Choice and Revealed Preference

Varian, Intermediate Microeconomics, 8e, Ch. 5 and Sections 7.1–7.7

Choice

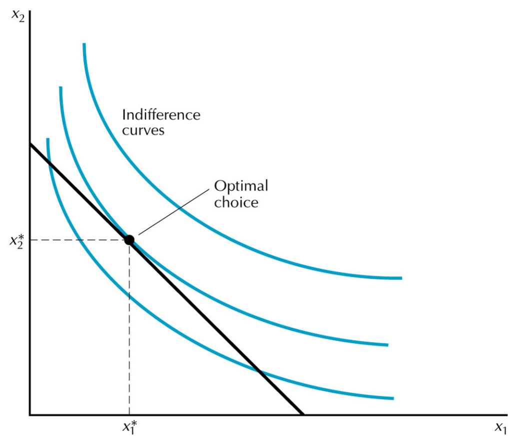
In this lecture explains:

- What will be the optimal choice of consumer.
- How this choice depends on the consumer's preferences.
- How can we estimate utility function from consumption choices we observe.
- Some of the implications of optimal choice.

Optimal Choice

The optimal choice is where the indifference curve is tangent to the budget line:

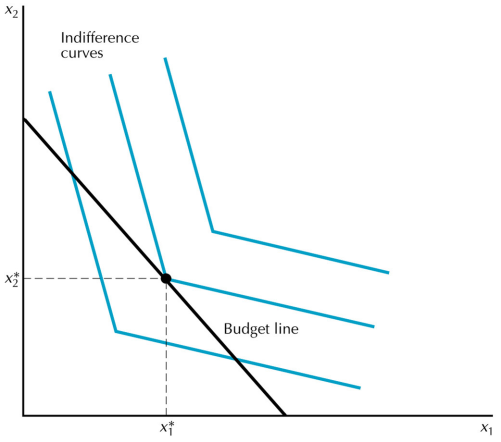
$$\text{MRS} = -\text{price ratio} = -\frac{p_1}{p_2}$$



Exceptions: Kinky Tastes

The IC doesn't cross the budget line, but there is no unique tangent line at the optimal point.

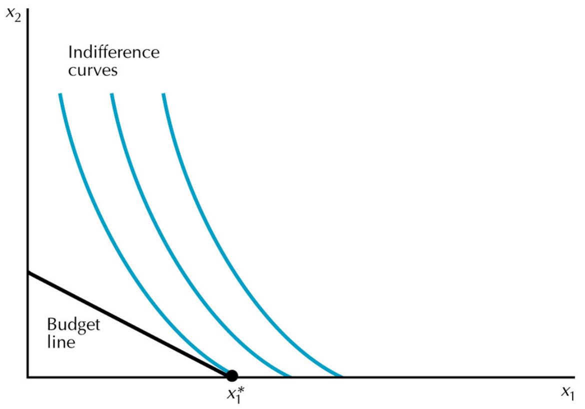
For tangency, we need smooth indifference curves.



Exceptions: Boundary Optimum

The IC doesn't cross the budget line, but the slopes are different.

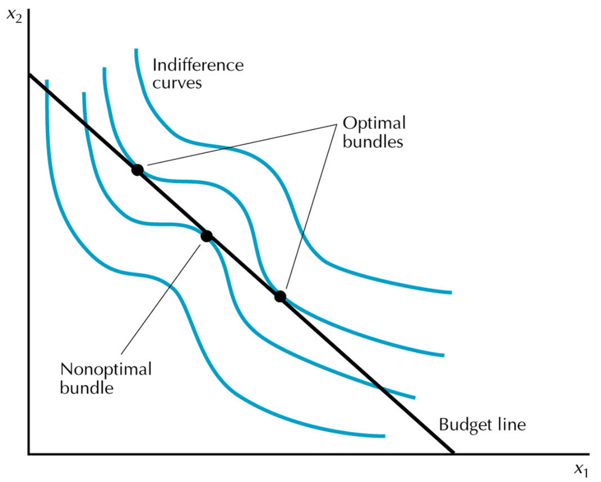
For tangency, we need an **interior optimum** = both goods are consumed.



Exceptions: More Than One Tangency

Tangency condition is only a *necessary* condition for optimal choice.

It is *sufficient* in the case of convex preferences.



Consumer Demand

Optimal choice is the consumer's **demanded bundle**.

As we vary prices and income, we get **demand functions**.

Demand functions will depend on both prices and income: $x_1(p_1, p_2, m)$ and $x_2(p_1, p_2, m)$.

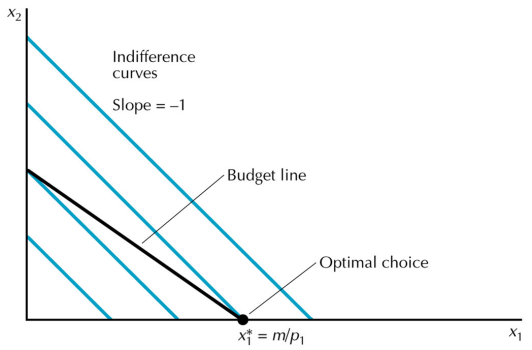
Different preferences lead to different demand functions

We want to study how the demanded bundle changes as price and income change.

Examples: Perfect Substitutes

If goods 1 and 2 are perfect substitutes (1:1 trade-off), the demand function for good 1 is

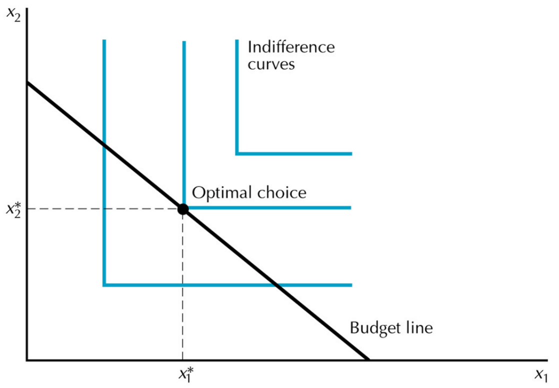
$$x_1 = \begin{cases} m/p_1 & \text{when } p_1 < p_2; \\ \text{any number between 0 and } m/p_1 & \text{when } p_1 = p_2; \\ 0 & \text{when } p_1 > p_2. \end{cases}$$



Examples: Perfect Complements

If goods 1 and 2 are perfect complements and consumer purchases the amount x of both goods (left and right shoe), the demand function can be derived from the budget constrant as follows:

$$p_1 x + p_2 x = m \iff x_1 = x_2 = x = \frac{m}{p_1 + p_2}.$$



Examples: Neutrals and Bads

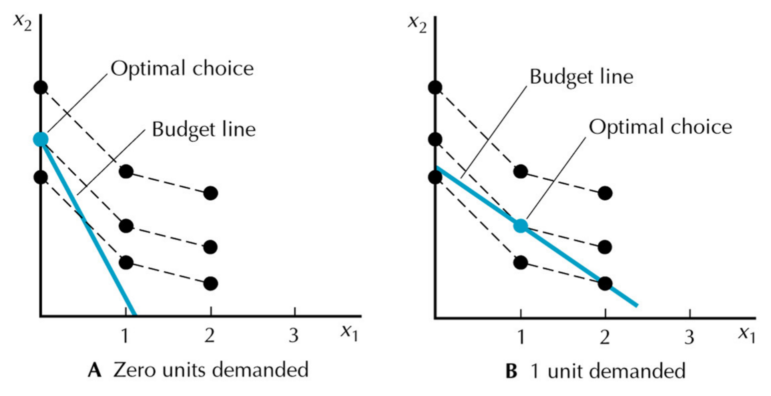
The consumer spends all money on the good and doesn't purchase the neutral good or the bad.

If commodity 1 is good and commodity 2 is neutral or bad, the demand function is

$$x_1 = \frac{m}{p_1} \text{ and } x_2 = 0.$$

Examples: Discrete Goods

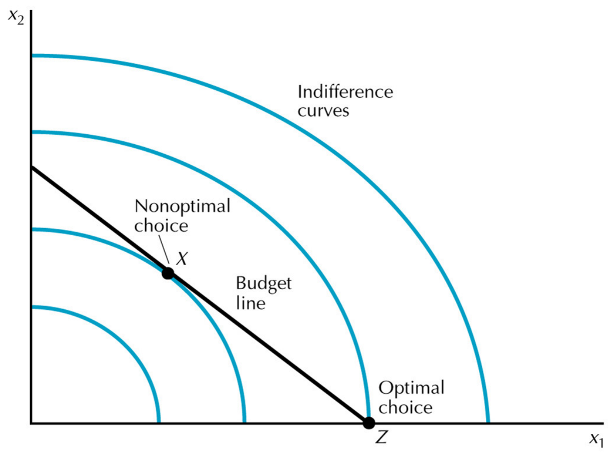
Good 1 is discrete good (integer units) and good 2 is money.
Consumption bundles: $(1, m - p_1)$, $(2, m - 2p_1)$, $(3, m - 3p_1)$, ...



Examples: Concave Preferences

Tangency doesn't work – like in the case of perfect substitutes.

E.g. olives and ice cream.



Examples: Cobb-Douglas Preferences

The Cobb-Douglas utility function is $u(x_1, x_2) = x_1^c x_2^d$.

It is convenient to use logs of the Cobb-Douglas utility function $u(x_1, x_2) = \ln(x_1^c x_2^d) = c \ln x_1 + d \ln x_2$.

The problem we want to solve is

$$\max\, c \ln x_1 + d \ln x_2 \quad \text{such that } p_1 x_1 + p_2 x_2 = m.$$

Using $MRS = -p_1/p_2$, we get two equations with two unknowns:

$$\frac{cx_2}{dx_1} = \frac{p_1}{p_2}, \text{ and } p_1 x_1 + p_2 x_2 = m.$$

Examples: Cobb-Douglas Preferences (cont'd)

The solution of the equations are the Cobb-Douglas demand functions

$$x_1 = \frac{c}{c+d}\frac{m}{p_1},\ x_2 = \frac{d}{c+d}\frac{m}{p_2}$$

Convenient property: The Cobb-Douglas consumer spends a fixed fraction of his income on each good:

$$\frac{p_1 x_1}{m} = \frac{p_1}{m}\frac{c}{c+d}\frac{m}{p_1} = \frac{c}{c+d}$$

$$\frac{p_2 x_2}{m} = \frac{p_2}{m}\frac{d}{c+d}\frac{m}{p_2} = \frac{d}{c+d}.$$

Convenient to use the Cobb-Douglas utility function

$$u(x_1, x_2) = x_1^a x_2^{1-a}$$

where the parameter a is the fraction of income spent on good 1.

Estimating Utility Function

Examine the consumption data and see if you can "fit" a utility function to it.

Year	p_1	p_2	m	x_1	x_2	s_1	s_2	Utility
1	1	1	100	25	75	.25	.75	57.0
2	1	2	100	24	38	.24	.76	33.9
3	2	1	100	13	74	.26	.74	47.9
4	1	2	200	48	76	.24	.76	67.8
5	2	1	200	25	150	.25	.75	95.8
6	1	4	400	100	75	.25	.75	80.6
7	4	1	400	24	304	.24	.76	161.1

The income shares (s_1, s_2) are more or less constant $\implies$ Cobb-Douglas utility function $u(x_1, x_2) = x_1^{1/4} x_2^{3/4}$ seems to fit these data well.

Estimating Utility Function (cont'd)

We can use the fitted utility function as guide to policy decisions.

Suppose a system of taxes that would result in prices (2,3) and the income of 200. At these prices, the demanded bundle is

$$x_1 = \frac{1}{4}\frac{200}{2} = 25$$

$$x_2 = \frac{3}{4}\frac{200}{3} = 50$$

The estimated utility of this bundle is $u(x_1, x_2) = 25^{1/4}50^{3/4} \approx 42$, which is more than in year 2 but less than in year 3.

In real life more complicated forms are used, but basic idea is the same.

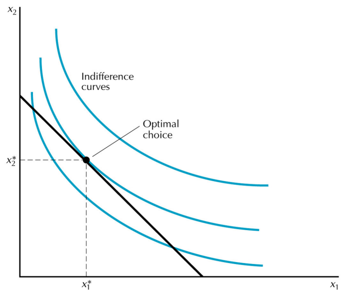
Implications of MRS Condition

Why do we care that MRS = −price ratio?

If everyone faces the same prices, then everyone has the same local trade-off between the two goods. This is independent of income and tastes.

Since everyone locally values the trade-off the same, we can make policy judgments. Is it worth sacrificing one good to get more of the other?

Prices serve as a guide to relative marginal valuations.



Application: Choosing a Tax

Which is better, a quantity tax or an income tax?

We can show an income tax is always better in the sense that given any quantity tax, there is an income tax that makes the consumer better off.

Application: Choosing a Tax (Outline of Argument)

Quantity tax

- original budget constraint: $p_1 x_1 + p_2 x_2 = m$
- budget constraint with tax: $(p_1 + t)x_1 + p_2 x_2 = m$
- optimal choice with tax: $(p_1 + t)x_1^* + p_2 x_2^* = m$
- tax revenue raised: tx_1^*.

Income tax that raises same amount of revenue leads to budget constraint: $p_1 x_1 + p_2 x_2 = m - tx_1^*$.

- This line has same slope as original budget line.
- also passes through (x_1^*, x_2^*) – proof: $p_1 x_1^* + p_2 x_2^* = m - tx_1^*$.
- this means that (x_1^*, x_2^*) is affordable under the income tax, so the optimal choice under the income tax must be even better than (x_1^*, x_2^*).

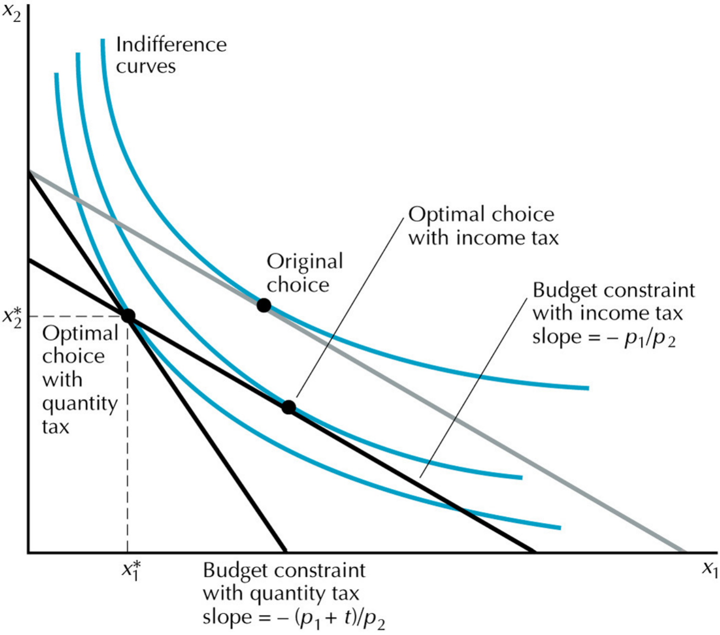


FIGURE 5.9 Income tax versus a quantity tax

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Application: Choosing a Tax (Caveats)

- Only applies for one consumer, no longer true if we have to have the same income tax for all consumers. A person that doesn't consume any of good 1 will certainly prefer the quantity tax to a uniform income tax.
- We have assumed that income is exogenous. If income responds to tax, e.g. tax dicourages people from earning income $\Longrightarrow$ problems.
- We left out supply response. A complete analysis would have to take supply response into account.

"It appears to be some sort of tax cut promise."

Application: The Cost of Christmas

Joel Valdfogel, "The Deadweight Loss of Christmas" (AER, 1993):

- „In the standard microeconomic framework of consumer choice, the best a gift-giver can do with, say, $10 is to duplicate the choice that the recipient would have made."(p. 1328)
- In most situations, the the recipient is worse off..
- Estimates cost of gift-giving using surveys given to Yale undergraduates.
- Gift-giving destroys between 10 % and 1/3 of the value of the gift: min. loss $4 billions.
- The worst gifts by extended family – also give money most often.

Summary

- The optimal choice is the bundle in the consumer's budget set that lies on the highest indifference curve.
- In optimum, the MRS is typically equal to the slope of budget line.
- We can estimate a utility function from consumption choices and use it to evaluate economic policies.
- If everyone faces the same prices of two goods, then everyone will have the same MRS between the two goods.

Revealed Preference

The second part of the lecture explains

- what does it mean if a bundle is revealed preferred to another,
- how can we recover preferences from the observed choices,
- what are the Weak and Strong Axioms of Revealed Preference.

The Motivation of Revealed Preference

Up until now we've started with preference and then described behavior. But in real life, preferences are not directly observable.

Revealed preference is "working backwards" — start with behavior and describe preferences.

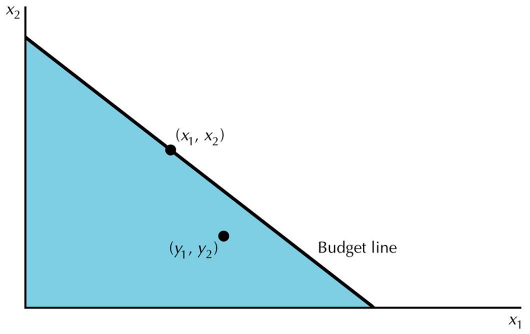
If we want to "recover" preferences from behavior of people, we have to assume that preferences don't change over time.

In this lecture, we also assume that the preferences are strictly convex – we get a *unique* demanded bundle (not necessary for the theory of revealed preferences, simplifies the exposition).

The Idea of Revealed Preference

If (x_1, x_2) is chosen when (y_1, y_2) is affordable, then we know that (x_1, x_2) is better than (y_1, y_2).

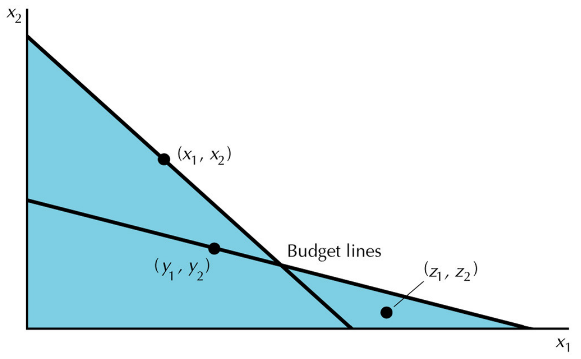
In equations: If (x_1, x_2) is chosen when prices are (p_1, p_2) and (y_1, y_2) is some other bundle such that $p_1 x_1 + p_2 x_2 \geq p_1 y_1 + p_2 y_2$, then if the consumer is choosing the most preferred bundle she can afford, we must have $(x_1, x_2) \succ (y_1, y_2)$.



The Idea of Revealed Preference (cont'd)

If $p_1 x_1 + p_2 x_2 \geq p_1 y_1 + p_2 y_2$, we say that (x_1, x_2) is **directly revealed preferred** to (y_1, y_2).

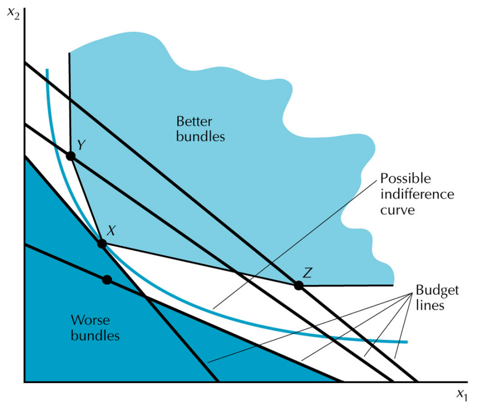
If X is revealed preferred to Y, and Y is revealed preferred to Z, then transitivity implies that X is **indirectly revealed preferred** to Z.



The Idea of Revealed Preference (cont'd)

The more choices, the more information about the consumer's preferences.

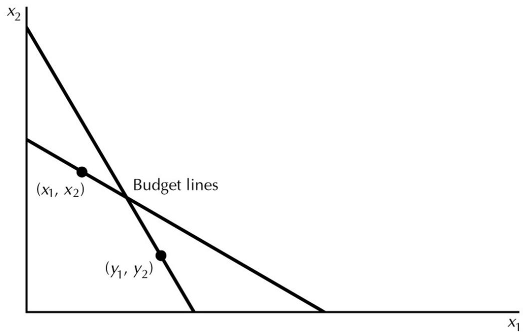
The figure uses the demanded bundles for recovering preferences – the indifference curve lies between the two shaded areas.



Choices Inconsistent with the Model of Consumer Choice

In the figure, (x_1, x_2) is directly revealed preferred to (y_1, y_2) and (y_1, y_2) is directly revealed preferred to (x_1, x_2).

Or for a bundle (x_1, x_2) purchased at prices (p_1, p_2) and a different bundle (y_1, y_2) purchased at prices (q_1, q_2), we have $p_1x_1 + p_2x_2 > p_1y_1 + p_2y_2$ and $q_1y_1 + q_2y_2 > q_1x_1 + q_2x_2$.



Weak Axiom of Revealed Preference

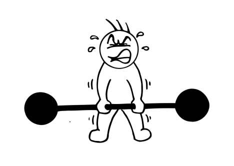
Weak Axiom of Revealed Preference (WARP): If (x_1, x_2) is directly revealed preferred to (y_1, y_2), then (y_1, y_2) cannot be directly revealed preferred to (x_1, x_2).

In other words, for a bundle (x_1, x_2) purchased at prices (p_1, p_2) and a different bundle (y_1, y_2) purchased at prices (q_1, q_2), if

$$p_1x_1 + p_2x_2 \geq p_1y_1 + p_2y_2,$$

then it must *not* be the case that

$$q_1y_1 + q_2y_2 \geq q_1x_1 + q_2x_2.$$



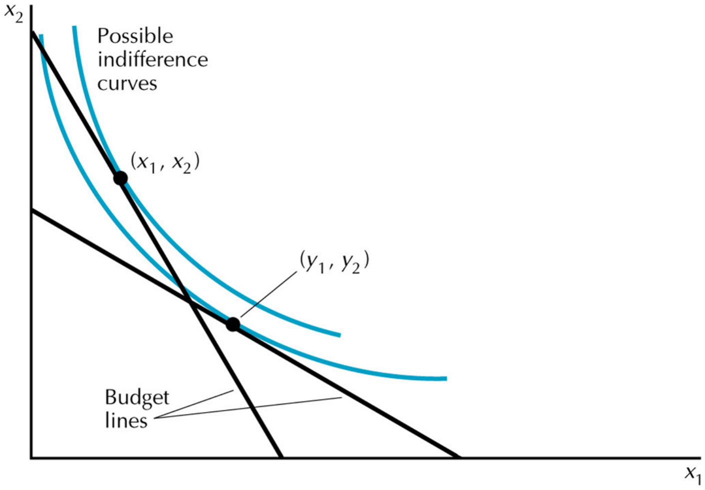


FIGURE 7.5 Satisfying WARP

How to Check WARP

How to test systematically WARP? Consider the following consumption data:

Observation	p_1	p_2	x_1	x_2
1	1	2	1	2
2	2	1	2	1
3	1	1	2	2

The table below shows cost of each bundle at each set of prices. The choices (e.g. bundle 1, prices 1) are directly revealed preferred to baskets in the same row with * (e.g. bundle 2, prices 1).

		Bundles		
		1	2	3
Prices	1	5	4*	6
	2	4*	5	6
	3	3*	3*	4

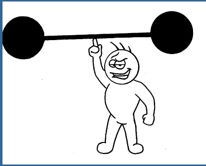
Strong Axiom of Revealed Preference

WARP is only a necessary condition for behavior to be consistent with utility maximization.

Strong Axiom of Revealed Preference (SARP): if (x_1, x_2) is directly or indirectly revealed preferred to (y_1, y_2), then (y_1, y_2) cannot be directly or indirectly revealed preferred to (x_1, x_2).

If his observed behavior is consistent with SARP, then we can always find well-behaved preferences (utility function) that explain the behavior of the consumer as maximizing behavior.

SARP is a necessary and sufficient condition for utility maximization.



How to Check SARP

The table below shows cost of each bundle at each set of prices. The choices are *indirectly* revealed preferred to baskets in the same row with $(*)$ (e.g. at prices 1, the bundle 1 is indirectly revealed preferred to bundle 3).

		Bundles		
		1	2	3
Prices	1	20	10*	22(*)
	2	21	20	15*
	3	12	15	10

This gives us a completely operational test for whether a particlular consumer's choices are consistent with economic theory.

We could use it also for units consisting of several people like households or universities.

Summary

- If one bundle is chosen when another bundle could have to be chosen, it is *revealed prefered* to the second.
- The consumer chooses the the most preferred bundle she can afford. Therefore, the chosen bundles must be preferred to the affordable bundles that were not chosen.
- We can estimate consumer's preferences using observed choices.
- The Strong Axiom of Revealed Preference (SARP) is a necessary and sufficient condition that consumer choices must obey if they are to be consistent with the economic model of optimizing choice.