ECON 4350: Growth and Investment
Excercises for seminar 3

Spring 2007

Discussion topic 1

There is not a clear-cut answer to the following questions. You should, however, think about them and increase your awareness to these issues.

1. Why is it more plausible that we can experience increasing returns to scale when we adopt a broader concept of capital that also incorporates human capital. Try finding some examples.

2. Assume both consumption, C, physical capital K and human capital H are produced from the inputs K, H and raw-labor L. Suppose that the production function for human-capital was

   $$H = K^{\alpha_h} H^{\eta_h} L^{1-\alpha_h-\eta_h}$$

   differed from that characterizing production of physical capital (out of Y)

   $$Y = K^{\alpha_y} H^{\eta_y} L^{1-\alpha_y-\eta_y}$$

   What do you think are reasonable assumptions concerning the factor-intensities in the two production functions (i.e. $\alpha_h, \eta_h, \alpha_y$ and η_y)?

Convergence

The following exercise takes you through various manipulations of the Solow-model in order to derive expressions that we will need when we turn to empirical studies of convergence.

1. Prove the following rule: If GDP per capita grows at g per cent per year, then it will double in approximately $70/g$ years.

2. Show that if

   $$\dot{z(t)} \equiv \frac{dz(t)}{dt} = \beta(z^* - z(t)) \tag{1}$$

then

$$z(t) - z(0) = (1 - e^{-\beta t})z^* + (1 - e^{-\beta t})z(0) \tag{2}$$

Explain the role of β.

3. Assume instead that

$$\dot{z(t)} \equiv \frac{dz(t)}{dt} = H(z(t))$$

where $H(z^*) = 0$. Under what conditions does the following approximation make sense?

$$\dot{z(t)} \equiv \frac{dz(t)}{dt} \simeq -H'(z^*)(z^* - z(t)) \tag{3}$$

4. In the text-book Solow model, Show that

$$\frac{d\ln(\hat{k})}{dt} = \frac{sf(e^{\ln \hat{k}})}{e^{\ln \hat{k}}} - (n + x + \delta) \equiv H(\ln \hat{k})$$

Where the last equality defines the function $H(\ln \hat{k})$.

5. Why is $H(\ln \hat{k}^*) = 0$? Show that

$$H'(\ln \hat{k}^*) = (1 - \alpha^*)(n + x + \delta) \equiv \beta$$

where $\alpha^* = f'(\hat{k}^*)\hat{k}^*/f(\hat{k}^*)$, i.e. the capital elasticity at $\hat{k}^*$, and the last equality defines the constant β.

6. Discuss the roles of the different parameters in β.

7. Find an estimate of the time it takes for $\ln(\hat{k})$ to go halfway to $\ln(\hat{k}^*)$.

8. Show that

$$\frac{d\ln(\hat{y}^*}{dt} = \alpha^* \frac{d\ln(\hat{k}^*)}{dt}$$

and that around the steady state we have approximately

$$\ln(\hat{y}) - \ln(\hat{y}^*) \simeq \alpha^*(\ln(\hat{k}) - \ln(\hat{k}^*))$$

9. Use these result to conclude that we use the approximation/linearization

$$\frac{d\ln(\hat{y}(t))}{dt} = \beta(\ln(\hat{y}^*) - \ln(\hat{y}(t))) \tag{4}$$

10. (This question is tedious, but straightforward. Only do it if you find the time) Show that in the augmented Solow-model with a CD-production function

$$\beta = (1 - \alpha - \eta)(n + x + \delta) \tag{5}$$

(Hint: Define the function $\frac{d\ln(\hat{y}(t))}{dt} = H(\ln \hat{k}, \ln \hat{h})$ and linearize around the steady-state).

11. Use these results to derive equation (16) in MRW (Mind the differences in notation).

12. Why was it convenient to choose $z = \ln k$, instead of $z = k$, when doing the approximation (3)?

13. If we are to use MRW (16) as a framework for regression, is it legitimate to set β equal across countries?