Budget Constraint, Preferences and Utility

Varian: Intermediate Microeconomics, 8e, Chapters 2, 3 and 4

Consumer Theory

Consumers choose the best bundles of goods they can afford.

- This is virtually the entire theory in a nutshell.
- But this theory has many surprising consequences.

Two parts to consumer theory

- "can afford" – **budget constraint**
- "best" – according to consumers' **preferences**

Consumer Theory (cont´d)

What do we want to do with the theory?

- Test it. See if it is adequate to describe consumer behavior.
- Predict how behavior changes as economic environment changes.
- Use observed behavior to estimate underlying values.

These values can be used for

- cost-benefit analysis,
- predicting impact of some policy.

Budget Constraint

The first part of the lecture explains

- what is the budget constraint and the budget line,
- how changes in income and prices affect the budget line,
- how taxes, subsidies and rationing affect the budget line.

Consumption Bundle

For goods 1 and 2, the consumption bundle (x_1, x_2) shows how much of each good is consumed.

Suppose that we can observe

- the prices of the two goods (p_1, p_2)
- and the amount of money the consumer has to spend m (income).

The **budget constraint** can be written as $p_1 x_1 + p_2 x_2 \leq m$.

The *affordable* consumption bundles are bundles that don't cost more than income.

The set of affordable consumption bundles is **budget set** of the consumer.

Two Goods

Theory works with more than two goods, but can't draw pictures.

We often think of good 2 (say) as a **composite good**, representing money to spend on other goods.

Budget constraint becomes $p_1 x_1 + x_2 \leq m$.

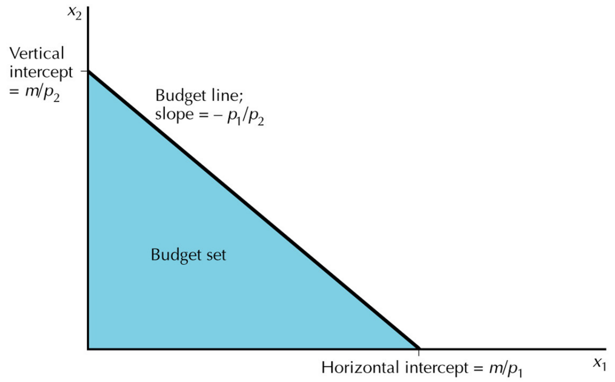
Money spent on good 1 ($p_1 x_1$) plus the money spent on good 2 (x_2) has to be less than or equal to the available income (m).

Budget Line

Budget line is $p_1x_1 + p_2x_2 = m$. It can be also written as

$$x_2 = \frac{m}{p_2} - \frac{p_1}{p_2}x_1.$$

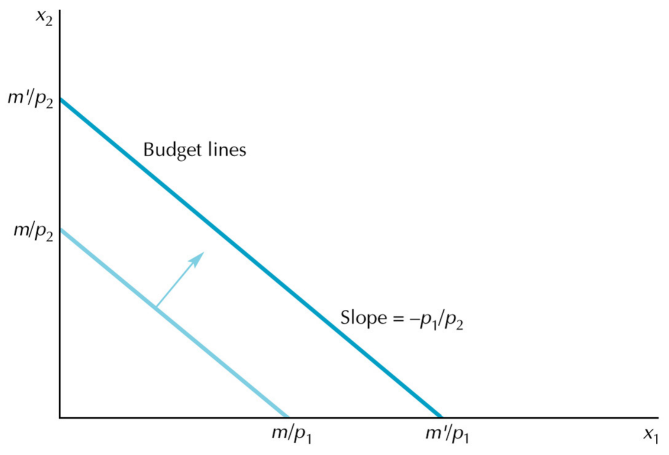
Slope of budget line = opportunity cost of good 1.



Change in Income

Increasing m makes parallel shift out.

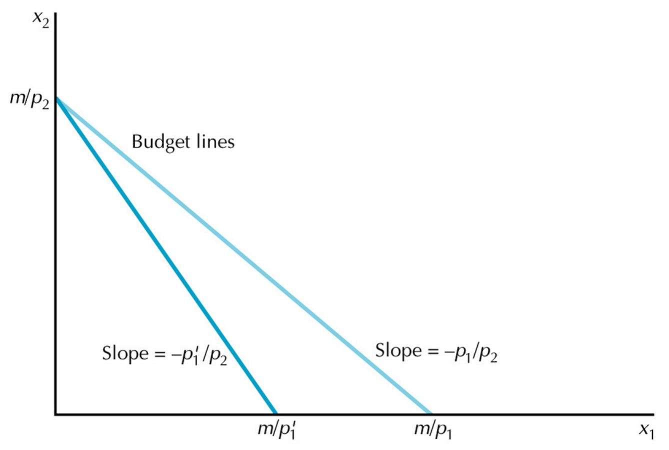
The vertical intercept increases and the slope remains the same.



Change in One Price

Increasing p_1 makes the budget line steeper.

The vertical intercept remains the same and the slope changes.



Changes in More Variables

Multiplying all prices by t is just like dividing income by t:

$$tp_1x_1 + tp_2x_2 = m \iff p_1x_1 + p_2x_2 = \frac{m}{t}.$$

Multiplying all prices and income by t doesn't change budget line:

$$tp_1x_1 + tp_2x_2 = tm \iff p_1x_1 + p_2x_2 = m.$$

A perfectly balanced inflation doesn't change consumption possibilities.

Numeraire

We can arbitrarily assign one price or income a value of 1 and adjust the other variables so as to describe the same budget set.

Budget line: $p_1x_1 + p_2x_2 = m$
The same budget line for $p_2 = 1$:

$$\frac{p_1}{p_2}x_1 + x_2 = \frac{m}{p_2}.$$

The same budget line for $m = 1$:

$$\frac{p_1}{m}x_1 + \frac{p_2}{m}x_2 = 1.$$

The price adjusted to 1 is called the **numeraire** price.

Useful when measuring relative prices; e.g. English pounds per dollar, 1987 dollars versus 1974 dollars, etc.

Taxes

Three types of taxes:

- **quantity** tax – consumer pays amount t for each unit she purchases.
  $\rightarrow$ Price of good 1 increases to $p_1 + t$.
- **value tax** (or *ad valorem* tax) – consumer pays a proportion of the price τ.
  $\rightarrow$ Price of good 1 increases to $p_1 + \tau p_1 = (1 + \tau)p_1$.
- **lump-sum** tax – amount of tax is independent of the consumer's choices.
  $\rightarrow$ The income of consumer decreases by the amount of the tax.

Subsidies

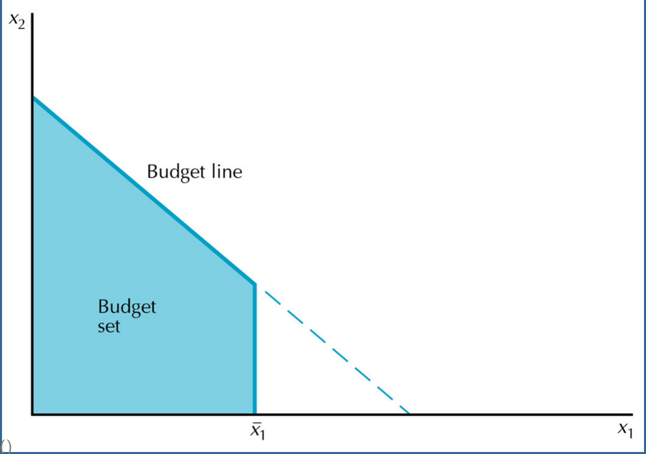
Subsidies – opposite effect than the taxes

- quantity subsidy of s on good 1
  $\rightarrow$ Price price of good 1 decreases to $p_1 - s$.
- *ad valorem* subsidy at a rate of σ on good 1
  $\rightarrow$ Price price of good 1 decreases to $p_1 - \sigma p_1 = (1 - \sigma)p_1$.
- lump-sum subsidy
  $\rightarrow$ The income increases by the amount of the subsidy.

Rationing

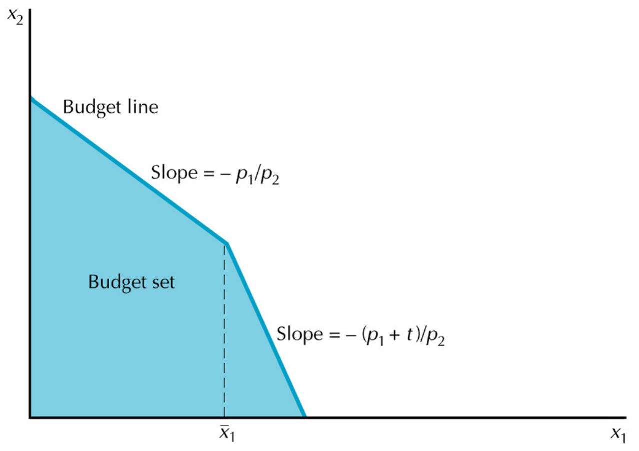
Rationing – can't consume more than a certain amount of some good.

Good 1 is rationed, no more than $\bar{x}$ units of good 1 can be consumed by any consumer.



Taxing Consumption Greater than $\bar{x}_1$

Taxed only consumption of good 1 in excess of $\bar{x}_1$, the budget line becomes steeper right of $\bar{x}_1$



The Food Stamp Program

Before 1979 was an ad valorem subsidy on food

- paid a certain amount of money to get food stamps which were worth more than they cost
- some rationing component — could only buy a maximum amount of food stamps

After 1979 got a straight lump-sum grant of food coupons. Not the same as a pure lump-sum grant since could only spend the coupons on food.

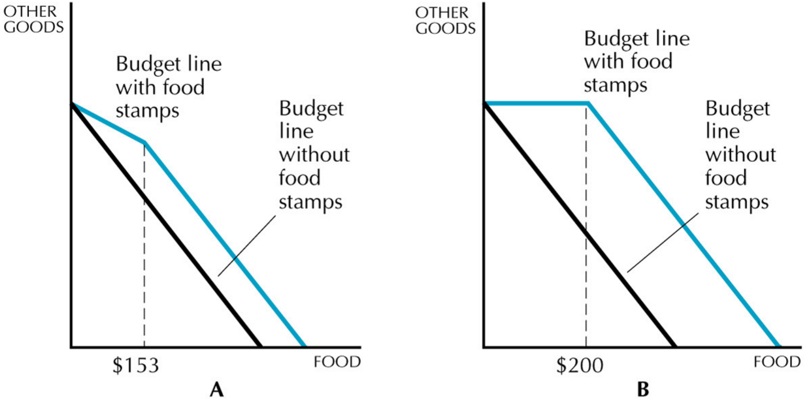


FIGURE 2.6 Food stamps

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Summary

- The budget set consists of bundles of goods that the consumer can afford at given prices and income. Typically assume only 2 goods – one of the goods might be composite good.
- The budget line can be written as $p_1x_1 + p_2x_2 = m$.
- Increasing income shifts the budget line outward. Increasing price of one good changes the slope of the budget line.
- Taxes, subsidies, and rationing change the position and slope of the budget line.

Preferences

The second part of the lecture explains

- what are consumer's preferences,
- what properties have well-behaved preferences,
- what is marginal rate of substitution.

Preferences - Introduction

Economic model of consumer behavior – people choose the best things they can afford

- up to now, we clarified "can afford"
- next, we deal with "best things"

Several observations about optimal choice from movements of budget lines

- perfectly balanced inflation doesn't change anybody's optimal choice
- after a rise of income, the same choices are available – consumer must be at least as well of as before

Preferences

Preferences are relationships between bundles.

- If a consumer chooses bundle (x_1, x_2) when (y_1, y_2) is available, then it is natural to say that bundle (x_1, x_2) is preferred to (y_1, y_2) by this consumer.
- Preferences have to do with the entire bundle of goods, not with individual goods.

Notation

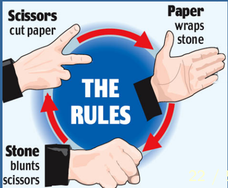
- $(x_1, x_2) \succ (y_1, y_2)$ means the x-bundle is strictly preferred to the y-bundle.
- $(x_1, x_2) \sim (y_1, y_2)$ means that the x-bundle is regarded as indifferent to the y-bundle.
- $(x_1, x_2) \succeq (y_1, y_2)$ means the x-bundle is at least as good as (or weakly preferred) the y-bundle.

Assumptions about Preferences

Assumptions about "consistency" of consumers' preferences:

- **Completeness** — any two bundles can be compared: $(x_1, x_2) \succeq (y_1, y_2)$, or $(x_1, x_2) \preceq (y_1, y_2)$, or both
- **Reflexivity** — any bundle is at least as good as itself: $(x_1, x_2) \succeq (x_1, x_2)$
- **Transitivity** — if the bundle X is at least as good as Y and Y at least as good as Z, then X is at least as good as Z: If $(x_1, x_2) \succeq (y_1, y_2)$, and $(y_1, y_2) \succeq (z_1, z_2)$, then $(x_1, x_2) \succeq (z_1, z_2)$

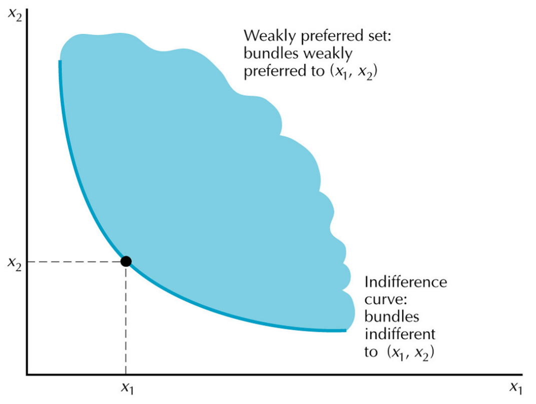
Transitivity necessary for theory of optimal choice. Otherwise, there could be a set of bundles for which there is no best choice.



Indifference Curves

Weakly preferred set are all consumption bundles that are weakly preffered to a bundle (x_1, x_2).

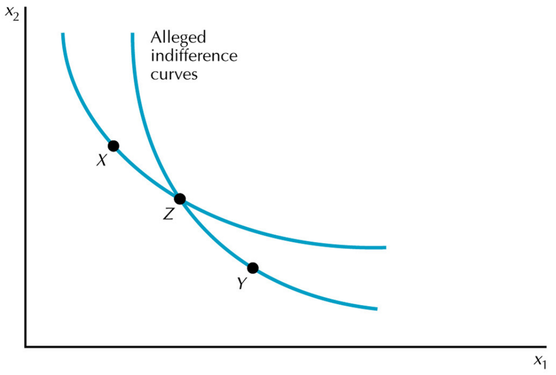
Indifference curve is formed by all consumption bundles for which the consumer is indifferent to (x_1, x_2) – like contour lines on a map.



Indifference Curves (cont'd)

Note that indifference curves describing two distinct levels of preference cannot cross.

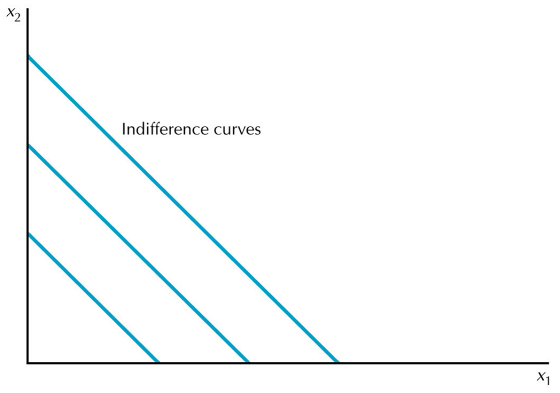
Proof — we know that $X \sim Z$ and $Z \sim Y$. Transitivity implies that $X \sim Y$. This contradicts the assumption that $X \succ Y$.



Examples: Perfect Substitutes

Perfect substitutes have constant rate of trade-off between the two goods; constant slope of the indifference curve (not necessarily -1).

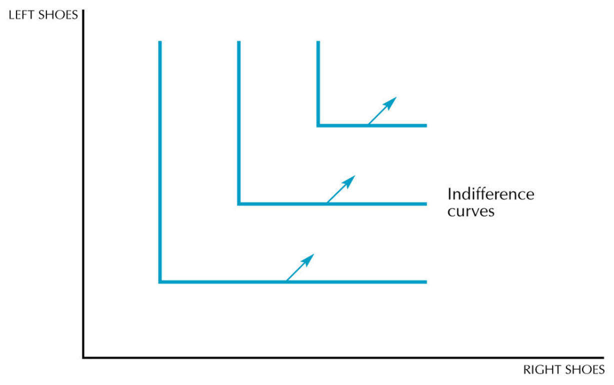
E.g. red pencils and blue pencils; pints and quarts.



Examples: Perfect Complements

Perfect complements are consumed in fixed proportion (not necessarily 1:1).

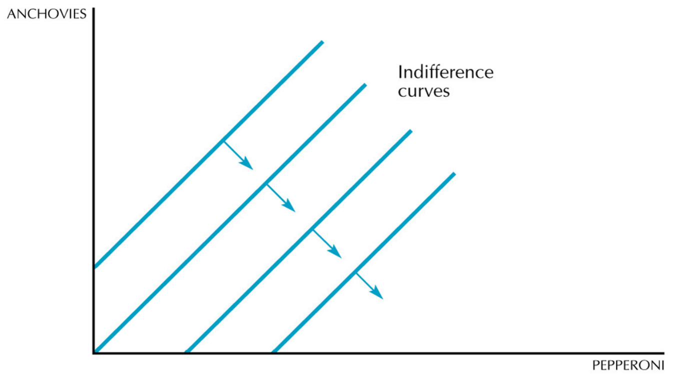
E.g. right shoes and left shoes; coffee and cream.



Examples: Bad Good

A **bad** is a commodity that the consumer doesn't like.

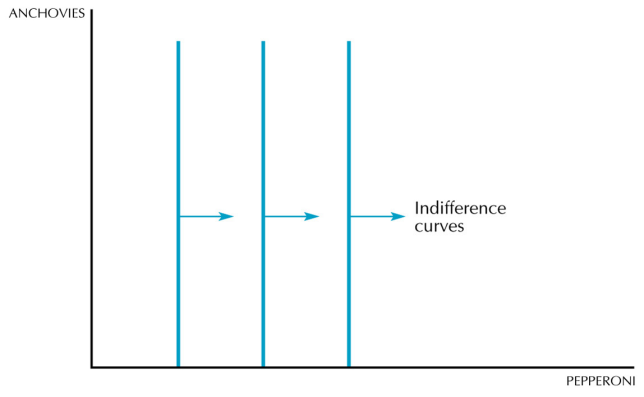
Suppose consumer is doesn't like anchovies and likes pepperoni.



Examples: Neutral Good

Consumer doesn't care about the **neutral good**.

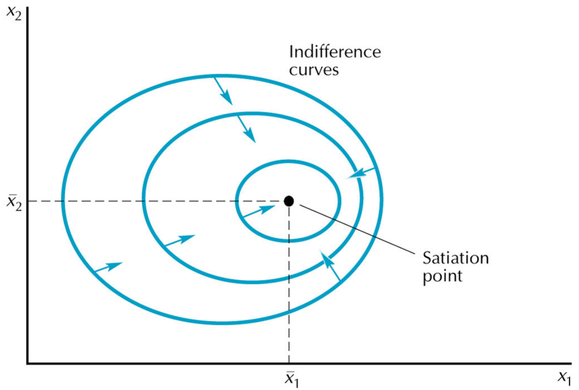
Suppose consumer is neutral about anchovies and likes pepperoni.



Examples: Satiation Point

Satiation or **bliss point** is the most preferred bundle $(\bar{x}_1, \bar{x}_2)$

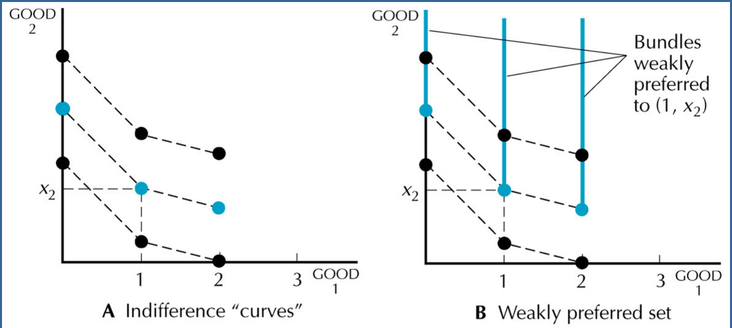
- When consumer has too much of good, it becomes a bad – reducing consumption of the good makes consumer better off.
- E.g. amount of *chocolate cake* and *ice cream* per week



Examples: Discrete Good

Discrete good is only available in integer amounts.

- Indiference "curves" – sets of discrete points; weakly preferred set – line segments.
- Important if consumer chooses only few units of the good per time period (e.g. cars).



Well-Behaved Preferences

Monotonicity – more is better (we have only goods, not bads) $\implies$ indifference curves have negative slope (see Figure 3.9):
If (y_1, y_2) has at least as much of both goods as (x_1, x_2) and more of one, then $(y_1, y_2) \succ (x_1, x_2)$.

Convexity – averages are preferred to extremes $\implies$ slope gets flatter as you move further to right (see Figure 3.10):
If $(x_1, x_2) \sim (y_1, y_2)$, then $(tx_1 + (1-t)y_1, tx_2 + (1-t)y_2) \succeq (x_1, x_2)$ for all $0 \leq t \leq 1$

- non convex preferences – olives and ice cream
- strict convexity – If the bundles $(x_1, x_2) \sim (y_1, y_2)$, then $(tx_1 + (1-t)y_1, tx_2 + (1-t)y_2) \succ (x_1, x_2)$ for all $0 \leq t \leq 1$

WELL-BEHAVED WOMEN SELDOM MAKE HISTORY
LAUREL THATCHER ULRICH

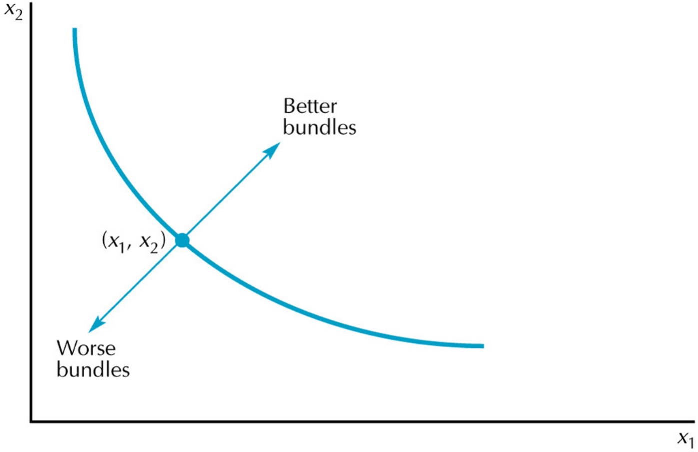


FIGURE 3.9 Monotonic preferences

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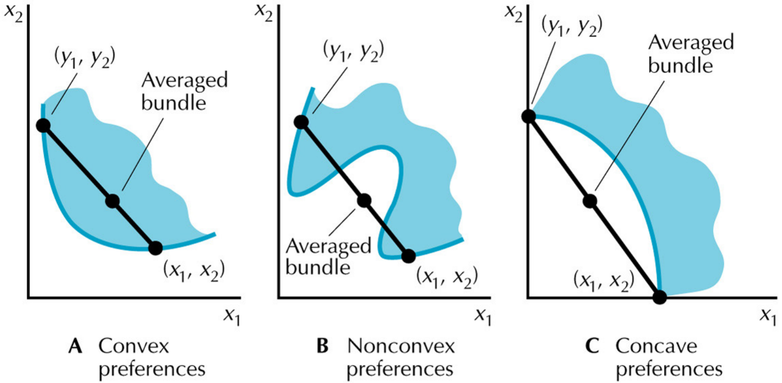

FIGURE 3.10 Various kinds of preferences

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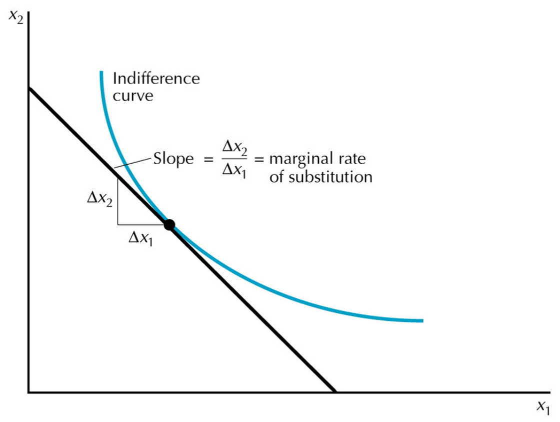
Marginal Rate of Substitution

Marginal rate of substitution (MRS) is the slope of the indifference curve: $MRS = \Delta x_2 / \Delta x_1 = dx_2 / dx_1$.

Sign problem — natural sign is negative, since indifference curves will generally have negative slope.



Marginal Rate of Substitution (cont'd)

MRS measures how the consumer is willing to trade off consumption of good 1 for consumption of good 2 (see Figure 3.12).

For strictly convex preferences, the indifference curves exhibit **diminishing marginal rate of sustitution**

Other interpretation: **marginal willingness to pay** – how much of good 2 is one willing to pay for a extra consumption of good 1.

If good 2 is a composite good, the willingness-to-pay interpretation is very natural.

Not the same as how much you have to pay.

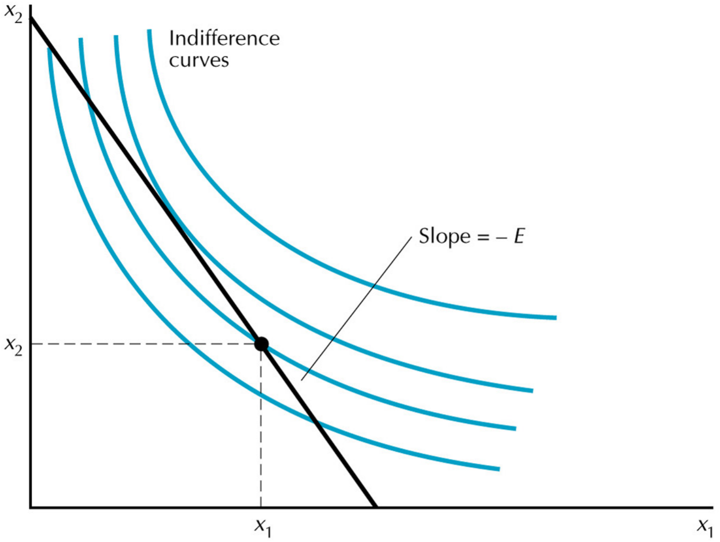


FIGURE 3.12 Trading at an exchange rate

Example: Slope of the Indifference Curve

1) Calculate the slope of the indifference curve $x_2 = 4/x_1$ at the point $(x_1, x_2) = (2, 2)$.

$$\text{Slope of the indifference curve} = \text{MRS} = \frac{dx_2}{dx_1} = \frac{-4}{x_1^2} = -1.$$

2) Calculate the slope of the indifference curve $x_2 = 10 - 6\sqrt{x_1}$ at the point $(x_1, x_2) = (4, 5)$.

$$\text{Slope of the indifference curve} = \text{MRS} = \frac{dx_2}{dx_1} = \frac{-3}{\sqrt{x_1}} = \frac{-3}{2}.$$

Summary

- Economists assume that a consumer can rank consumption bundles. The ranking describes the consumer's preferences.
- The preferences are assumed to be complete, reflexive and transitive.
- Well-behaved preferences are monotonic and convex.
- MRS measures the slope of the indifference curve. MRS can be interpreted as how much of good 2 is one willing to pay for an extra consumption of good 1.

Utility

The third part of the lecture explains

- what is utility,
- what is a utility function,
- what is a monotonic tranformation of a utility function,
- how can we use utility function to calculate MRS.

Utility

Two ways of viewing utility:

Old way - measures how “satisfied” you are (cardinal utility)

- not operational
- many other problems

New way - summarizes preferences, only the ordering of bundles counts (**ordinal utility**)

- operational
- gives a complete theory of demand

Ordinal Utility

A utility function assigns a number to each bundle of goods so that more preferred bundles get higher numbers.

If $(x_1, x_2) \succ (y_1, y_2)$, then $u(x_1, x_2) > u(y_1, y_2)$.

Three ways to assign utility that represent the same preferences:

Bundle	U_1	U_2	U_3
A	3	17	–1
B	2	10	–2
C	1	.002	–3

Utility Function is Not Unique

A **positive monotonic transformation** $f(u)$ is any increasing function.

Examples: $f(u) = 3u$, $f(u) = u + 3$, $f(u) = u^3$.

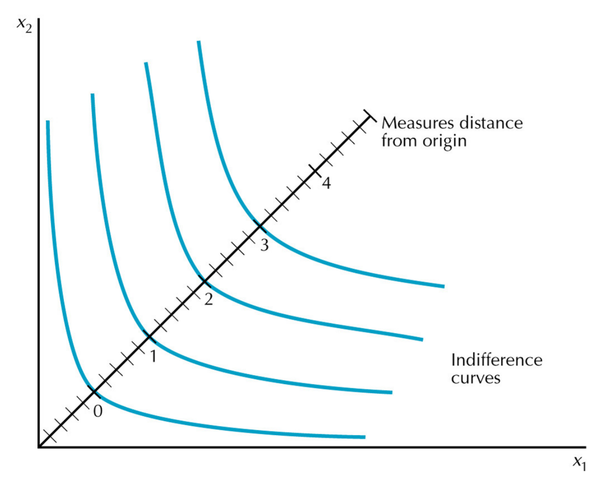
If $u(x_1, x_2)$ is a utility function that represents some preferences, then $f(u(x_1, x_2))$ represents the same preferences.

Why? Because $u(x_1, x_2) > u(y_1, y_2)$ only if $f(u(x_1, x_2)) > f(u(y_1, y_2))$.

$\Delta f(x) \geqslant 0$	Nondecreasing	
$\Delta f(x) \leqslant 0$	Nonincreasing	
$\Delta f(x) > 0$	Increasing	
$\Delta f(x) < 0$	Decreasing	

Constructing Utility Functions

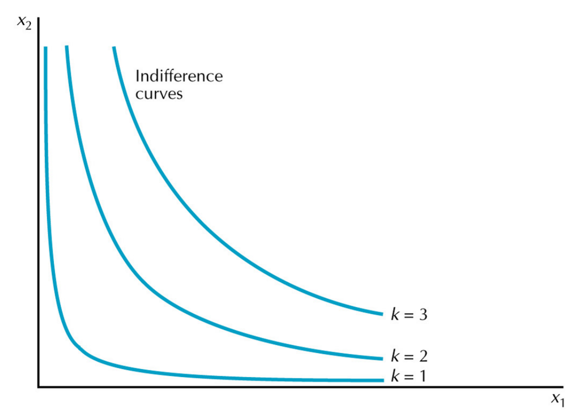
Mechanically using the indifference curves.



Examples: Utility to Indifference Curves

Easy — just plot all points where the utility is constant

Utility function $u(x_1, x_2) = x_1 x_2$;
Indifference curves: $k = x_1 x_2 \iff x_2 = \frac{k}{x_1}$



Examples: Indifference Curves to Utility

More difficult - given the preferences, what combination of goods describes the consumer's choices.

Perfect substitutes

- All that matters is total number of pencils, so $u(x_1, x_2) = x_1 + x_2$ does the trick.
- Can use any monotonic transformation of this as well, such as $\ln(x_1 + x_2)$.

Perfect complements

- What matters is the minimum of the left and right shoes you have, so $u(x_1, x_2) = \min\{x_1, x_2\}$ works.
- In general, if it not 1:1, the utility function is $u(x_1, x_2) = \min\{ax_1, bx_2\}$, where a and b are positive numbers.

Examples: Indifference Curves to Utility (cont'd)

Quasilinear preferences

- Indifference curves are vertically parallel (see Figure 4.4). Not particularly realistic, but easy to work with.
- Utility function has form $u(x_1, x_2) = v(x_1) + x_2$
- Specific examples: $u(x_1, x_2) = \sqrt{x_1} + x_2$ or $u(x_1, x_2) = \ln x_1 + x_2$

Cobb-Douglas preferences

- Simplest algebraic expression that generates well-behaved preferences.
- Utility function has form $u(x_1, x_2) = x_1^b x_2^c$ (See Figure 4.5).
- Convenient to take transformation $f(u) = u^{\frac{1}{b+c}}$ and write $x_1^{\frac{b}{b+c}} x_2^{\frac{c}{b+c}}$ or $x_1^a x_2^{1-a}$, where $a = b/(b+c)$.

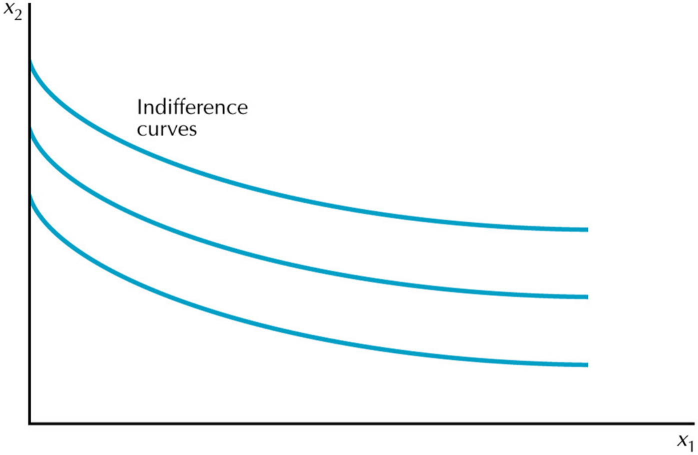


FIGURE 4.4 Quasilinear preferences

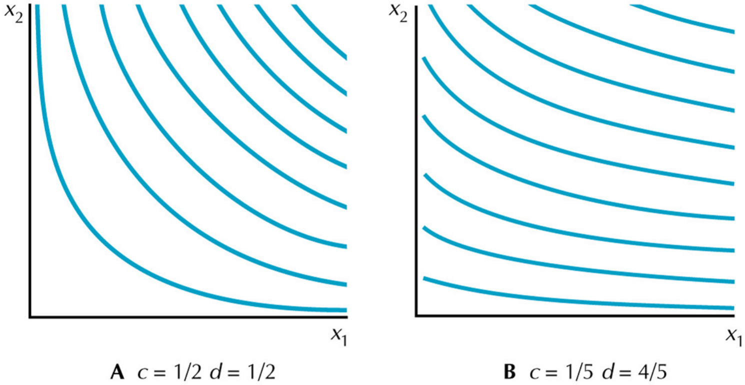


FIGURE 4.5 Cobb-Douglas indifference curves

Marginal Utility

Marginal utility (MU) is extra utility from some extra consumption of one of the goods, holding the other good fixed.

A *partial derivative* – this just means that you look at the derivative of $u(x_1, x_2)$ keeping x_2 fixed — treating it like a constant.
Examples:

- if $u(x_1, x_2) = x_1 + x_2$, then $MU_1 = \partial u / \partial x_1 = 1$
- if $u(x_1, x_2) = x_1^a x_2^{1-a}$, then $MU_1 = \partial u / \partial x_1 = a x_1^{a-1} x_2^{1-a}$

Note that marginal utility depends on which utility function you choose to represent preferences.

- If you multiply utility 2x, you multiply marginal utility 2x $\Longrightarrow$ it is not an operational concept.
- However, MU is closely related to MRS, which is an operational concept.

Relationship between MU and MRS

An indifference curve $u(x_1, x_2) = k$, where k is a constant.

We want to measure slope of indifference curve, the MRS.

So consider a change (Δx_1, Δx_2) that keeps utility constant. Then,

$$MU_1 \Delta x_1 + MU_2 \Delta x_2 = 0$$

$$\frac{\partial u}{\partial x_1} \Delta x_1 + \frac{\partial u}{\partial x_2} \Delta x_2 = 0.$$

Hence,

$$\frac{\Delta x_2}{\Delta x_1} = -\frac{MU_1}{MU_2}.$$

So we can compute MRS from knowing the utility function.

Example: Utility for Commuting

Question: Take a bus or take a car to work?

Each way of transport represents bundle of different characteristics: Let x_1 be the time of taking a car, y_1 be the time of taking a bus. Let x_2 be cost of car, etc.

Suppose utility function takes linear form $U(x_1, ..., x_n) = \beta_1 x_1 + ... + \beta_n x_n$.

We can observe a number of choices and use statistical tech- niques to estimate the parameters β_i that best describe choices.

Example: Utility for Commuting (con't)

Domenich a McFadden (1975) report a utility function

$$U(TE, TT, C) = -0.147TW - 0.0411TT - 2.24C,$$

where

TW = total walking time to and from bus or car in minutes
TT = total time of trip in minutes
C = total cost of trip in dollars.

Once we have the utility function we can do many things with it:

- Calculate the marginal rate of substitution between two characteristics. How much money would the average consumer give up in order to get a shorter travel time?
- Forecast consumer response to proposed changes.
- Estimate whether proposed change is worthwhile in a benefit-cost sense.

Summary

- A utility function is a way to represent a preference ordering. The numbers assigned to different utility levels have no intrinsic meaning.
- Any monotonic transformation of a utility function will represent the same preferences.
- The marginal rate of substitution is equal to MRS $= \Delta x_2/\Delta x_1 = -MU_1/MU_2$.