

ECON 4350: Growth and Investment

Lecture note 6

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14 Growth through increasing variety

Required reading: Romer (1990)

Secondary reading: BSiM 6

14.1 Ideas: Rivalry, excludability and nonconvexities

- Are knowledge/ideas public goods?
- We should distinguish between two aspects: Rivalry and excludability.
- Rivalry is a technological attribute of the good itself.
- An idea is in general non-rival.
- That is of course not to say that e.g. one firm's sales of a product produced using the idea is not competing with (is rival to) another firm's sales.
- Note further that there is a fundamental difference between knowledge in the form of ideas/designs (our A) and knowledge in the form of the human capital of individuals.
- The former is in general non-rival and only partially excludable, the latter has a high degree of both rivalry and excludability.
- Note further that nonrival knowledge can be accumulated without bound on a per capita basis, while human capital can not because it is constrained by the life time of each separate individual.

- If a nonrival input has productive value this inevitably violates the replication argument underlying constant returns to scale and leads to nonconvexities.
- If K and L are all rival inputs and A is a nonrival input such as knowledge and the production function is $F(A, K, L)$, the standard replication argument leads to CRS in K, L :

$$F(A, sK, sL) = sF(A, K, L) \quad (1)$$

which also implies (as a special case of Euler's formula)

$$F(A, K, L) = KF_K + LF_L \quad (2)$$

But if A is productive we must have

$$F(sA, sK, sL) > sF(A, K, L) \quad (3)$$

and

$$F(A, K, L) < AF_A + KF_K + LF_L \quad (4)$$

- Note that (4) implies that a firm having this production function could not survive as price taker remunerating all inputs at their marginal product.
- Thus, under perfect competition we can not explain why firms remunerate A and hence not explain why private agents do research.
- We have looked at two approaches: Either treating A as a proper public good (provided by the government) or letting production of A follow as a by-product such as through learning by doing.
- We now look at a new approach: Letting knowledge be partially excludable and introducing market power.

14.2 Monopolistic competition in horizontally differentiated products

14.2.1 Production of the consumption good

- The following is a somewhat simplified version of the model in Romer (1990).

- The production function for the representative firm producing the final good Y is

$$Y = L_1^{1-\alpha} X$$

where L_1 is labor input and X is an index of the use of the N other inputs $\{x_i\}_{i=1}^N$ (different capital goods)

$$X = \sum_{i=1}^N x_i^\alpha$$

which inserting gives:

$$Y = L_1^{1-\alpha} \sum_{i=1}^N x_i^\alpha \quad (5)$$

- Note that the production function exhibits CRS with respect to all inputs L_1 and $\{x_i\}_{i=1}^N$. Thus the assumption of a single representative price-taking firm is legitimate.
- The marginal product of input x_i is then

$$MP_{x_i} = \alpha L_1^{1-\alpha} x_i^{\alpha-1}$$

which is infinite at $x_i = 0$. Hence the firm will want to use all some of all N inputs available.

- Note that if we regard x_i as different capital goods (5) assumes that these have additively separable effects on production. The marginal product of capital good i (tractor) does not depend on the amount used of other capital goods (e.g. computers)
- This contrasts with the traditional case where we measure the total use of capital goods simply as the sum ($X = (\sum_{i=1}^N x_i)^\alpha = K^\alpha$). This assumes perfect substitutability between tractors and computers.
- Assume for reasons that will become clearer below that all inputs x_i are measured in the same unit and employed in equal amounts $x_i = x$, then

$$Y = L_1^{1-\alpha} N x^\alpha = L_1^{1-\alpha} (N x)^\alpha N^{1-\alpha} \quad (6)$$

Hence, if we associate $K = N x$ with the total amount of resources used of the inputs $\{x_i\}_{i=1}^N$ we see that the production function exhibits CRS in L_1 and K .

- Technological progress in this model is captured by an increase in the number/variety of available inputs to production, that is, through increases in N .
- Note that for given levels of L_i and K production increases in N . An interpretation: As a broader diversity of inputs become available production can increase by using more specialized inputs.

14.2.2 Production of intermediate goods

- The demand for intermediate good x_i follows from the profit-maximization of the representative producer of Y , that is

$$\max_{L_1, \{x_i\}_{i=1}^N} \sum_{i=1}^N (L_1^{1-\alpha} x_i^\alpha - p_i x_i)$$

- Which readily gives the inverse demand function from the FOC

$$\alpha L_1^{1-\alpha} x_i^{\alpha-1} = p_i(x_i) \quad (7)$$

- Each input x_i is supplied by a single firm that has monopoly power in the production of the intermediate good x_i .
- This monopoly power is assumed to be based on the ownership/renting of a patented design for the production of x_i .
- To simplify we assume that the firm producing intermediate output x_i can do so simply by converting a unit of capital/consumption good Y into x_i . In this sense, all x_i have the same production function as Y .
- This implies that the x_i is produced with a fixed unit price equal to the interest rate r .
- Thus, inserting for (7), each monopolist faces the profit maximizing problem

$$\pi = \max_{x_i} p(x_i)x_i - rx_i = \max_{x_i} \alpha L_1^{1-\alpha} x_i^{\alpha-1} x_i - rx_i$$

giving

$$\alpha L_1^{1-\alpha} x_i^{\alpha-1} = r/\alpha \quad (8)$$

or

$$p_i = r/\alpha \quad (9)$$

- Due to the symmetry of inputs x_i in (5), we must have $x_i = x$ and $p_i = p$ for all i in equilibrium.
- The profit for the monopolists producing intermediate inputs is hence

$$\pi = \frac{r}{\alpha}x - rx = \frac{1 - \alpha}{\alpha}rx > 0 \quad (10)$$

14.2.3 Research and development

- Designs for new intermediate goods are developed through research and development. The production of new ideas for designs is characterized by the deterministic production function

$$\dot{A} = BL_2A \quad (11)$$

where L_2 is the number of workers in R&D, satisfying the constraint $L_1 + L_2 = L$, where the total supply of labor is fixed.

- For notational convenience we measure A such that $A \equiv N$.
- Note that ideas, A , are partially excludable. Inventors own the patent for the designs on how to produce x_i , but the ideas underlying each design is non-excludable in the production of new ideas and designs. Hence, A is a public good in (11).
- We do not need to be explicit as to who conduct the R&D activities (be it the monopolistic producers of intermediate goods or someone else) as long as the patent can be rented.
- The value V_a of inventing one new design is thus equal to the net present value of receiving the profit in (10) in all future periods, or

$$V_a = \sum_{t=0}^{\infty} \frac{1}{(1+r)^t} \pi = \frac{\pi}{r} = \frac{(1-\alpha)}{\alpha}x \quad (12)$$

- One more unit of labor devoted to research gives rise to BA more new ideas, each having value V_a . Hence, the marginal product of labor in the R&D sector is BAV_a .
- Equating the marginal products of labor in the two sectors we then have (remember that $A = N$)

$$BAV_a = (1-\alpha)L_1^{-\alpha}x^\alpha A \implies V_a = \frac{1-\alpha}{B}L_1^{-\alpha}x^\alpha$$

which together with (8) and (12) gives

$$r = B\alpha L_1 \tag{13}$$

i.e. the interest rate as a function of L_1 .

14.2.4 Consumers

- To close the model we need to add the consumer side. We adopt the usual case of intertemporal utility maximization for given market prices (w and r), using a CRRA functional form.
- Note that we can as usual define an accounting stock $K = \sum_{i=1}^N x_i$ (since x_i are produced by converting one unit of capital). In equilibrium $x_i = x$ for all i , so

$$K = K = \sum_{i=1}^N x_i = Nx = Ax$$

- For simplicity we neglect depreciation, so

$$\dot{K}(t) = Y(t) - C(t)$$

- We skip the details of familiar utility maximization problem, turning directly to the Euler equation

$$\frac{\dot{C}}{C} = \frac{r - \rho}{\theta} \tag{14}$$

where ρ and θ are the now common parameters of the life-time utility expression.

14.2.5 The balanced growth path

- We will now show that we get a balanced growth path where r and L_1 are constant.
- We do not solve for the transitional dynamics because this is difficult, and not all that interesting.
- Start by assuming that r is constant. Then L_1 is also constant according to (13), and hence x is also constant according to (8). It then follows from (6) that the growth rate of Y is the same as the growth rate of A (or N).

- Further, since $K = Ax$ and x is constant K also grows at the same constant rate as A . Hence also K/Y is constant.
- It then follows that

$$\frac{C}{Y} = \frac{Y - \dot{K}}{Y} = 1 - \frac{\dot{K}}{K} \frac{K}{Y}$$

is constant since \dot{K}/K and K/Y are both constant.

- We then finally have

$$\gamma = \frac{\dot{C}}{C} = \frac{\dot{Y}}{Y} = \frac{\dot{K}}{K} = \frac{\dot{A}}{A} = BL_2 = BL - BL_1 = BL - \frac{r}{\alpha} \quad (15)$$

- Thus, the existence of this balanced growth path depends on r being constant. But the constancy of r was what we assumed at the outset and showed that it implied this balanced growth path. That there indeed is consistency between these two is seen by setting (14) and (15) equal which gives

$$r = \frac{\alpha}{\alpha + \theta} (\theta BL + \rho)$$

i.e. a constant as required.

- Thus, there exists a balanced growth path

$$\gamma = \frac{\dot{C}}{C} = \frac{\dot{Y}}{Y} = \frac{\dot{K}}{K} = \frac{\dot{A}}{A} = \frac{\alpha BL - \rho}{\alpha + \theta} \quad (16)$$

- Note the following about the steady state growth rate:
 1. There is positive and constant growth in equilibrium (The model is misbehaved if $\alpha BL < \rho$).
 2. This growth is driven by private agents doing research ($L_1 > 0$), so we have endogenized technological progress.
 3. The growth depends on the willingness to save reflected by ρ and θ . Hence there is room for policy to affect the long run growth rate.
 4. The growth rate increases in population L . Hence there is a scale effect.
 5. It can be shown that the two last properties relies heavily on the properties of the R&D production function (11), or more specifically from \dot{A} being linear in A .

14.3 Implications for policy

- We now look at the allocation a social planner would have chosen in the setting with an increasing variety of intermediate inputs.
- Since the aggregate capital indicator X is concave in each x_i , and the price of each capital good x_i is one unit of foregone consumption, the social planner will always choose $x_i = x$ for all i .
- Hence production is given by

$$Y = L_1^{1-\alpha} N x^\alpha = L_1^{1-\alpha} A x^\alpha = A^{1-\alpha} L_1^{1-\alpha} K^\alpha$$

where the second equality follows from using the measurement convention $A = N$. In the third equality we have defined $K = N x = A x$ as the total amount of resources used on capital goods.

- This definition of the accounting stock K ('capital') has the following law of motion:

$$\dot{K} = Y - C = A^{1-\alpha} L_1^{1-\alpha} K^\alpha - C$$

- We now can formulate the problem of finding a socially optimal balanced growth path (i.e. we again ignore transitional dynamics) by solving the social planners problem:

$$\max \int_0^\infty \frac{C^{1-\theta}}{1-\theta} e^{-\rho t} dt \quad (17)$$

$$\dot{K} = A^{1-\alpha} L_1^{1-\alpha} K^\alpha - C \quad (18)$$

$$\dot{A} = B L_2 A \quad (19)$$

$$L_1 + L_2 \leq L \quad (20)$$

- Notice that (19) is the R&D production function.
- The control variables are C and L_2 , and there are two state variables K and A . The extension of optimal control theory to this two dimensional case is straightforward.
- The current-value Hamiltonian for the problem is

$$\mathcal{H} = \frac{C^{1-\theta}}{1-\theta} + p[A^{1-\alpha}(L - L_2)^{1-\alpha} K^\alpha - C] + q B L_2 A \quad (21)$$

- The co-state variables p and q must satisfy

$$\dot{p} = \rho p - \frac{\partial \mathcal{H}}{\partial K} \quad (22)$$

$$\dot{q} = \rho q - \frac{\partial \mathcal{H}}{\partial A} \quad (23)$$

- While we have the usual first order conditions for the control variables

$$\frac{\partial \mathcal{H}}{\partial C} = C^{-\theta} - p = 0 \quad (24)$$

$$\frac{\partial \mathcal{H}}{\partial L_2} = -(1 - \alpha)pA^{1-\alpha}(L - L_2)^{-\alpha}K^\alpha + qBA = 0 \quad (25)$$

- Since we are only looking at balanced growth path solutions we must have $\gamma^* = \frac{\dot{C}}{C} = \frac{\dot{A}}{A}$. Since (19) gives $\frac{\dot{A}}{A} = BL_2$ we have $\gamma^* = BL_2$ and for determining the common growth rate γ^* we need only to determine the share of labor used for research L_2 .
- Note that (dividing by A) (25) implies

$$(1 - \alpha)pA^{-\alpha}(L - L_2)^{-\alpha}K^\alpha = qB$$

and since

$$\frac{\partial \mathcal{H}}{\partial A} = (1 - \alpha)pA^{-\alpha}(L - L_2)^{1-\alpha}K^\alpha + qBL_2$$

we have

$$\frac{\partial \mathcal{H}}{\partial A} = qB(L - L_2) + qBL_2 = qBL$$

- Joining this with (23) we have

$$\frac{\dot{q}}{q} = \rho - BL \quad (26)$$

- As usual (24) gives

$$-\theta\left(\frac{\dot{C}}{C}\right) = \frac{\dot{p}}{p}$$

- Since on a balanced growth path we have $\frac{\dot{C}}{C} = \frac{\dot{A}}{A}$ and must also have $\frac{\dot{q}}{q} = \frac{\dot{p}}{p}$, this translates to

$$-\theta\left(\frac{\dot{A}}{A}\right) = \frac{\dot{q}}{q}$$

Inserting for $\frac{\dot{A}}{A} = BL_2$ and (26) we get

$$-\theta BL_2 = \rho - BL$$

which we can finally solve for L_2

$$L_2 = \frac{L}{\theta} - \frac{\rho}{B\theta} \quad (27)$$

which in turn implies that the growth rate on the balanced growth path is

$$\gamma^* = \frac{\dot{C}}{C} = \frac{\dot{A}}{A} = \frac{BL - \rho}{\theta} \quad (28)$$

- Since $\alpha \in (0, 1)$ it follows that the growth rate in the decentralized solution we found last time (γ) falls short of the growth rate in the socially optimal solution (γ^*). This, of course, follows from too little research being done in the decentralized solution.
- There are two reasons why there is too little research in the decentralized solution
 1. There are positive externalities in research. The private agents do not take into consideration that new inventions also makes it more easy to come up with yet newer inventions (that is, by doing research they contribute to A which is a public good in the R&D production function (19)).
 2. There is monopolistic price setting for the intermediate goods. Thus, mark-up pricing forces a wedge between the marginal social product of a new invention/design and its market compensation. Thus, the producer which maximizes the monopoly profit does not have as strong incentives to come up with a new design as does a social planner maximizing the social surplus.
- Note that market power is necessary for there to be private incentives to invest, but that monopoly pricing gives rise to sub-optimal growth.

15 Growth through increasing quality

Required reading: Aghion and Howitt (1998), Chapter 2.1–2.3.

15.1 The Schumpeterian perspective

- Rather than assuming monopolistic competition in a variety of intermediate inputs, we now look at a case where innovations lead to new and better intermediate inputs which replace the old ones.
- As long as the good is the best one available, its innovator has a monopoly for sales of inputs (enforced by a patent). But since the new good only survives until it itself is replaced, a new innovation only gives a temporary monopoly.
- The profit enjoyed while in the monopoly position secures incentives for doing R&D.
- This framework reflects Schumpeter's ideas about creative destruction. New innovations are creative in increasing productivity. However, they are destructive in removing the ground for older products. But then again the fact that one can benefit from a monopoly position after having destroyed ones competitors is what induces the creative behavior in the first place.

15.2 A simple model without capital accumulation

- We now formulate a simple model of Aghion and Howitt (1998).
- Output of the consumption good per unit of calendar time is given by

$$Y_t = A_t x_t^\alpha \quad (29)$$

where x_t is the amount used of the newest intermediate input and is the only input to production.

- Note that the index t does not denote calendar time but keeps track of the number of innovations.
- There is a given stock of labor L which is either used to do research (n) or to produce x . One unit of labor produces one unit of the intermediate input. Thus we have labor market clearing given by

$$L = n + x \quad (30)$$

- When n units of labor are used in research this gives an expected λn new innovations per unit of calendar time. A new innovation

gives rise to a new intermediate input x_{t+1} with a higher productivity $A_{t+1} = \gamma A_t$, where $\gamma > 1$ is a parameter characterizing the magnitude of innovations.

- The most essential relation of the model is the arbitrage equation

$$w_t = \lambda V_{t+1} \quad (31)$$

This says that labor must have the same expected value in its two uses. It can be used in producing the intermediate good giving the wage w_t per hour, or to do research which gives an expected λ new innovations per extra hour. Each new innovation leads to the creation of a monopoly market for the new intermediate input x_{t+1} which gives rise to an income stream with net present value V_{t+1} .

- Since we do neglect capital accumulation in the model the interest rate, r , is exogenous.
- The discounted value V_{t+1} must then obey

$$rV_{t+1} = \pi_{t+1} - \lambda n_{t+1} V_{t+1} \quad (32)$$

A patent on good number $t + 1$ is an asset with expected return rV_{t+1} per unit calendar time. On the right hand side we have the profit from production and monopoly sales of x_{t+1} given by π_{t+1} per unit calendar time. At some future point the asset will become worthless, since the good is obsolete after a yet newer innovation. This occurs with probability λn_{t+1} per unit of calendar time, so $\lambda n_{t+1} V_{t+1}$ is the expected capital loss per unit of calendar time. Equality between the two sides is thus a standard asset equation.

- Equation (32) can be reorganized to

$$V_{t+1} = \frac{\pi_{t+1}}{r + \lambda n_{t+1}} \quad (33)$$

that is, the income stream is discounted by a factor which is larger than the interest rate r because at some future time (characterized by the probability λn_{t+1}) the income stream will stop.

- We now have to determine the profit flow π_t and the allocation of labor x_t . This is found from the profit maximization of the monopolist producer of the intermediate good

$$\pi_t = \max_{x_t} [p_t(x_t)x_t - w_t x_t]$$

- Since the final good sector is competitive it follows that the price of x_t will equal its marginal product in production of Y , giving an inverse demand function for the input x_t

$$p_t(x_t) = A_t \alpha x_t^{\alpha-1}$$

Note that α is the elasticity of this demand curve, thus also characterizing the market power of the intermediate monopolist.

- The monopolist then chooses

$$x_t = \left(\frac{\alpha^2}{w_t/A_t} \right)^{1/(1-\alpha)} \quad (34)$$

and hence

$$\pi_t = \left(\frac{1}{\alpha} - 1 \right) w_t x_t = A_t \tilde{\pi}(\omega_t) \quad (35)$$

where $\omega_t = w_t/A_t$ is the productivity adjusted wage rate, and $\tilde{\pi}(\omega_t) \equiv (1/\alpha - 1)A_t \alpha^{2/(1-\alpha)} \omega_t^{-\alpha/(1-\alpha)}$, so $\tilde{\pi}'(\omega_t) < 0$. Note from (34) that also $x_t'(\omega_t) < 0$.

- Inserting in (33) and rearranging (remember that $A_{t+1} = \gamma A_t$) we get

$$\omega_t = \lambda \frac{\gamma \tilde{\pi}(\omega_{t+1})}{r + \lambda n_{t+1}} \quad (36)$$

which together with equilibrium in the labor market

$$L = n_t + \tilde{x}(\omega_t) \quad (37)$$

determine the model by determining the paths of ω_t and n_t .

- We focus on the steady state where $\omega_t = \omega$ and $n_t = n$ for all t . Since (36) is downward sloping and (37) upward sloping in the (n, ω) -plane we can determine a unique \hat{n} .

- From straightforward algebra it follows that the steady state value \hat{n} satisfies

$$1 = \lambda \frac{\gamma^{\frac{1-\alpha}{\alpha}} (L - \hat{n})}{r + \lambda \hat{n}} \quad (38)$$

- Notice that the amount of research (n) determines the growth. Aghion and Howitt (1998) (Section 2.2.2).

15.3 Incentives for R&D and implications for policy

- We have the following results
 1. An decrease in the interest rate raises the benefit to research (future income more valuable) and raises \hat{n} .
 2. An increase in available labor reduces the wage and hence the marginal cost of research as well as raising the expected monopoly profit (increased demand).
 3. The more likely are new innovations the less costly is research. But the less worth is a new innovation because it increases creative destruction. As specified here, the former effect is dominant so an increased λ leads to a rise in \hat{n} .
 4. Increasing the size of each innovation (γ) raises the next intervals monopoly profit relative to today's productivity and hence raises the incentives to do research.
 5. The amount of research decreases in the elasticity of the demand curve facing the monopolist (α). So product market competition is bad for growth.
- When comparing the decentralized solution with that chosen by a social planner there are three effects.
 1. *The intertemporal spillover effect.* When evaluating the stream of future profit from a new innovation a private agent does not take into consideration that all future innovations are derived by standing on his shoulders. Hence, by having pushed the technology one step forward future monopolists are enabled to earn a yet higher profit with the yet better technology. That is, the private agent discounts π_t too heavily. This effects lead to too little private research.

2. *The appropriability effect.* As in the Romer model, the private monopolist can not appropriate the entire increase in social surplus from the new intermediate good and hence has too weak incentives for research.
 3. *The business-stealing effect.* The private firm does not internalize that the monopolist it replaces loses its entire profit. This leads to too strong incentives for research.
- Which effect dominates is an empirical question. However, this model has an important difference from the Romer model in that it opens up the possibility that laissez-faire growth can be too high.

References

- [1] **Aghion, P., Howitt, P.** *Endogenous growth theory*, Cambridge and London: MIT Press, 1998, Chapter 2.
- [2] **Romer, Paul M.** Endogenous Technological Change, *Journal of Political Economy*, October 1990, 98 (5) . Pp. 71-102. Part 2.