth in the long run.
- Remembering that A might in turn depend on parameters of policy, we see that in this framework there is a much more important role for policy.

References

- [1] **Romer, Paul, M.** Endogenous Technological Change, *Journal of Political Economy*, October 1990, 98 (5), 71-102, Part 2.