Demand and Slutsky Equation

Varian: Intermediate Microeconomics, 8e, Chapters 6 and 8

Demand

The first part of the lecture explains

- what does the consumer's demand function for a good depend on,
- what are normal and inferior goods, ordinary and Giffen goods, and substitutes and complements,
- what is the inverse demand function.

Introduction

Demand functions — relate prices and income to choices:

$$x_1 = x_1(p_1, p_2, m)$$
$$x_2 = x_2(p_1, p_2, m)$$

Comparative statics – comparing two situations before and after a change without being concerned about any adjustment process.

Comparative statics in consumer theory – how demand changes when *income* and *prices* change.

Changes in Income

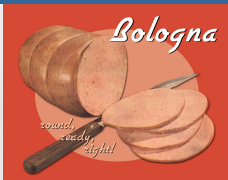A rise in income shifts the budget line out.

Normal good – increase in income increases demand (Figure 6.1):

$$\frac{\Delta x_1}{\Delta m} > 0.$$

Inferior good – increase in income decreases demand (Figure 6.2):

$$\frac{\Delta x_1}{\Delta m} < 0.$$

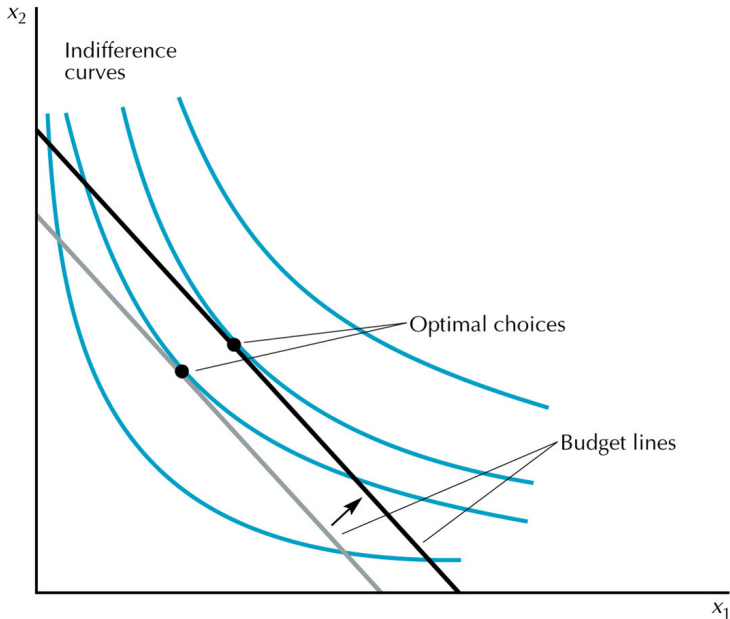Examples of inferior goods: nearly any kind low quality good like gruel, bologna, shacks.
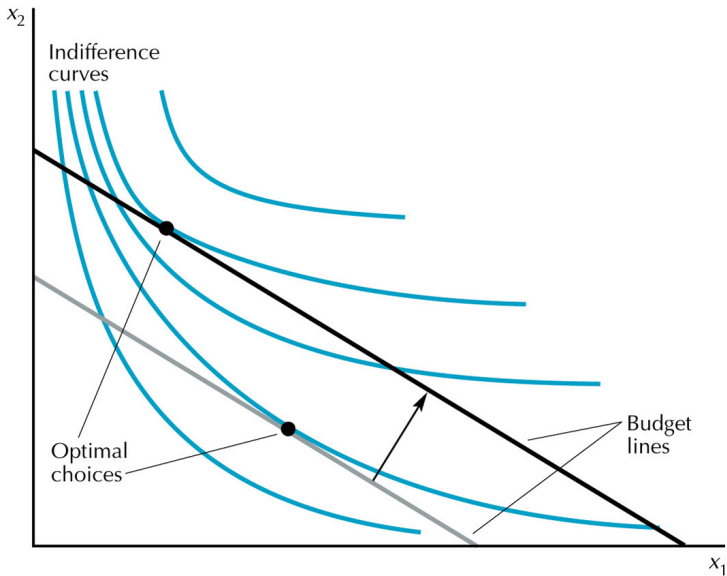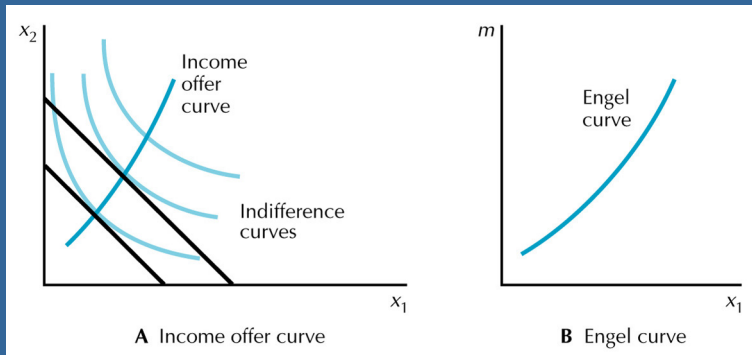
FIGURE 6.1 **Normal goods**

x_2

Indifference curves

Optimal choices

Budget lines

x_1

FIGURE 6.2 An inferior good

As income changes, the optimal choice moves along the **income offer curve** or **income expansion path**.

The relationship between the optimal choice and income, with prices fixed, is called the **Engel curve**.
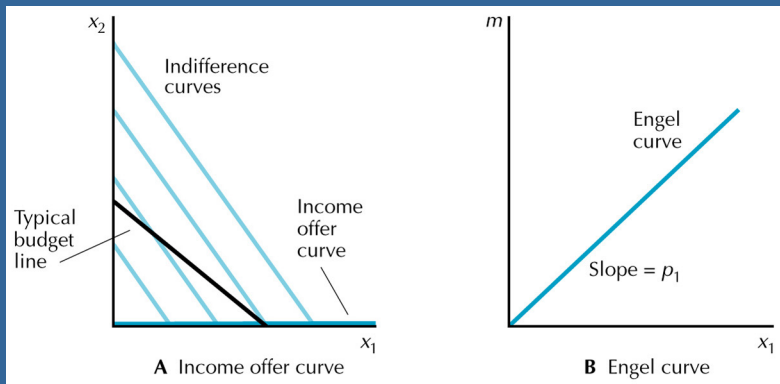

A Income offer curve   **B** Engel curve

Examples: Perfect Substitutes

Suppose that $p_1 < p_2$: Consumer is specializing in consuming good 1 $\implies$ horizontal income offer curve.

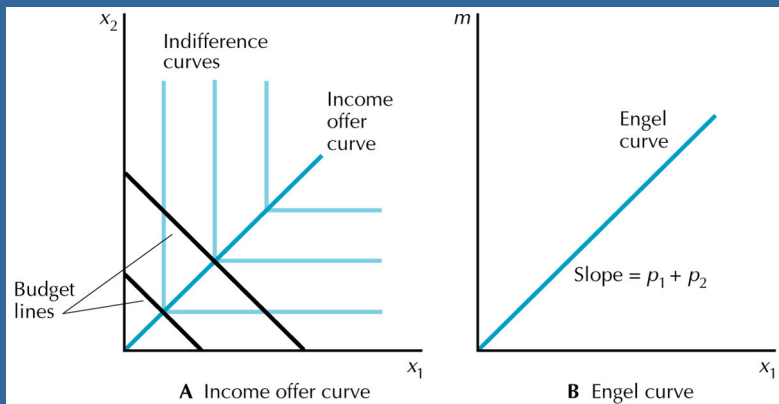Demand for good 1 is $x_1 = m/p_1$
Engel curve is a straight line: $m = p_1 x_1$.

A Income offer curve

B Engel curve

Examples: Perfect Complements

Demand for good 1 is $x_1 = m/(p_1 + p_2)$.

Engel curve is a straight line: $m = (p_1 + p_2)x_1$.

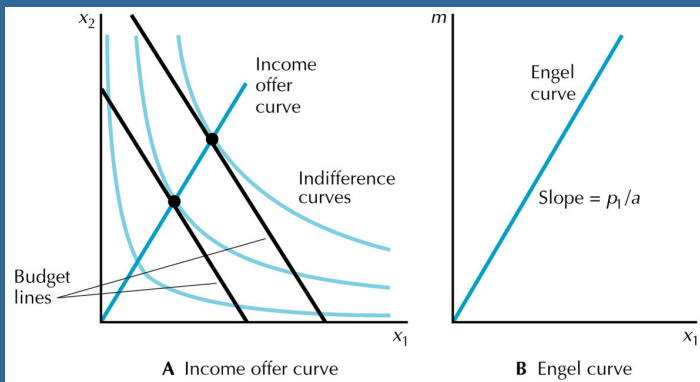**A** Income offer curve

B Engel curve

Examples: Cobb-Douglas Preferences

For $u(x_1, x_2) = x_1^a x_2^{1-a}$, the demand for good 1 $x_1 = am/p_1$ and the demand for good 2 is $x_2 = (1-a)m/p_2$.

The income offer curve is a straight line: $x_2 = \frac{(1-a)p_1}{ap_2} x_1$.

Engel curve is a straight line: $m = p_1 x_1 / a$.

A Income offer curve   **B** Engel curve

Examples: Homothetic Preferences

The consumer has **homothetic preferences**, if the demand for good goes up by the same proportion as income. Or if $(x_1, x_2) \succ (y_1, y_2)$, then $(tx_1, tx_2) \succ (ty_1, ty_2)$ for any positive value of t.

If the consumer has homothetic preferences, the income offer curves and Engel curves are straight lines. Thus perfect substitutes, perfect complements and Cobb-Douglas are homothetic preferences.

Homothetic preferences are not very realistic. The other options:
- **luxury good** – demand increases by a greater proportion than income.
- **necessary good** – demand increases by a lesser proportion than income.

Examples: Quasilinear Preferences

The indifference curves shift in a parallel way. $\implies$ It is tangent to the budget line at a bundle (x_1^*, x_2^*), then another indifference curve must be tangent at $(x_1^*, x_2^* + k)$ for any constant k.

Real-life example: choice between single good that is a small part of the consumer's budget (e.g. salt or toothpaste) and all other goods.

A Income offer curve   **B** Engel curve
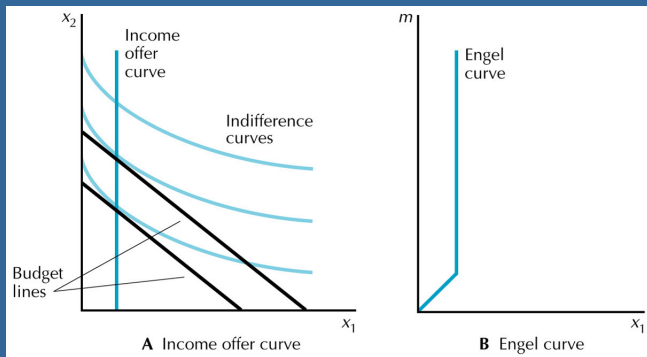
Changes in Price

Changes in price lead to a tilts or pivots of the budget line.

Ordinary good – decrease in price increases demand (see Figure 6.9):

$$\frac{\Delta x_1}{\Delta p_1} < 0.$$

Giffen good – decrease in price decreases demand (see Figure 6.10):

$$\frac{\Delta x_1}{\Delta p_1} > 0.$$

Intuituion behind Giffen good: Suppose you consume a little of meat and a lot of potatoes. A reduction of price of potatoes gives you extra money for meat. Then you might need less potatoes.
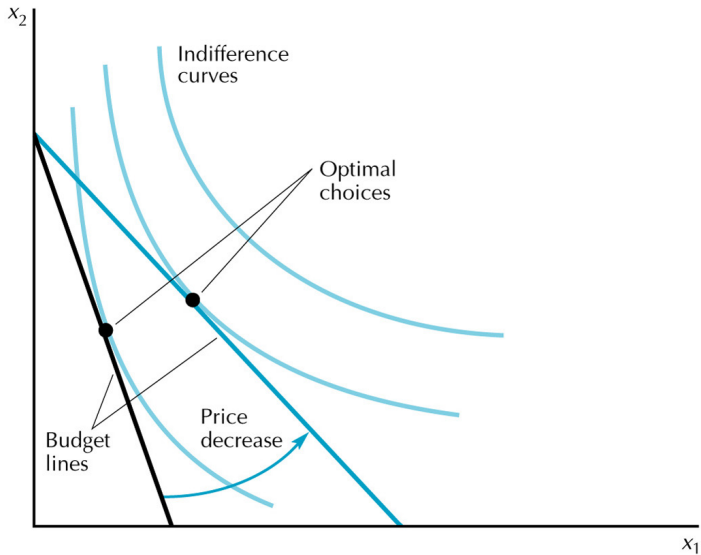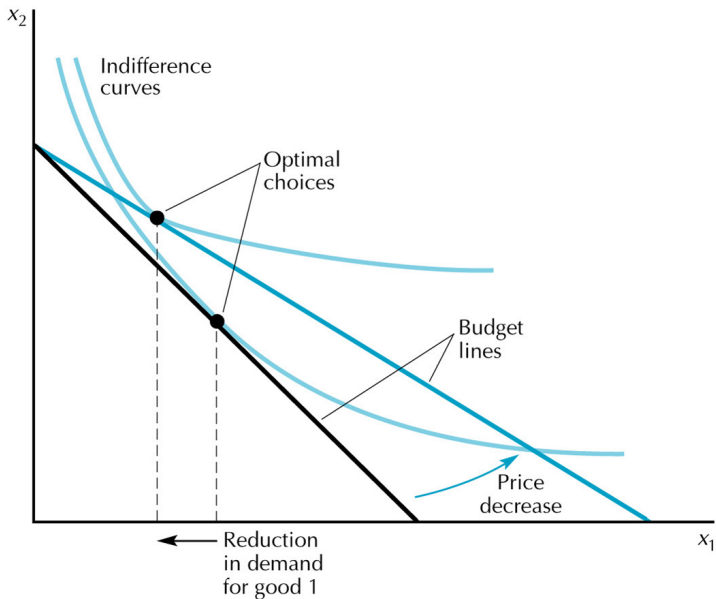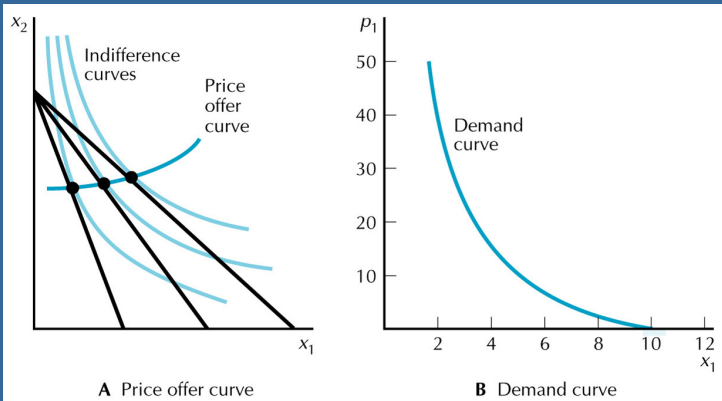
FIGURE 6.9 An ordinary good

FIGURE 6.10 A Giffen good

Changes in Prices (cont'd)

As price changes the optimal choice moves along the **price offer curve**.

The relationship between the optimal choice and a price, with income and the other price fixed, is called the **demand curve**.

A Price offer curve **B** Demand curve

Examples: Perfect Substitutes

The demand function for good 1 is

$$x_1 = \begin{cases} m/p_1 & \text{when } p_1 < p_2; \\ \text{any number between 0 and } m/p_1 & \text{when } p_1 = p_2; \\ 0 & \text{when } p_1 > p_2. \end{cases}$$

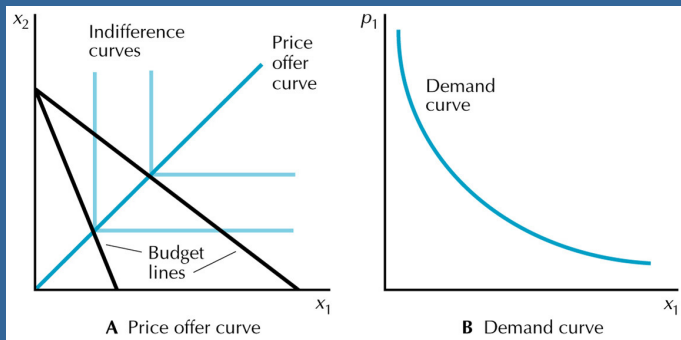**A** Price offer curve   **B** Demand curve
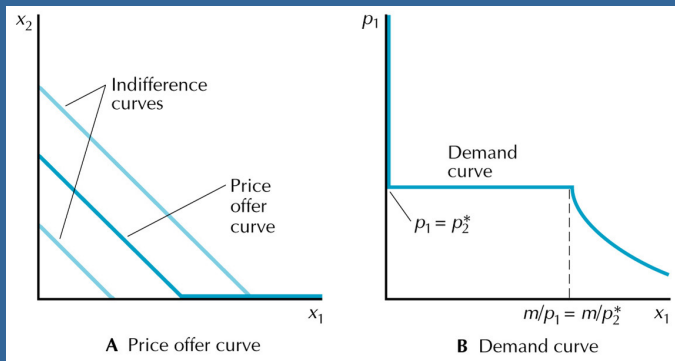
Examples: Perfect Complements

Consumed in fixed proportions $\implies$ price offer curve is a straight line.

If consumed in 1:1 proportion, the demand for good 1 is given by
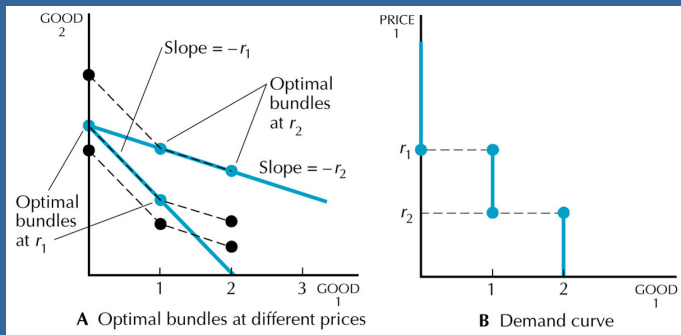
$$x_1 = \frac{m}{p_1 + p_2}.$$

A Price offer curve

B Demand curve

Examples: Discrete Goods

Reservation price r_n – price where consumer is just indifferent between consuming and not consuming unit n of good.

For the price r_1 the utility from 0 and 1 units is the same:
$$u(0, m) = u(1, m - r_1).$$

For the price r_2, $u(1, m - r_2) = u(2, m - 2r_2).$

A Optimal bundles at different prices **B** Demand curve

Examples: Discrete Goods (cont'd)

Special case: quasilinear preferences $u(x_1, x_2) = v(x_1) + x_2$.

$u(0, m) = u(1, m - r_1)$ can be written as $v(0) + m = v(1) + m - r_1$.
$u(1, m - r_2) = u(2, m - 2r_2)$ as $v(1) + m - r_2 = v(2) + m - 2r_2$.

If $v(0) = 0$, then $v(0) + m = v(1) + m - r_1 \iff r_1 = v(1)$.

Similarly, $v(1) + m - r_2 = v(2) + m - 2r_2 \iff r_2 = v(2) - v(1)$.

Reservation prices just measure marginal utilities.

WWW.FTIPKY.CZ

Substitutes and Complements

Blue and red pencils are perfect subsitutes, what about pencils and pens?
Substitutes – increase in p_2 increases demand for x_1:

$$\frac{\Delta x_1}{\Delta p_2} > 0.$$

Left and right shoes are perfect complements, what about shoes and socks?
Complements – increase in p_2 decreases demand for x_1:
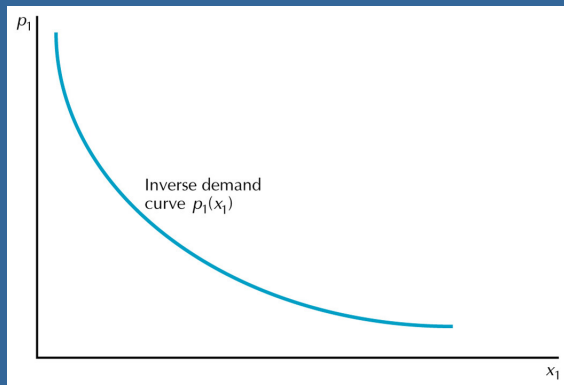
$$\frac{\Delta x_1}{\Delta p_2} < 0.$$

The Inverse Demand Curve

Usually think of demand curve as measuring quantity as a function of price – e.g. the Cobb-Douglas demand for good 1 is $x_1 = am/p_1$.

We can also think of price as a function of quantity = the **inverse demand function**. The Cobb-Douglas indirect demand function is $p_1 = am/x_1$.

Inverse demand curve $p_1(x_1)$

The Inverse Demand Curve: Economic Interpretation

If both goods are consumed in positive amounts,

$$|\text{MRS}| = \frac{p_1}{p_2} \iff p_1 = p_2|\text{MRS}|.$$

Price of good 1 is proportional to the absolute value of MRS between good 1 and 2.

Suppose $p_2 = 1$, then p_1 measures how much of good 2 would the consumer trade for one additional unit of good 1.

If good 2 is money, p_1 measures marginal willingness to pay:

How many dollars is the consumer willing to give up for one additional unit of good 1.

Summary

- The consumer's demand function for a good depends on the prices of all goods and income.
- Normal good – demand increases when income increases X inferior good
- Ordinary good – demand decreases when its price increases X Giffen good
- Good 1 is substitute (complement) for good 2 if the demand for good 1 increases (decreases) when the price of good 2 increases.
- The height of the inverse demand function at any given level of consumption measures the marginal willingness to pay for an additional unit of good.
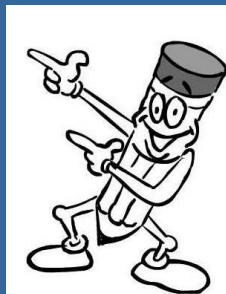
Slutsky Equation

The second part of the lecture explains

- what are the substitution and income effects,
- what is the Slutsky equation,
- what does the Law of Demand say.
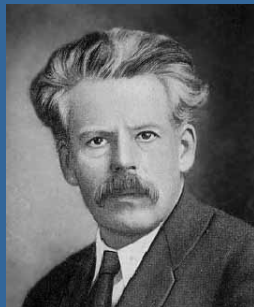
Slutsky Equation

We want a way to decompose the effect of a price change into "simpler" pieces.

That's what analysis is all about – break up into simple pieces to determine behavior of whole.

With this analytical tool, we will be able to answer the following questions:

- Does a reduction in price always increase the demand for the good? What if we have a Giffen good?
- Does an increase in wage induce people to work more? What if the wage increases from $10 to $100 an hour? What if it increases from $100 to $10.000 an hour?

Slutsky Equation (con't)

Break up price change into a **pivot** and a **shift**.

These are hypothetical changes. We can examine each change in isolation and look at sum of two changes.
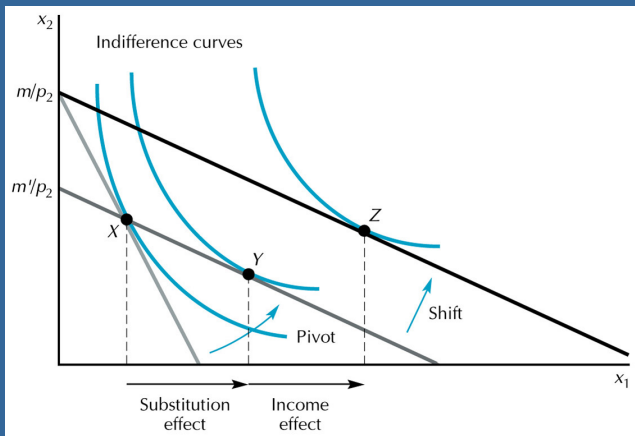
The Pivoted Line

The pivoted line has the same slope (relative prices) like the final budget line, but the purchasing power is adjusted so that the original consumption bundle (x_1, x_2) is just affordable.

How much we have to adjust the income to keep (x_1, x_2) just affordable?
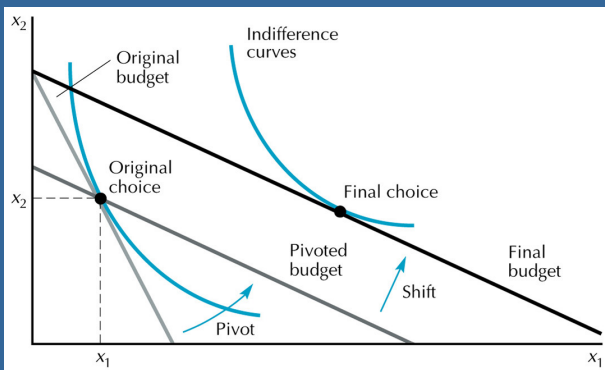
The Pivoted Line (con't)

Let m' be the income that will make (x_1, x_2) just affordable and p'_1 the final price of good 1. Then

$$m' = p'_1 x_1 + p_2 x_2.$$
$$m = p_1 x_1 + p_2 x_2.$$

Substracting the second equation from the first, we have

$$m' - m = x_1[p'_1 - p_1].$$

Using $\Delta m = m' - m$ and $\Delta p_1 = p'_1 - p_1$, we get

$$\Delta m = x_1 \Delta p_1.$$

Example: 20 candy bars, price increase from 50 to 60 cents a piece: 20 candy bars are just affordable if the income increases by

$$\Delta m = 20 \times 0.1 = \$2.00.$$

Slutsky Substitution Effect

Substitution effect is change in demand due to pivot.

Slutsky substitution effect measures how demand changes when we change prices, keeping purchasing power fixed:
$$\Delta x_1^s = x_1(p_1', m') - x_1(p_1, m).$$

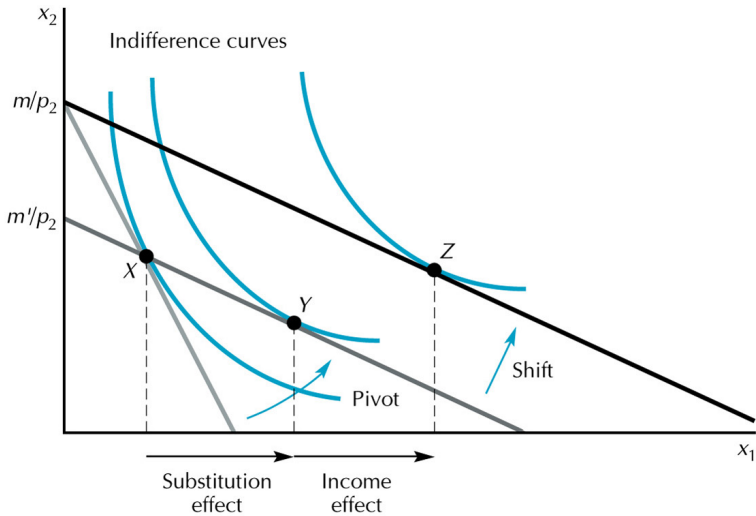Sometimes called the change in **compensated demand**: The consumer is compensated for the effect of a price change by Δm.

FIGURE 8.2 Substitution effect and income effect

Sign of the Substitution Effect

Substitution effect is negative – "negative" means quantity moves opposite the direction of price.

The bundles on the pivoted budget line to the left of X (with less of good 1 than x_1) were affordable at the old prices.

If the consumer chooses the best bundles he can afford, then X must be preferred to all the bundles on the pivoted line inside the original budget set (revealed preference).

Similarly we could prove that if p_1 increases, x_1 goes down.

Example: Calculating the Substitution Effect

Suppose $m = \$120$, $p_1 = \$3$ per quart and the demand for milk is

$$x_1 = 10 + \frac{m}{10p_1}.$$

The demand for milk is $10 + 120/(10 \times 3) = 14$ quarts per week.

Now the price of milk falls to $p_1' = \$2$ per quart. The demand for milk is $10 + 120/(10 \times 2) = 16$ quarts per week.

In order to calculate the substitution effect, we need to calculate the income necessary to keep the purchasing power constant

$$m' = m + \Delta m = m + x_1 \Delta p_1 = 120 + 14 \times (2 - 3) = 106.$$

The substitution effect is

$$\Delta x_1^s = x_1(p_1', m') - x_1(p_1, m) = x_1(2, 106) - x_1(3, 120) = 1.3.$$

Income Effect

Income effect is change in demand due to shift.

This measures the change in demand for good 1 when we change the income from m' to m and keep the prices constant at (p_1', p_2):
$$\Delta x_1^n = x_1(p_1', m) - x_1(p_1', m').$$

Income effect is negative for normal goods (a rise in price reduces the income which reduces the demand) and positive for inferior good.
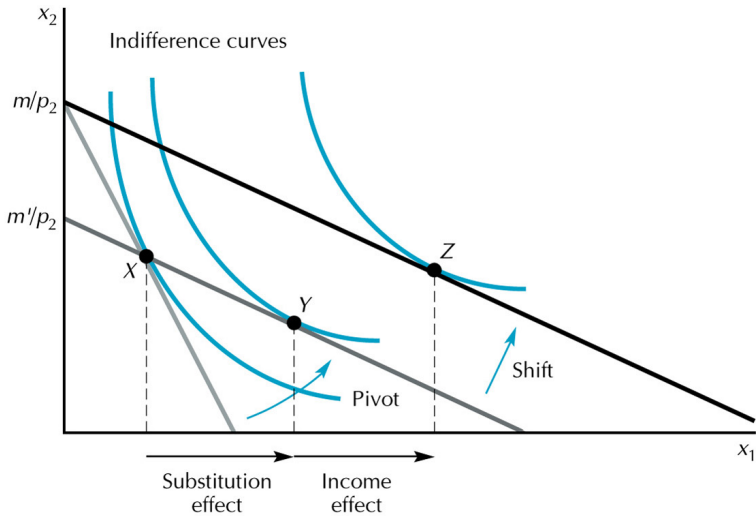
FIGURE 8.2 Substitution effect and income effect

The Total Change in Demand

Total change in demand is substitution effect plus the income effect. This equation is called **Slutsky indentity**:

$$\Delta x_1 = \Delta x_1^s + \Delta x_1^n.$$

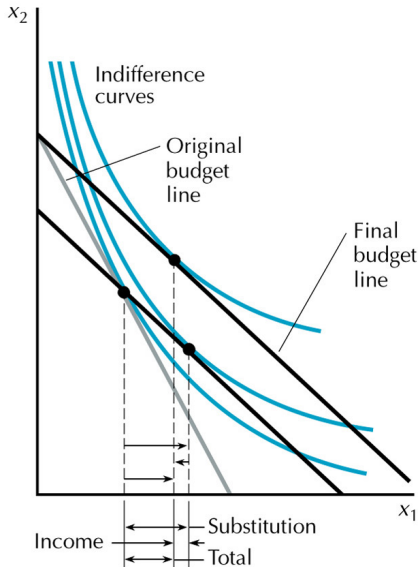$$x_1(p_1', m) - x_1(p_1, m) = x_1(p_1', m') - x_1(p_1, m) + x_1(p_1', m) - x_1(p_1', m').$$

It is an identity – the first and fourth terms cancel out.

If good is normal, the substitution effect and the income effect are negative. The total effect is negative.

If good is inferior, the substitution effect is negative and the income effect is positive. The total effect is ambiguous (see Figure 8.3).
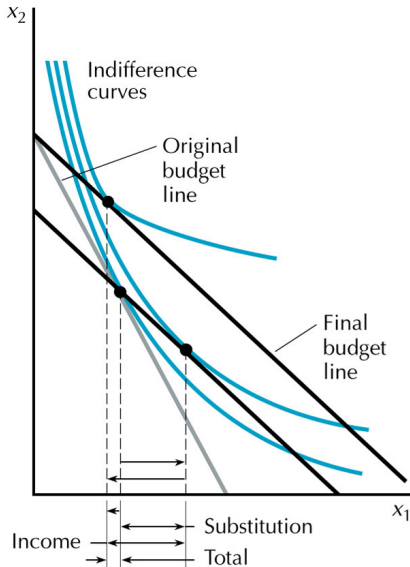
FIGURE 8.3 Inferior goods

The Rates of Change

If we want to express Slutsky identity in terms rates of change, it is convenient to define negative income effect:

$$\Delta x_1^m = x_1(p_1', m') - x_1(p_1', m) = -\Delta x_1^n.$$

Then the Slutsky identity is

$$\Delta x_1 = \Delta x_1^s - \Delta x_1^m.$$

Dividing the identity by Δp_1, we get

$$\frac{\Delta x_1}{\Delta p_1} = \frac{\Delta x_1^s}{\Delta p_1} - \frac{\Delta x_1^m}{\Delta p_1}.$$

From $\Delta m = x_1 \Delta p_1$, we get $\Delta p_1 = \frac{\Delta m}{x_1}$. Subsituting into (38), we get

$$\frac{\Delta x_1}{\Delta p_1} = \frac{\Delta x_1^s}{\Delta p_1} - \frac{\Delta x_1^m}{\Delta m} x_1.$$

The Law of Demand

The Law of Demand – *If the demand for a good increases when income increases, then the demand for that good must decrease when its price increases.*
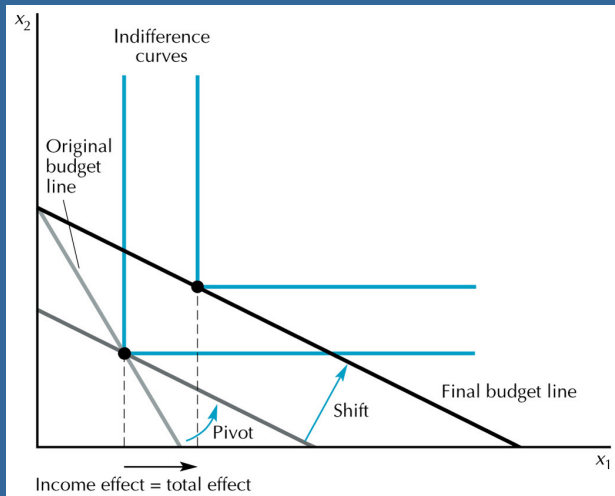
Follows directly from the Slutsky equation:
- If demand increases when income increases, it is a normal good.
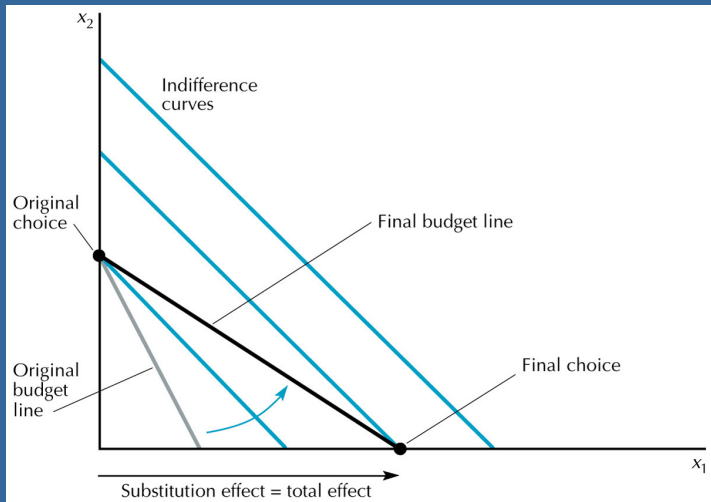- If we nave a normal good, then the total effect of a price change is negative.

Examples: Perfect Complements

Substitution effect is zero: The entire change in demand is due to income effect.
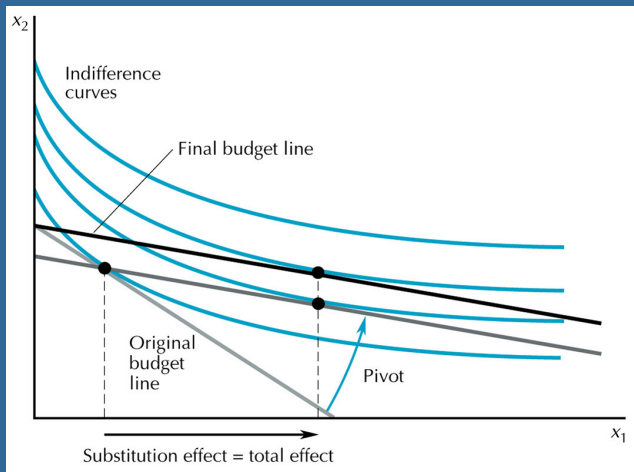
Examples: Perfect Substitutes

Income effect is zero: The entire change in demand is due to substitution effect.

Examples: Quasilinear Preferences

Quasilinear preferences imply that shift in income causes no change in demand: The entire change in demand is due to substitution effect.

Application: Rabating a Tax

In 1974, OPEC instituted an oil embargo against the US and was able to stop oil shipments for several weeks.

Many plans to reduce US dependency on foreign oil.

One of the plans:
- put a tax on gasoline, in order to reduce the consumption of oil
- return the revenues, so that consumers are not hit by the tax.

Critics: Consumers will use the rebated money to purchase more gasoline.

What does economic analysis say about the plan?

Application: Rabating a Tax (cont'd)

Original budget constraint: $px + y = m$

After tax budget constraint: $(p + t)x' + y' = m + tx' \iff px' + y' = m$

So (x', y') was affordable originally and rejected in favor of $(x, y) \implies$ consumer must be worse off.

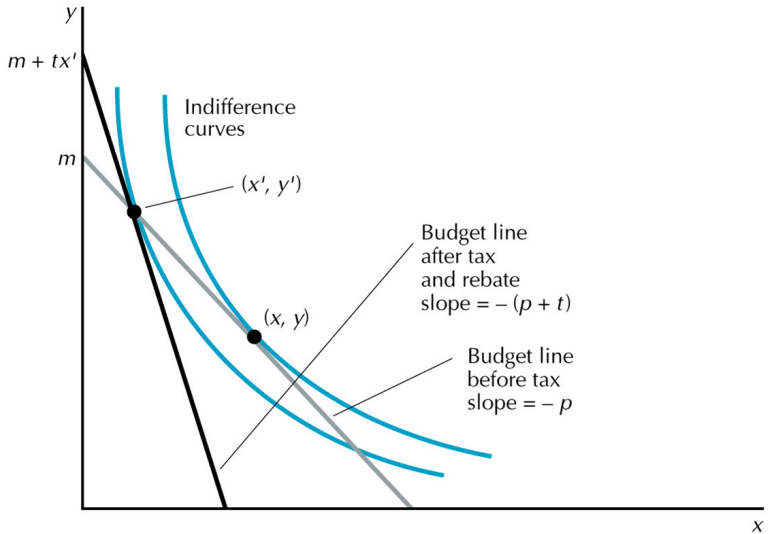The consumer will choose less gasoline and more of other goods (see Figure 8.7).
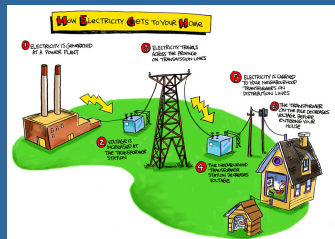
FIGURE 8.7 Rebating a tax

Application: Voluntary Real Time Pricing

Electricity producers have extreme capacity problem: easy to produce up to capacity, impossible to produce more.

Increasing capacities is expensive $\implies$ reducing the use of electricity during peak demand is attractive.

Peak demand due to weather is easy to forecast (in hot weather in Georgia – 30 % of usage due to air conditioning).

How to set up pricing system so that people have incentives to reduce consumption in peak demand.

One solution: Real Time Pricing (RTP)

RTP: industrial users equipped with special meters – the price of electricity can vary from minute to minute depending on the total demand of electricity.

Georgia Power Company – largest RTP program in the world: In 1999 reduced demand by 750 MW on high-price day by inducing some large customers to cut their demand by as much as 60 %.
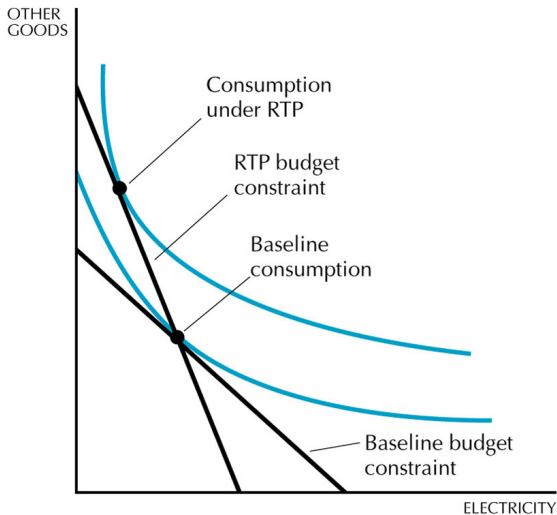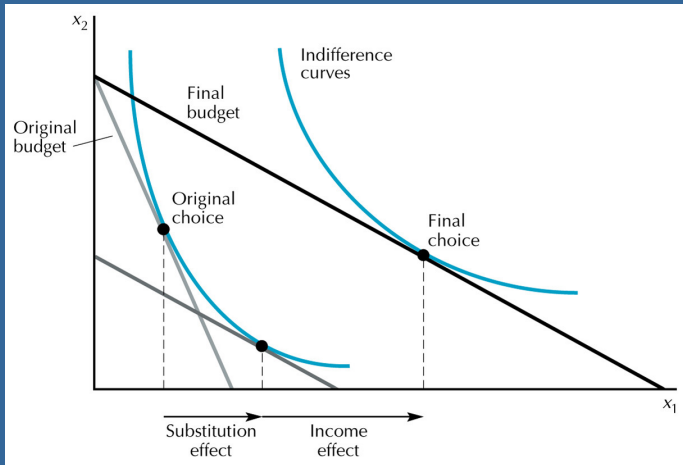
FIGURE 8.8 Voluntary real time pricing

The Hicks Substitution Effect

Hicks substitution effect a change in demand for a good due to a change in the relative prices while keeping the utility of the consumer constant.

The Sign of Hicks Substitution Effect

Negative – proof by revealed preferences:

(x_1, x_2) is demanded bundle at prices (p_1, p_2) and (y_1, y_2) is demanded bundle at prices (q_1, q_2).

Since $(x_1, x_2) \sim (y_1, y_2)$, it is *not* true that
$$p_1 x_1 + p_2 x_2 > p_1 x_1 + p_2 x_2 \text{ and } q_1 y_1 + q_2 y_2 > q_1 y_1 + q_2 y_2.$$

Hence, these inequalities *are* true:
$$p_1 x_1 + p_2 x_2 \leq p_1 x_1 + p_2 x_2 \text{ and } q_1 y_1 + q_2 y_2 \leq q_1 y_1 + q_2 y_2.$$

Adding and rearrangig the inequalities gives
$$(q_1 - p_1)(y_1 - x_1) + (q_2 - p_2)(y_2 - x_2) \leq 0.$$

Using $p_2 = q_2$ (change only in p_1), we get $(q_1 - p_1)(y_1 - x_1) \leq 0$.
The change in price and quantity demanded must have the opposite sign.

Summary

- There are two effects of a change in price: substitution and income effect.
- The **substitution effect** is the change in demand due to the change in relative prices when purchasing power is held constant.
- The income effect is the change in demand due to the change in purchasing power.
- Slutsky equation says that the total change in demand is the sum of the substitution and the income effect.
- The Law of Demand says that normal goods must have downward-sloping demand.