

INTERMEDIATE

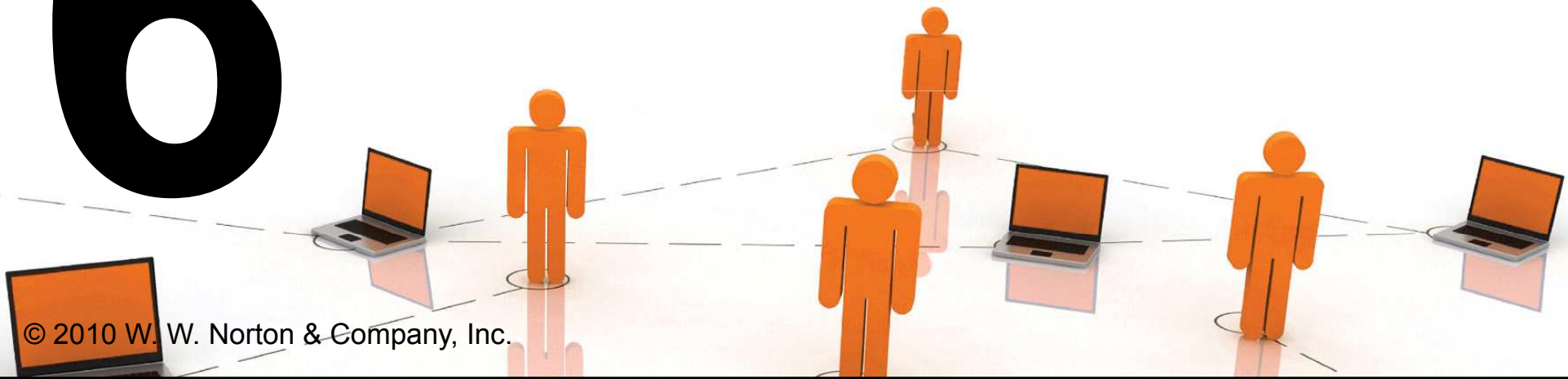
8TH EDITION

MICROECONOMICS

HAL R. VARIAN

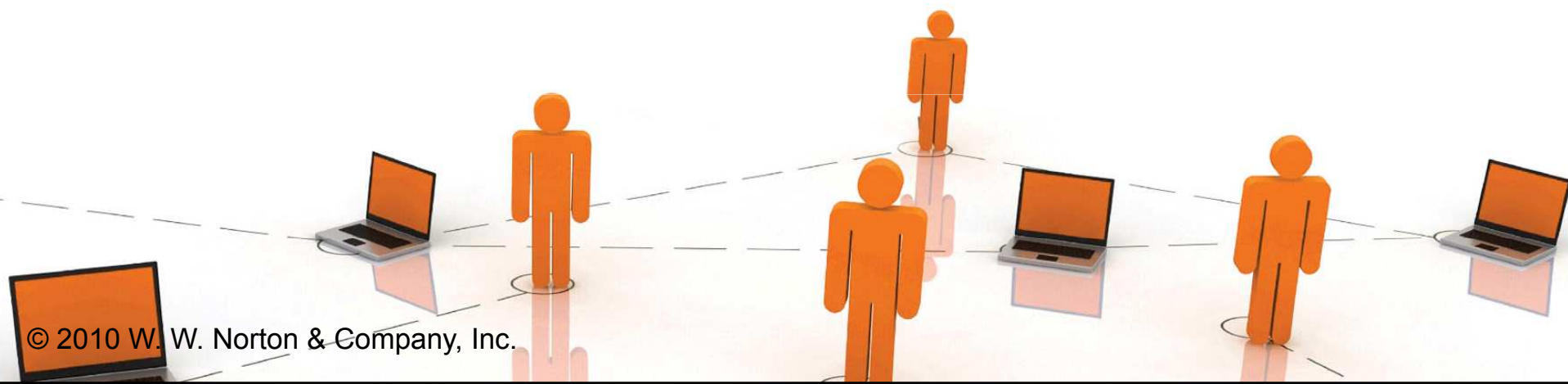
6

Demand



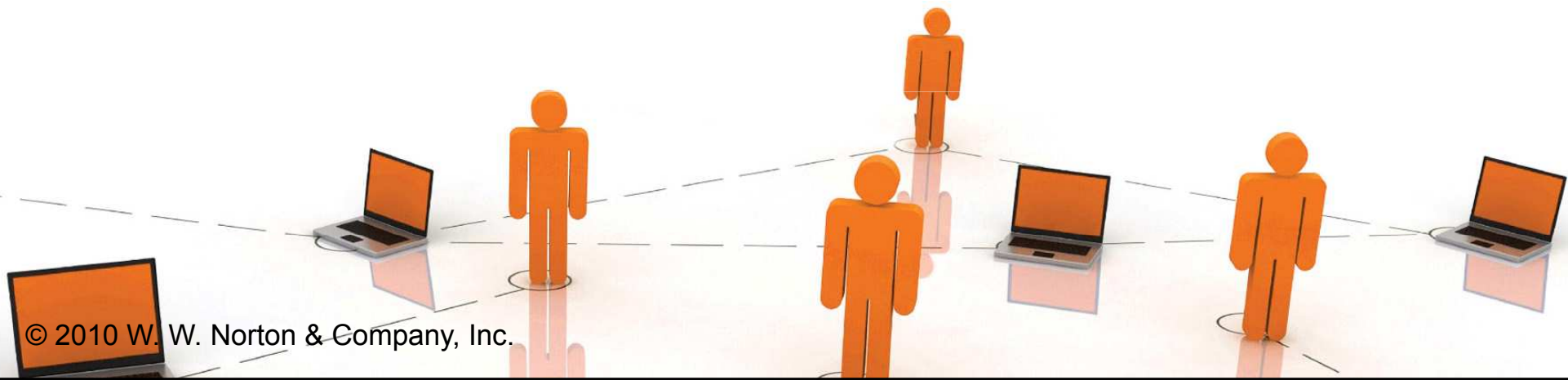
Properties of Demand Functions

- ◆ **Comparative statics analysis of ordinary demand functions -- the study of how ordinary demands $x_1^*(p_1, p_2, y)$ and $x_2^*(p_1, p_2, y)$ change as prices p_1 , p_2 and income y change.**



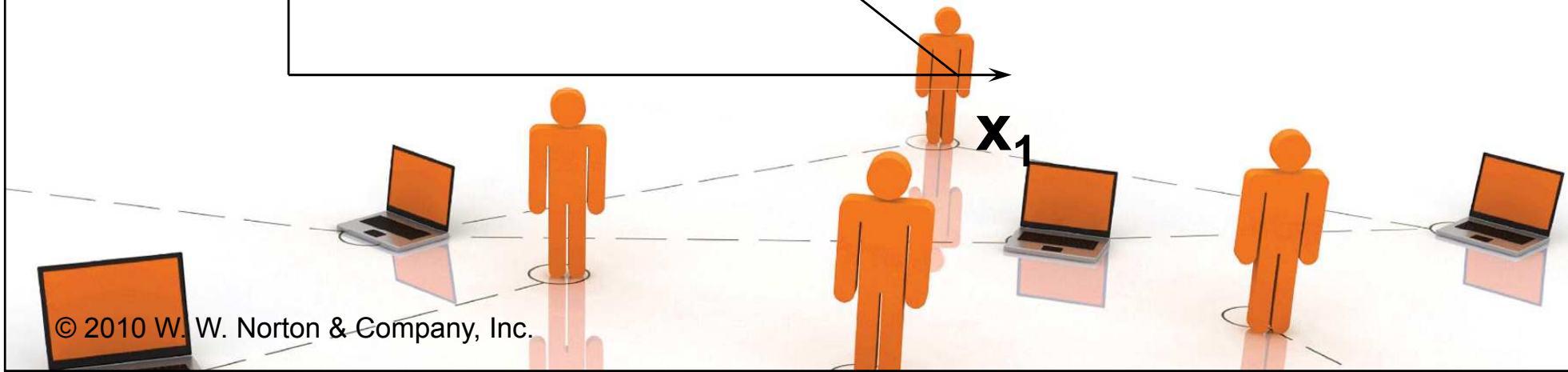
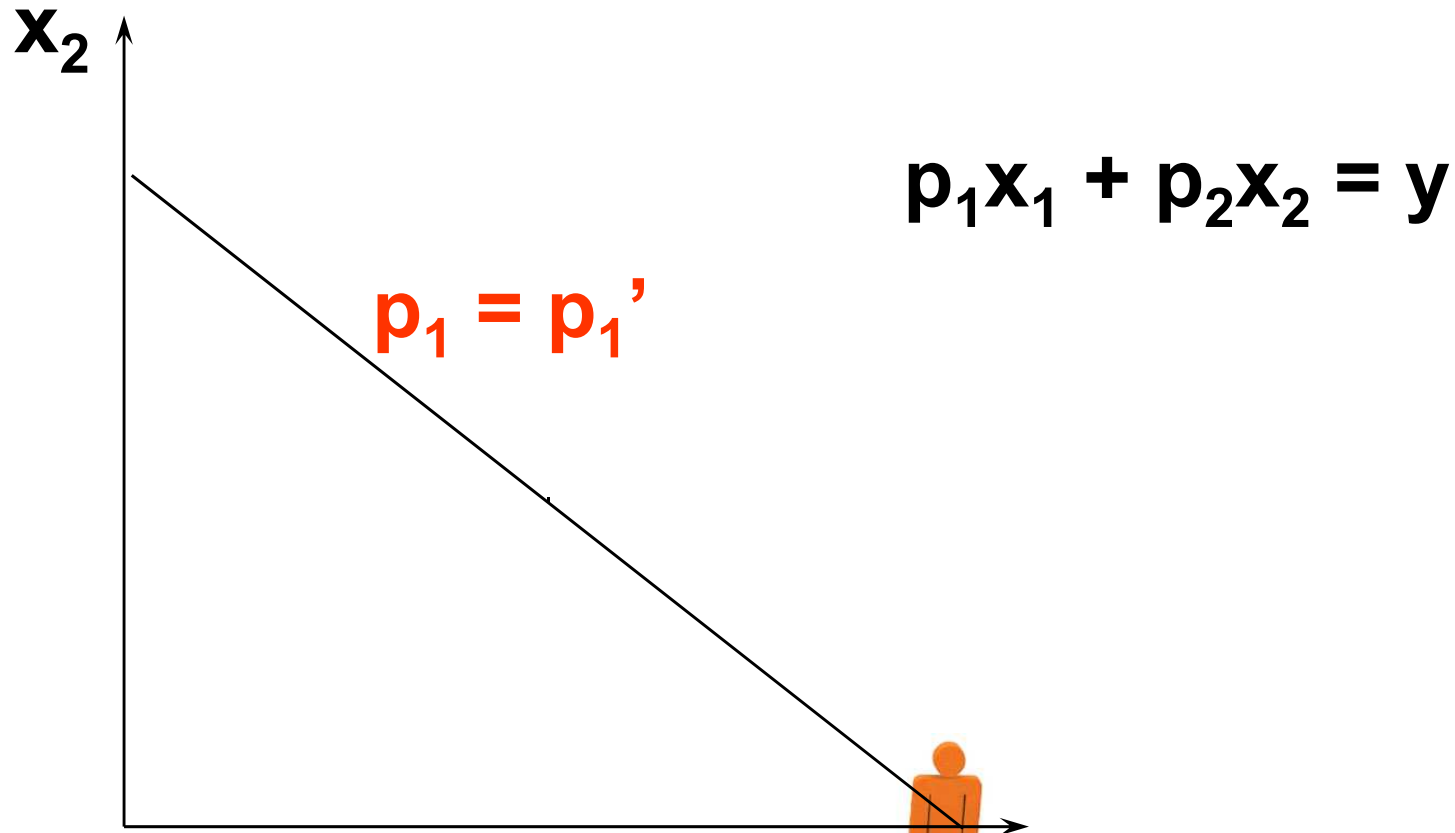
Own-Price Changes

- ◆ How does $x_1^*(p_1, p_2, y)$ change as p_1 changes, holding p_2 and y constant?
- ◆ Suppose only p_1 increases, from p_1' to p_1'' and then to p_1''' .



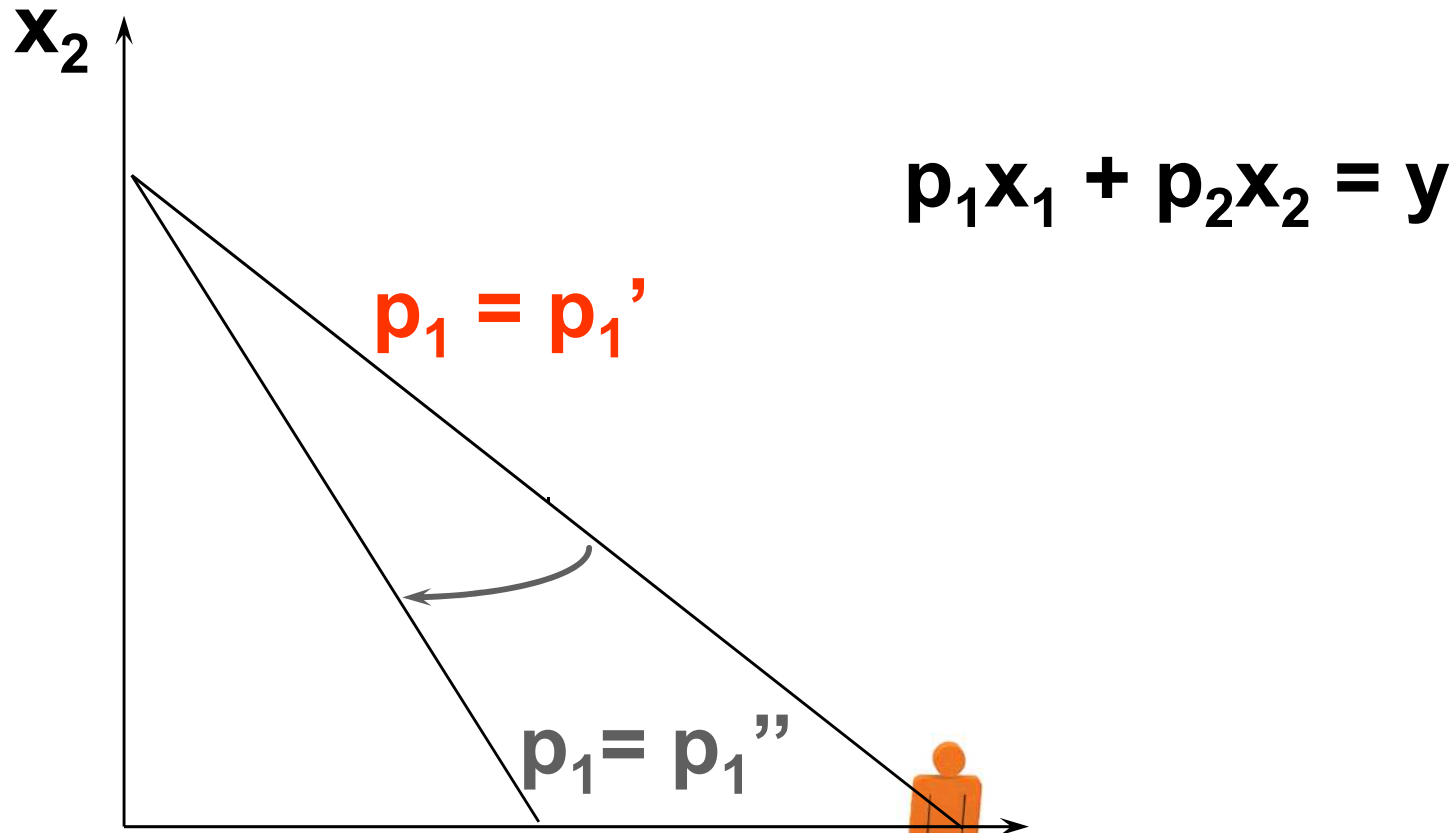
Own-Price Changes

Fixed p_2 and y .



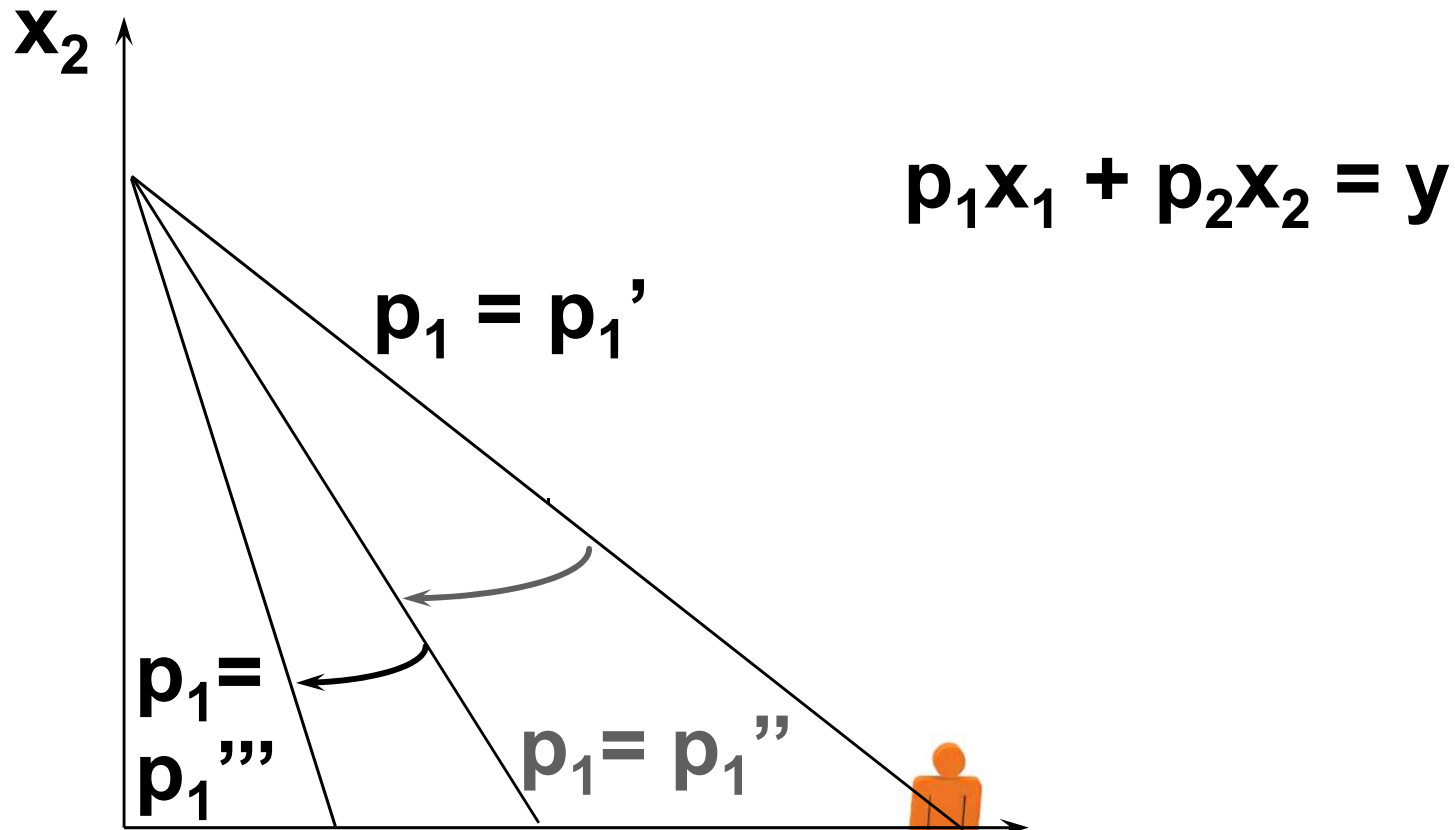
Own-Price Changes

Fixed p_2 and y .



Own-Price Changes

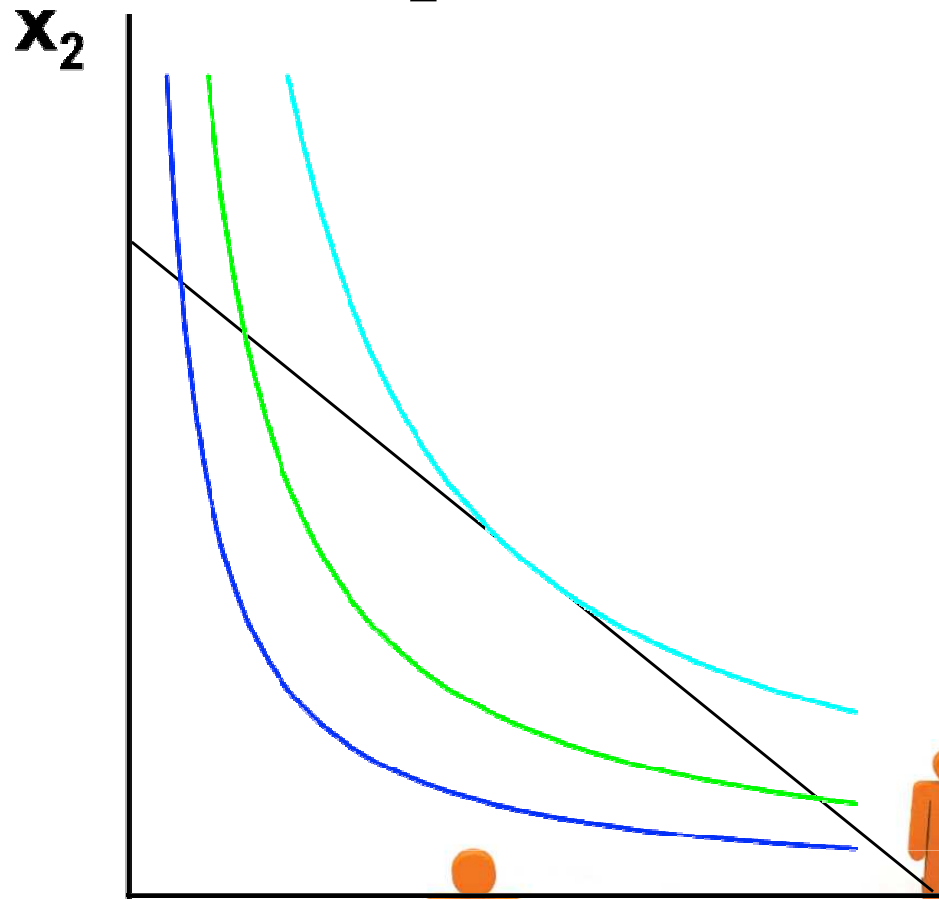
Fixed p_2 and y .



x_1

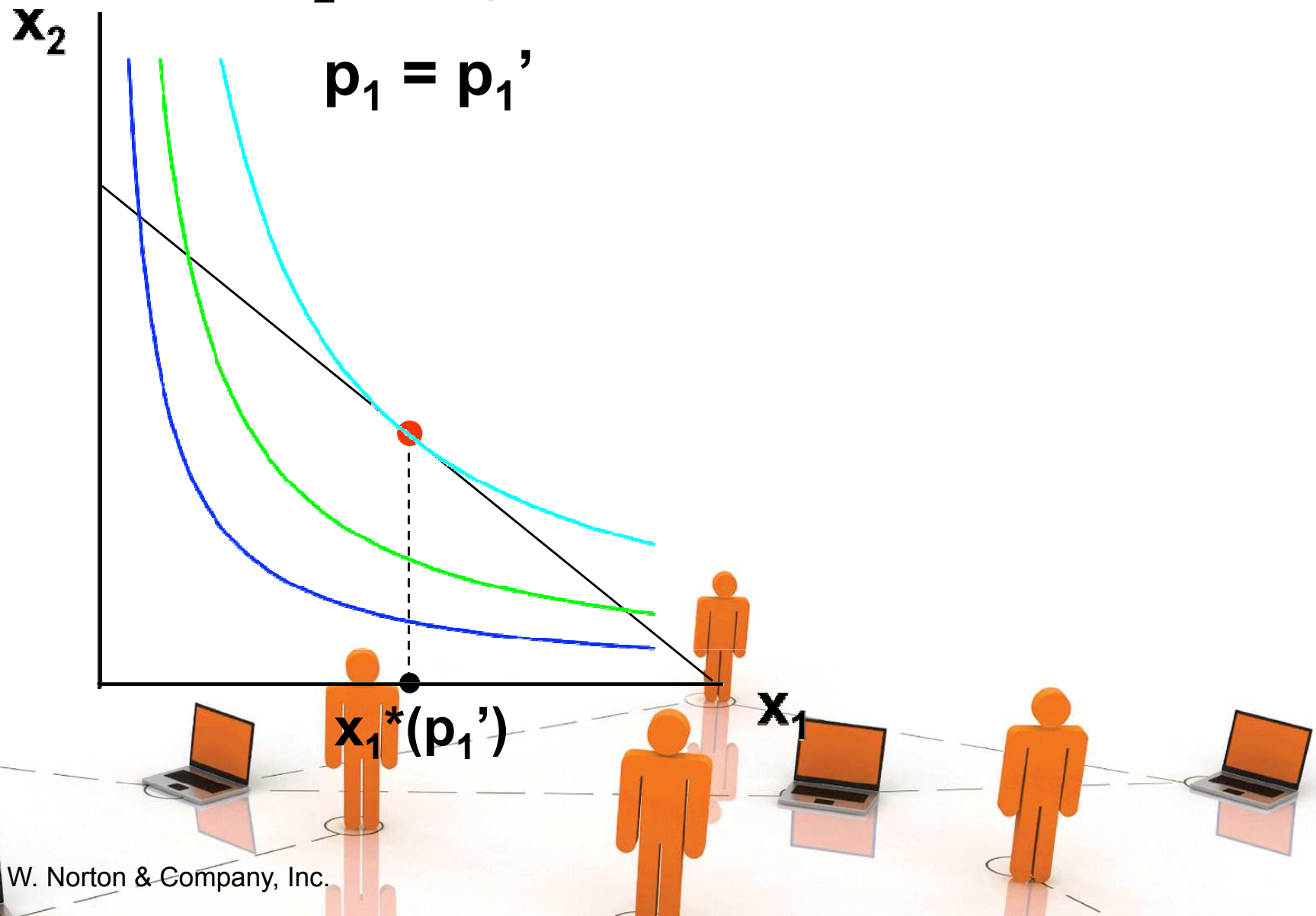
Own-Price Changes

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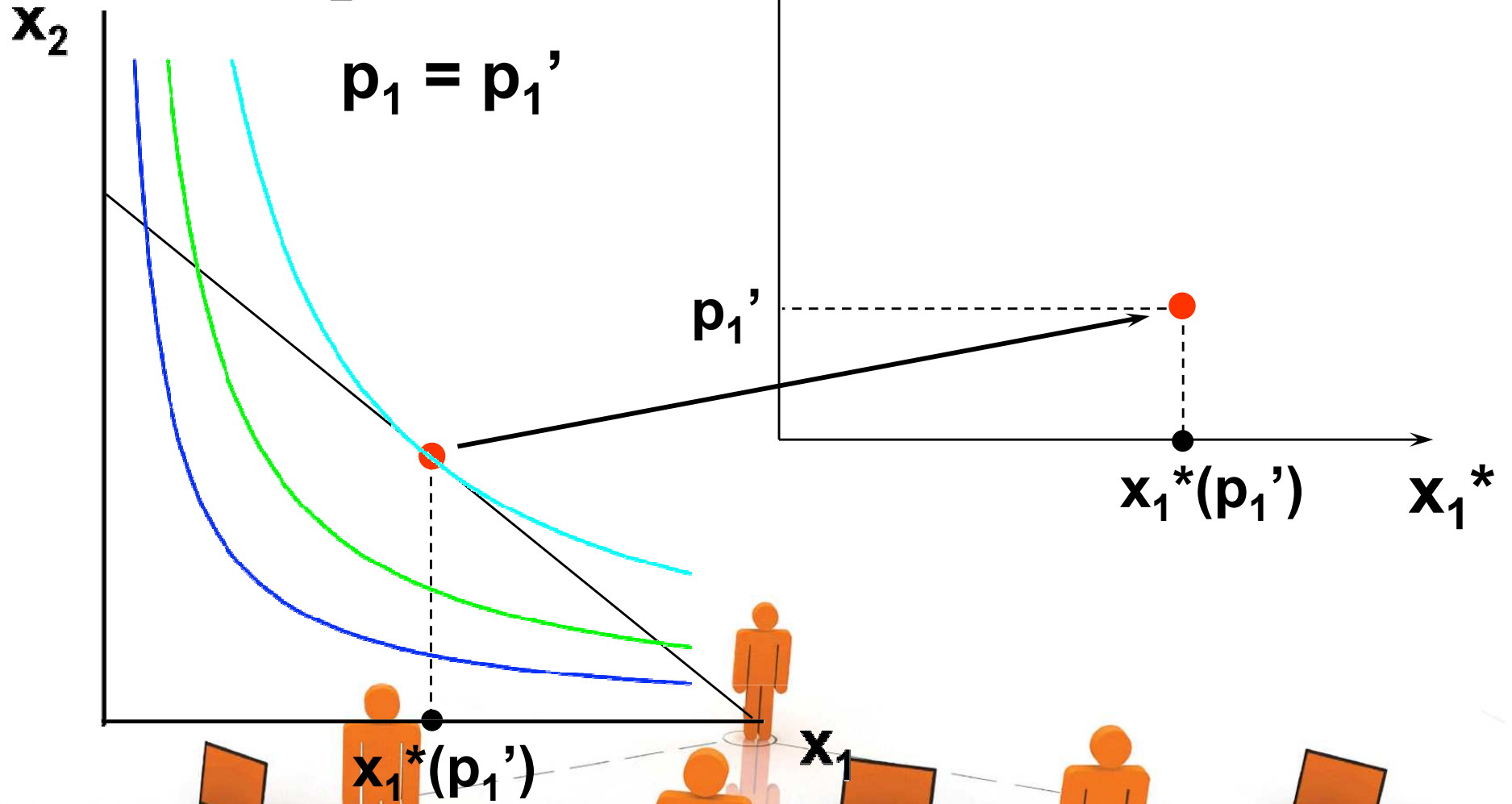
Own-Price Changes

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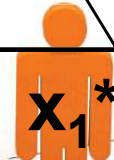
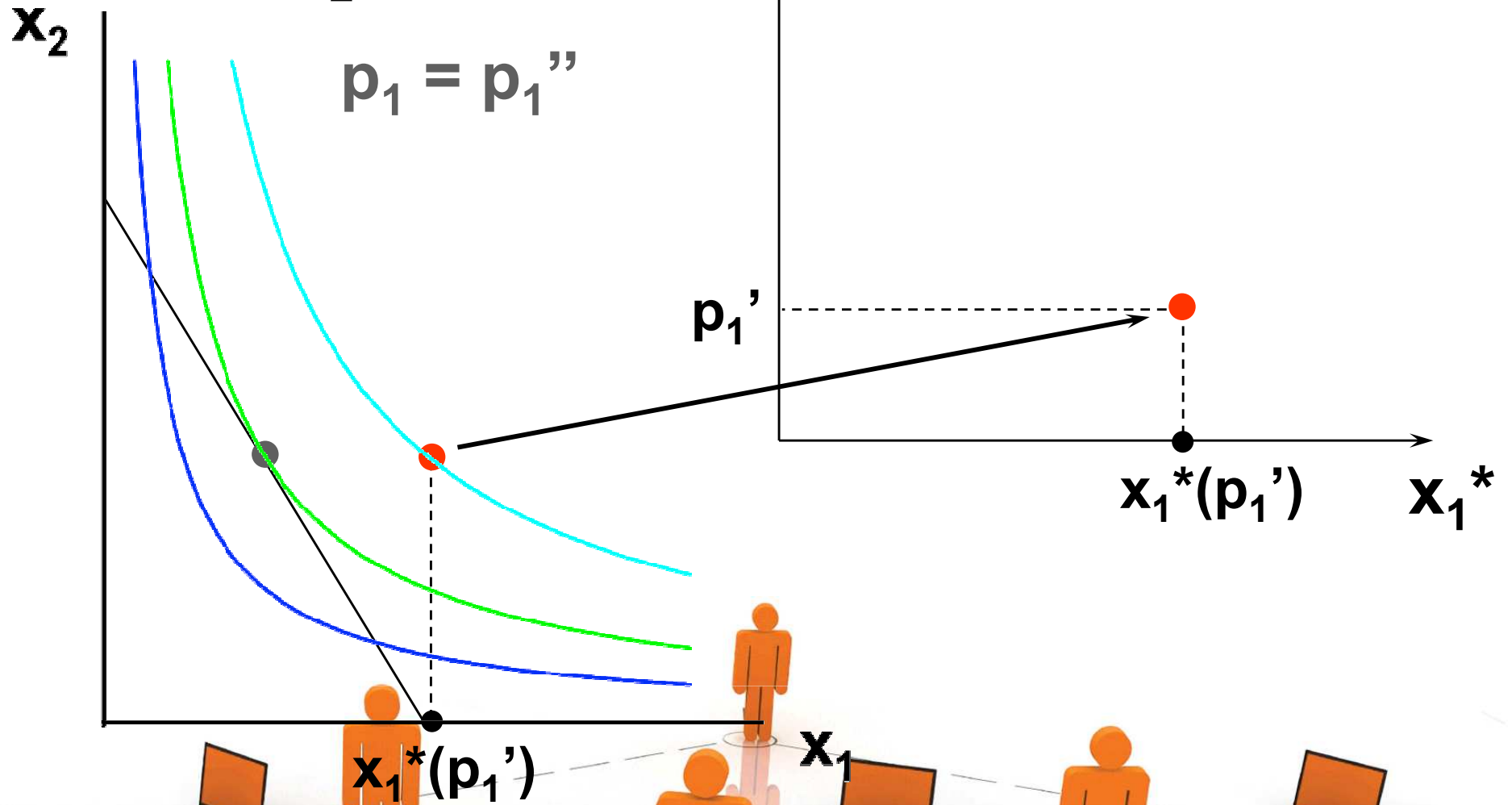
Own-Price Changes

Fixed p_2 and y .



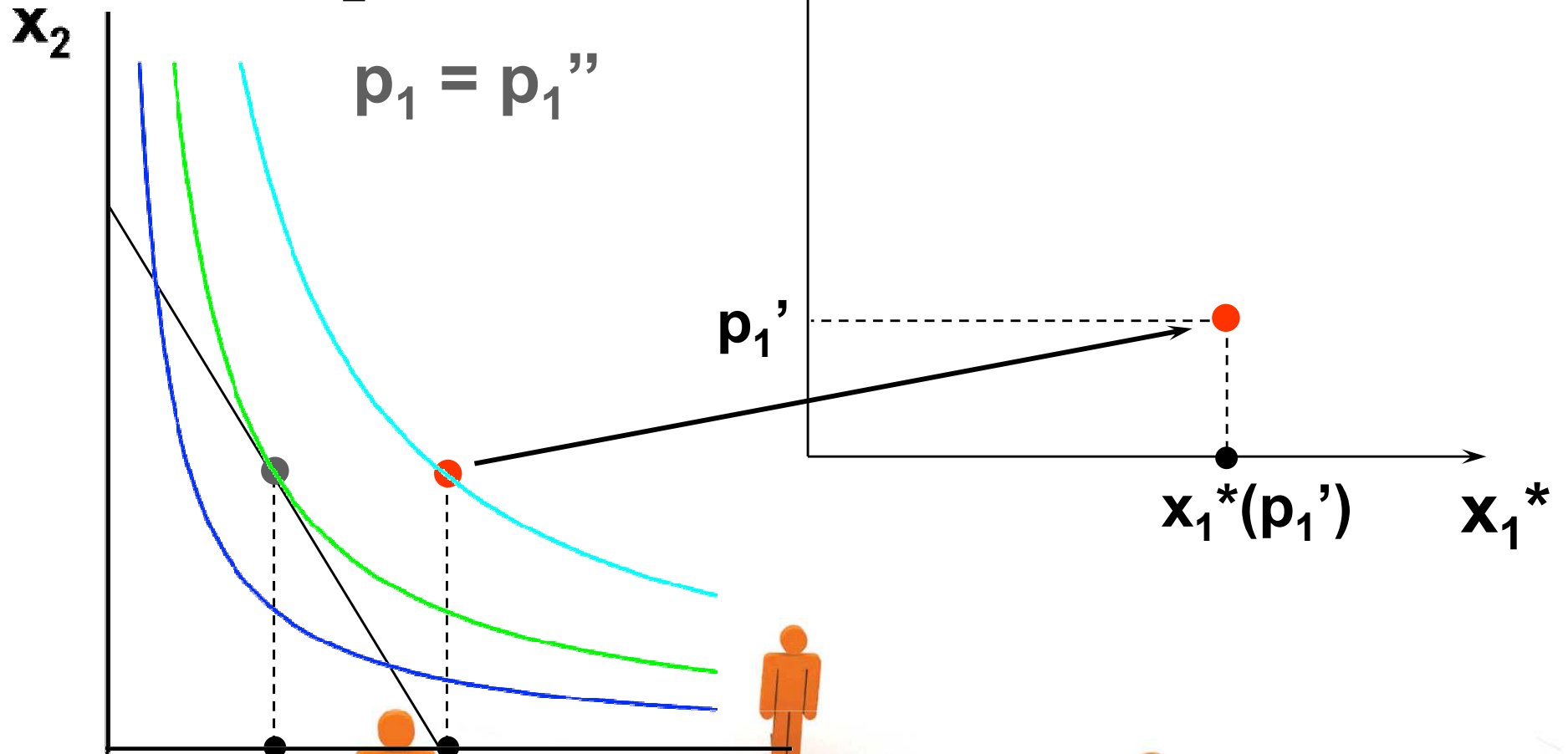
Own-Price Changes

Fixed p_2 and y .



Own-Price Changes

Fixed p_2 and y .



$p_1 = p_1''$

p_1'

$x_1^*(p_1')$

x_1^*

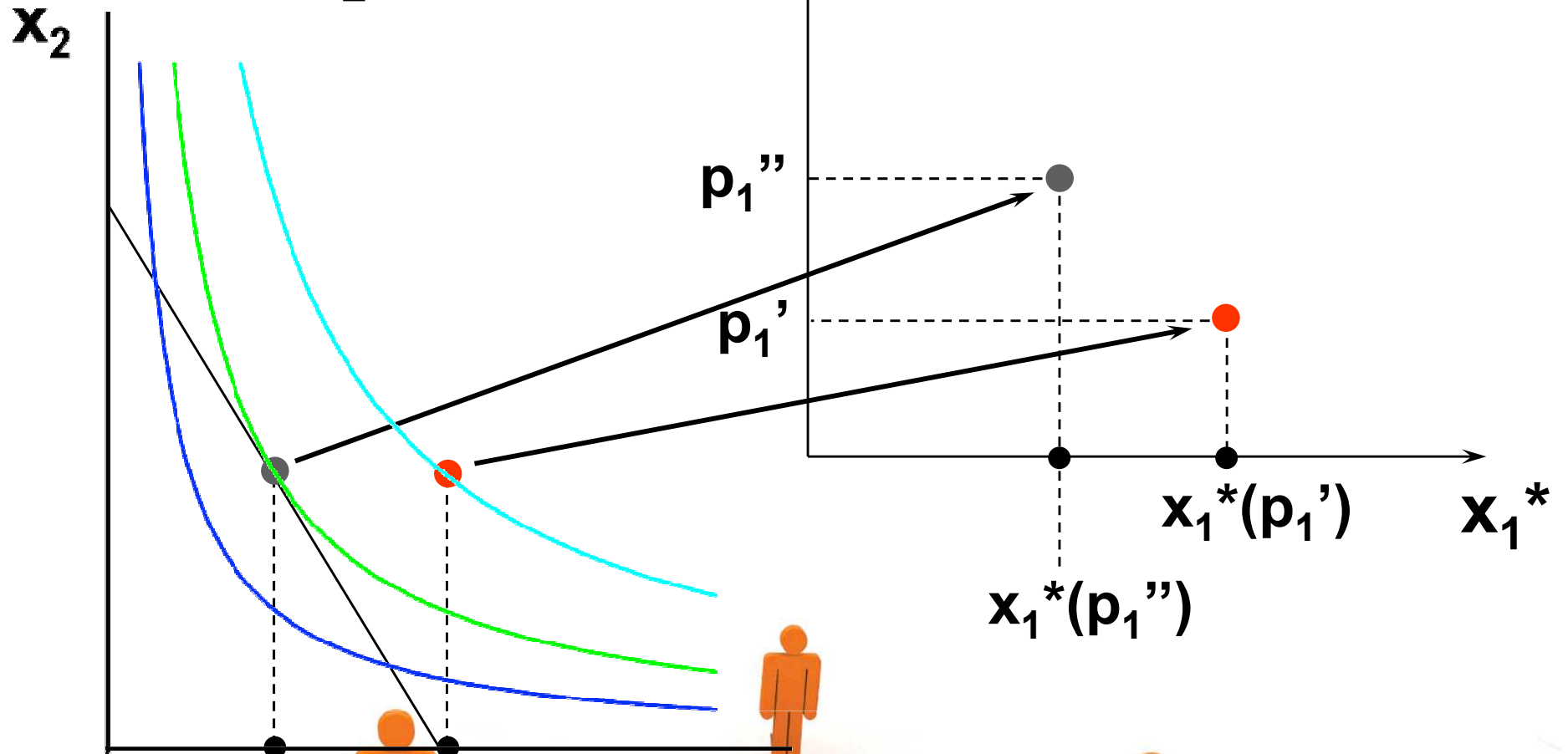
$x_1^*(p_1')$

$x_1^*(p_1'')$

x_1

Own-Price Changes

Fixed p_2 and y .

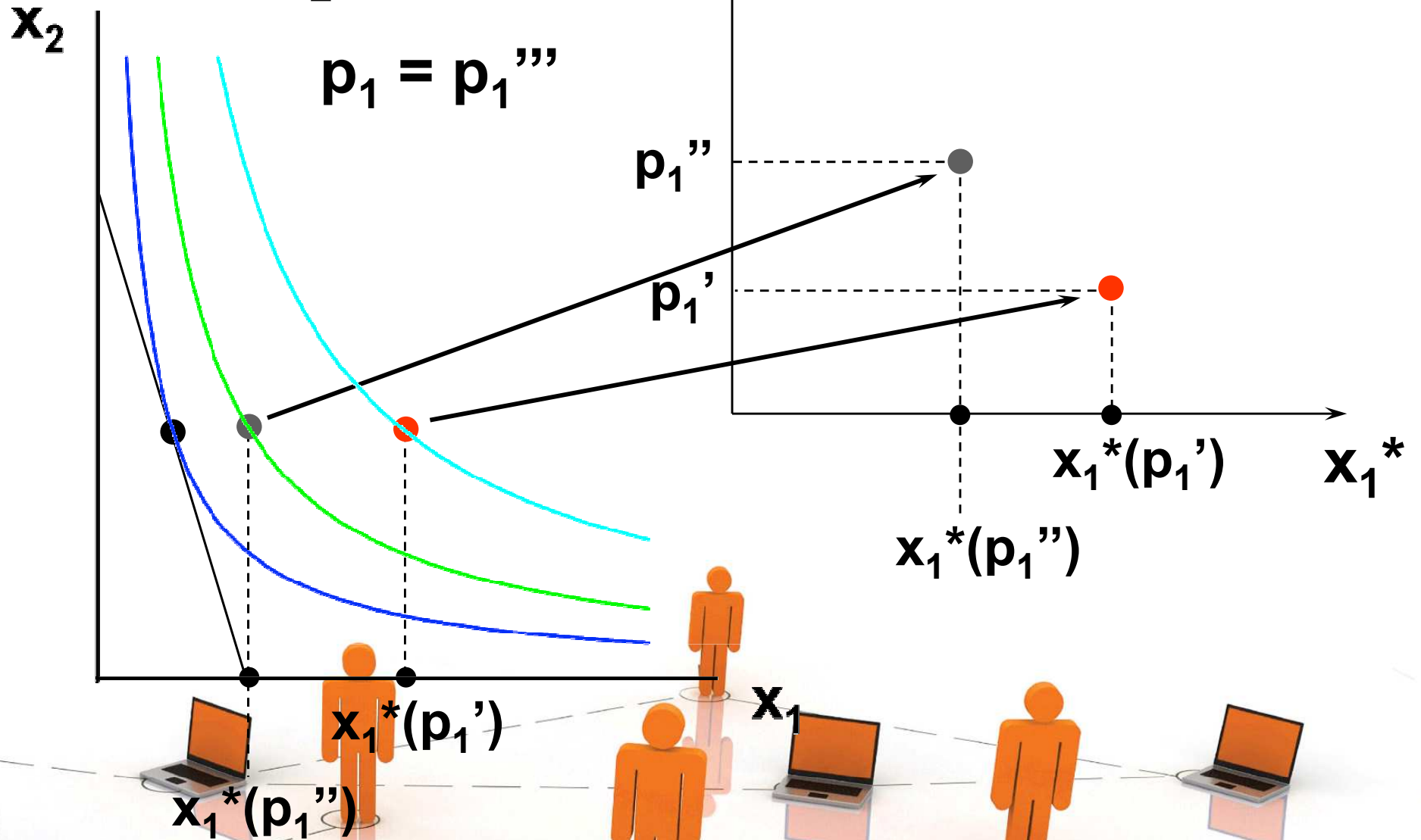


$x_1^*(p_1')$
 $x_1^*(p_1'')$

x_1

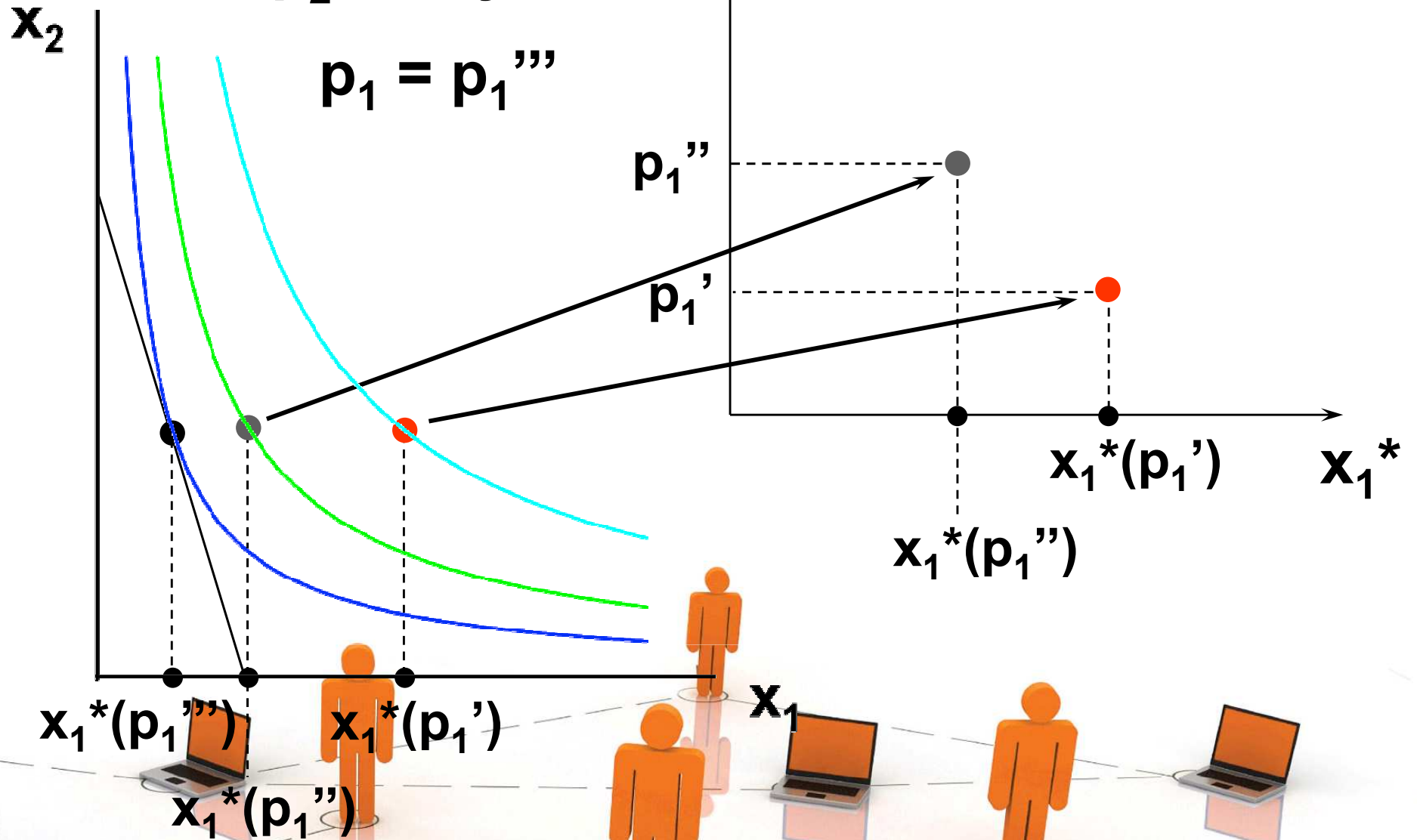
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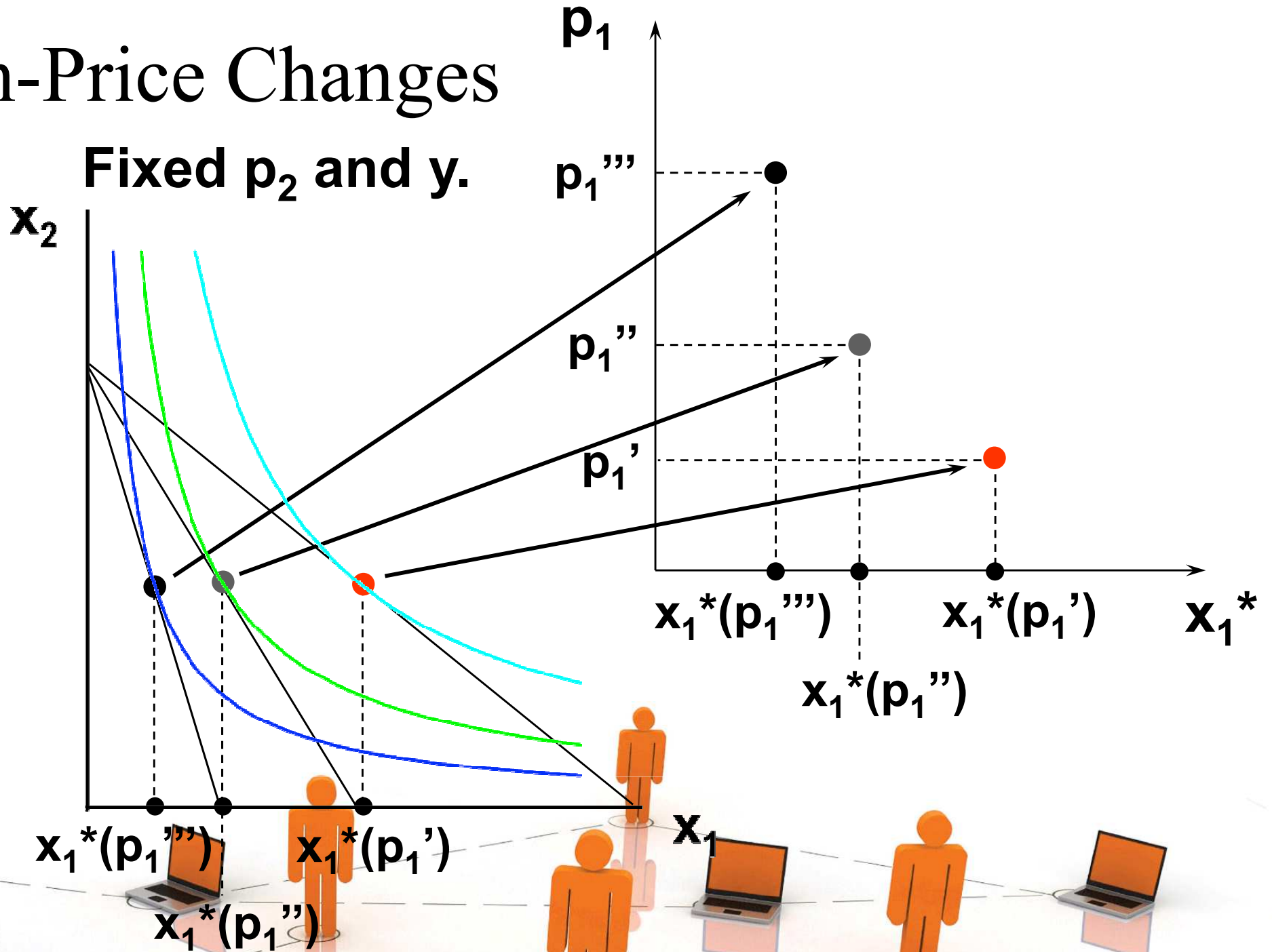
Own-Price Changes

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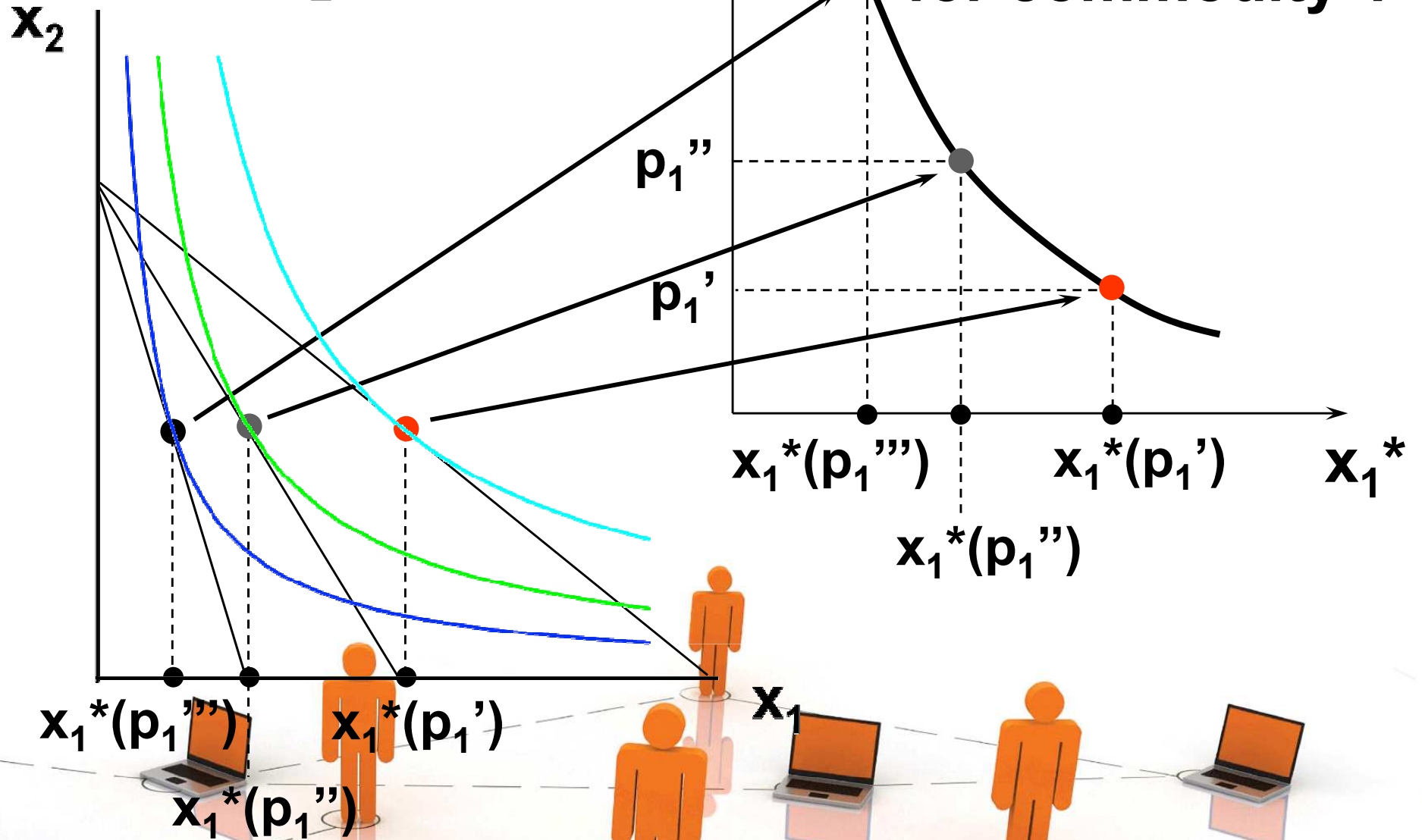
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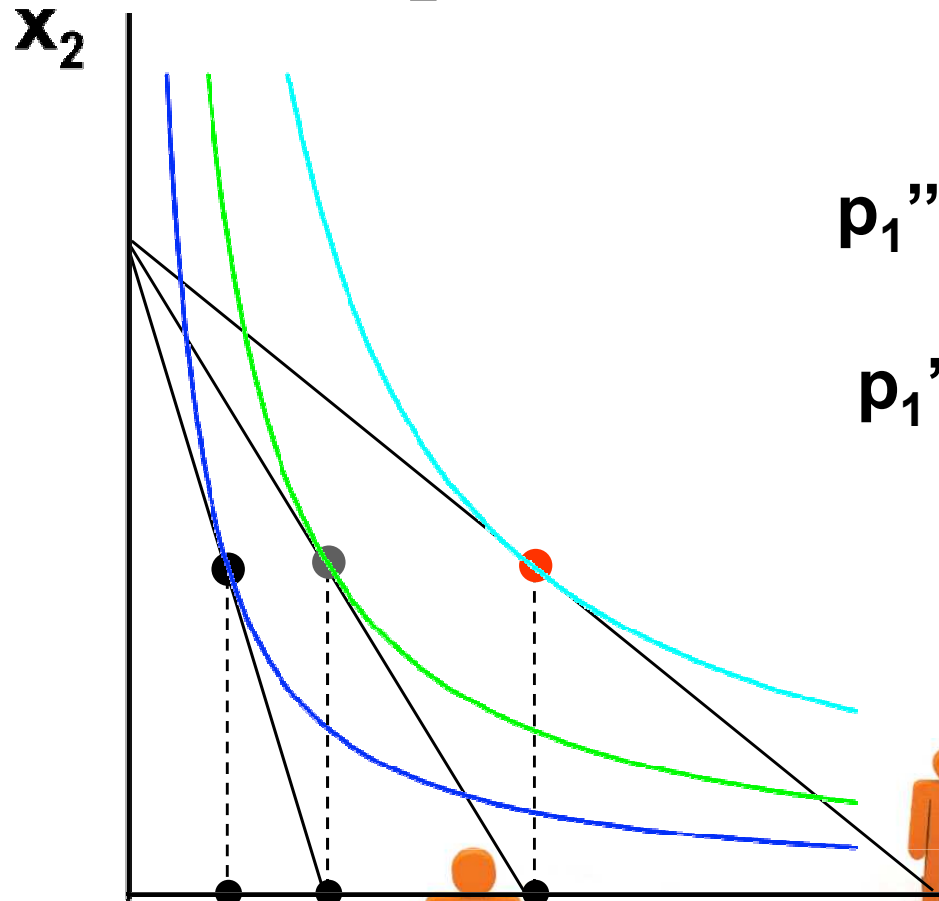
Own-Price Changes

Fixed p_2 and y .

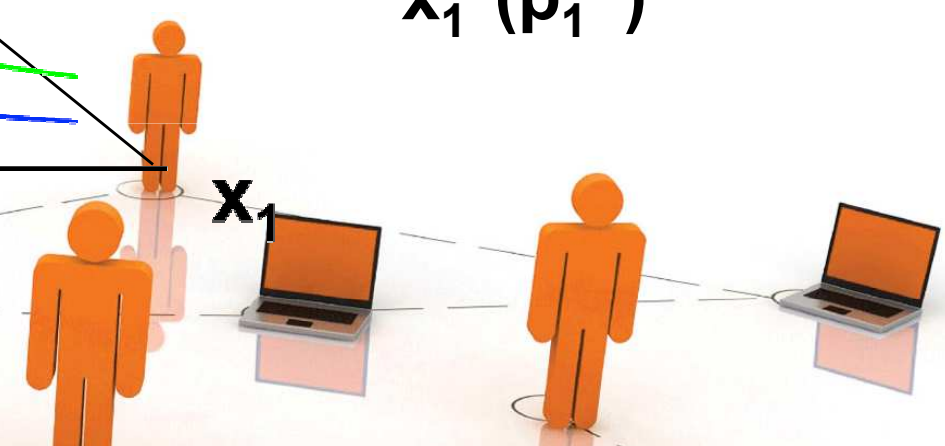
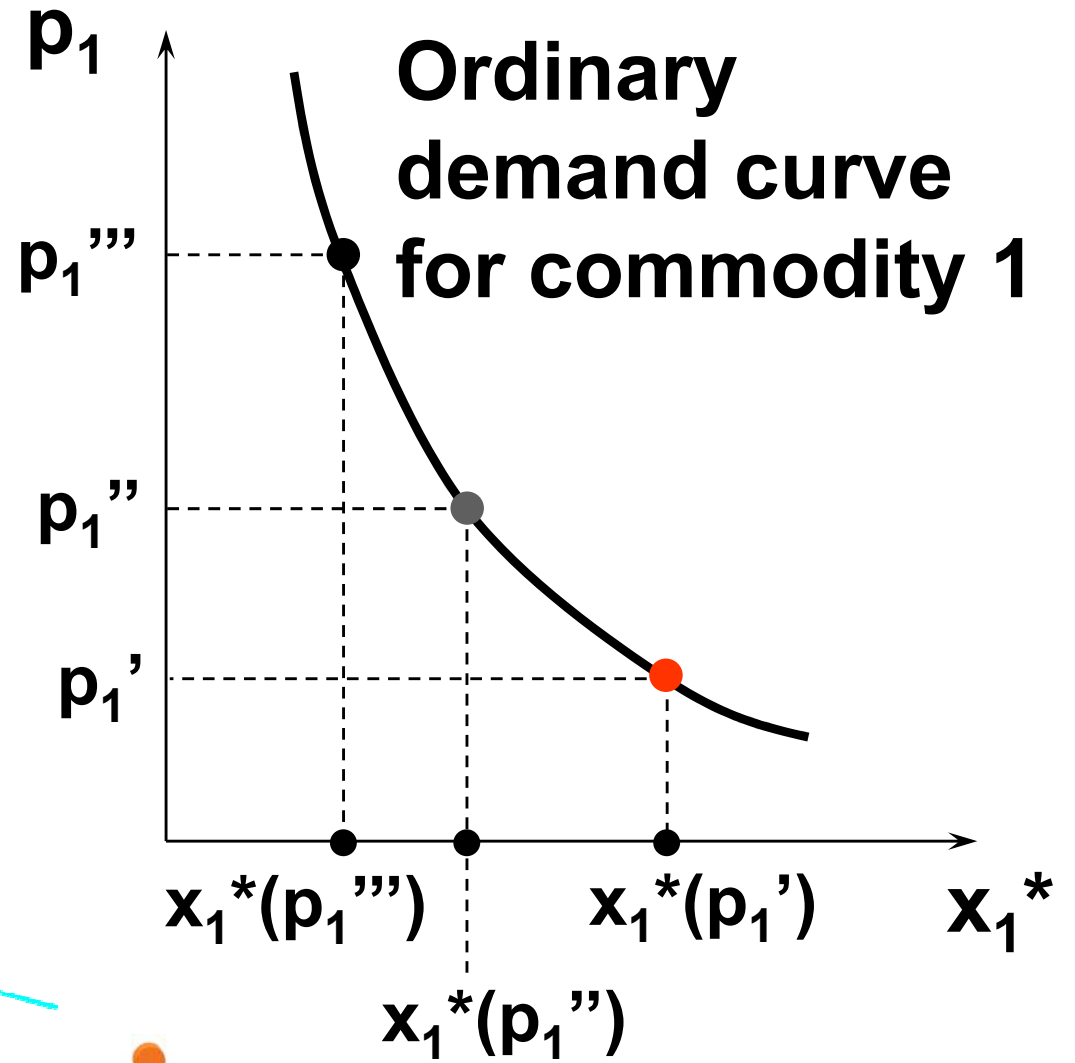


Own-Price Changes

Fixed p_2 and y .

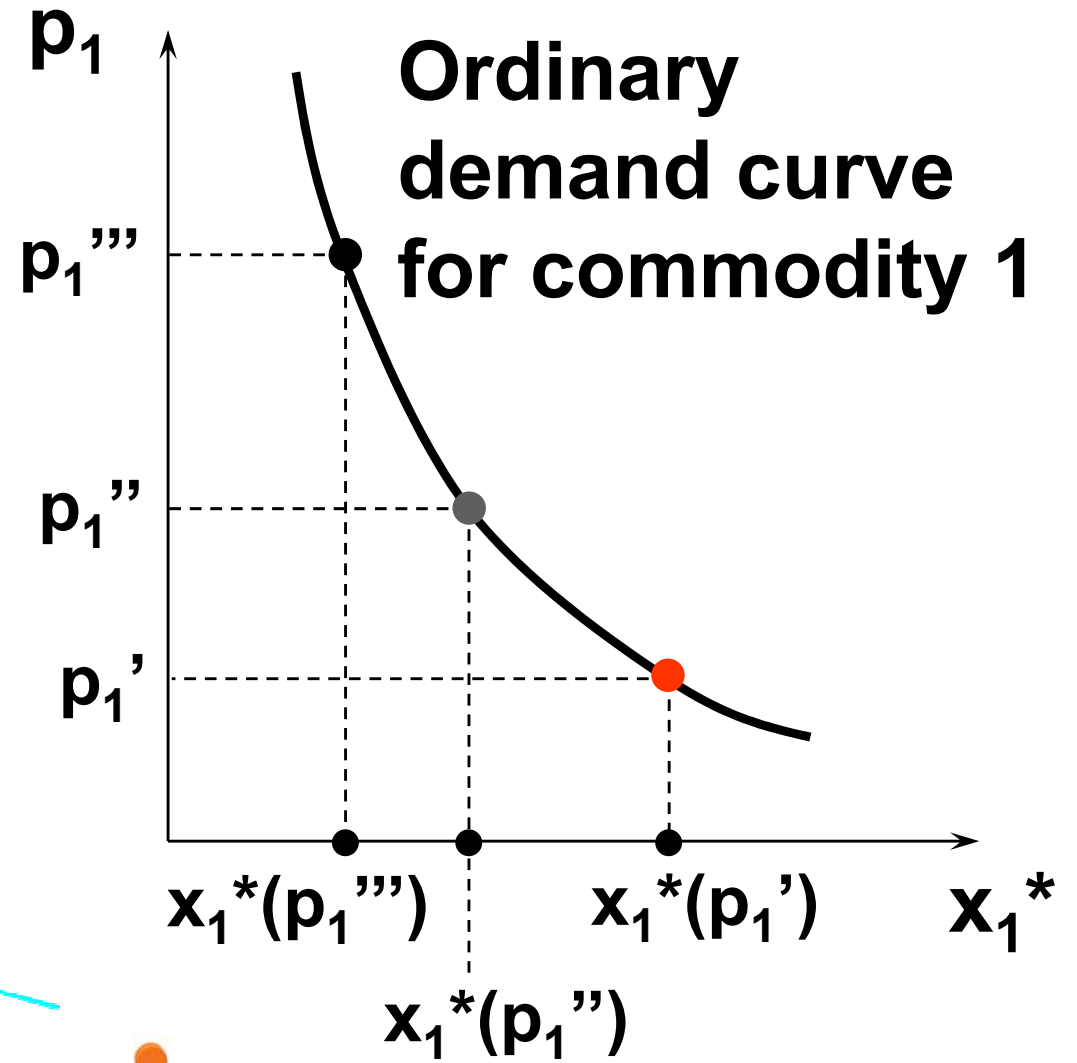
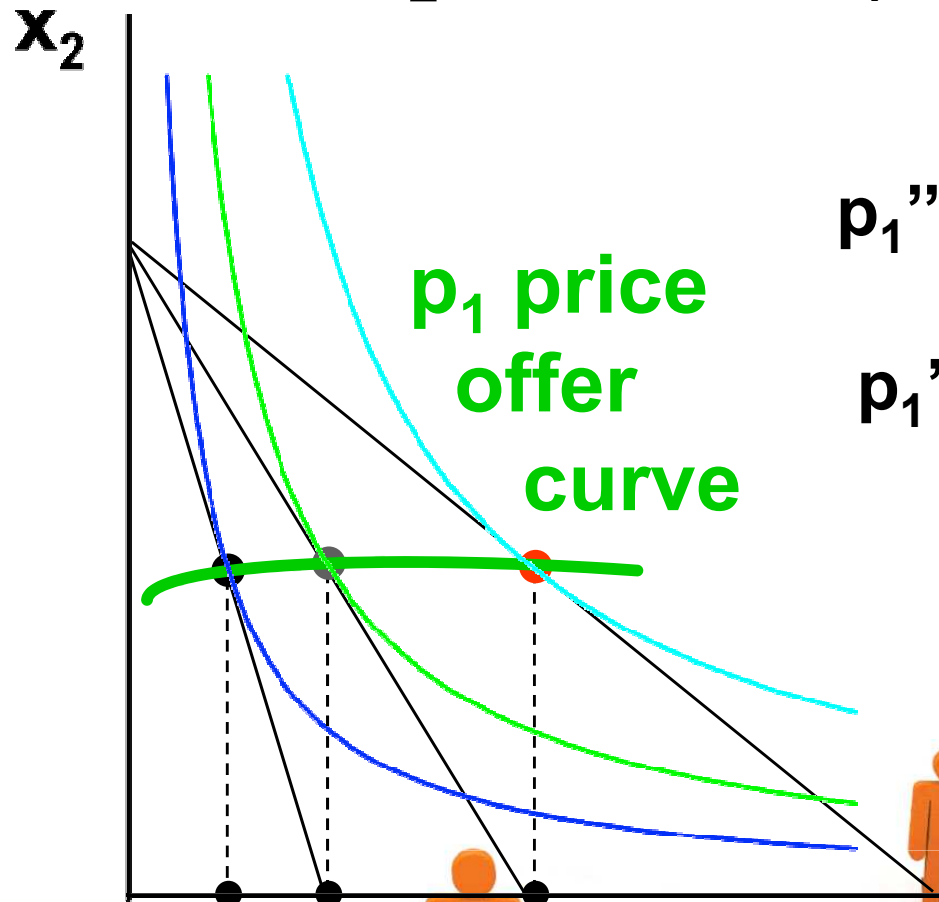


$x_1^*(p_1''')$
 $x_1^*(p_1')$
 $x_1^*(p_1'')$



Own-Price Changes

Fixed p_2 and y .



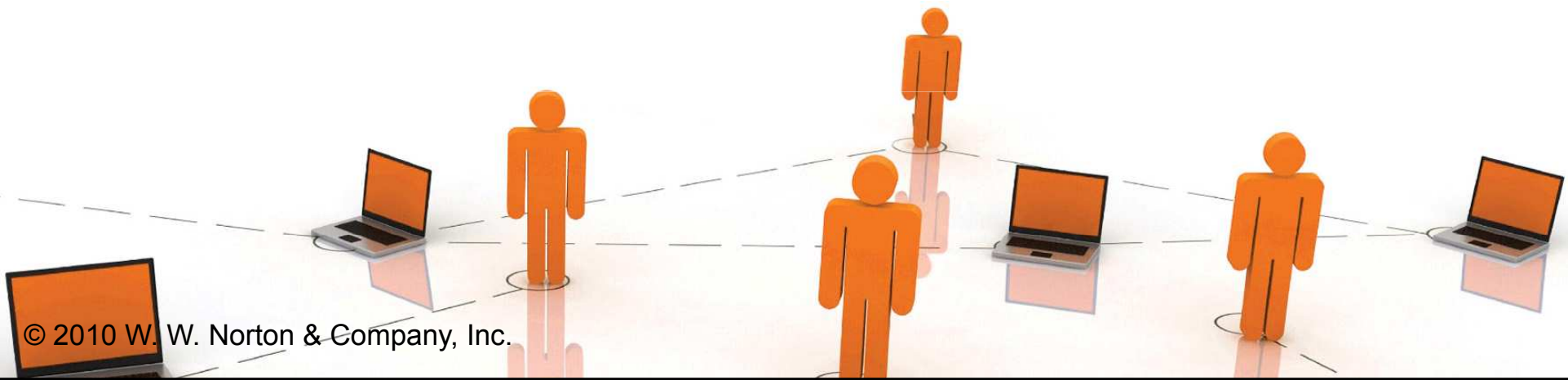
Own-Price Changes

- ◆ The curve containing all the utility-maximizing bundles traced out as p_1 changes, with p_2 and y constant, is the **p_1 -price offer curve**.
- ◆ The plot of the x_1 -coordinate of the p_1 -price offer curve against p_1 is the ordinary demand curve for commodity 1.



Own-Price Changes

- ◆ What does a p_1 price-offer curve look like for Cobb-Douglas preferences?



Own-Price Changes

- ◆ What does a p_1 price-offer curve look like for Cobb-Douglas preferences?
- ◆ Take

$$U(x_1, x_2) = x_1^a x_2^b.$$

Then the ordinary demand functions for commodities 1 and 2 are



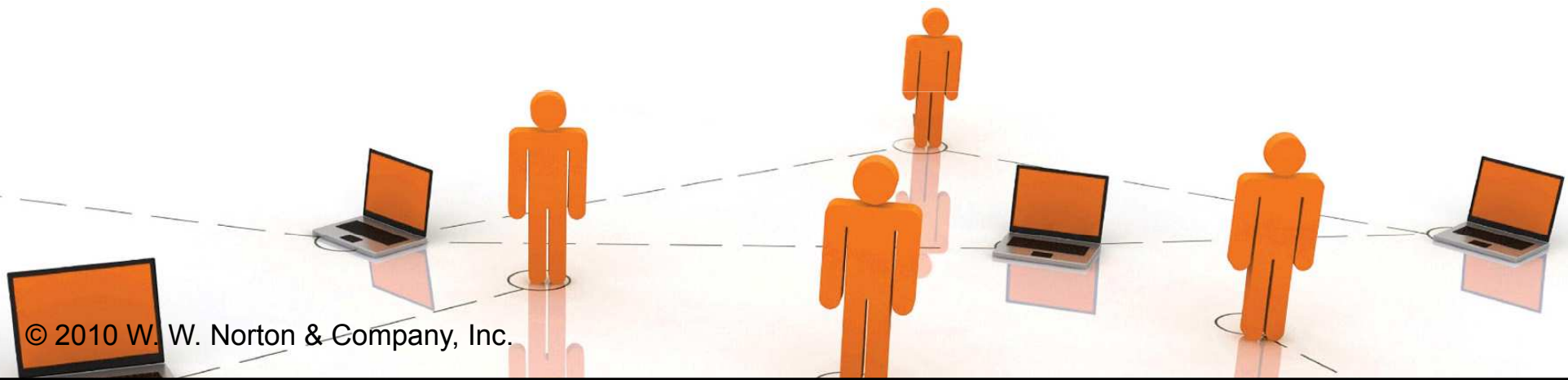
Own-Price Changes

$$x_1^*(p_1, p_2, y) = \frac{a}{a+b} \times \frac{y}{p_1}$$

and

$$x_2^*(p_1, p_2, y) = \frac{b}{a+b} \times \frac{y}{p_2}.$$

Notice that x_2^* does not vary with p_1 so the p_1 price offer curve is



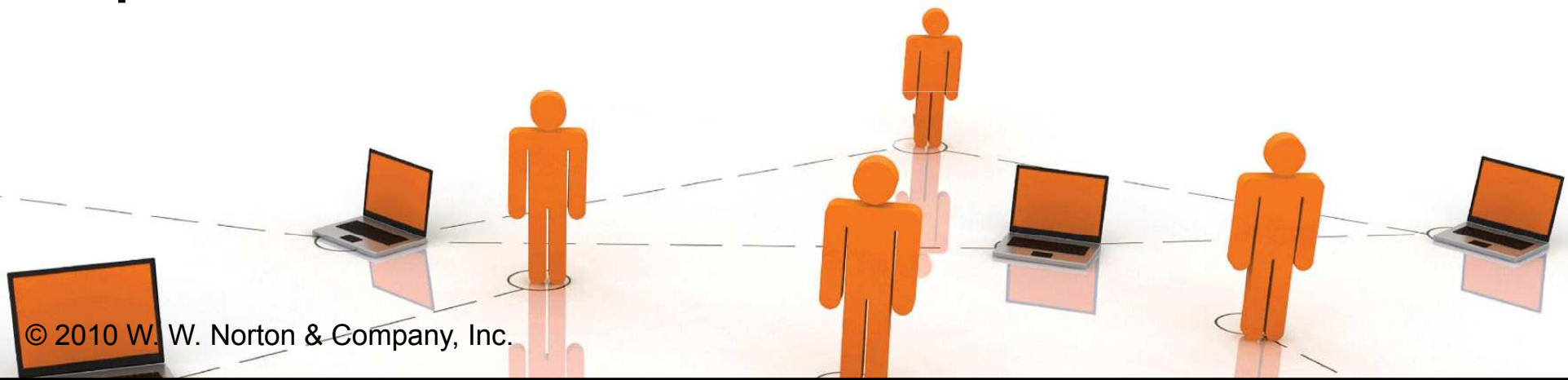
Own-Price Changes

$$x_1^*(p_1, p_2, y) = \frac{a}{a+b} \times \frac{y}{p_1}$$

and

$$x_2^*(p_1, p_2, y) = \frac{b}{a+b} \times \frac{y}{p_2}.$$

Notice that x_2^* does not vary with p_1 so the p_1 price offer curve is flat



Own-Price Changes

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and

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Notice that x_2^* does not vary with p_1 so the p_1 price offer curve is flat and the ordinary demand curve for commodity 1 is a



Own-Price Changes

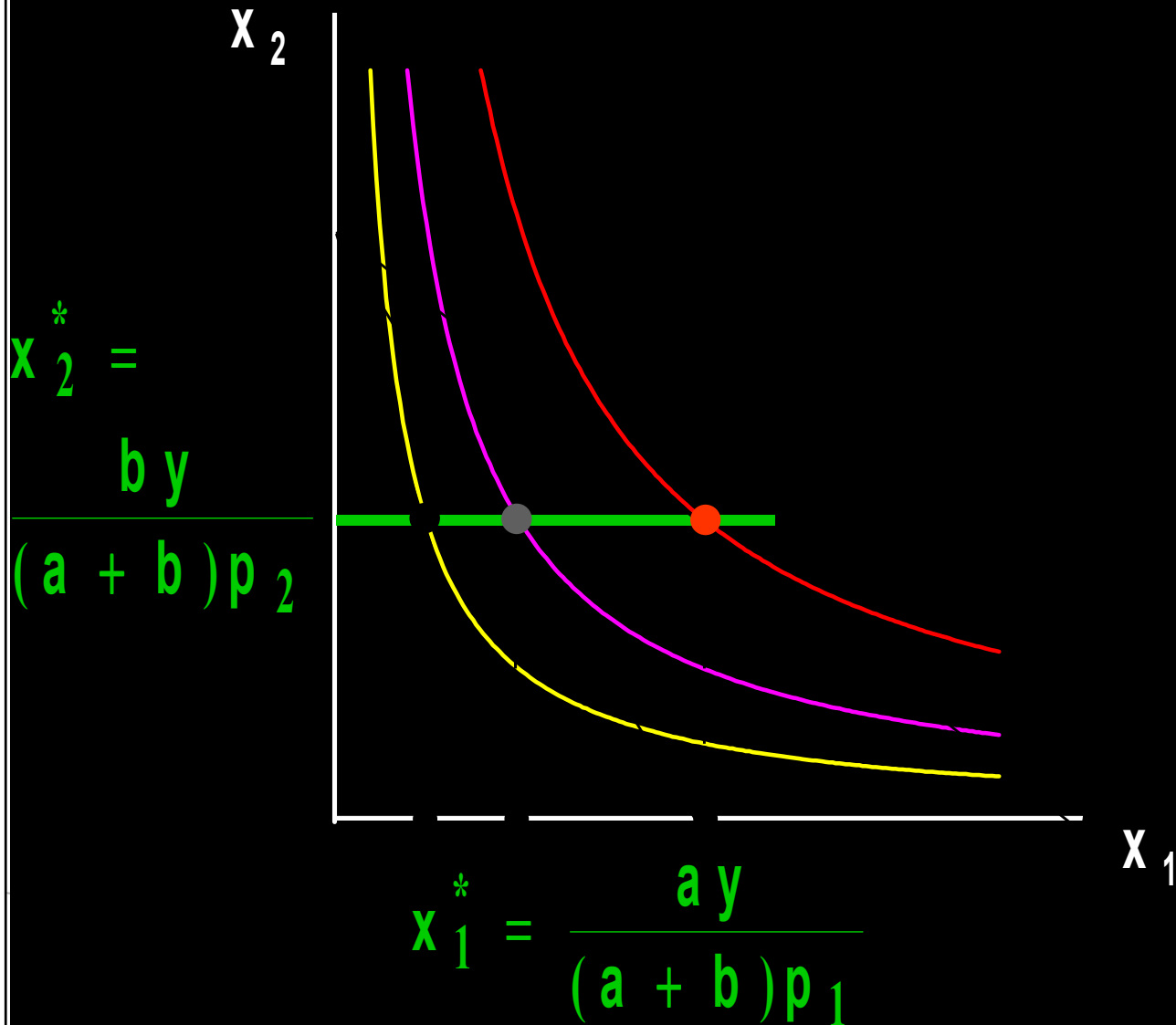
$$x_1^*(p_1, p_2, y) = \frac{a}{a+b} \times \frac{y}{p_1}$$

and

$$x_2^*(p_1, p_2, y) = \frac{b}{a+b} \times \frac{y}{p_2}.$$

Notice that x_2^* does not vary with p_1 so the p_1 price offer curve is flat and the ordinary demand curve for commodity 1 is a rectangular hyperbola.

Own Price Changes

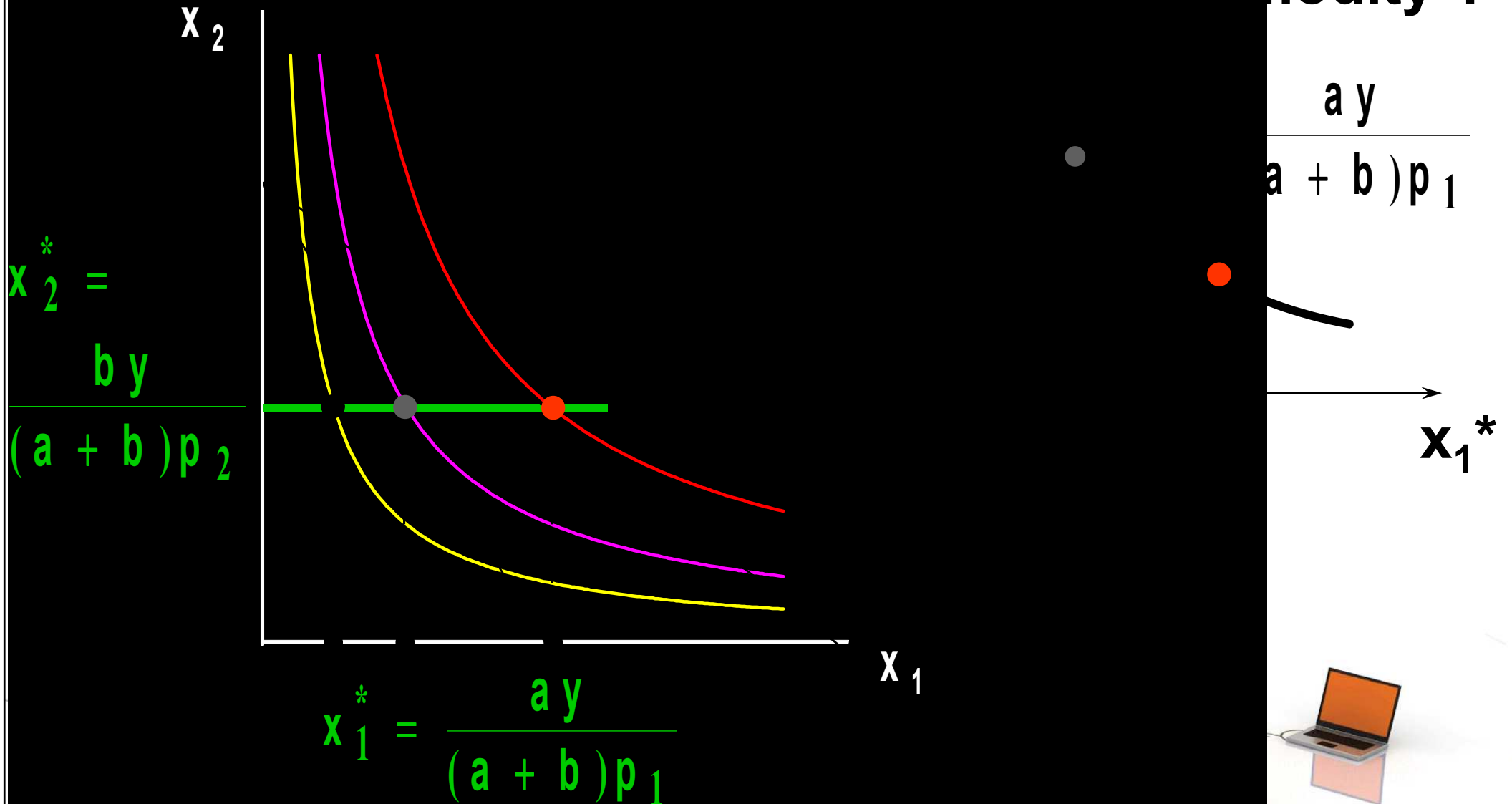


Own Price Changes

p_1 ↑

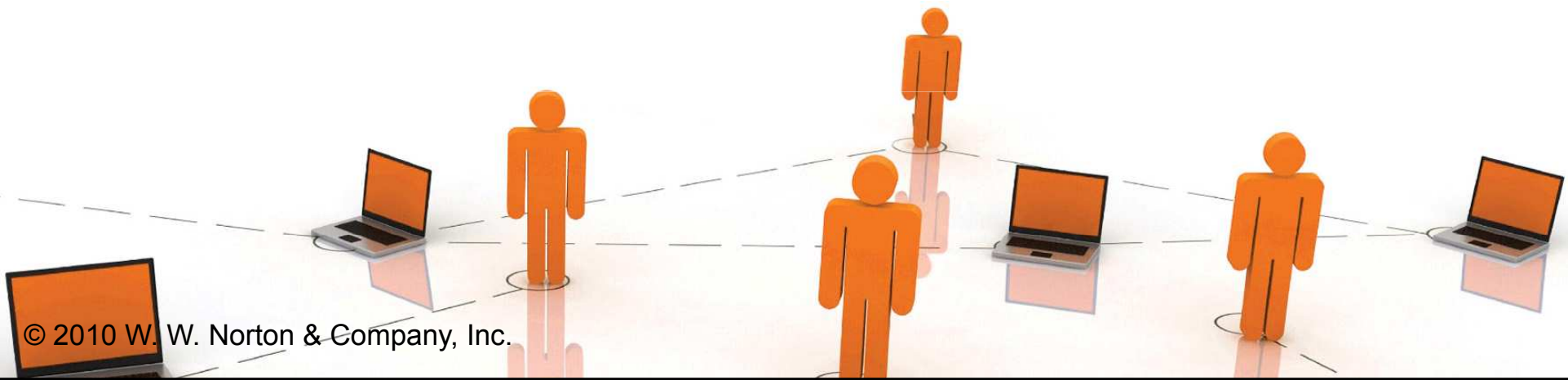
Ordinary

curve
commodity 1



Own-Price Changes

- ◆ **What does a p_1 price-offer curve look like for a perfect-complements utility function?**



Own-Price Changes

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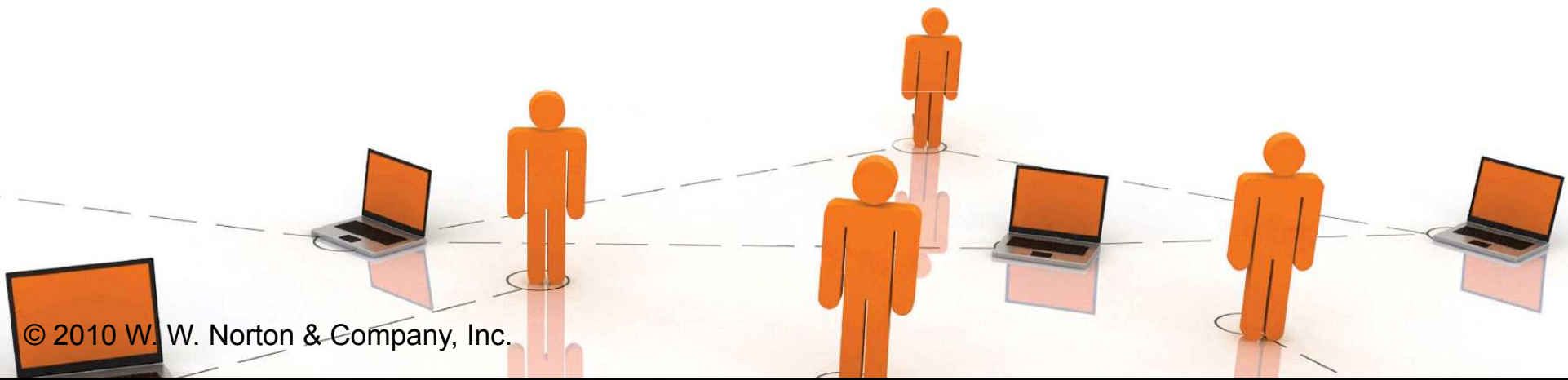
$$U(x_1, x_2) = \min\{x_1, x_2\}.$$

Then the ordinary demand functions for commodities 1 and 2 are



Own-Price Changes

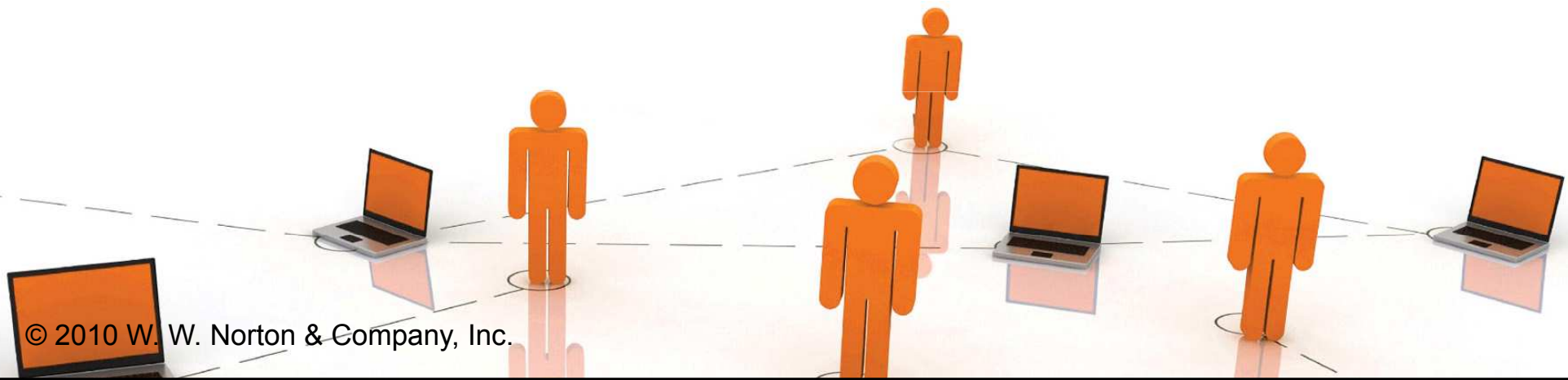
$$x_1^*(p_1, p_2, y) = x_2^*(p_1, p_2, y) = \frac{y}{p_1 + p_2}.$$



Own-Price Changes

$$x_1^*(p_1, p_2, y) = x_2^*(p_1, p_2, y) = \frac{y}{p_1 + p_2}.$$

With p_2 and y fixed, higher p_1 causes smaller x_1^* and x_2^* .

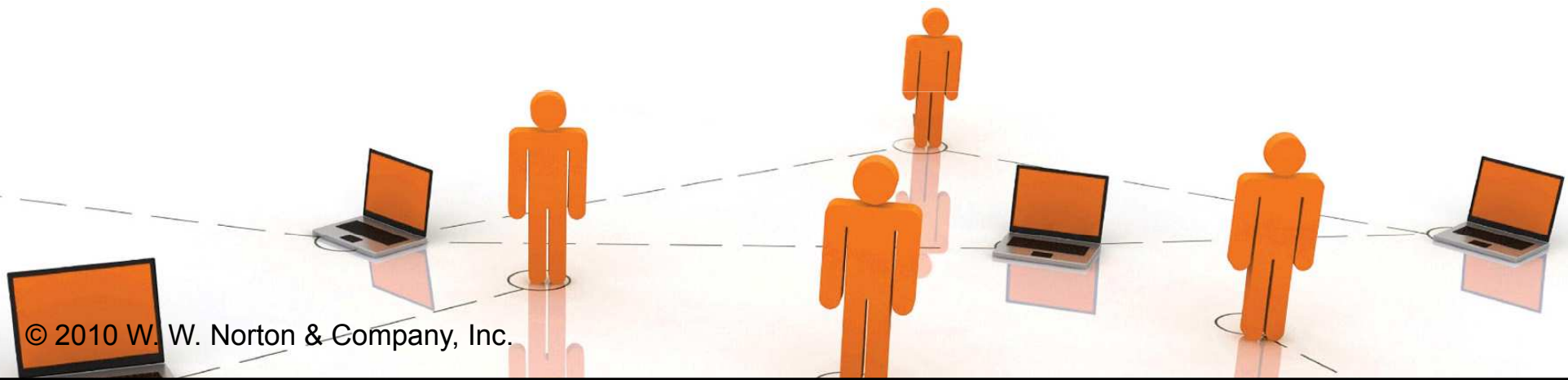


Own-Price Changes

$$x_1^*(p_1, p_2, y) = x_2^*(p_1, p_2, y) = \frac{y}{p_1 + p_2}.$$

With p_2 and y fixed, higher p_1 causes smaller x_1^* and x_2^* .

As $p_1 \rightarrow 0$, $x_1^* = x_2^* \rightarrow \frac{y}{p_2}$.



Own-Price Changes

$$x_1^*(p_1, p_2, y) = x_2^*(p_1, p_2, y) = \frac{y}{p_1 + p_2}.$$

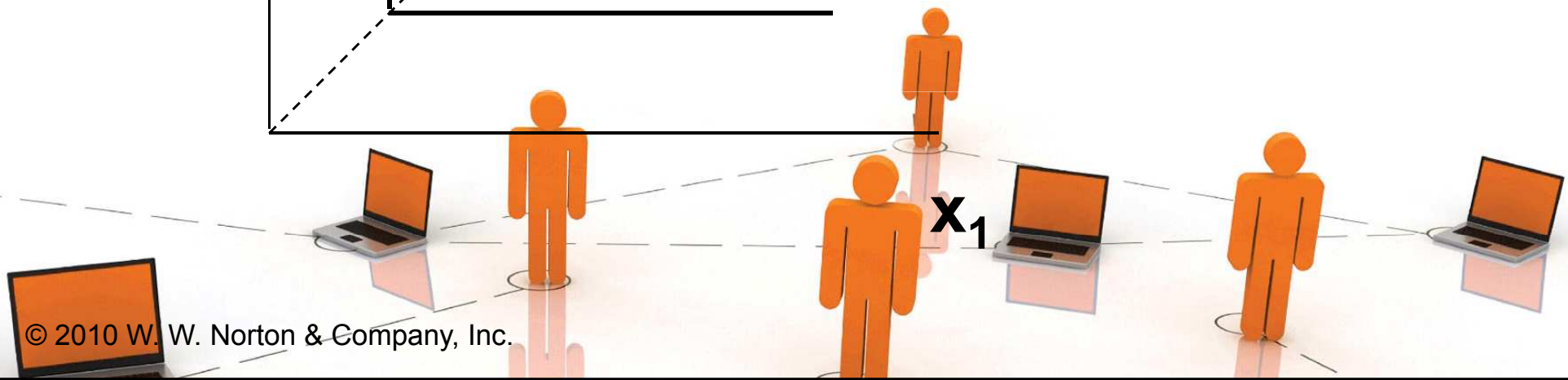
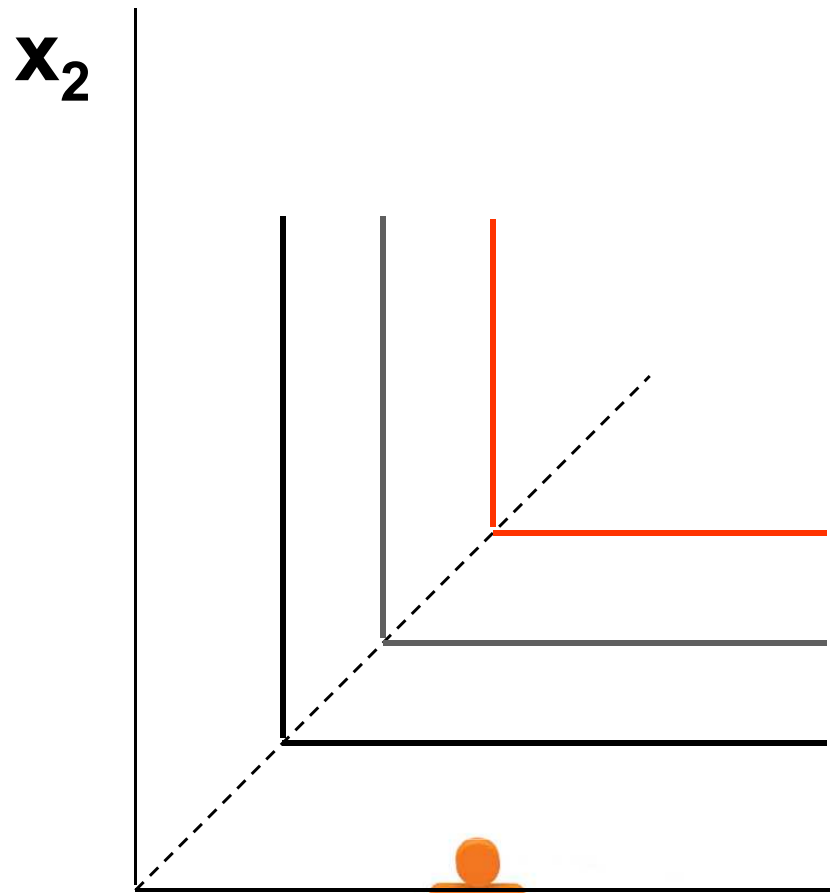
With p_2 and y fixed, higher p_1 causes smaller x_1^* and x_2^* .

$$\text{As } p_1 \rightarrow 0, \quad x_1^* = x_2^* \rightarrow \frac{y}{p_2}.$$

$$\text{As } p_1 \rightarrow \infty, \quad x_1^* = x_2^* \rightarrow 0.$$

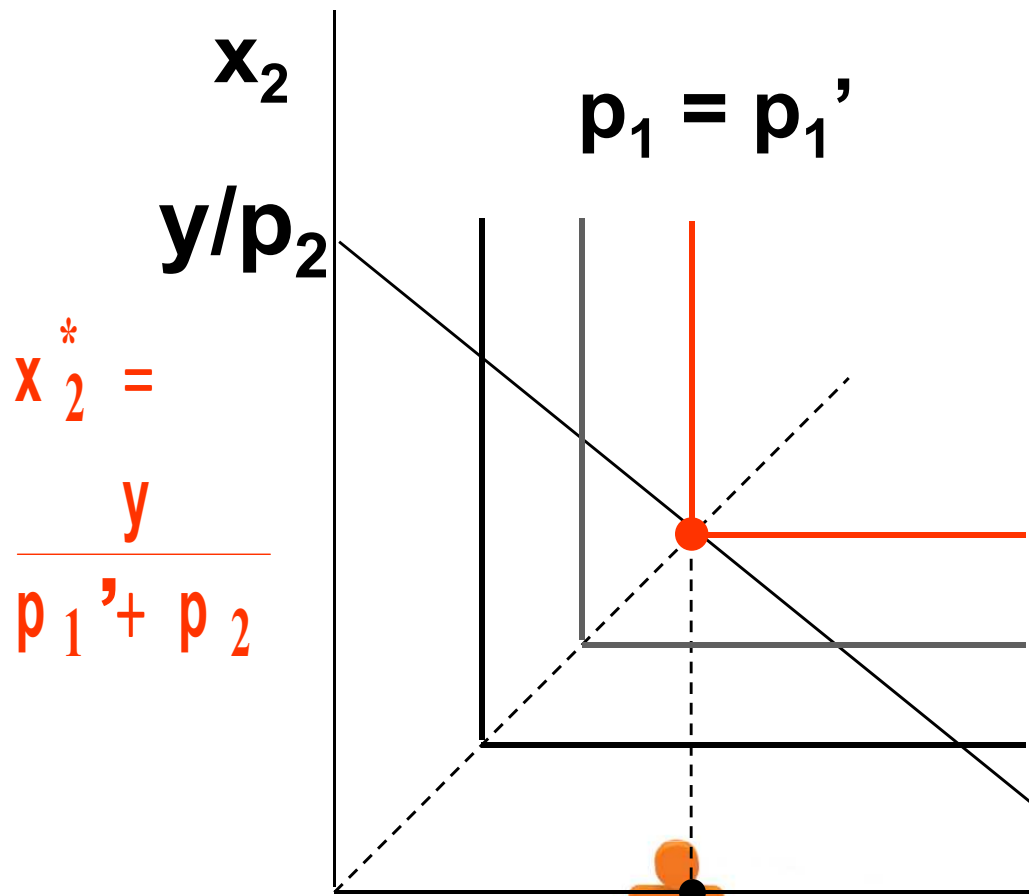
Own-Price Changes

Fixed p_2 and y .



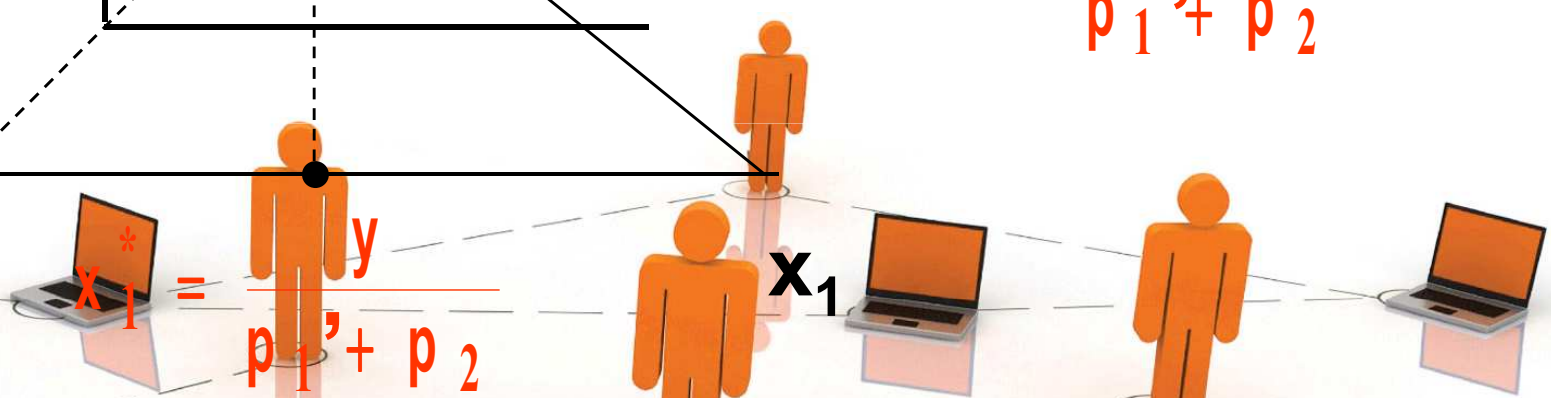
Own-Price Changes

Fixed p_2 and y .



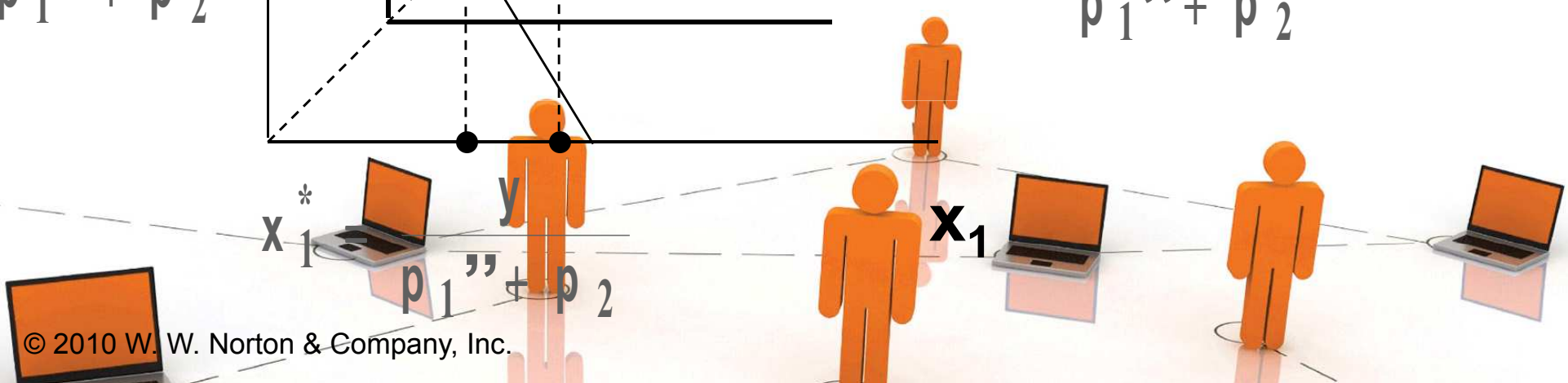
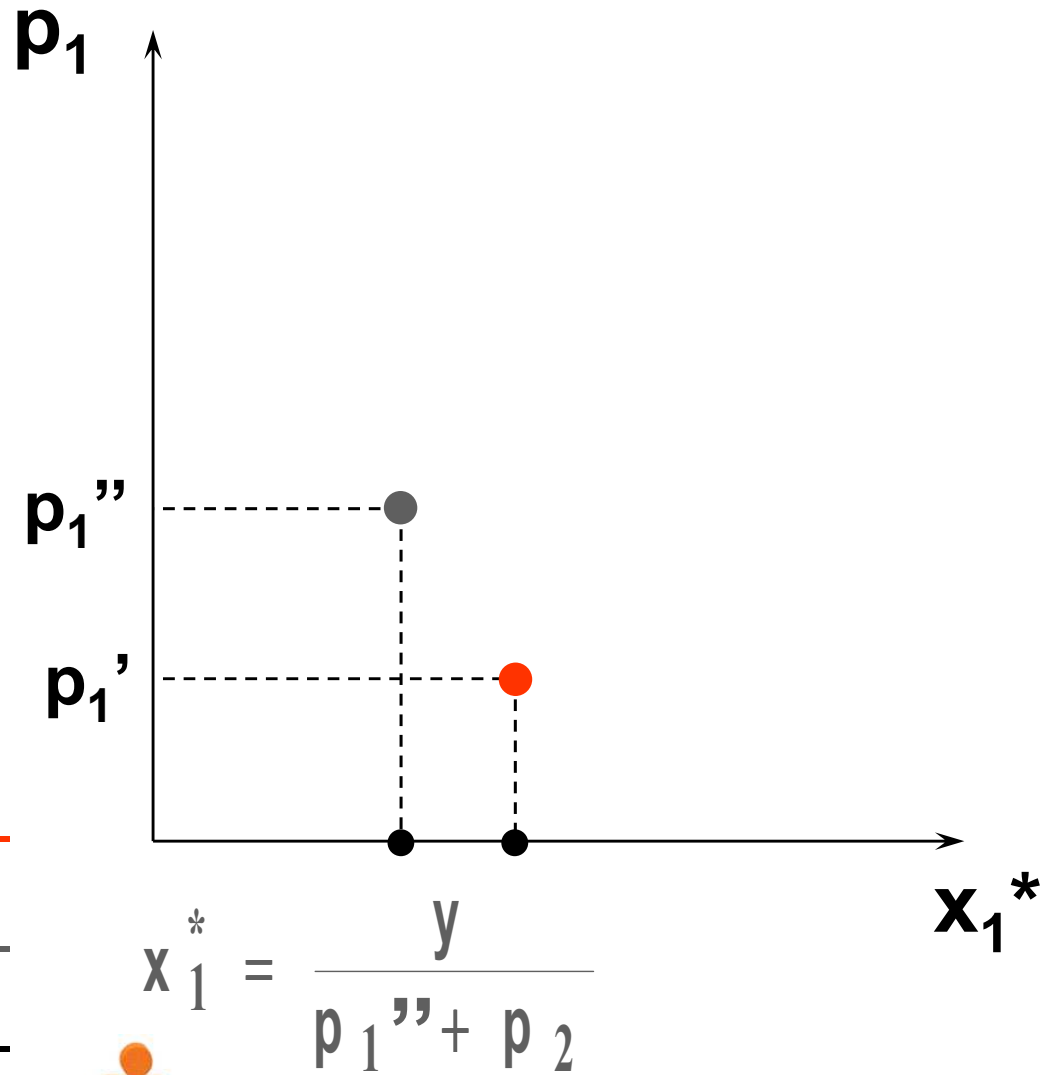
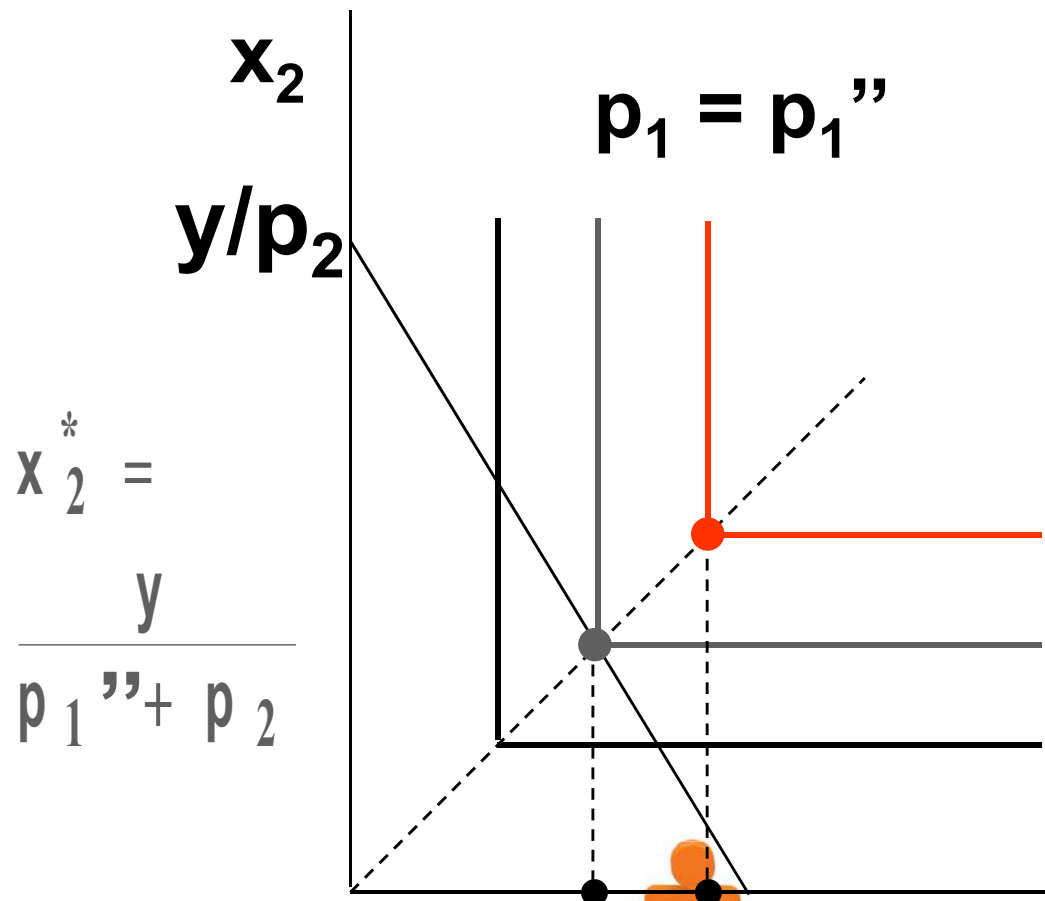
$$x_2^* = \frac{y}{p_1' + p_2}$$

$$x_1^* = \frac{y}{p_1' + p_2}$$



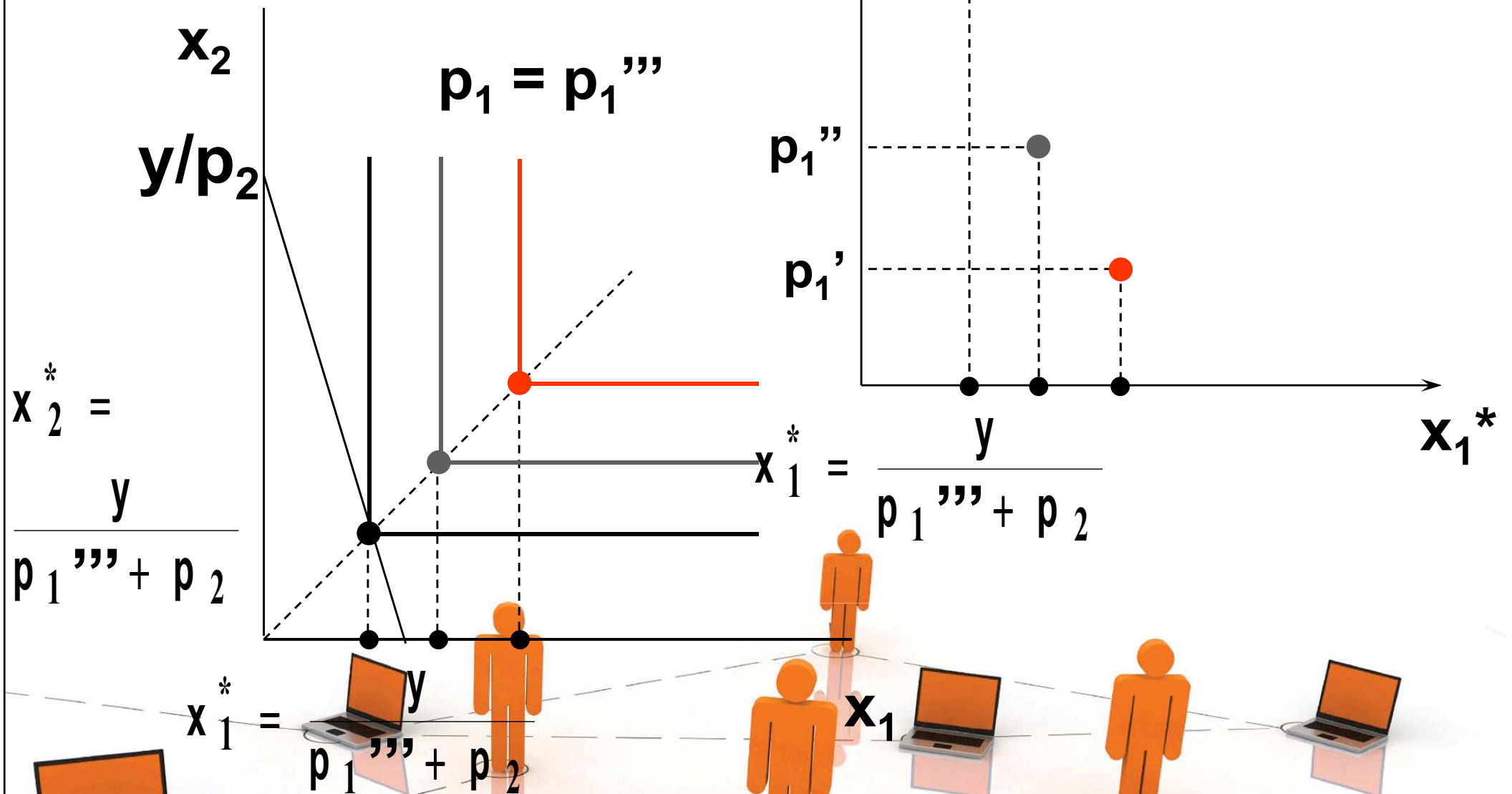
Own-Price Changes

Fixed p_2 and y .



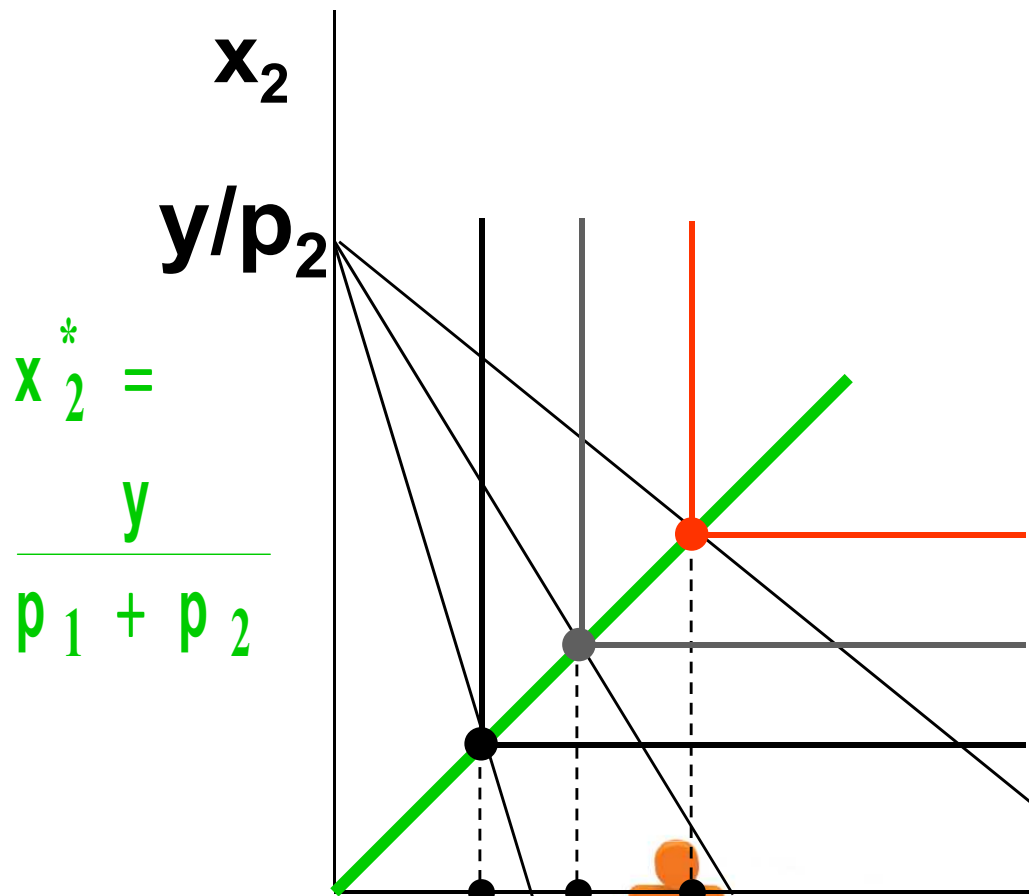
Own-Price Changes

Fixed p_2 and y .

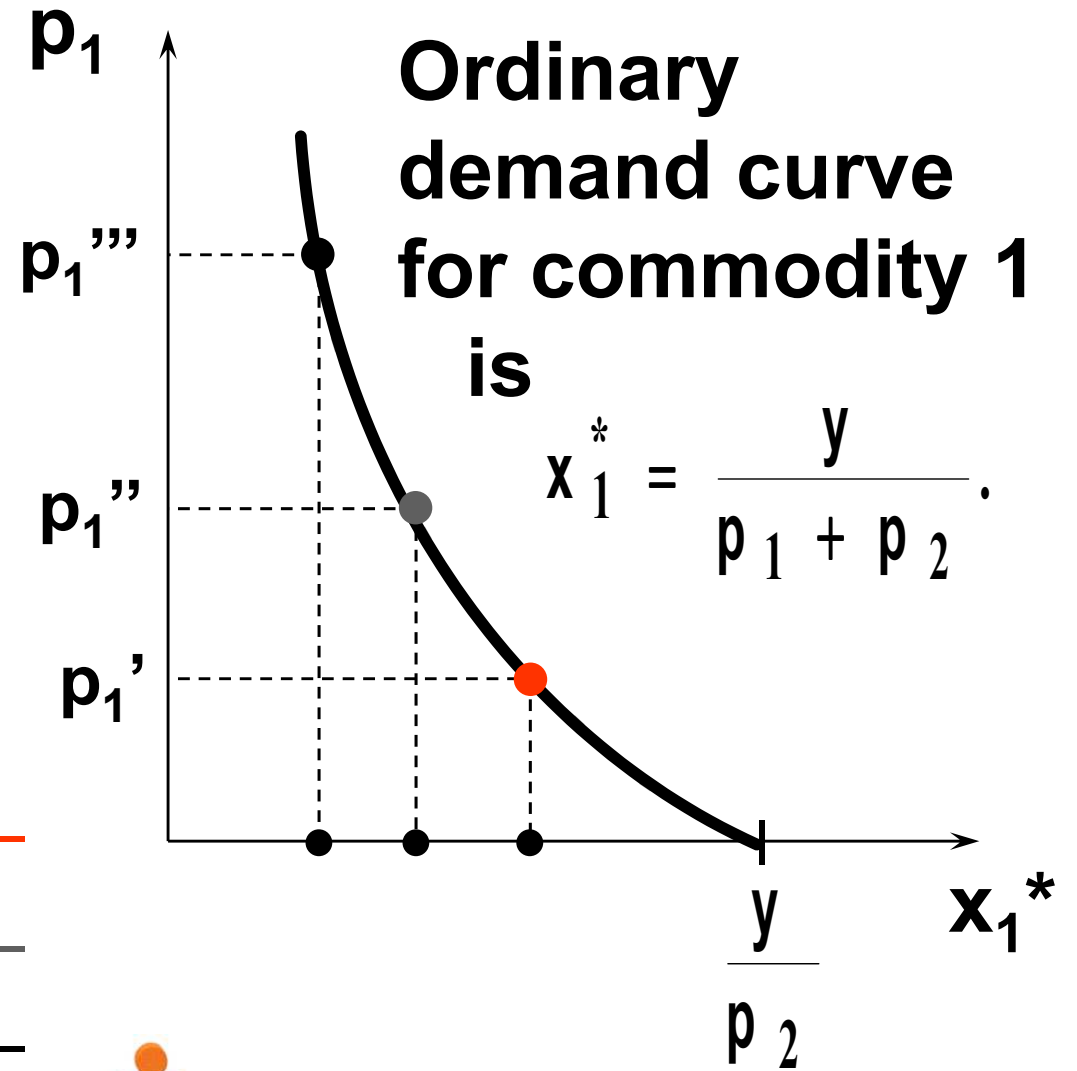


Own-Price Changes

Fixed p_2 and y .

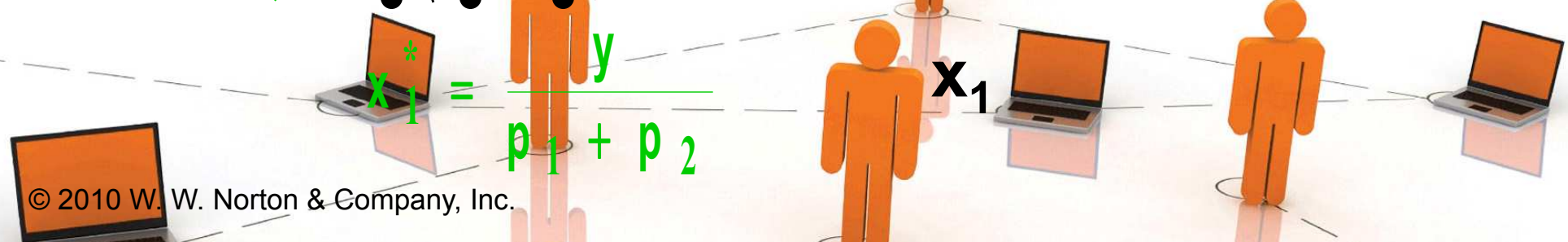


$$x_2^* = \frac{y}{p_1 + p_2}$$



Ordinary demand curve for commodity 1 is

$$x_1^* = \frac{y}{p_1 + p_2}$$



Own-Price Changes

- ◆ **What does a p_1 price-offer curve look like for a perfect-substitutes utility function?**

$$U(x_1, x_2) = x_1 + x_2.$$

Then the ordinary demand functions for commodities 1 and 2 are

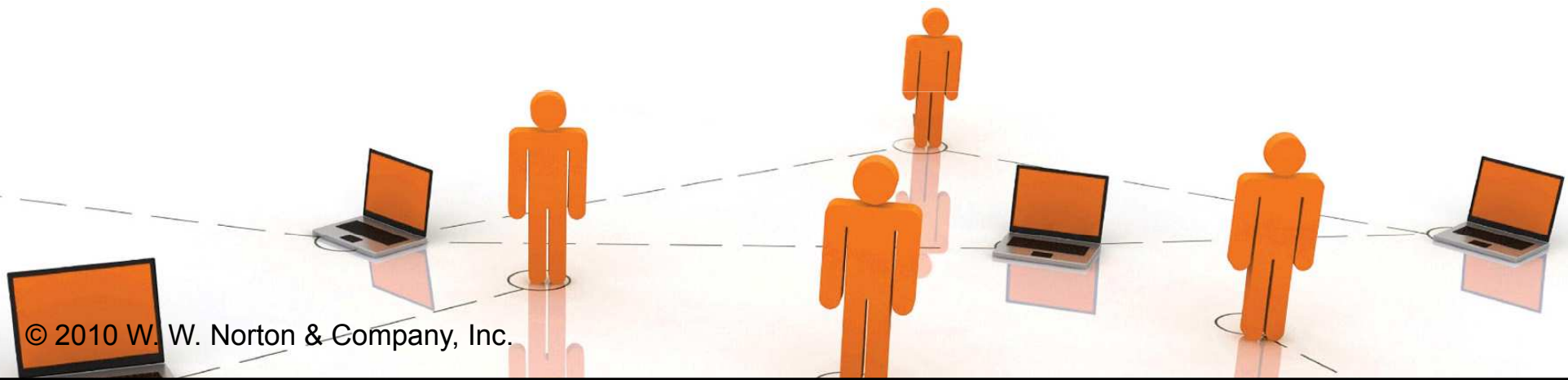


Own-Price Changes

$$x_1^*(p_1, p_2, y) = \begin{cases} 0 & , \text{ if } p_1 > p_2 \\ y / p_1 & , \text{ if } p_1 < p_2 \end{cases}$$

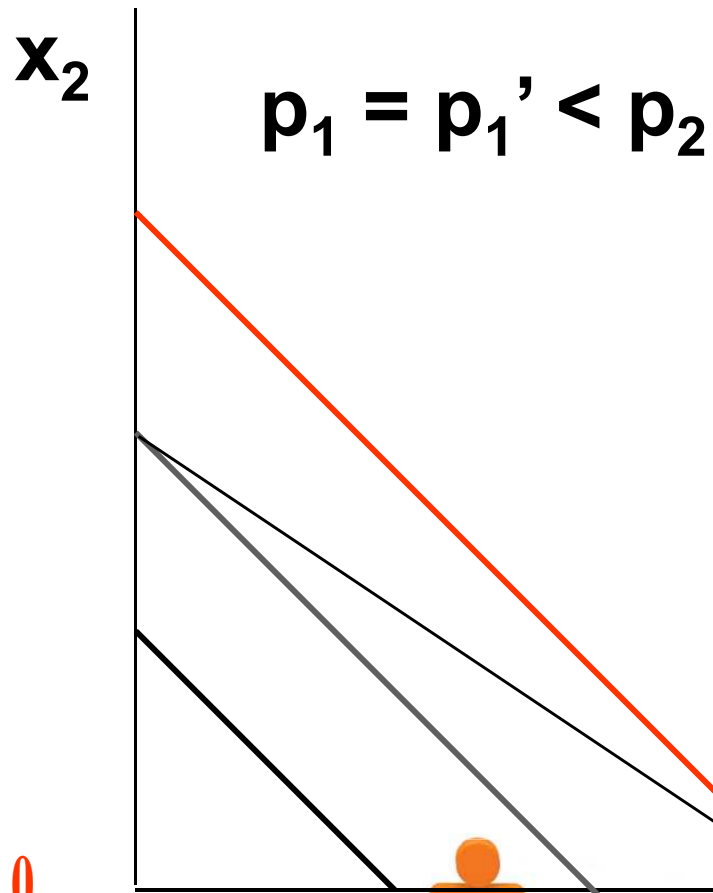
and

$$x_2^*(p_1, p_2, y) = \begin{cases} 0 & , \text{ if } p_1 < p_2 \\ y / p_2 & , \text{ if } p_1 > p_2 . \end{cases}$$



Own-Price Changes

Fixed p_2 and y .



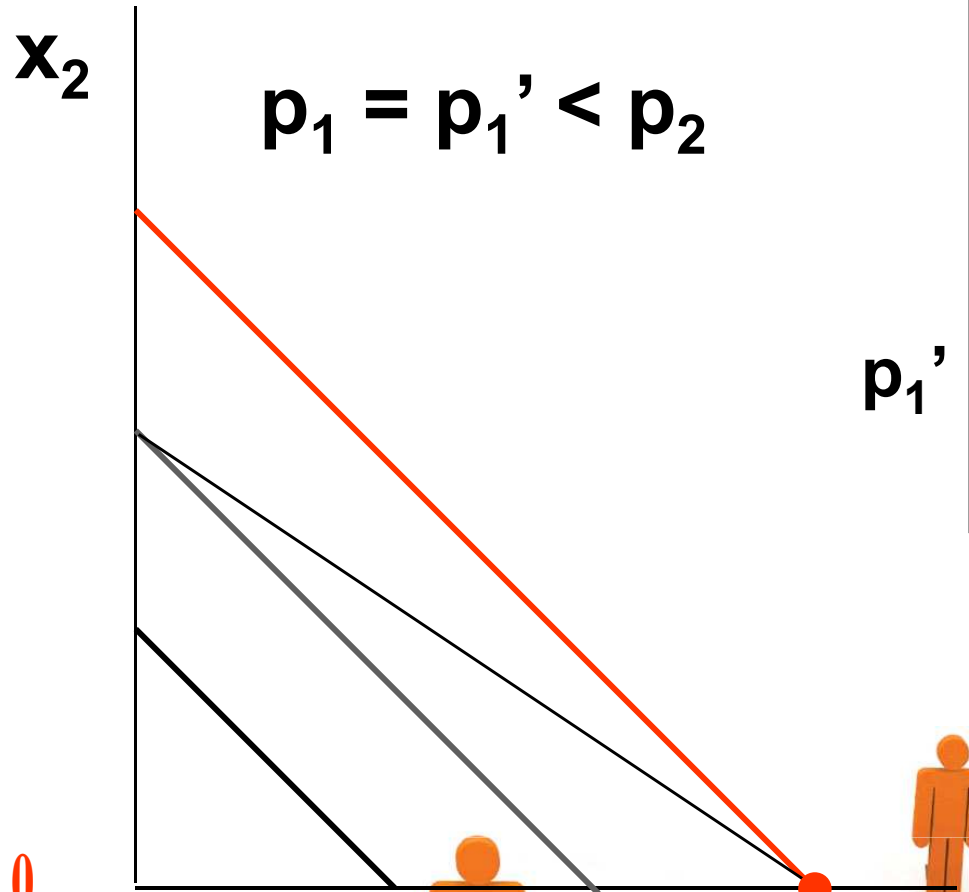
$$x_2^* = 0$$

$$x_1^* = y$$

x_1

Own-Price Changes

Fixed p_2 and y .



$$x_1^* = \frac{y}{p_1'} x_1^*$$

$$x_2^* = 0$$

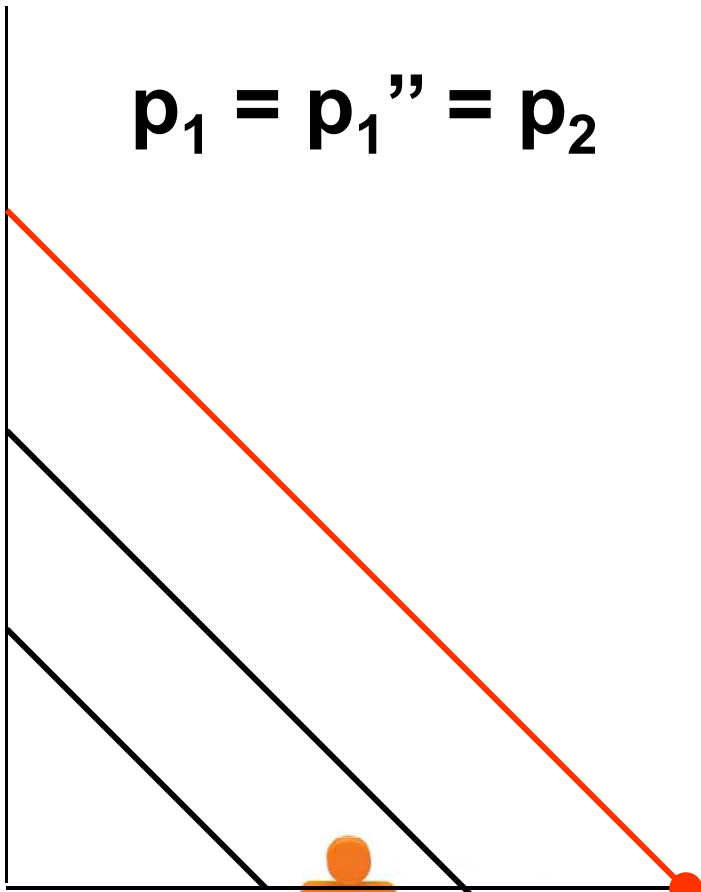
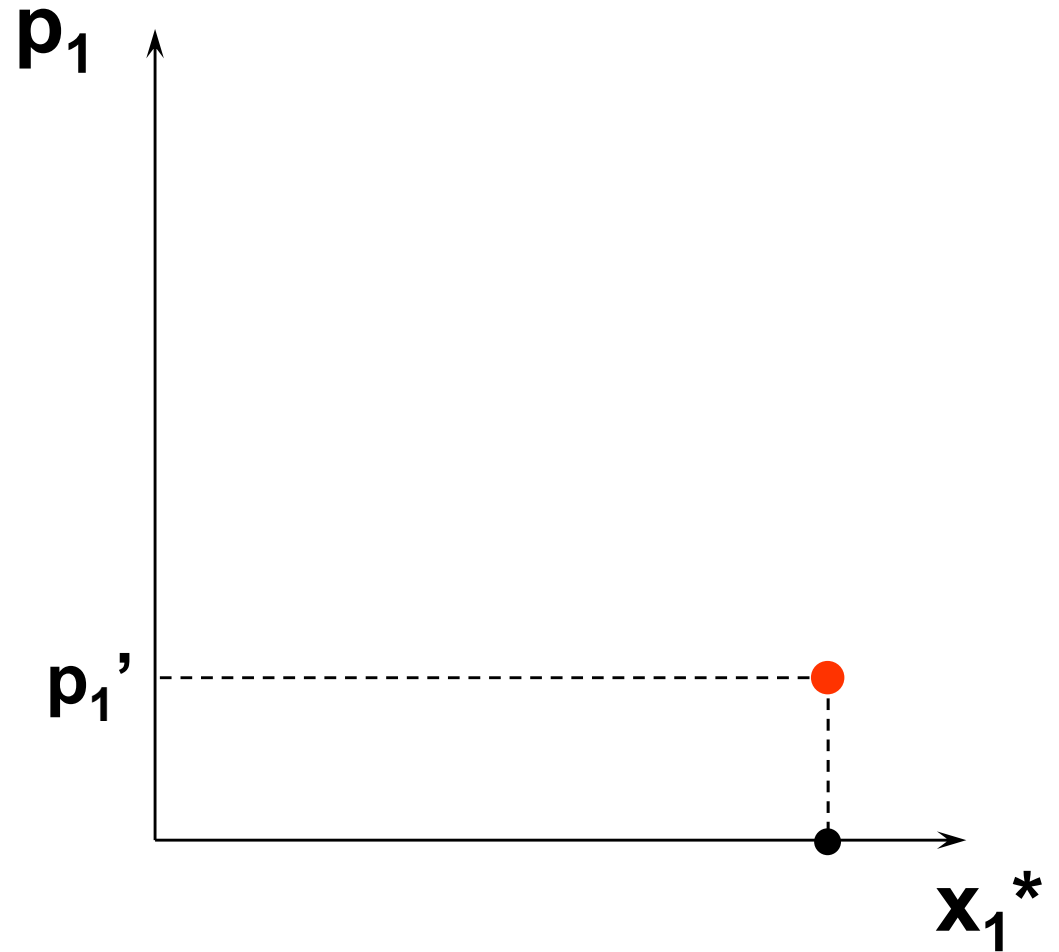
$$x_1^* = \frac{y}{p_1'}$$

x_1

Own-Price Changes

Fixed p_2 and y .

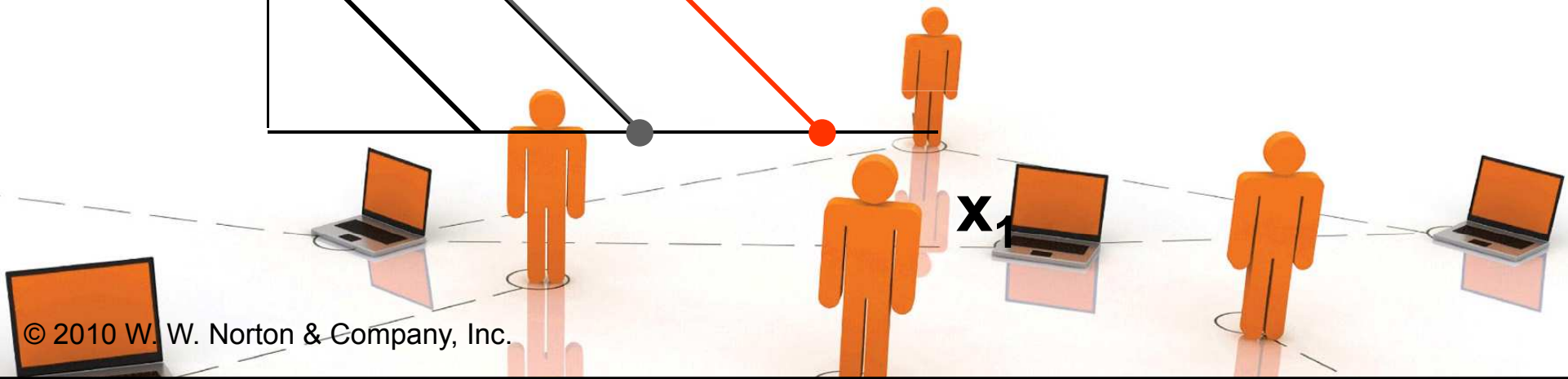
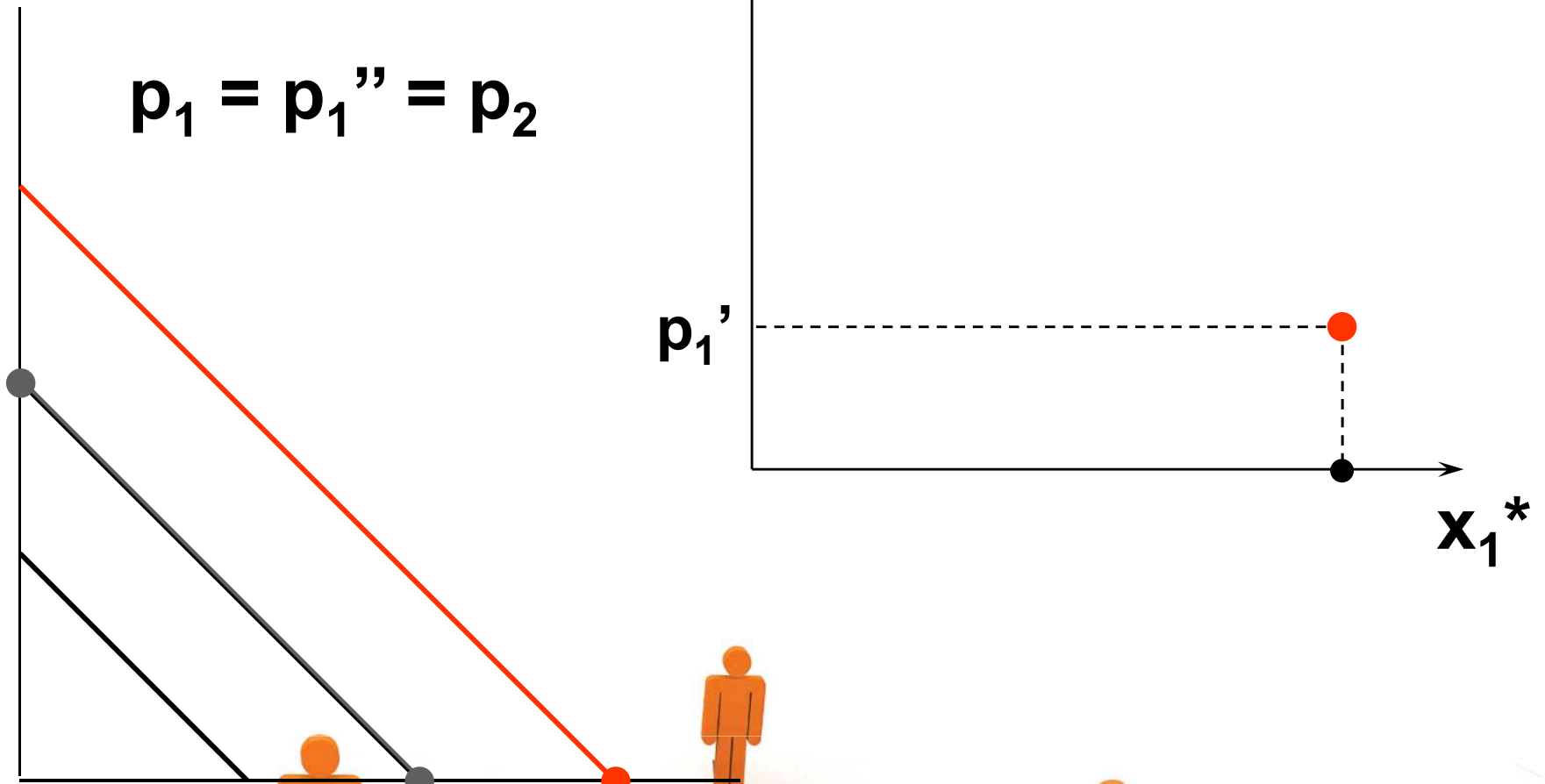
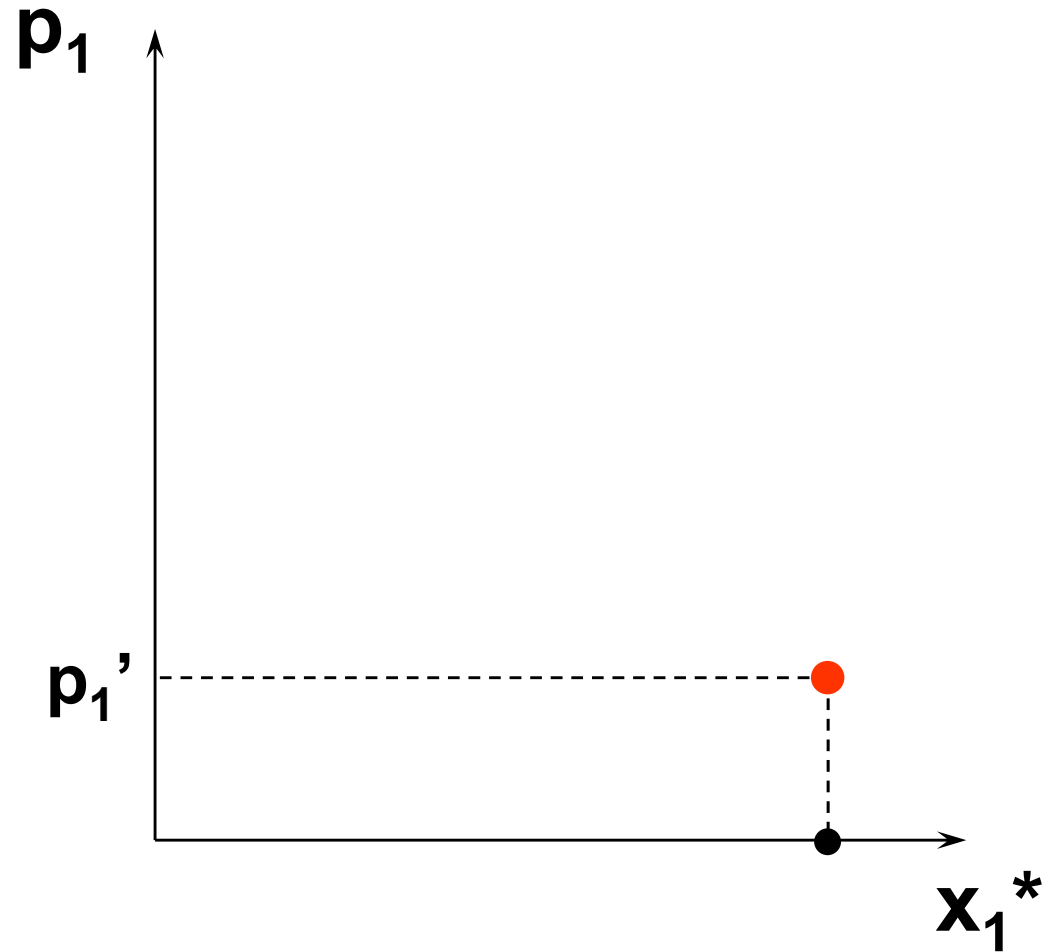
x_2
 $p_1 = p_1'' = p_2$



Own-Price Changes

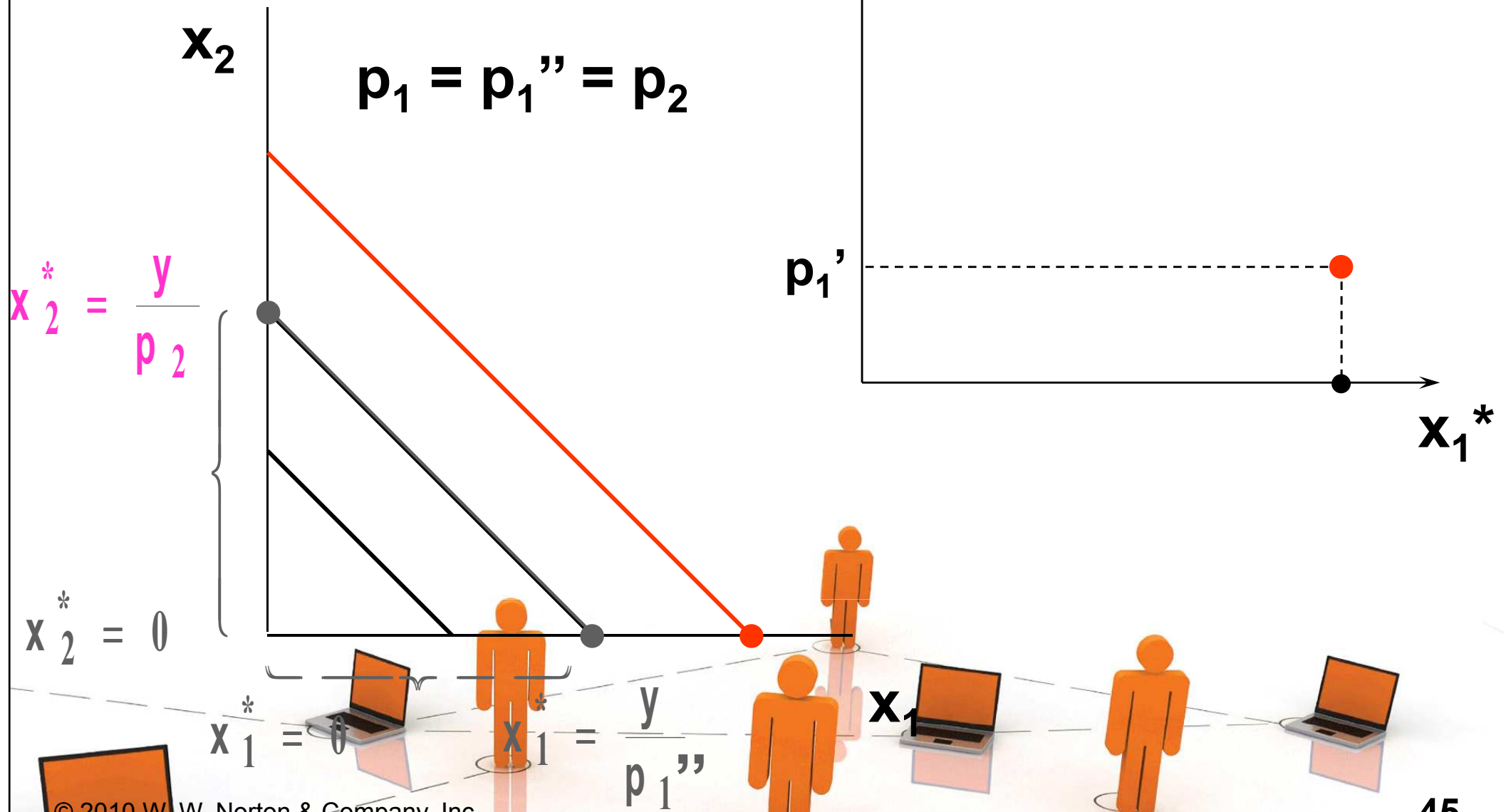
Fixed p_2 and y .

x_2
 $p_1 = p_1'' = p_2$



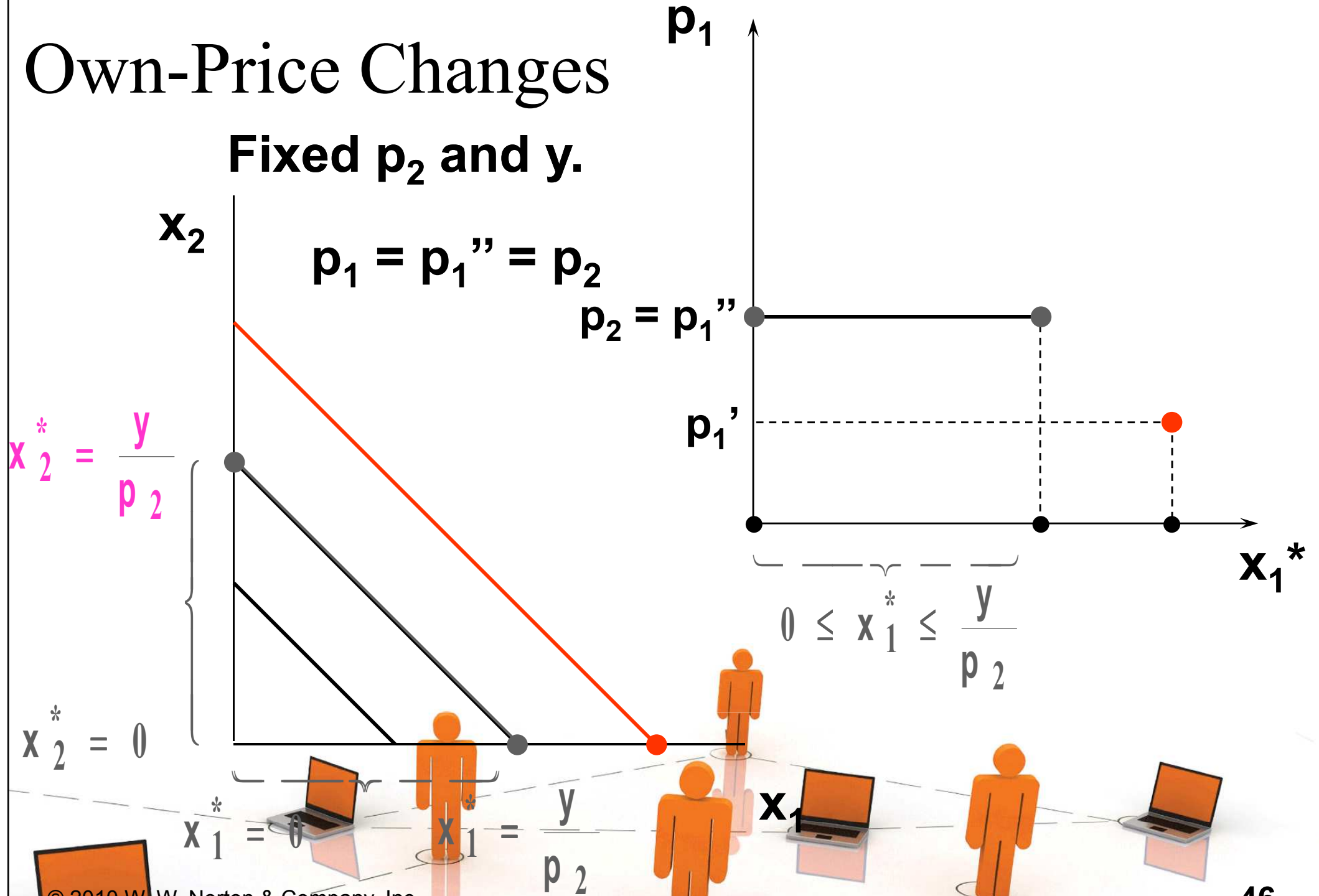
Own-Price Changes

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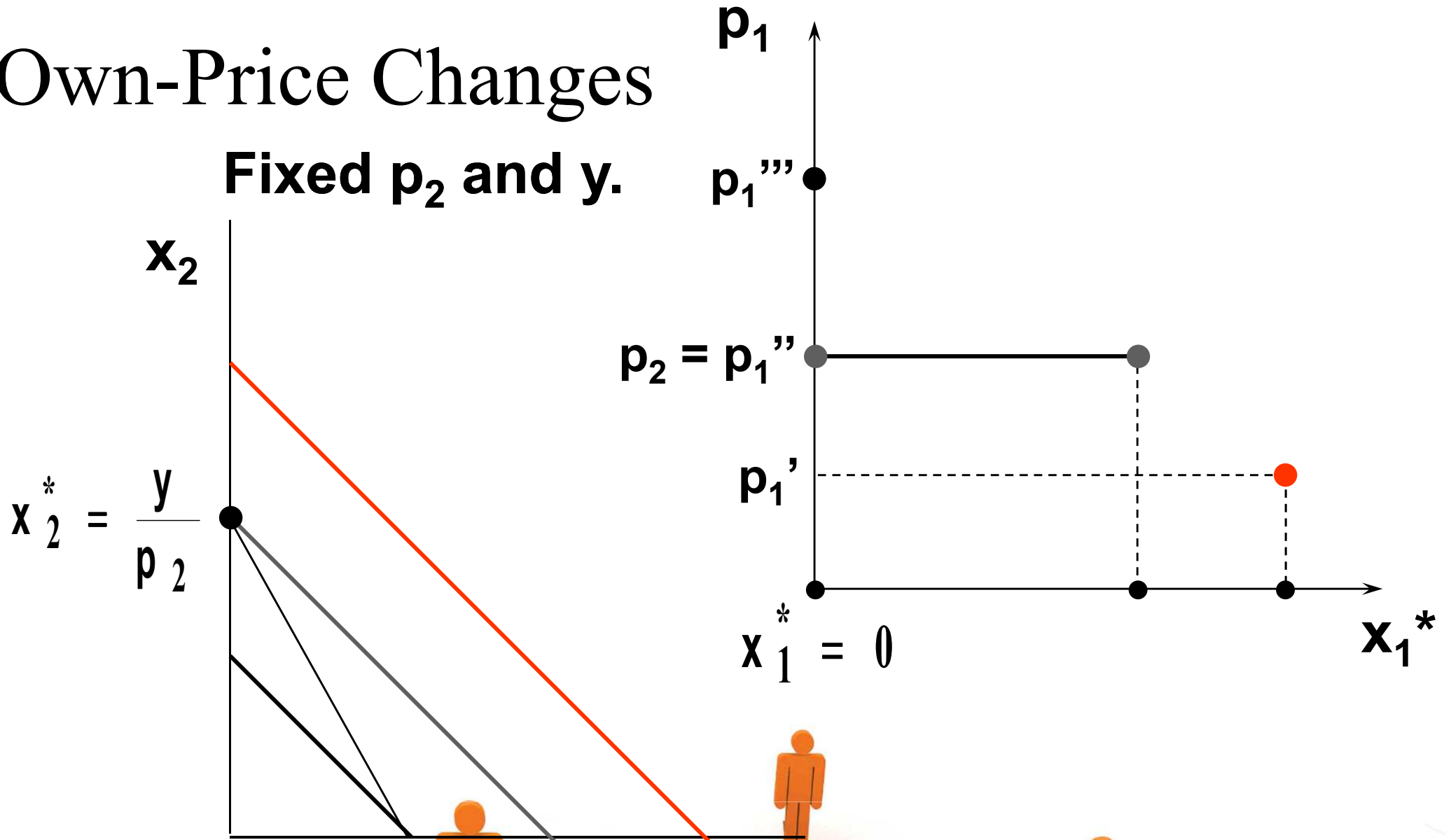
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Own-Price Changes

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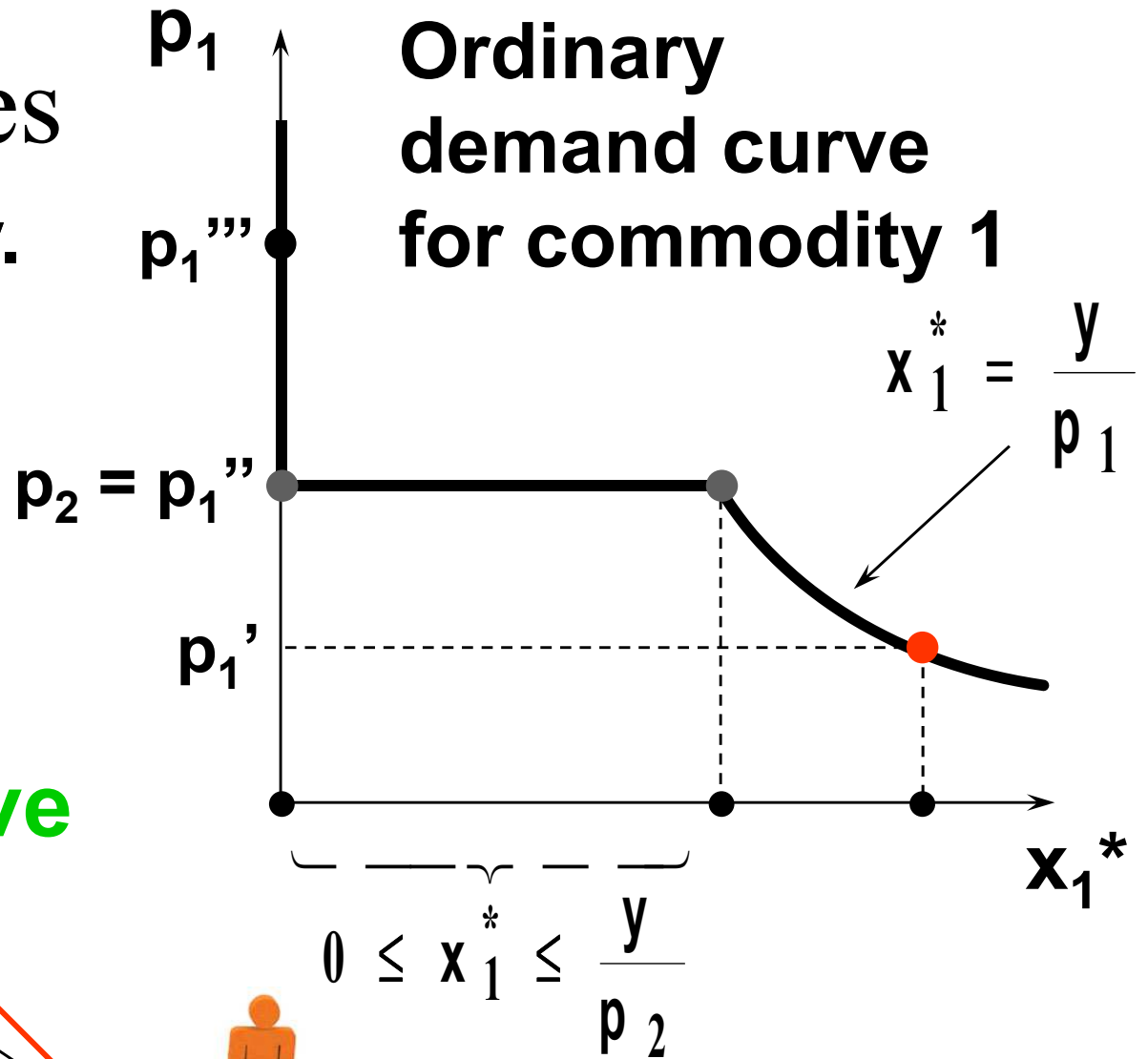
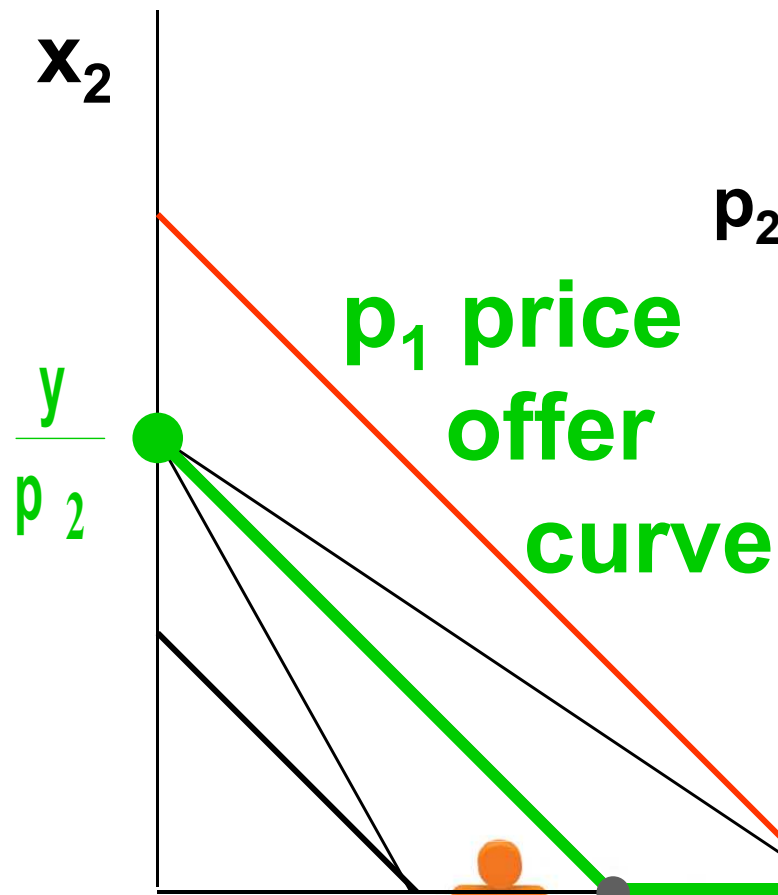


$$x_1^* = 0$$

$$x_1^*$$

Own-Price Changes

Fixed p_2 and y .

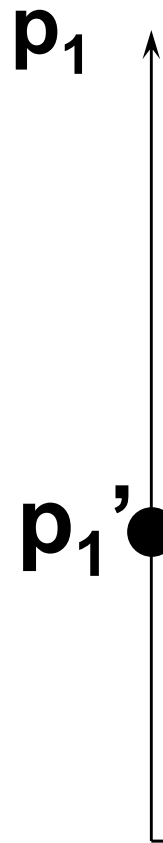


Own-Price Changes

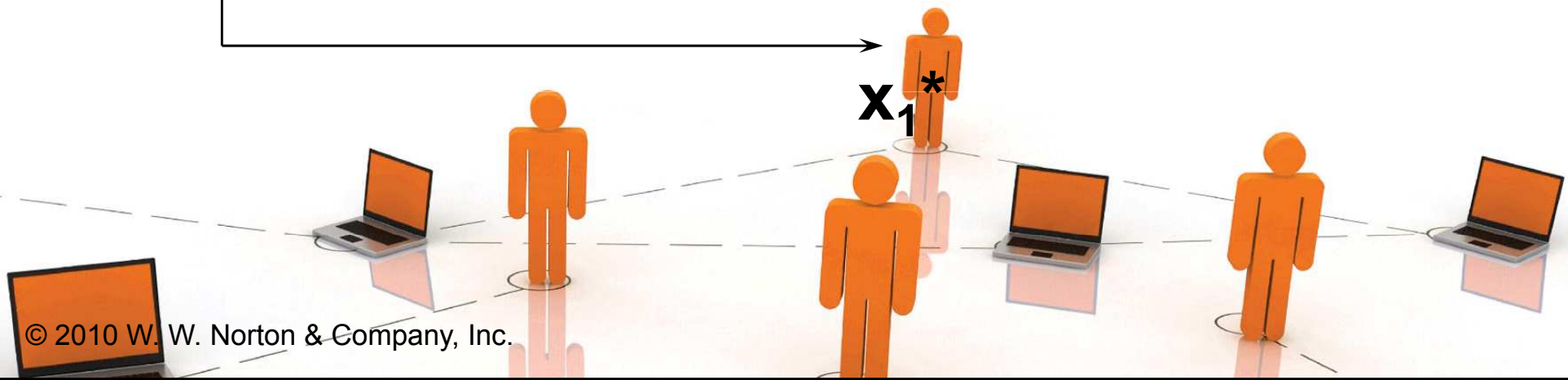
- ◆ Usually we ask “Given the price for commodity 1 what is the quantity demanded of commodity 1?”
- ◆ But we could also ask the inverse question “At what price for commodity 1 would a given quantity of commodity 1 be demanded?”



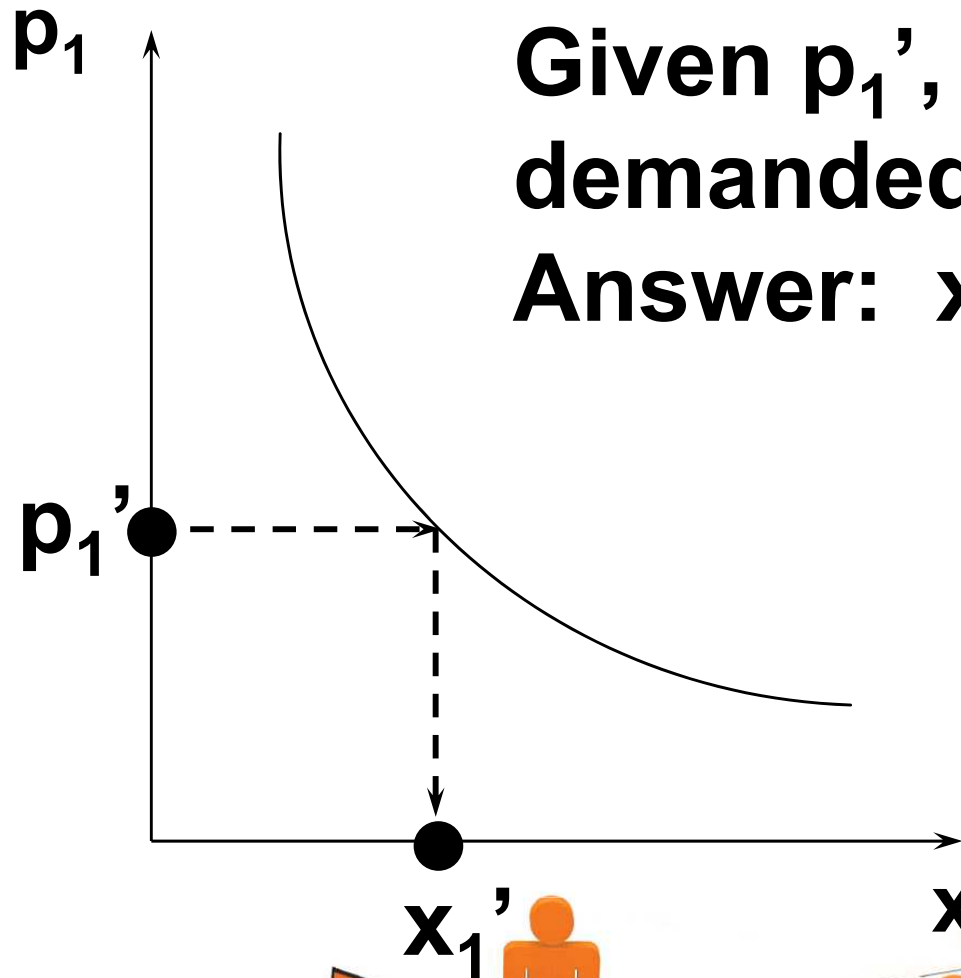
Own-Price Changes



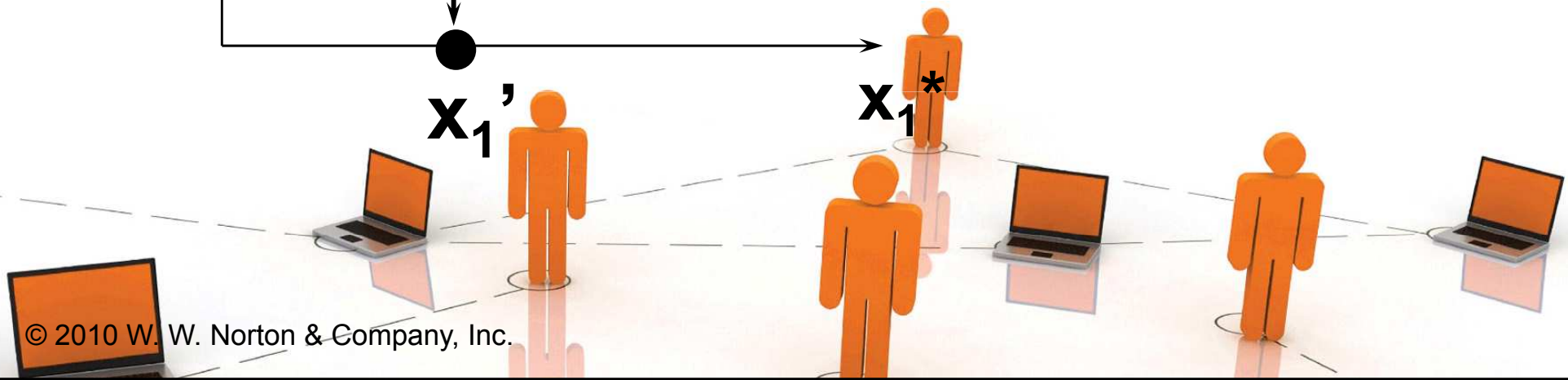
Given p_1' , what quantity is demanded of commodity 1?



Own-Price Changes



Given p_1' , what quantity is demanded of commodity 1?
Answer: x_1' units.



Own-Price Changes

p_1

**Given p_1' , what quantity is demanded of commodity 1?
Answer: x_1' units.**

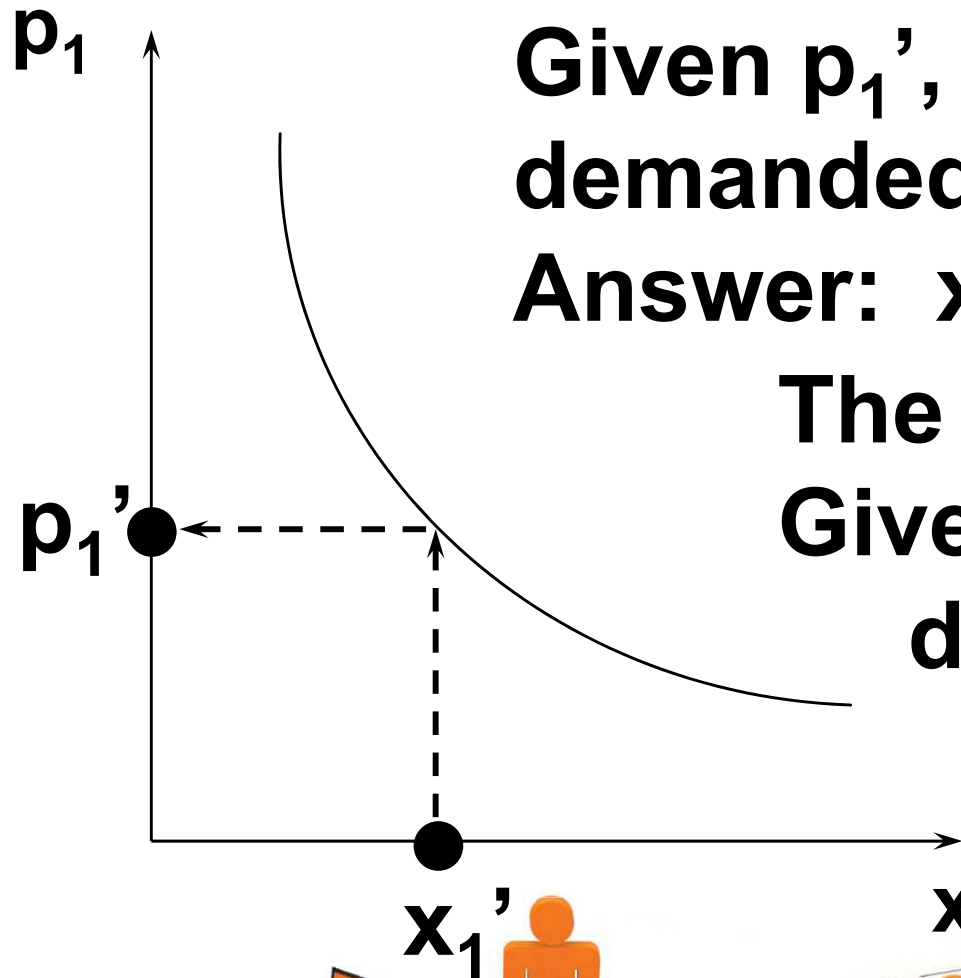
**The inverse question is:
Given x_1' units are demanded, what is the price of commodity 1?**

x_1'

x_1^*



Own-Price Changes

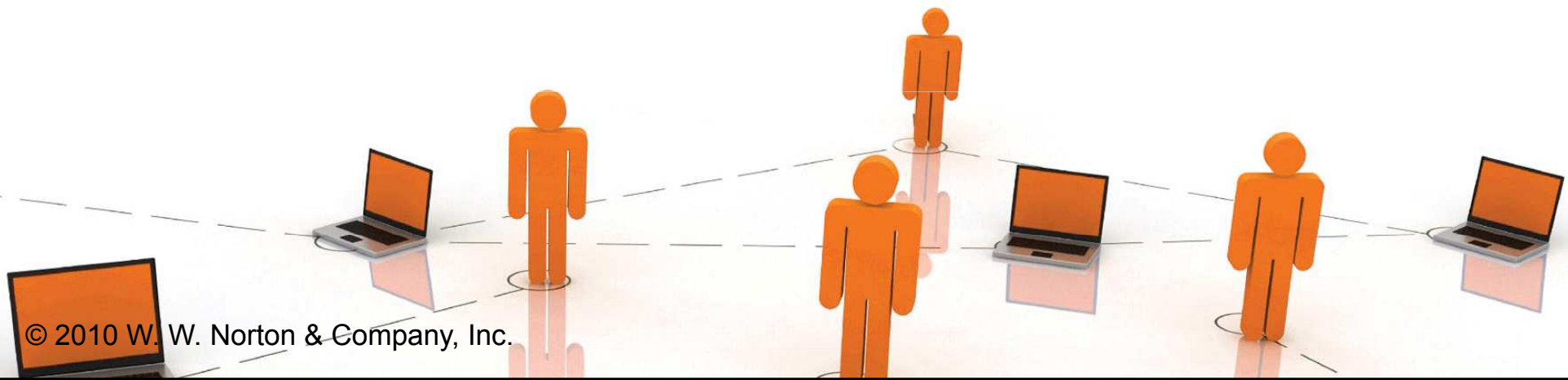


**Given p_1' , what quantity is demanded of commodity 1?
Answer: x_1' units.**

**The inverse question is:
Given x_1' units are demanded, what is the price of commodity 1?
Answer: p_1'**

Own-Price Changes

- ◆ **Taking quantity demanded as given and then asking what must be price describes the inverse demand function of a commodity.**



Own-Price Changes

A Cobb-Douglas example:

$$x_1^* = \frac{a y}{(a + b) p_1}$$

is the ordinary demand function and

$$p_1 = \frac{a y}{(a + b) x_1^*}$$

is the inverse demand function.

Own-Price Changes

A perfect-complements example:

$$x_1^* = \frac{y}{p_1 + p_2}$$

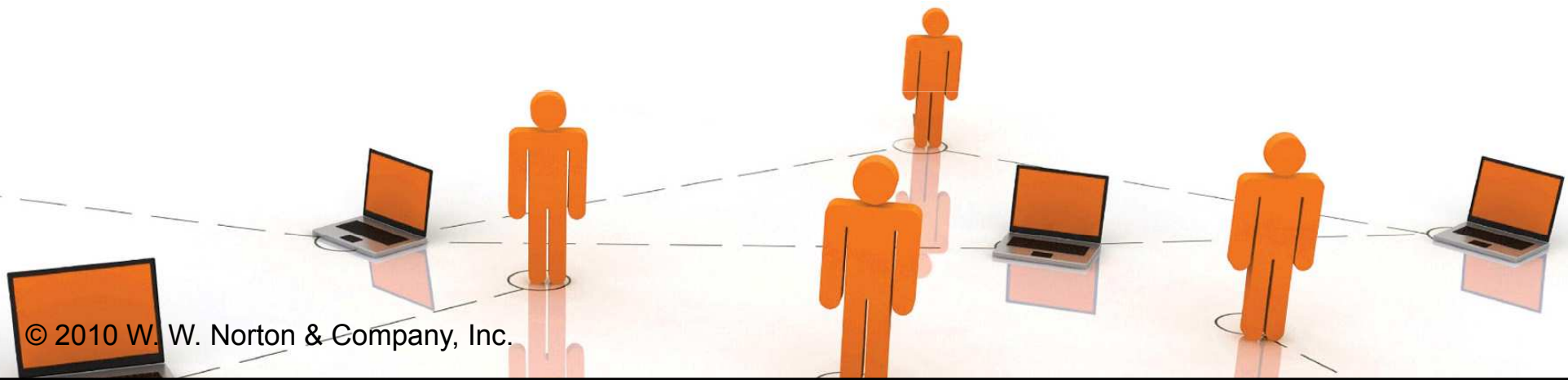
is the ordinary demand function and

$$p_1 = \frac{y}{x_1^*} - p_2$$

is the inverse demand function.

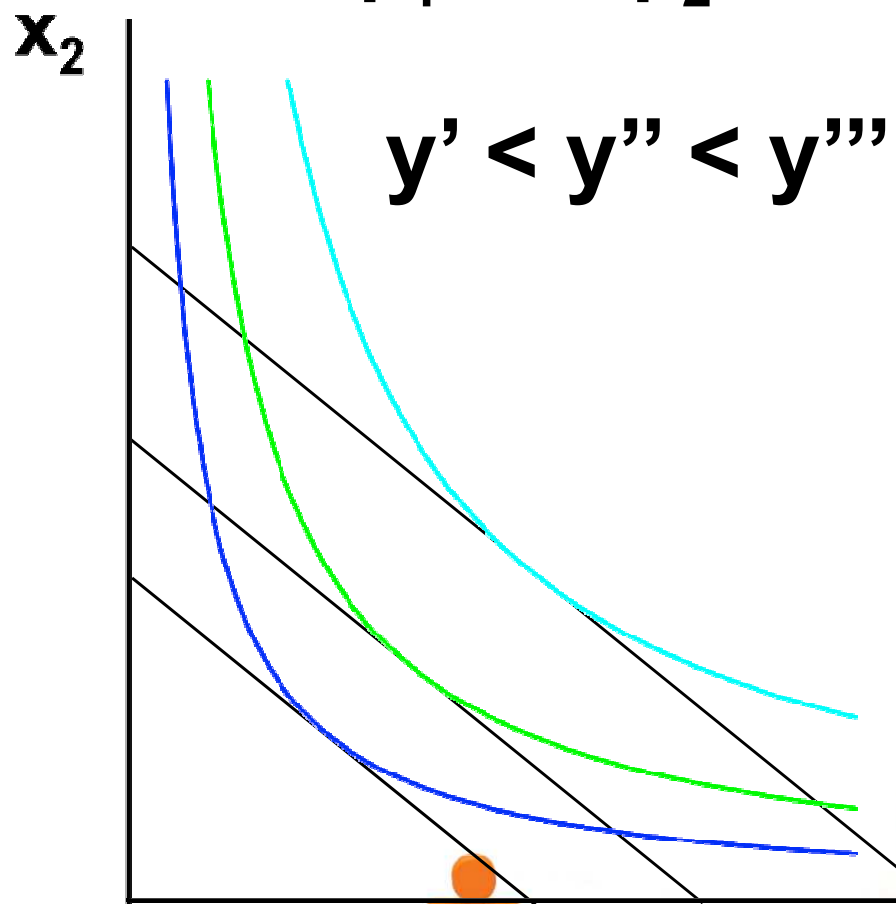
Income Changes

- ◆ How does the value of $x_1^*(p_1, p_2, y)$ change as y changes, holding both p_1 and p_2 constant?



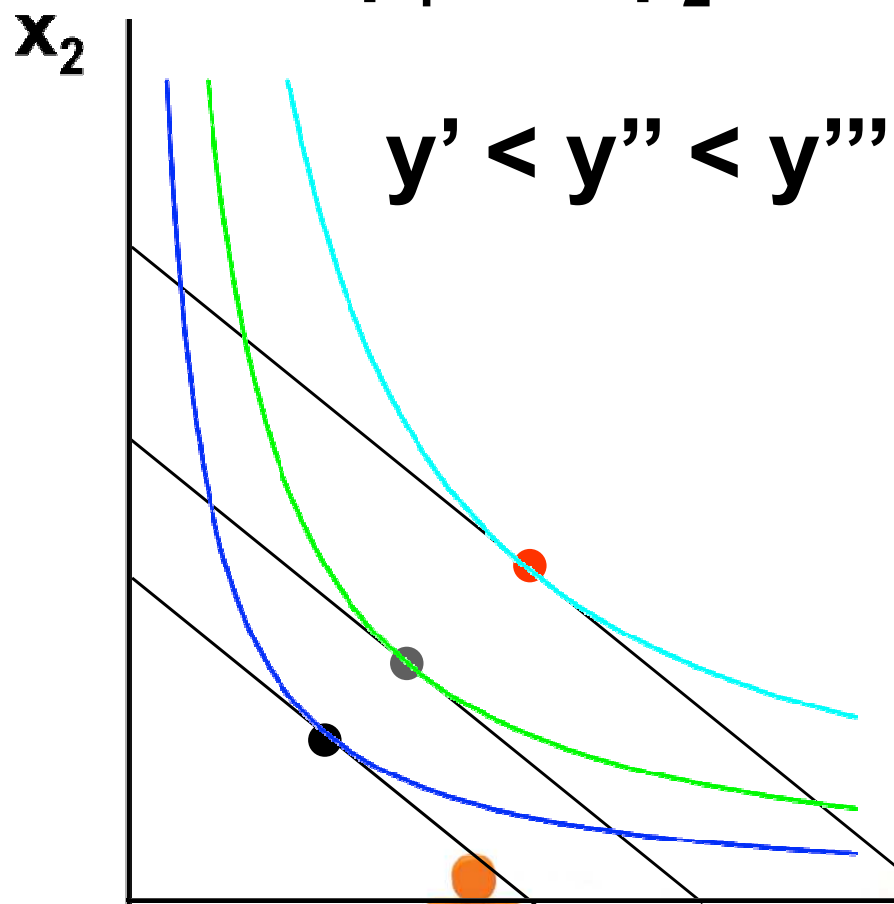
Income Changes

Fixed p_1 and p_2 .



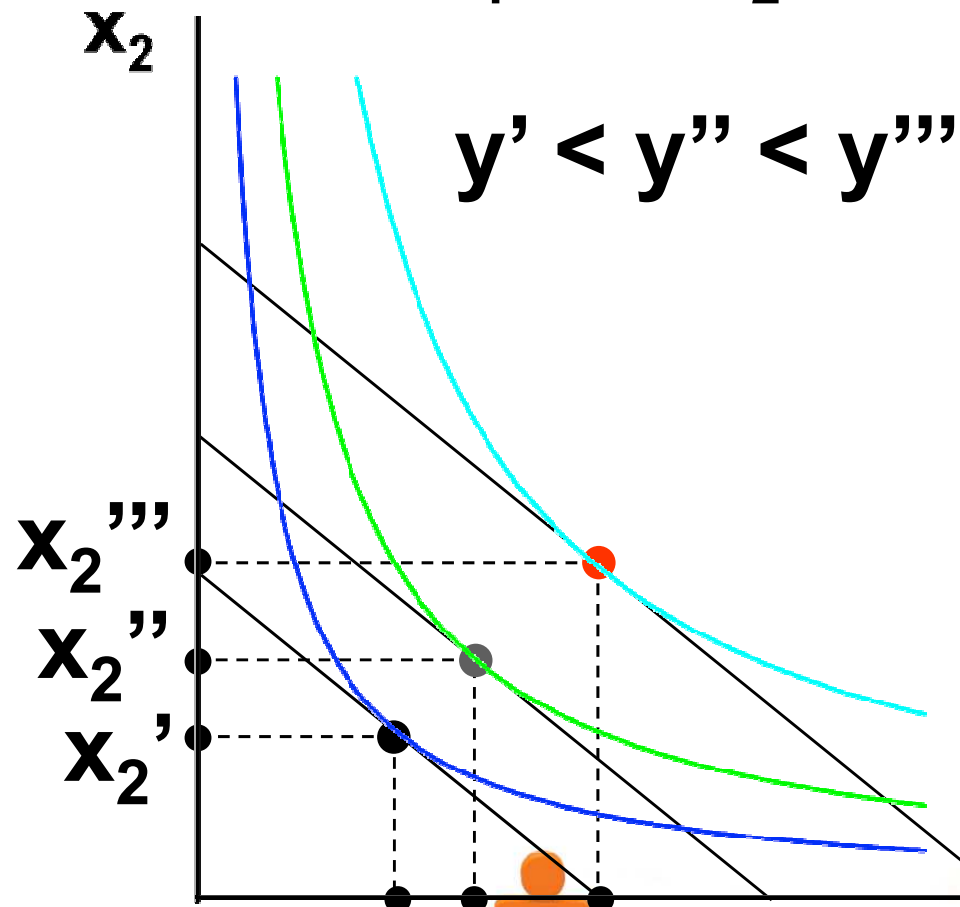
Income Changes

Fixed p_1 and p_2 .



Income Changes

Fixed p_1 and p_2 .



x_1'

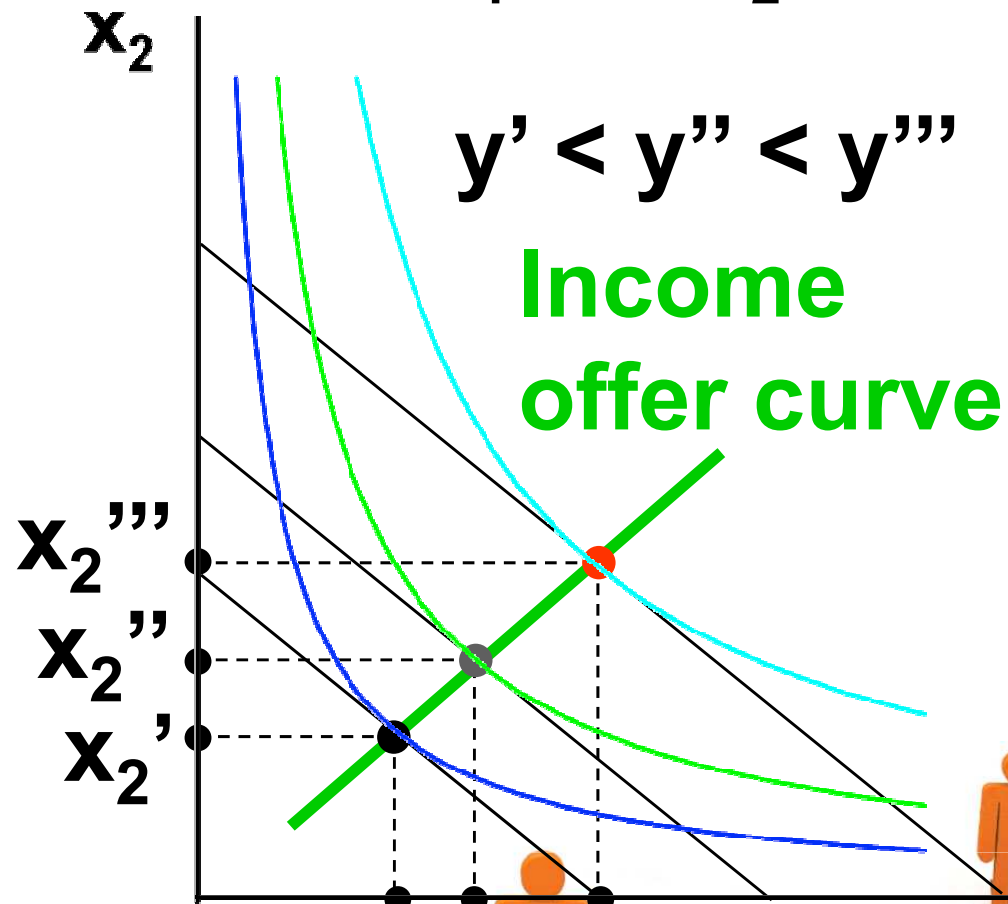
x_1''

x_1'''

x_1

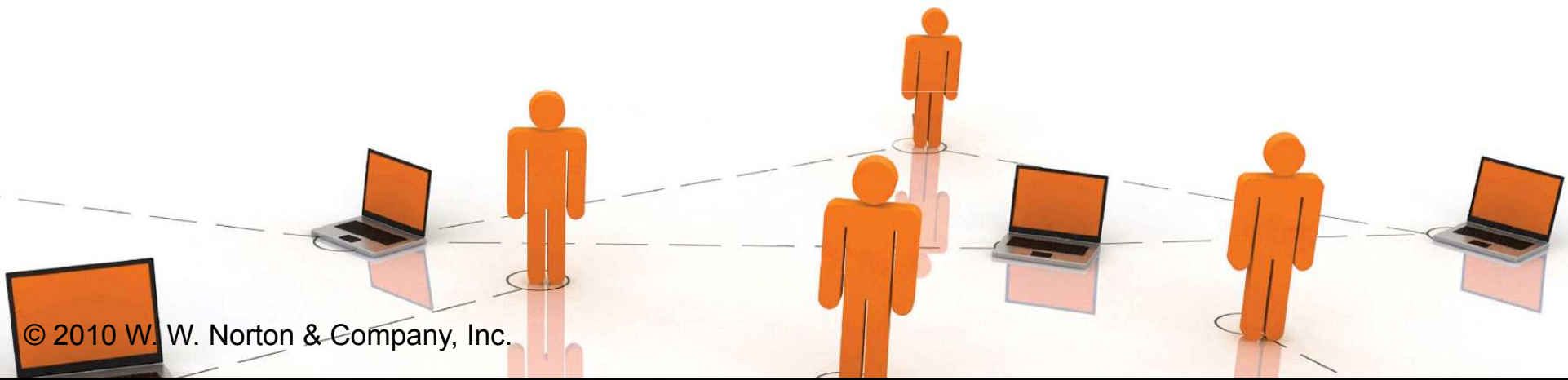
Income Changes

Fixed p_1 and p_2 .



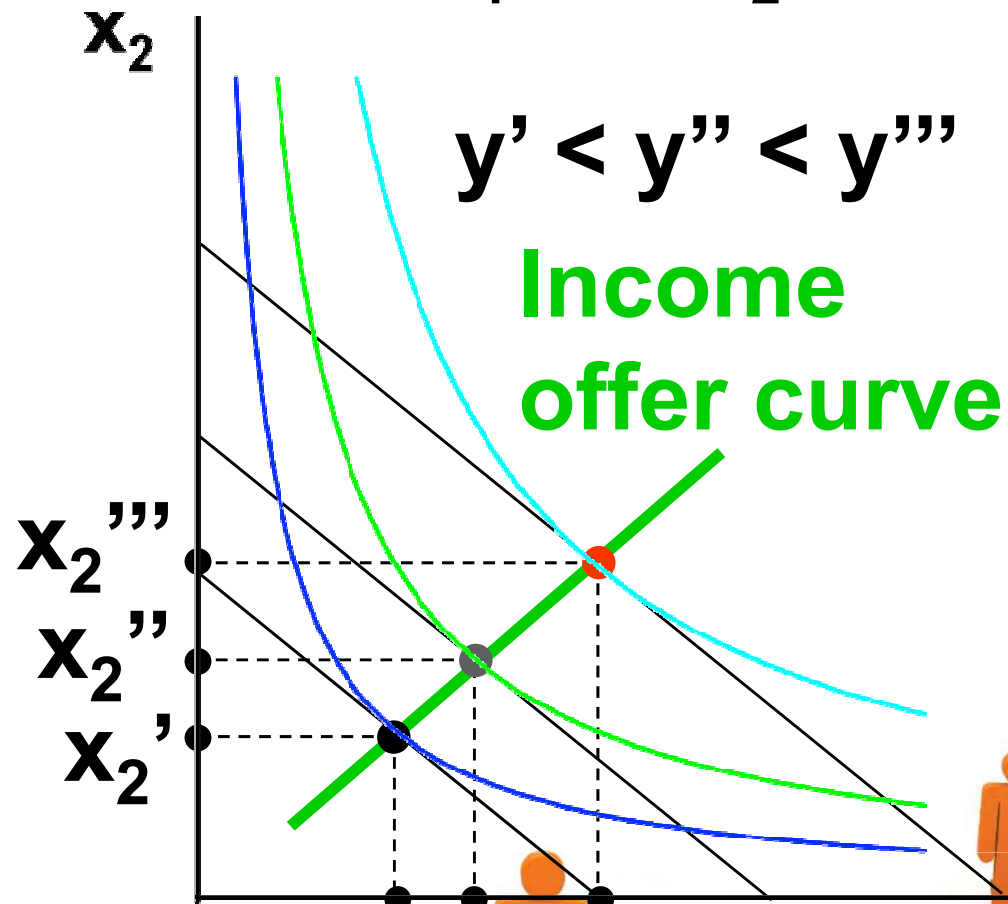
Income Changes

- ◆ **A plot of quantity demanded against income is called an Engel curve.**



Income Changes

Fixed p_1 and p_2 .



x_1'



x_1''

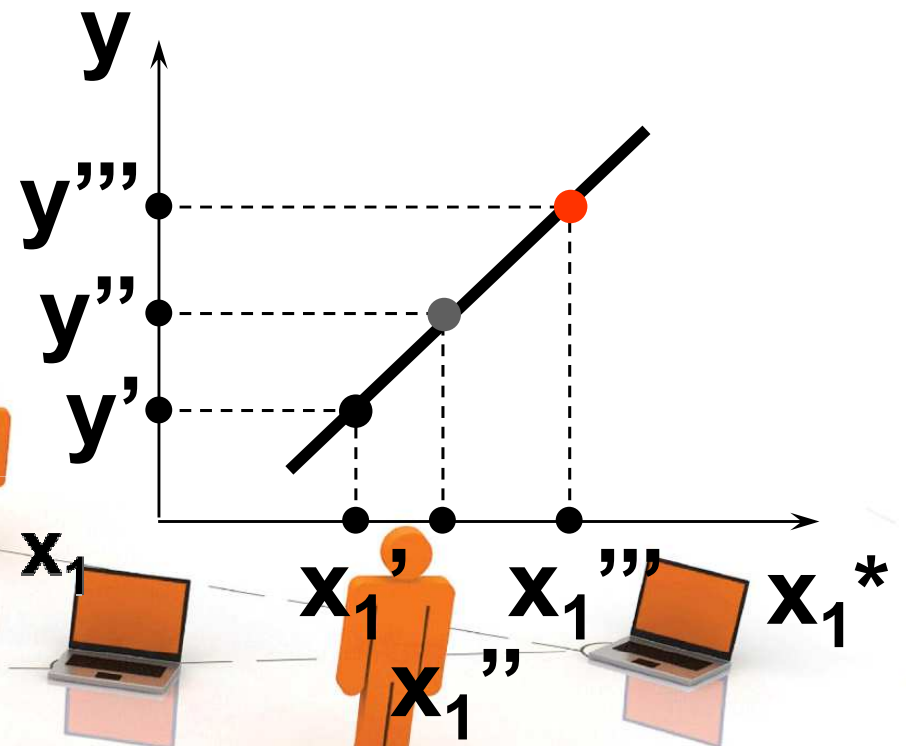
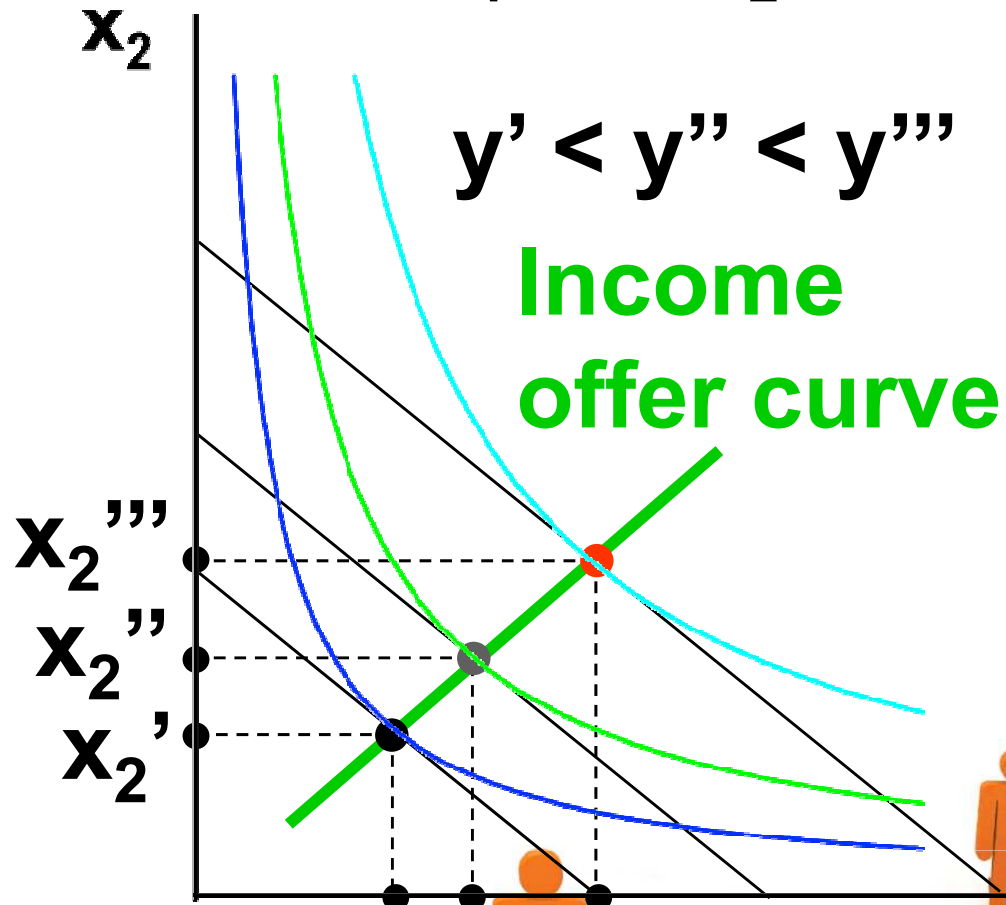


x_1'''



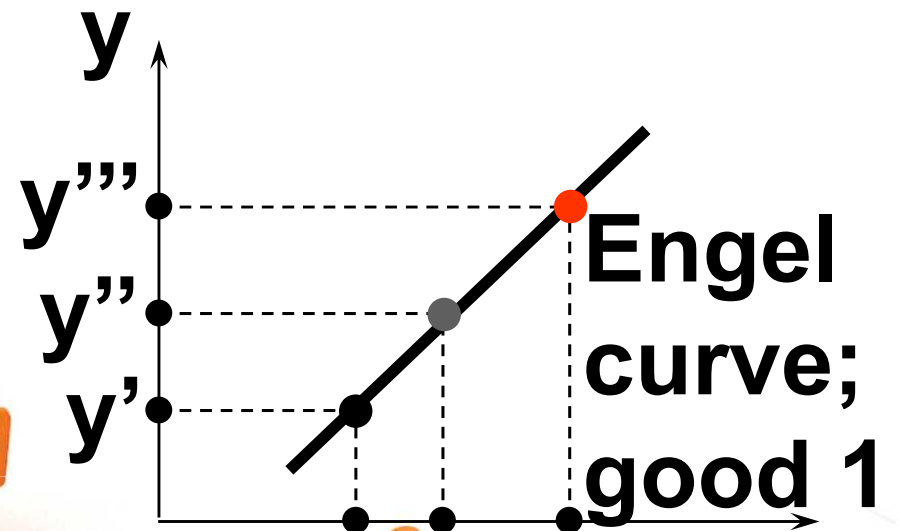
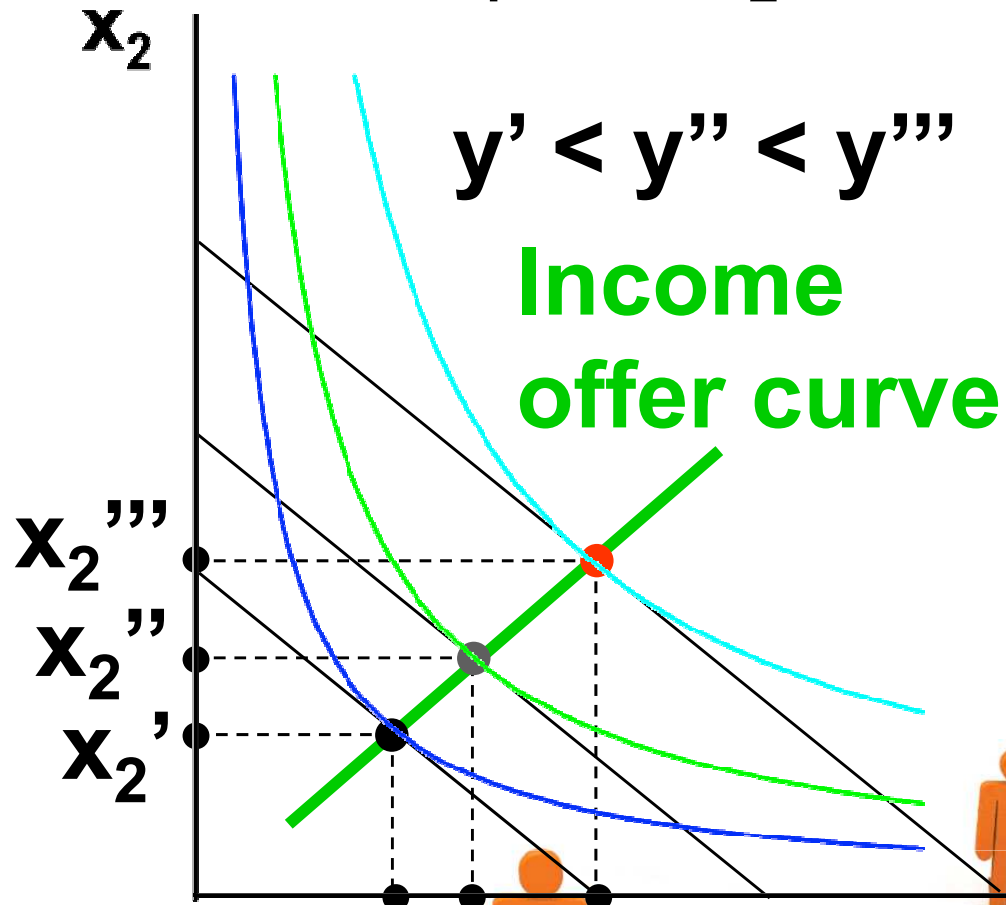
Income Changes

Fixed p_1 and p_2 .



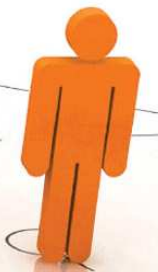
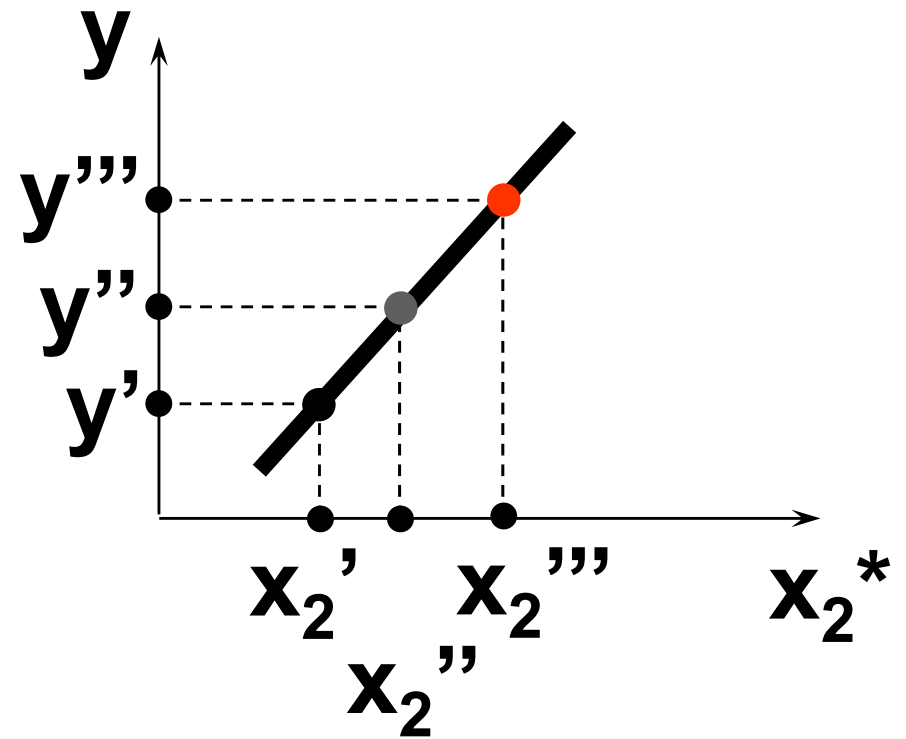
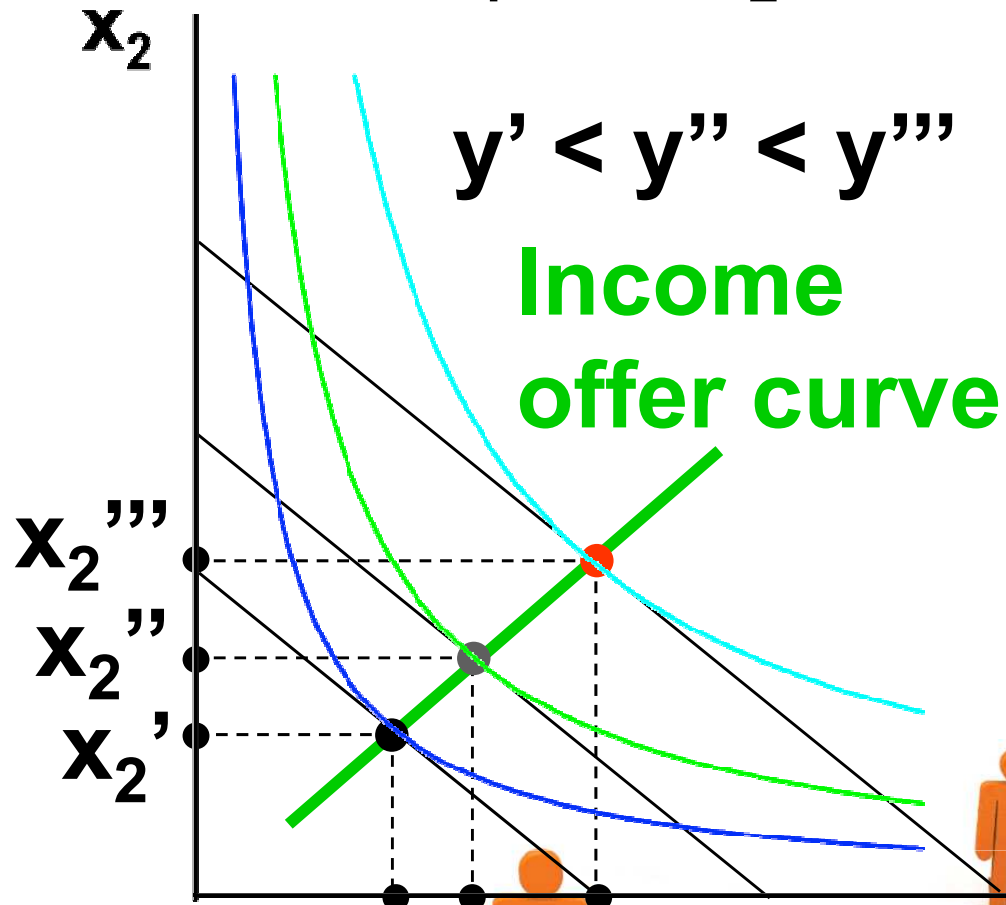
Income Changes

Fixed p_1 and p_2 .



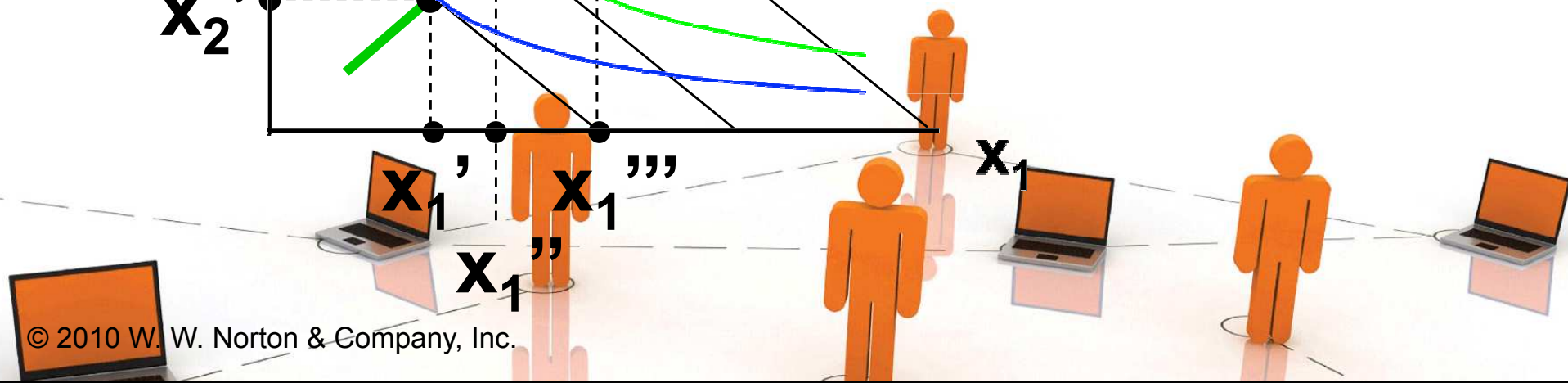
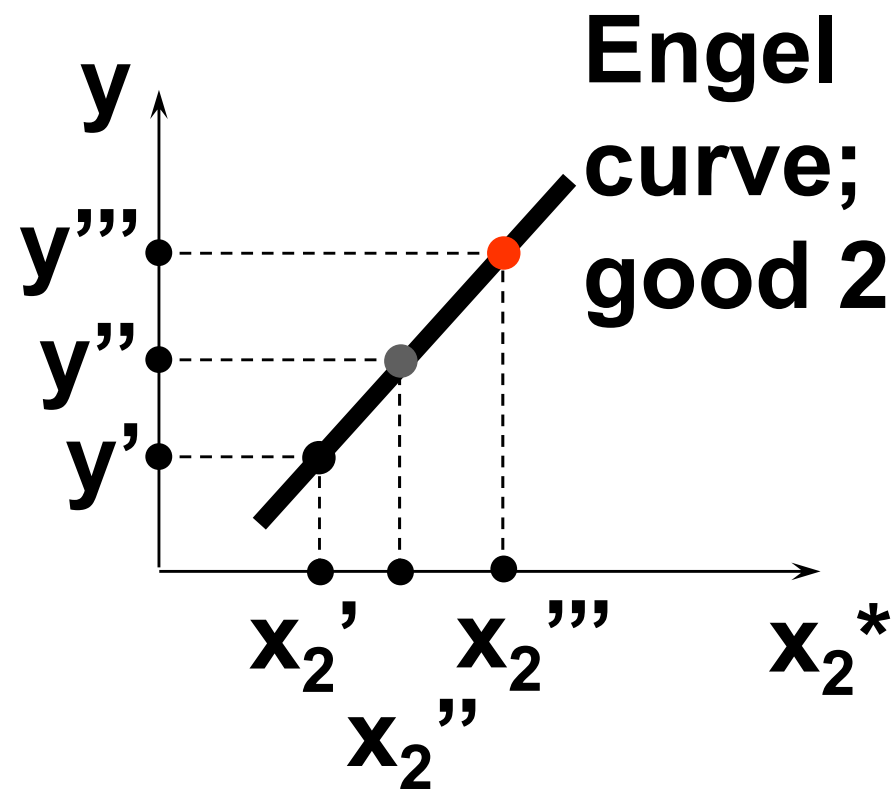
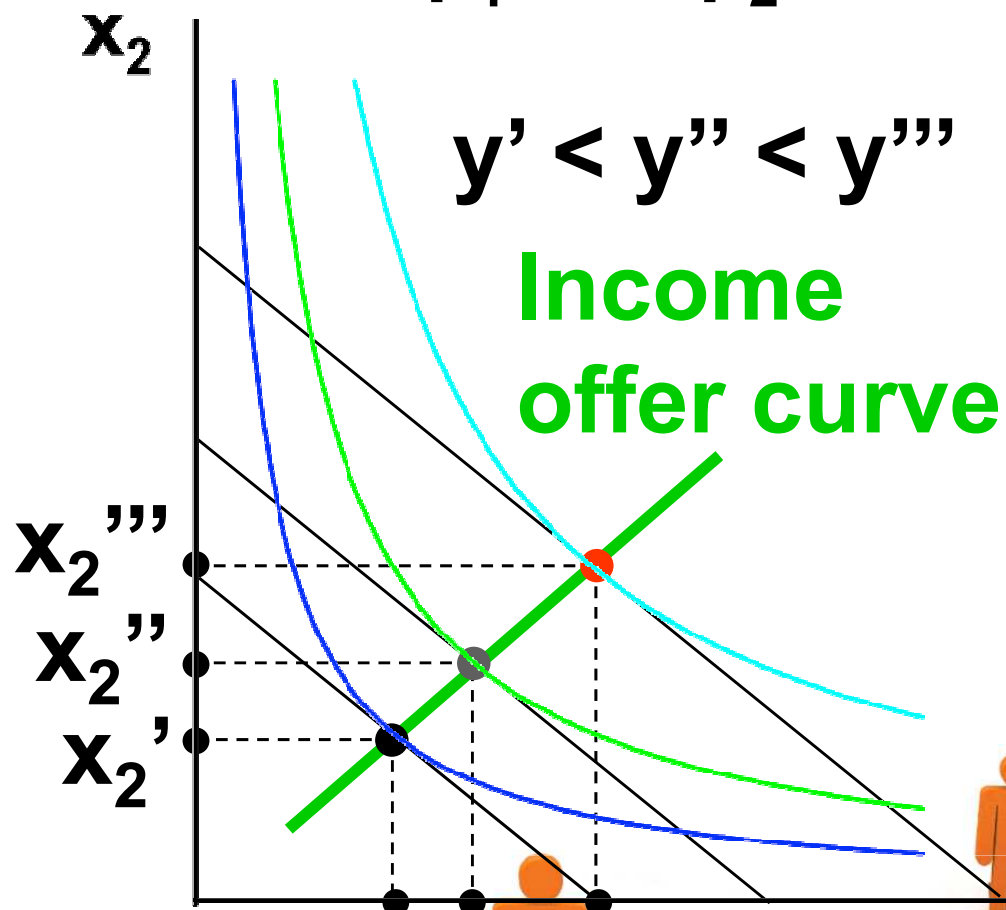
Income Changes

Fixed p_1 and p_2 .



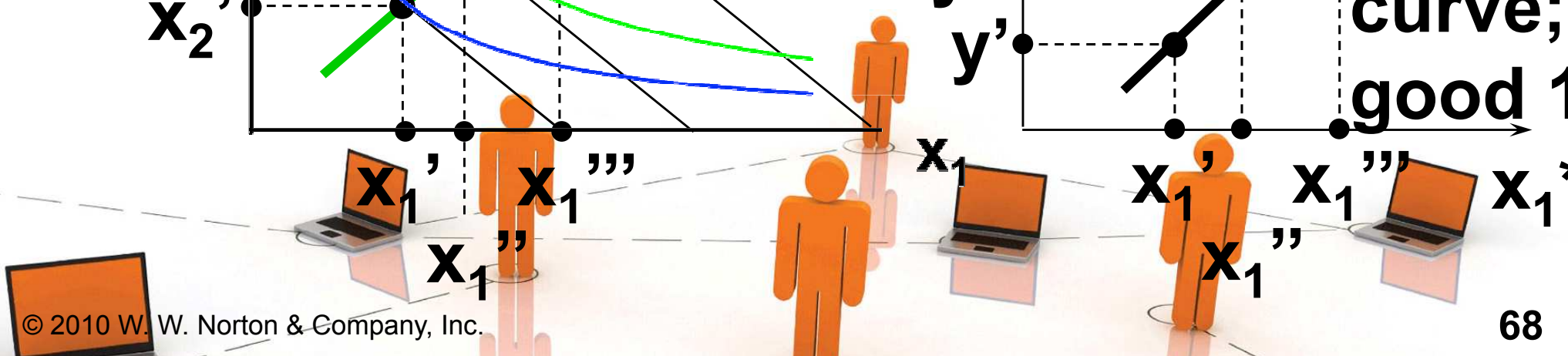
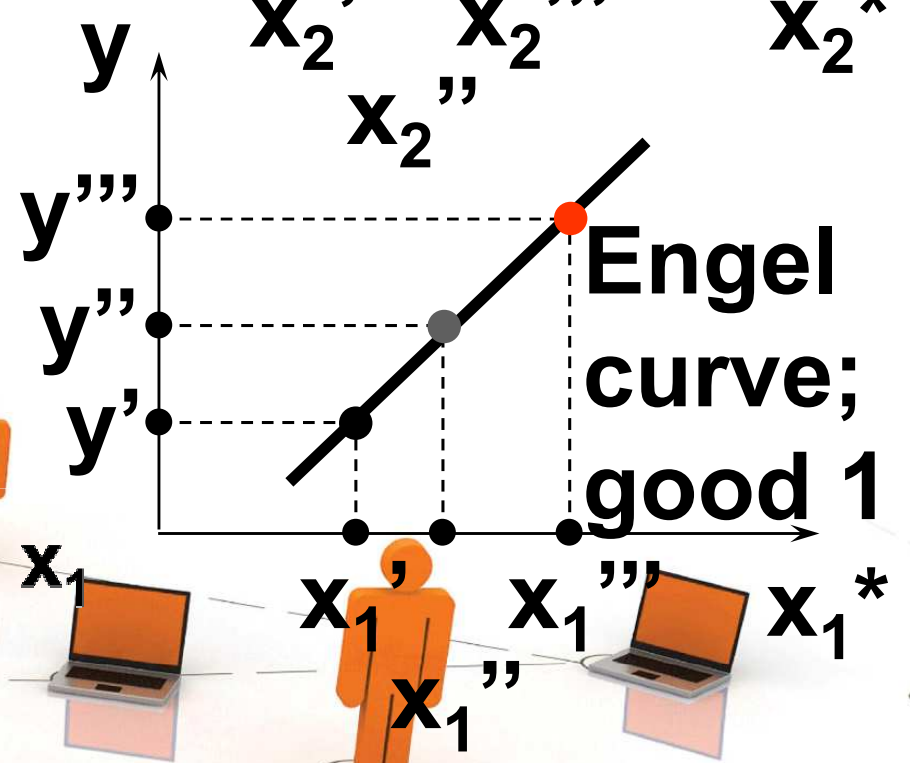
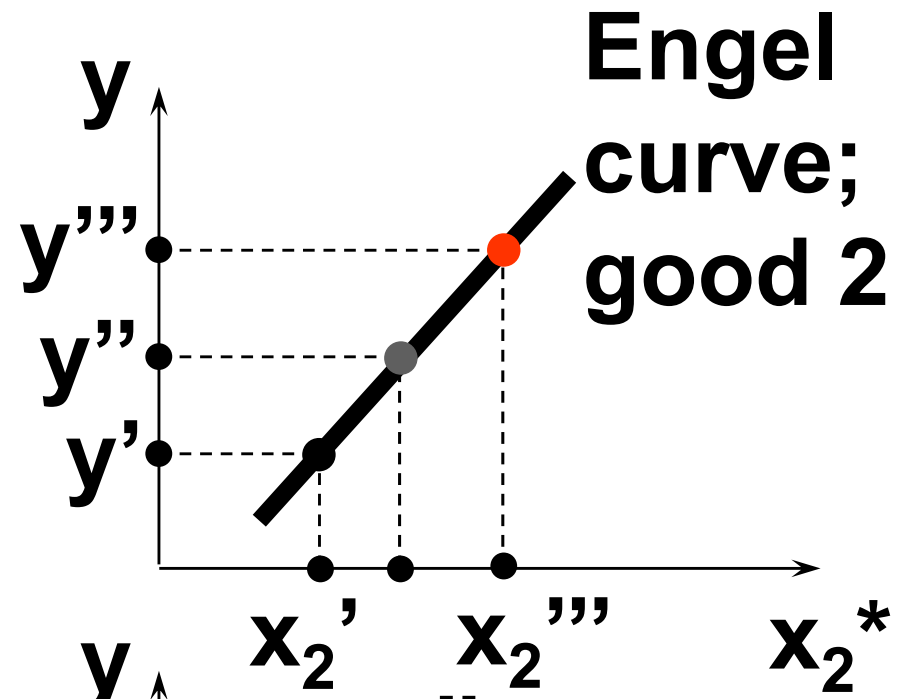
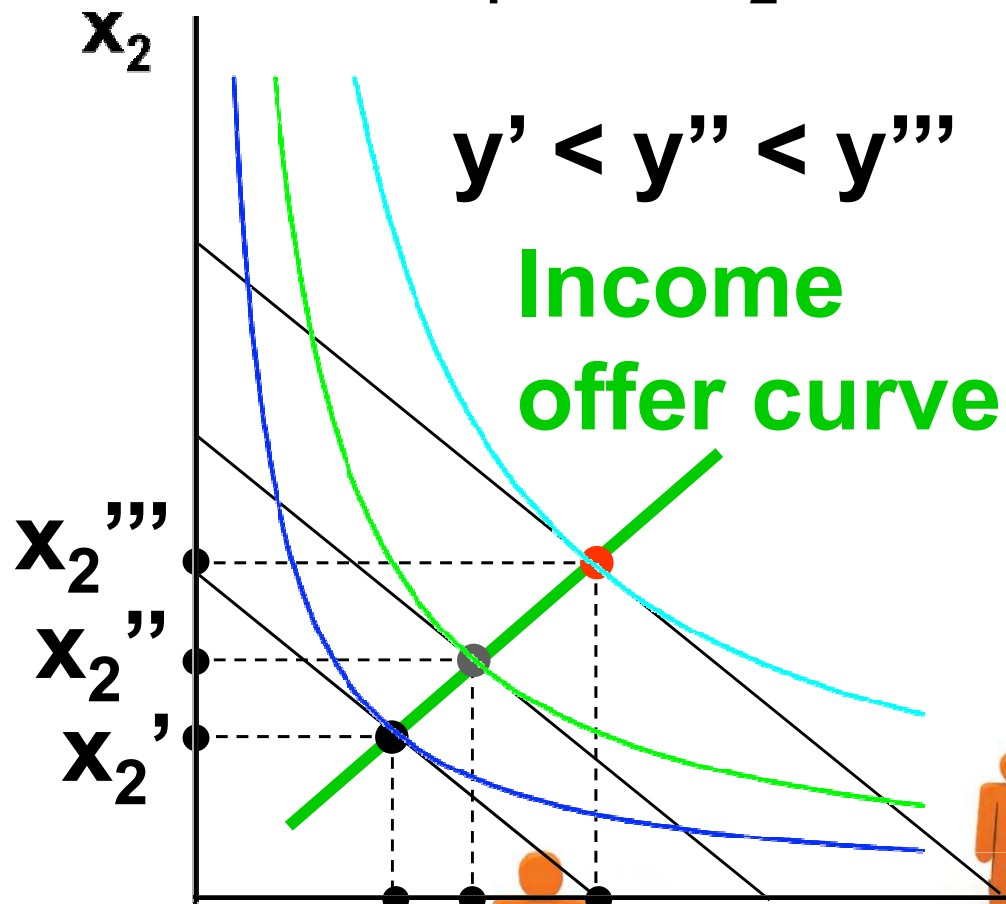
Income Changes

Fixed p_1 and p_2 .



Income Changes

Fixed p_1 and p_2 .



Income Changes and Cobb-Douglas Preferences

- ◆ An example of computing the equations of Engel curves; the Cobb-Douglas case.

$$U(x_1, x_2) = x_1^a x_2^b.$$

- ◆ The ordinary demand equations are

$$x_1^* = \frac{a y}{(a + b) p_1}; \quad x_2^* = \frac{b y}{(a + b) p_2}.$$

Income Changes and Cobb-Douglas Preferences

$$x_1^* = \frac{a y}{(a + b) p_1}; \quad x_2^* = \frac{b y}{(a + b) p_2}.$$

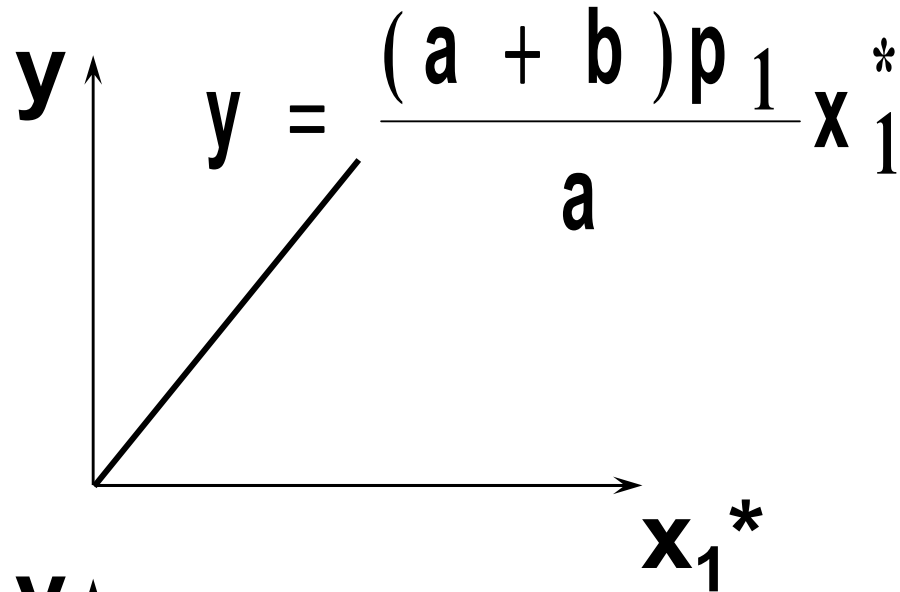
Rearranged to isolate y , these are:

$$y = \frac{(a + b) p_1}{a} x_1^* \quad \text{Engel curve for good 1}$$

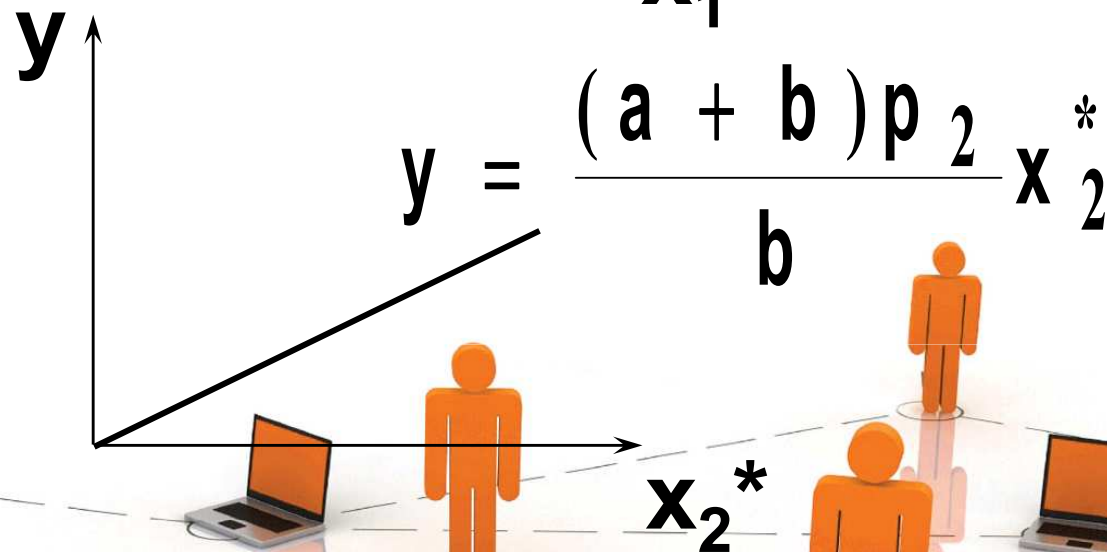
$$y = \frac{(a + b) p_2}{b} x_2^* \quad \text{Engel curve for good 2}$$



Income Changes and Cobb-Douglas Preferences



**Engel curve
for good 1**



**Engel curve
for good 2**

Income Changes and Perfectly-Complementary Preferences

- ◆ Another example of computing the equations of Engel curves; the perfectly-complementary case.

$$U(x_1, x_2) = \min\{x_1, x_2\}.$$

- ◆ The ordinary demand equations are

$$x_1^* = x_2^* = \frac{y}{p_1 + p_2}.$$



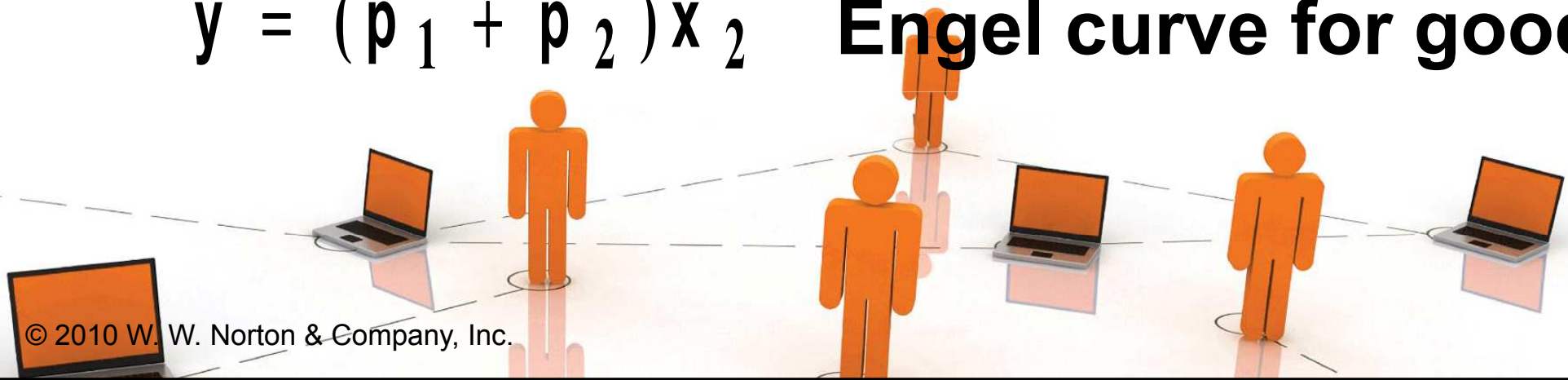
Income Changes and Perfectly-Complementary Preferences

$$x_1^* = x_2^* = \frac{y}{p_1 + p_2}.$$

Rearranged to isolate y , these are:

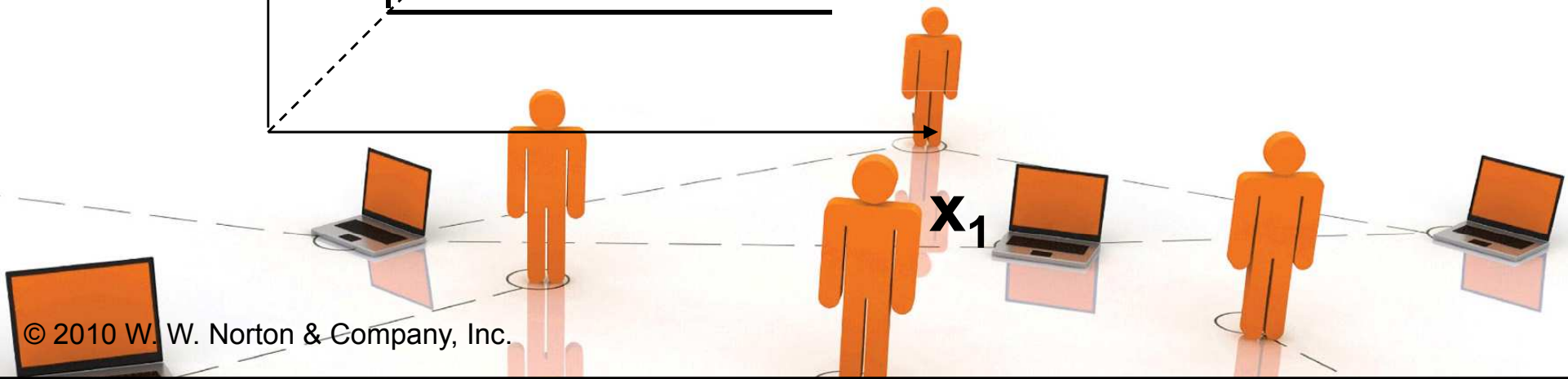
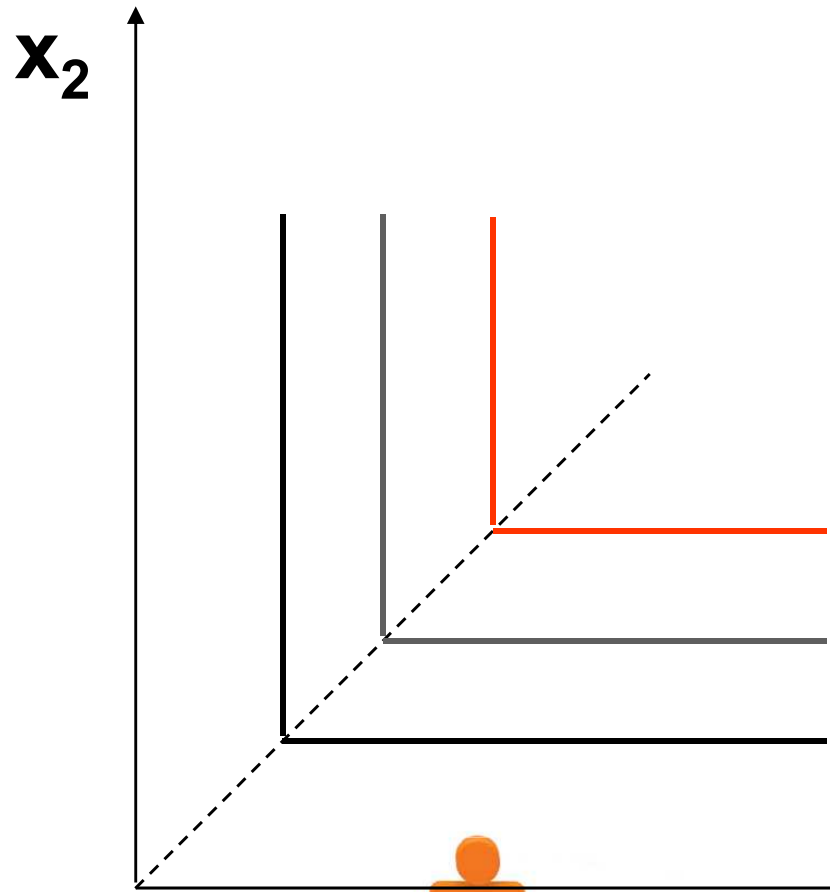
$$y = (p_1 + p_2) x_1^* \quad \text{Engel curve for good 1}$$

$$y = (p_1 + p_2) x_2^* \quad \text{Engel curve for good 2}$$



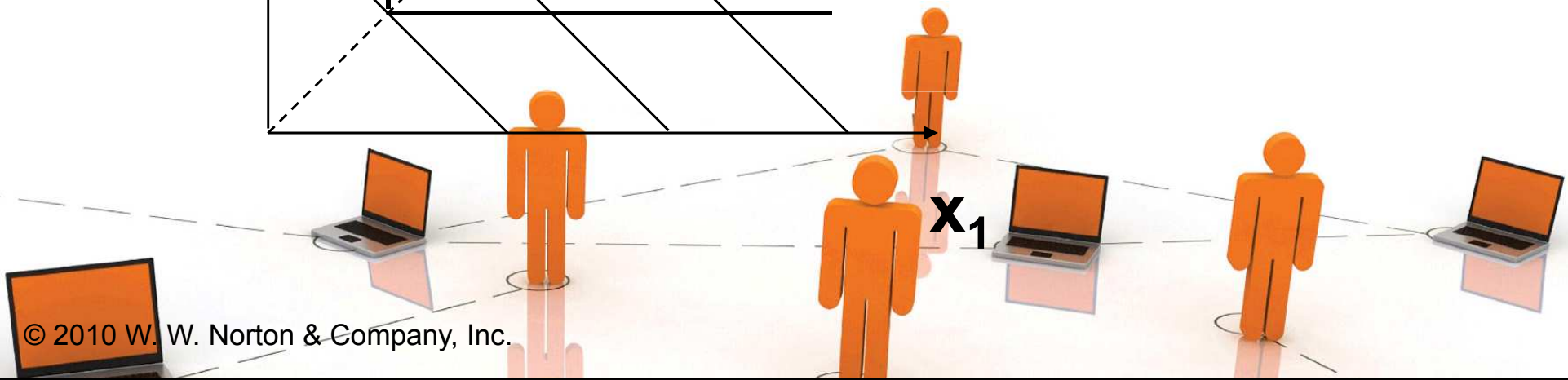
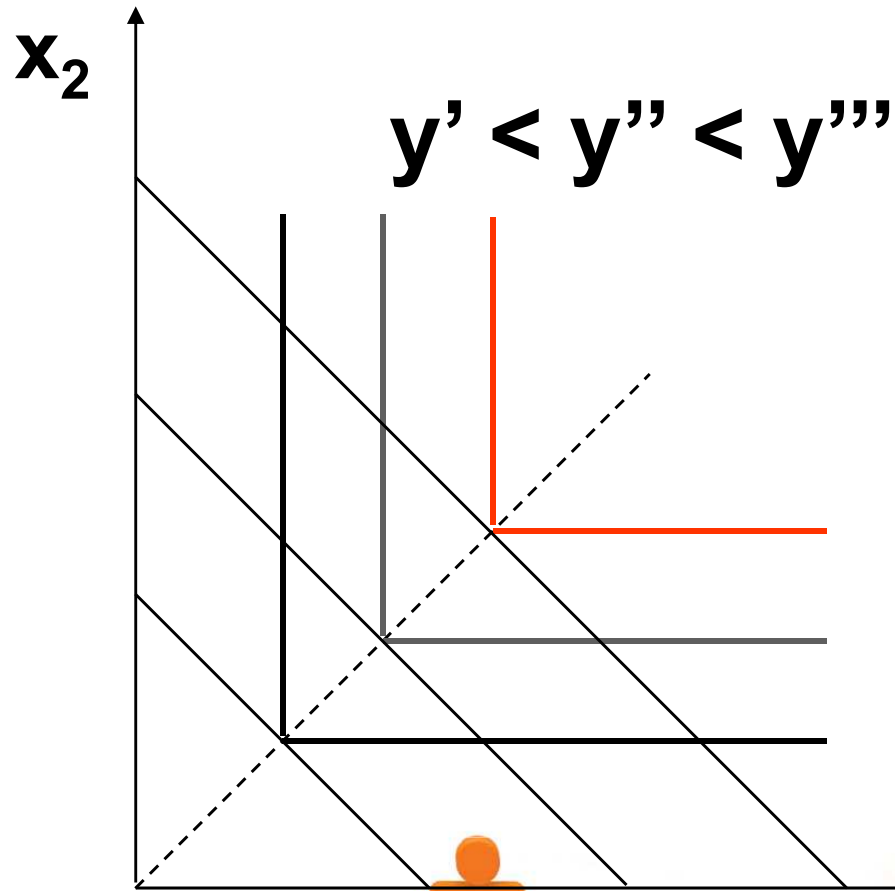
Income Changes

Fixed p_1 and p_2 .



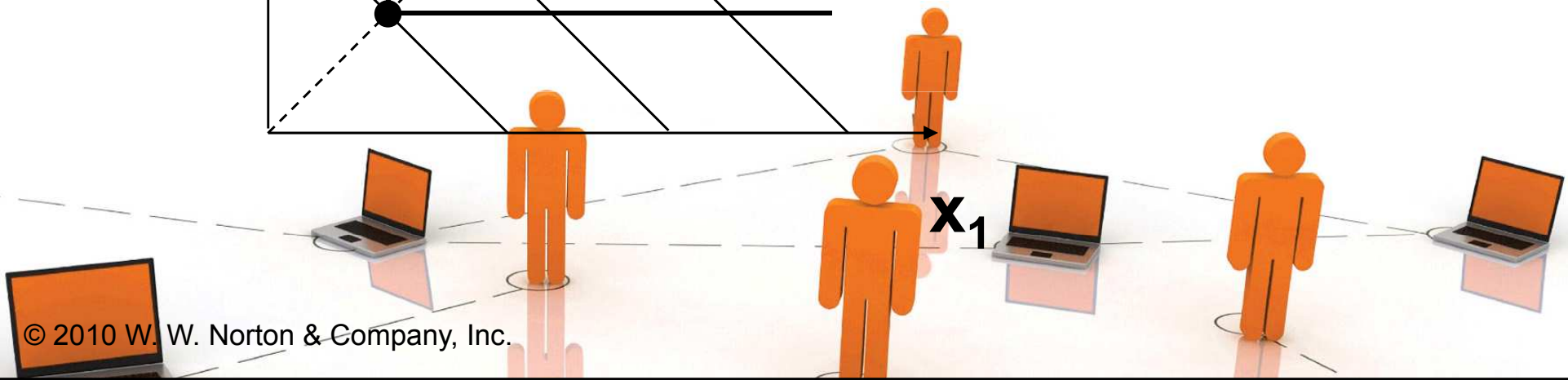
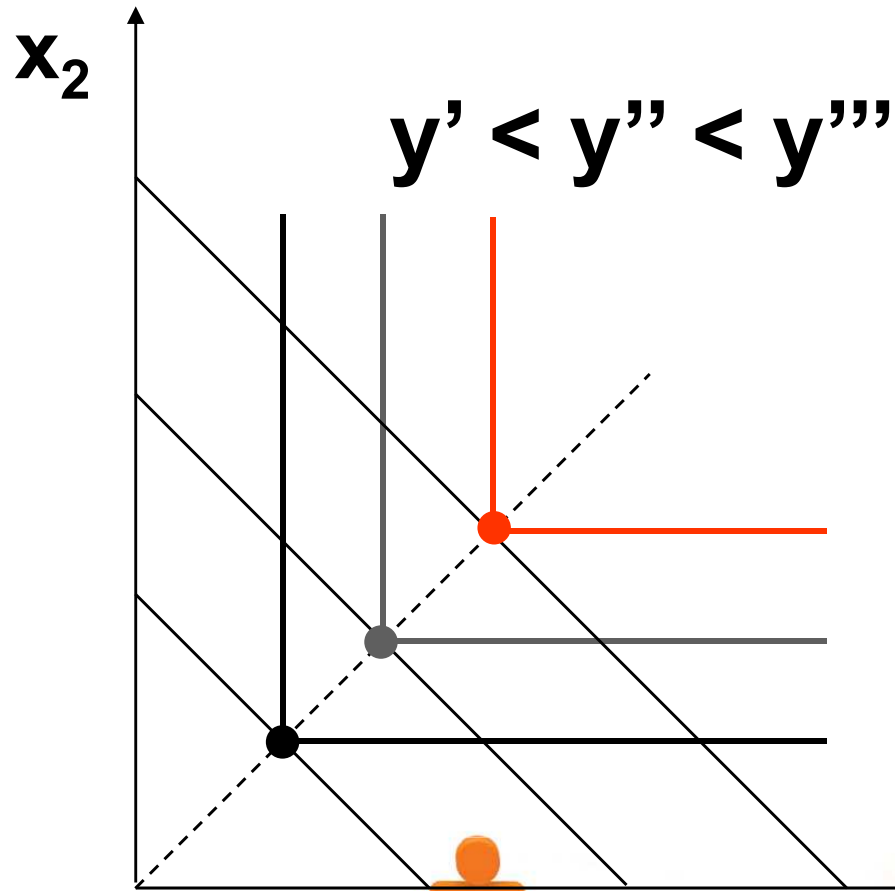
Income Changes

Fixed p_1 and p_2 .



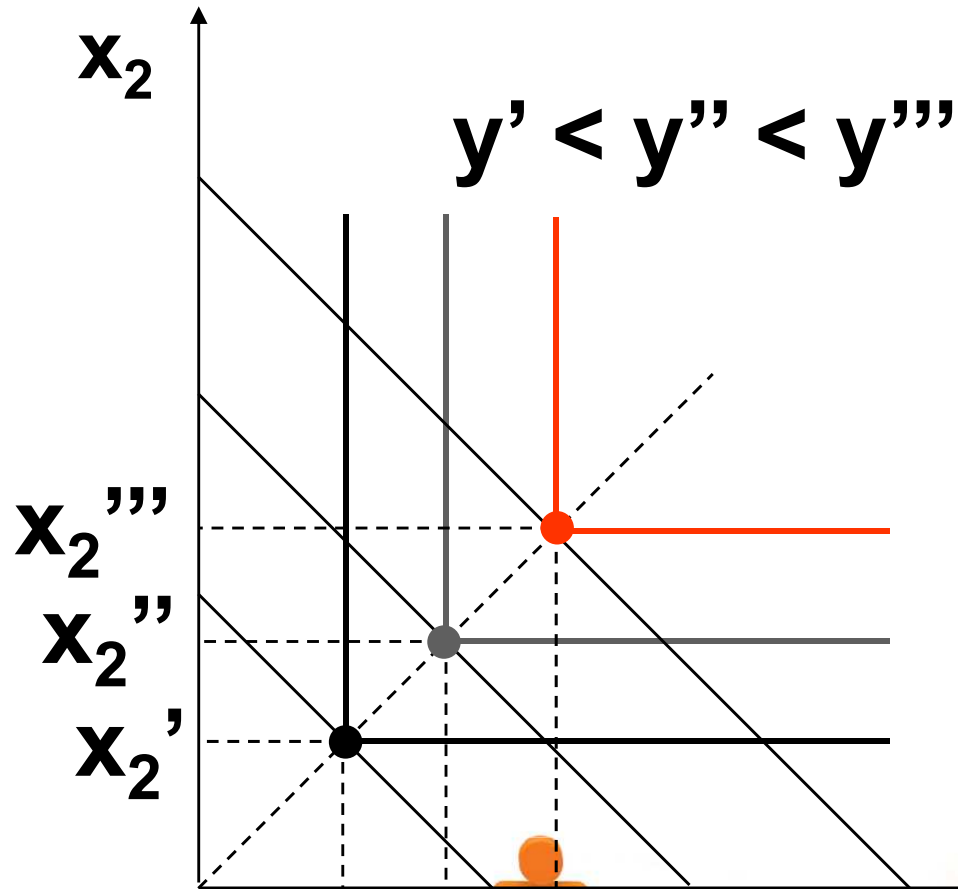
Income Changes

Fixed p_1 and p_2 .



Income Changes

Fixed p_1 and p_2 .



x_2'''
 x_2''
 x_2'

$y' < y'' < y'''$

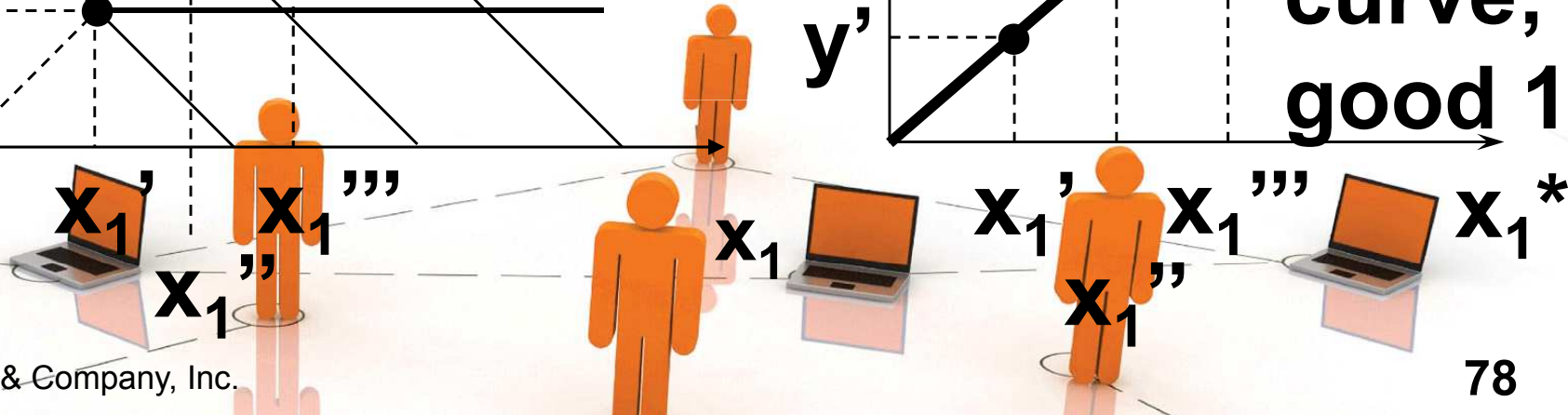
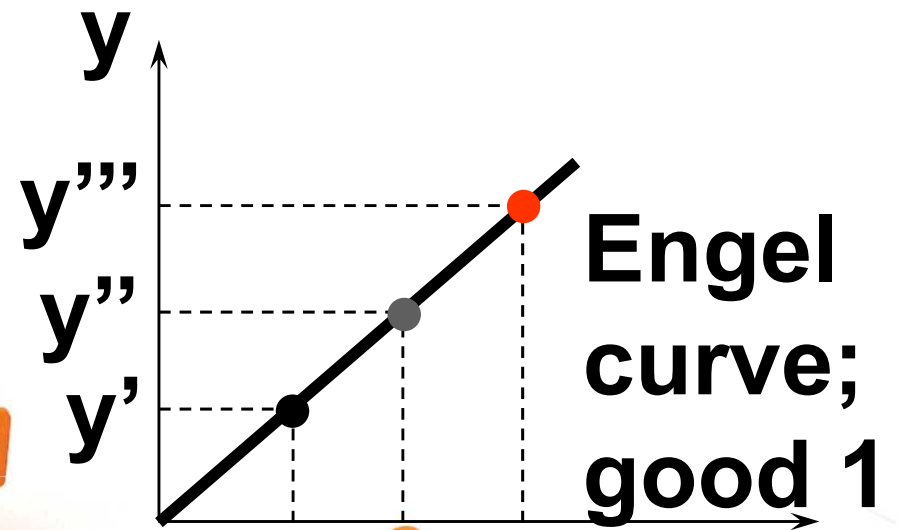
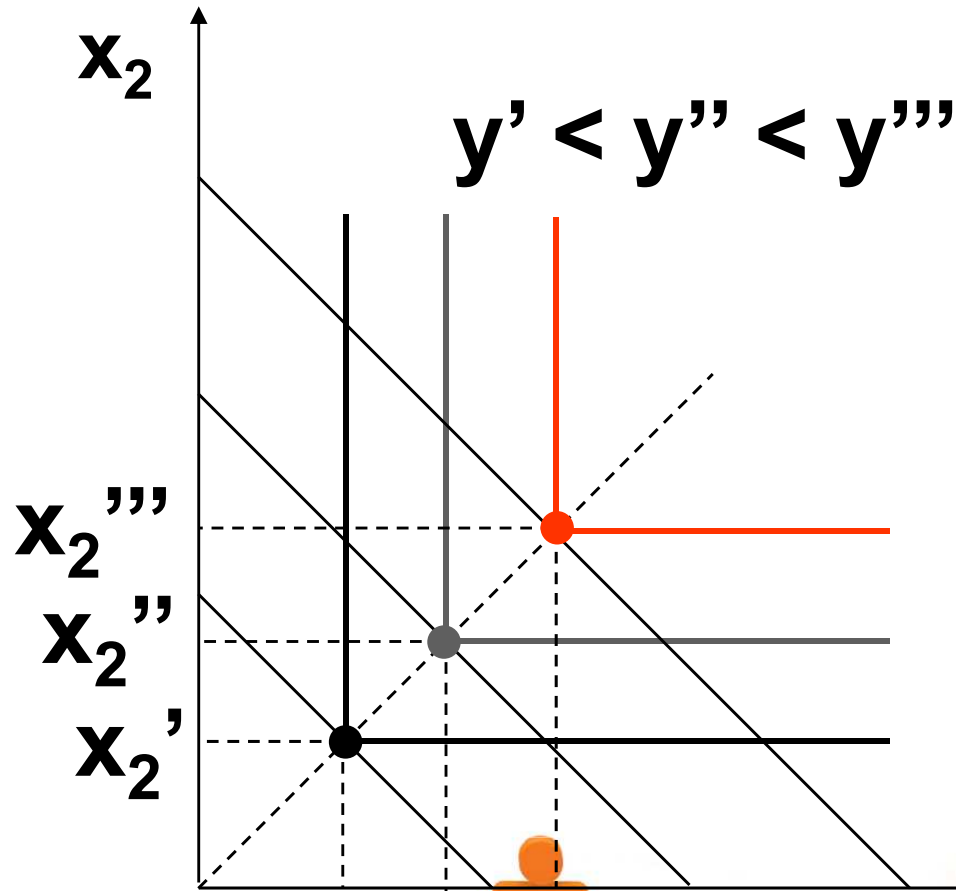
x_1' x_1'' x_1'''

x_1



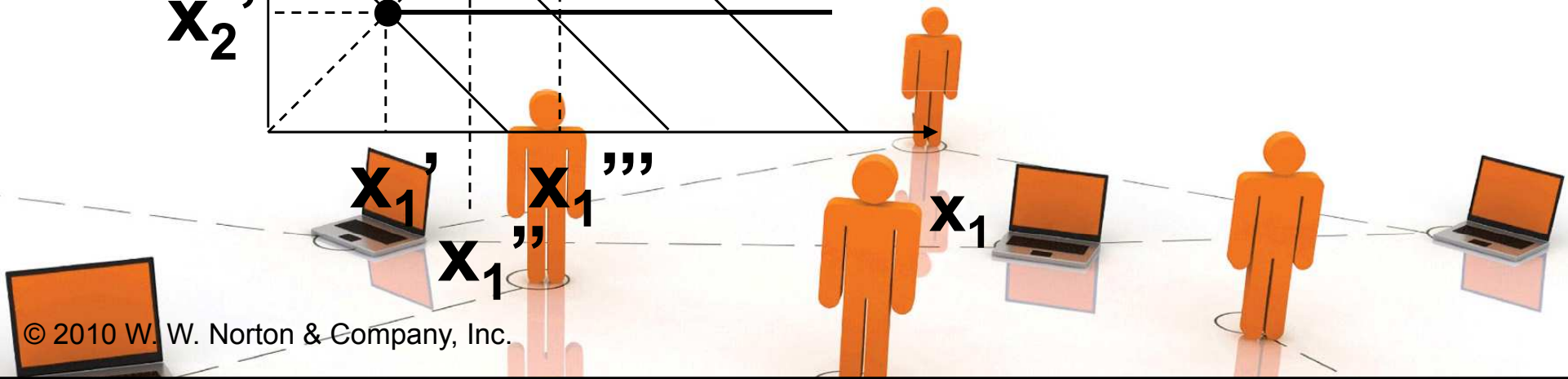
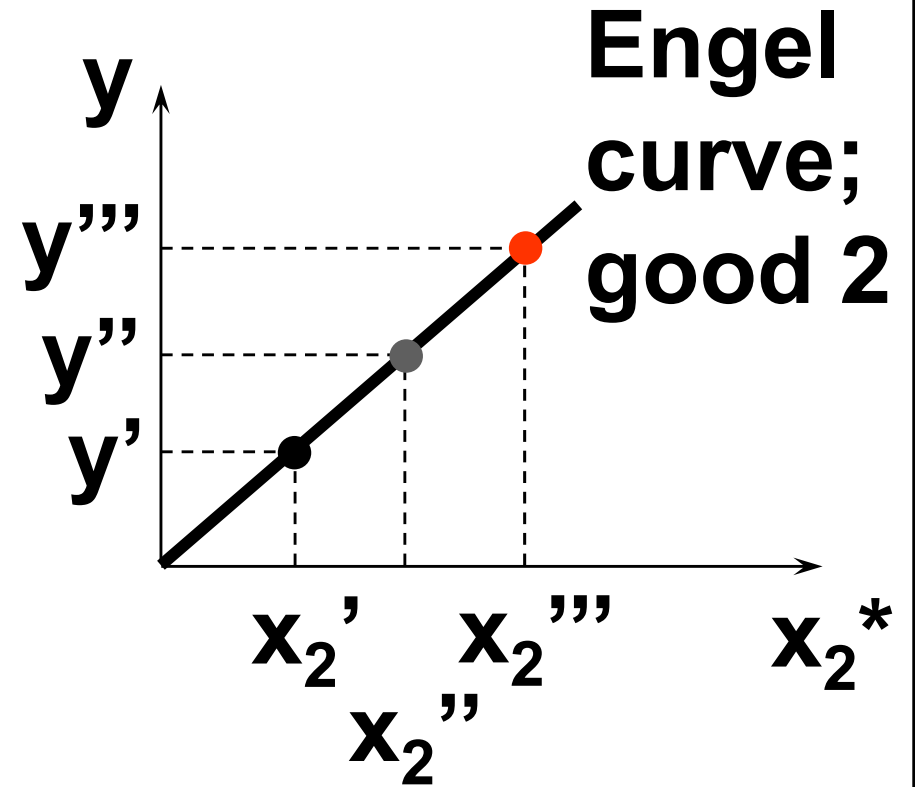
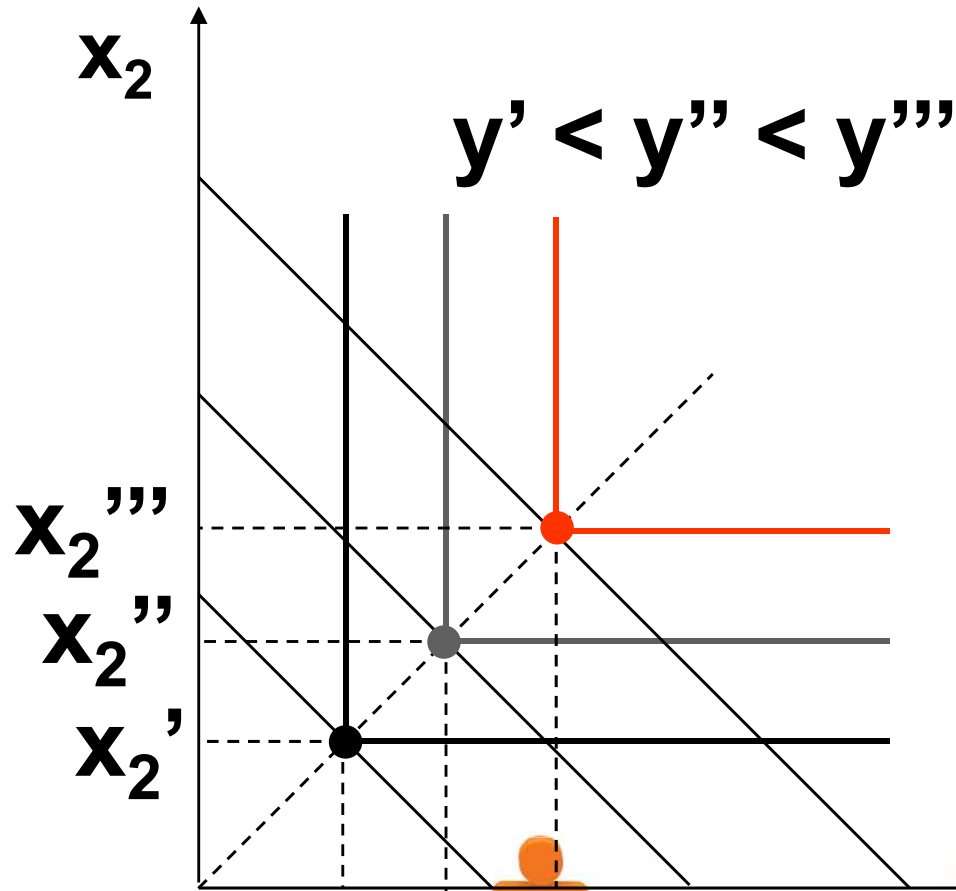
Income Changes

Fixed p_1 and p_2 .



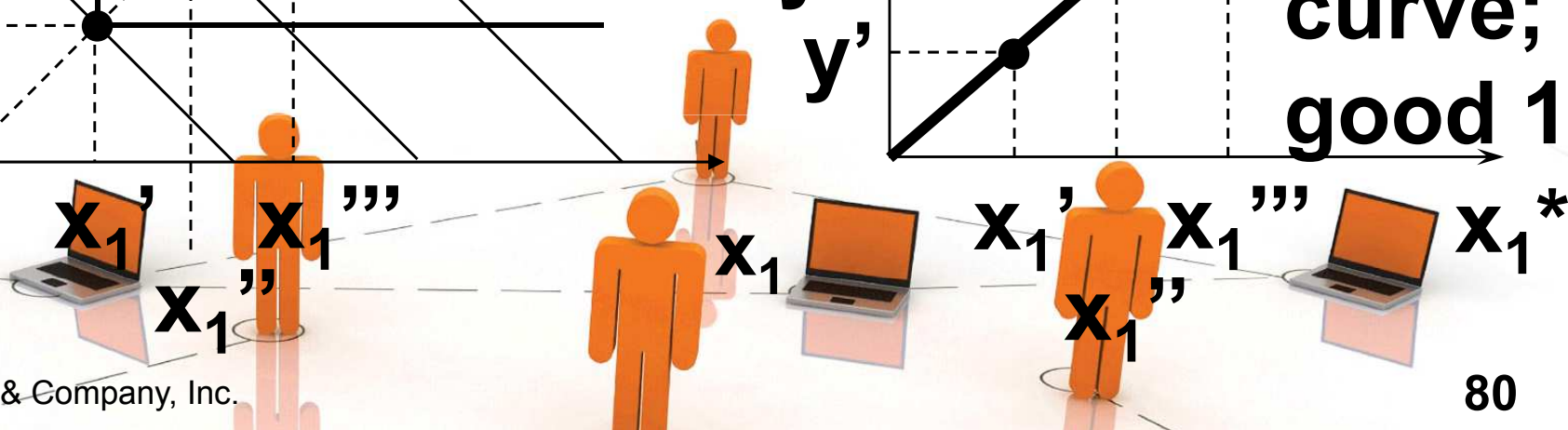
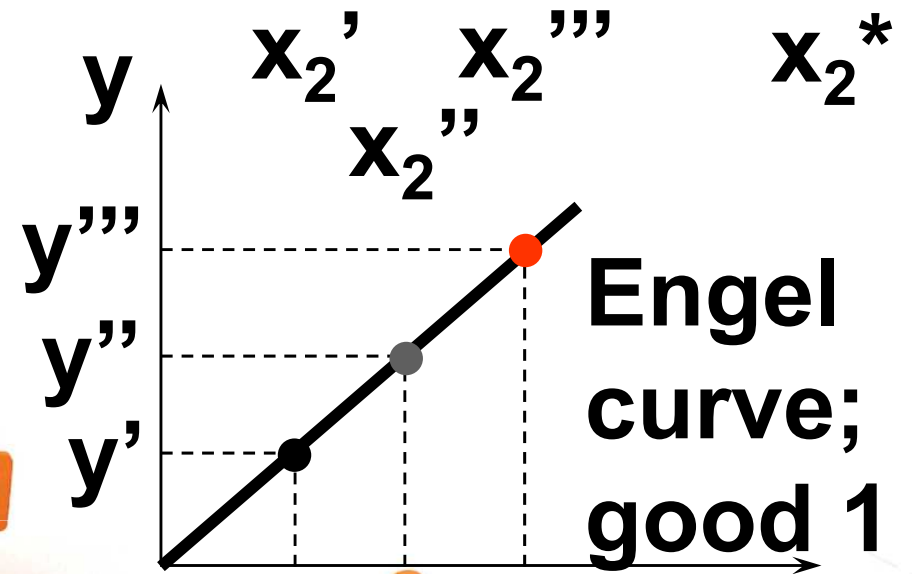
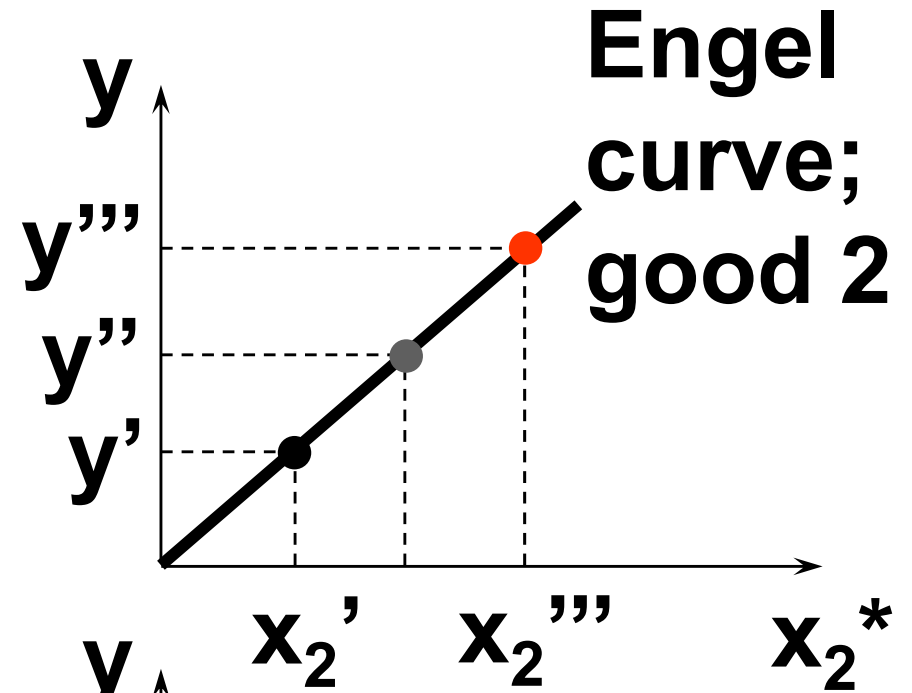
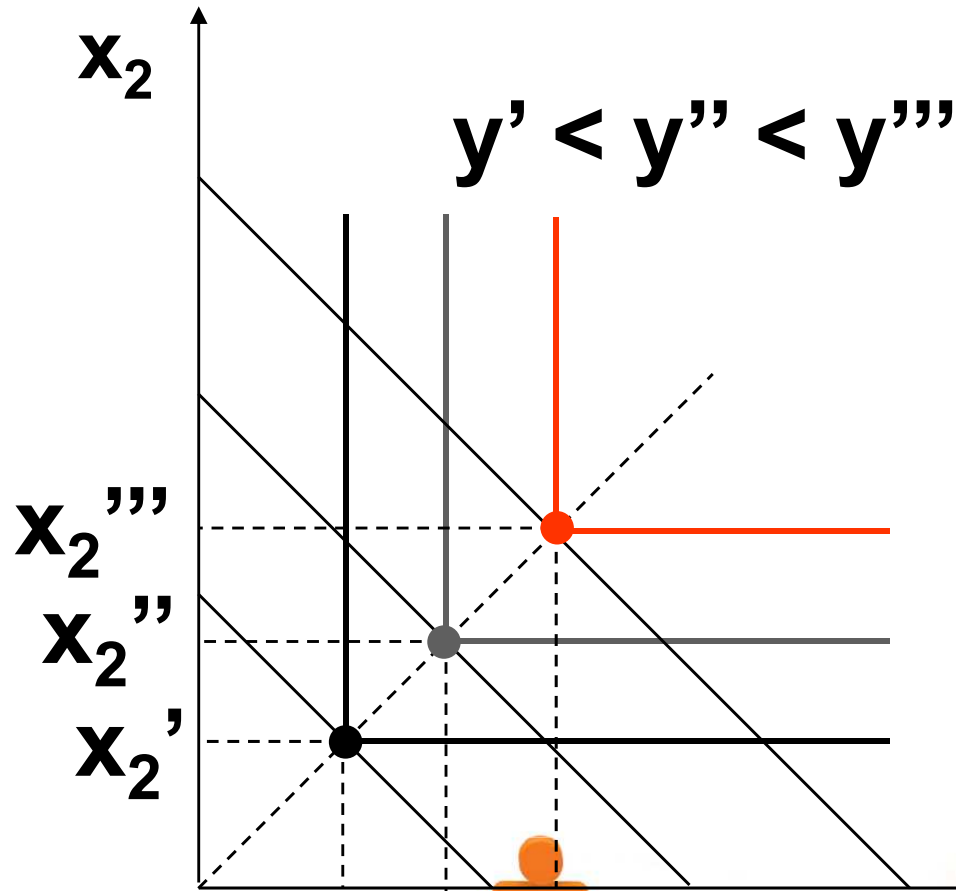
Income Changes

Fixed p_1 and p_2 .



Income Changes

Fixed p_1 and p_2 .

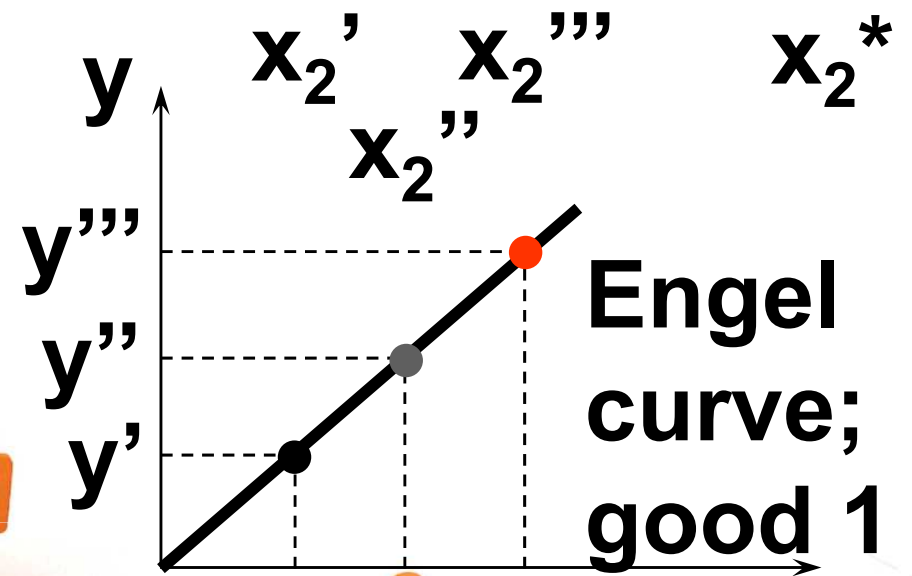
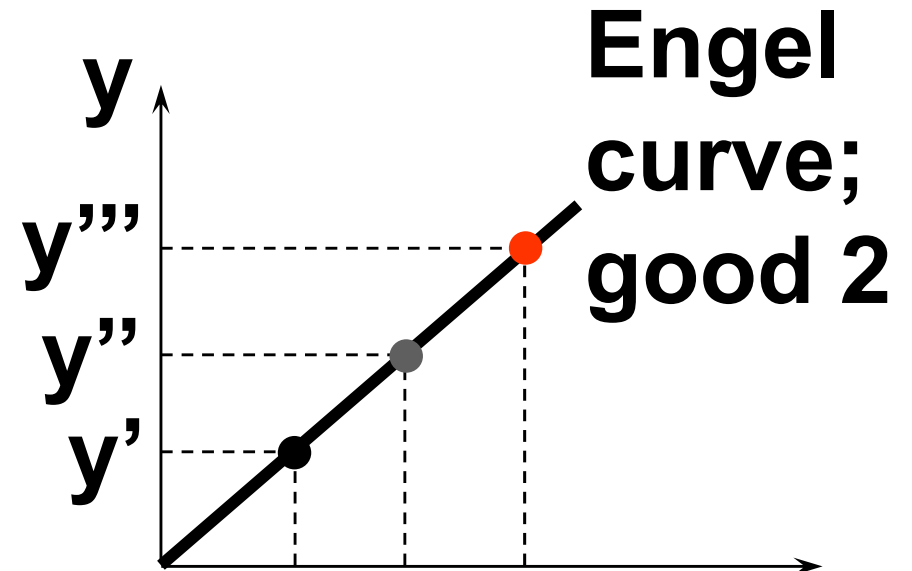


Income Changes

Fixed p_1 and p_2 .

$$y = (p_1 + p_2) x_2^*$$

$$y = (p_1 + p_2) x_1^*$$

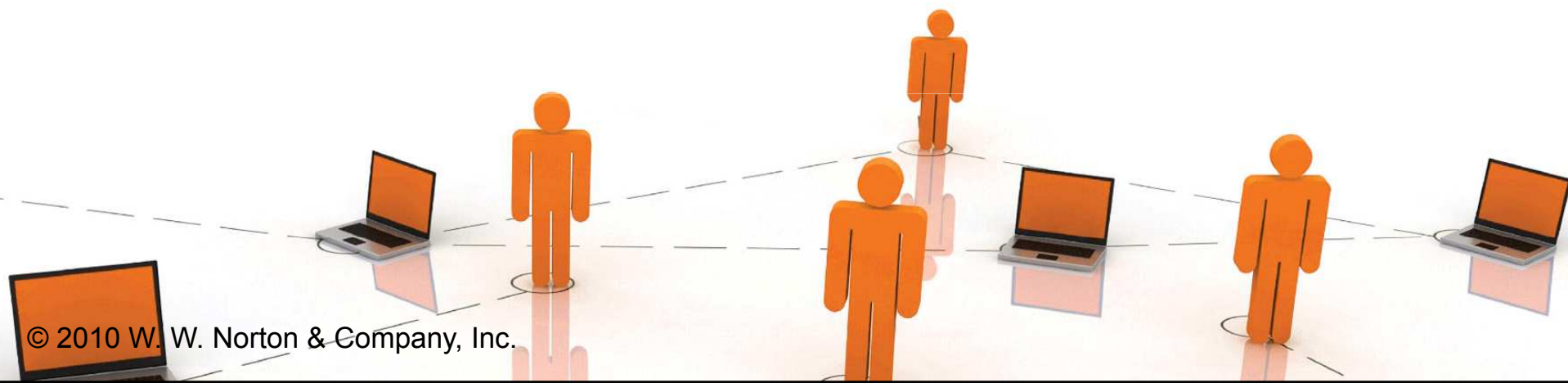


Income Changes and Perfectly-Substitutable Preferences

- ◆ Another example of computing the equations of Engel curves; the perfectly-substitution case.

$$U(x_1, x_2) = x_1 + x_2.$$

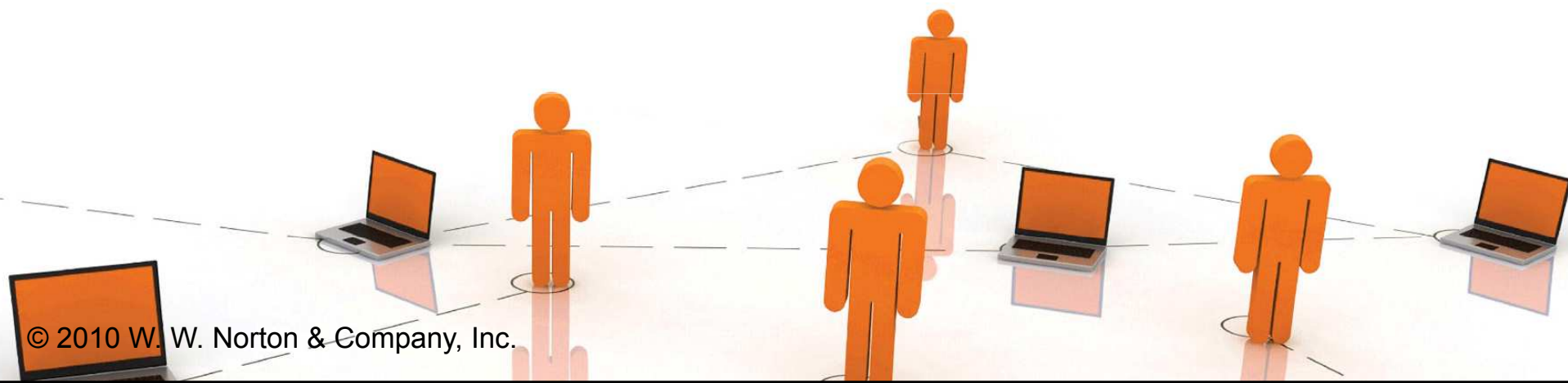
- ◆ The ordinary demand equations are



Income Changes and Perfectly-Substitutable Preferences

$$x_1^*(p_1, p_2, y) = \begin{cases} 0 & , \text{ if } p_1 > p_2 \\ y / p_1 & , \text{ if } p_1 < p_2 \end{cases}$$

$$x_2^*(p_1, p_2, y) = \begin{cases} 0 & , \text{ if } p_1 < p_2 \\ y / p_2 & , \text{ if } p_1 > p_2 . \end{cases}$$

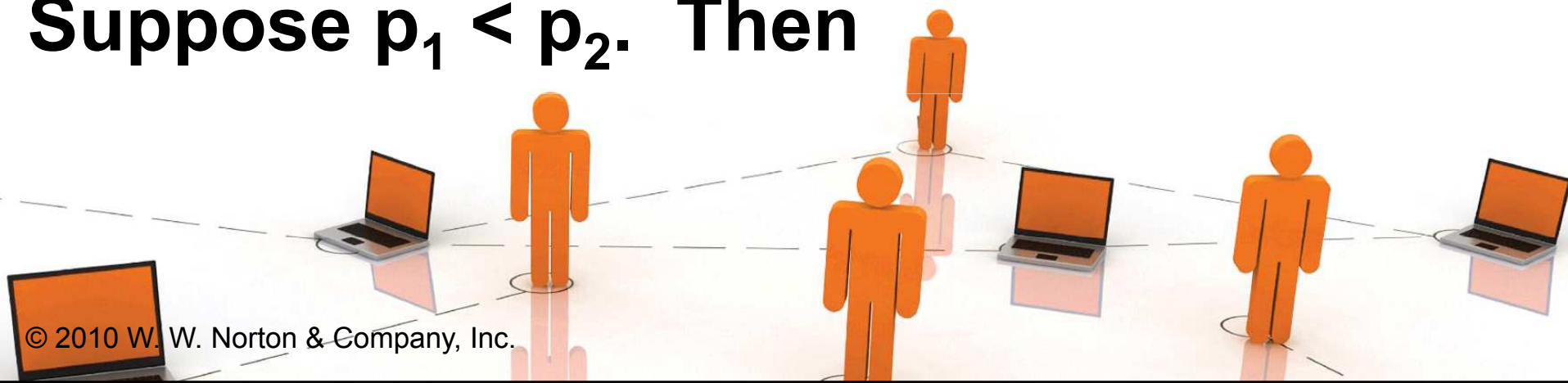


Income Changes and Perfectly-Substitutable Preferences

$$x_1^*(p_1, p_2, y) = \begin{cases} 0 & , \text{ if } p_1 > p_2 \\ y / p_1 & , \text{ if } p_1 < p_2 \end{cases}$$

$$x_2^*(p_1, p_2, y) = \begin{cases} 0 & , \text{ if } p_1 < p_2 \\ y / p_2 & , \text{ if } p_1 > p_2 . \end{cases}$$

Suppose $p_1 < p_2$. Then



Income Changes and Perfectly-Substitutable Preferences

$$x_1^*(p_1, p_2, y) = \begin{cases} 0 & , \text{ if } p_1 > p_2 \\ y / p_1 & , \text{ if } p_1 < p_2 \end{cases}$$

$$x_2^*(p_1, p_2, y) = \begin{cases} 0 & , \text{ if } p_1 < p_2 \\ y / p_2 & , \text{ if } p_1 > p_2 . \end{cases}$$

Suppose $p_1 < p_2$. Then $x_1^* = \frac{y}{p_1}$ and $x_2^* = 0$

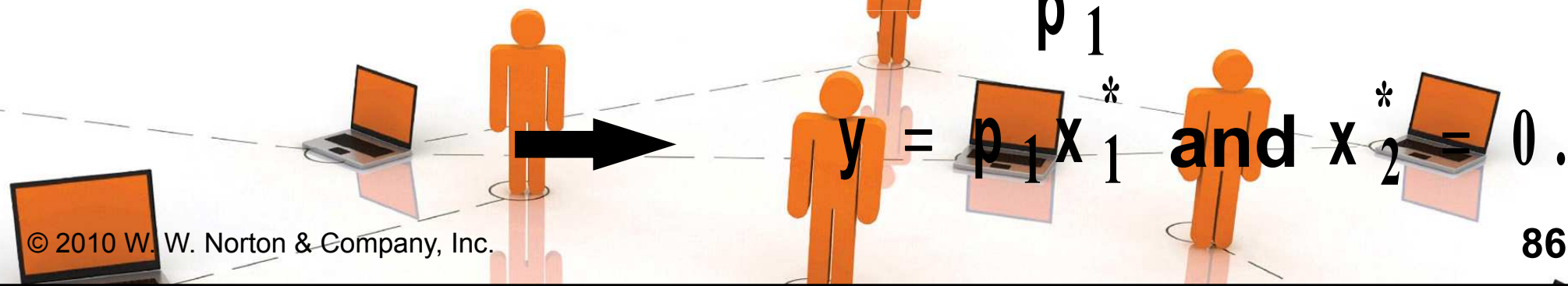


Income Changes and Perfectly-Substitutable Preferences

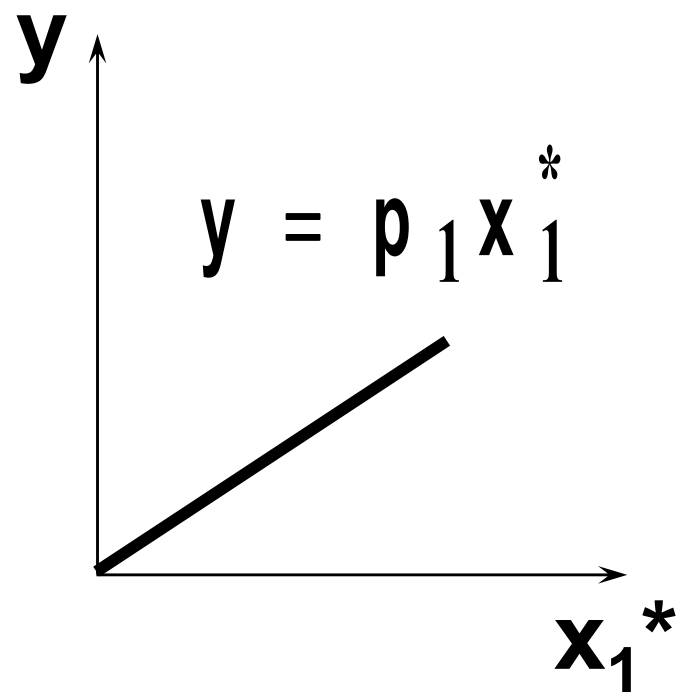
$$x_1^*(p_1, p_2, y) = \begin{cases} 0 & , \text{ if } p_1 > p_2 \\ y / p_1 & , \text{ if } p_1 < p_2 \end{cases}$$

$$x_2^*(p_1, p_2, y) = \begin{cases} 0 & , \text{ if } p_1 < p_2 \\ y / p_2 & , \text{ if } p_1 > p_2 . \end{cases}$$

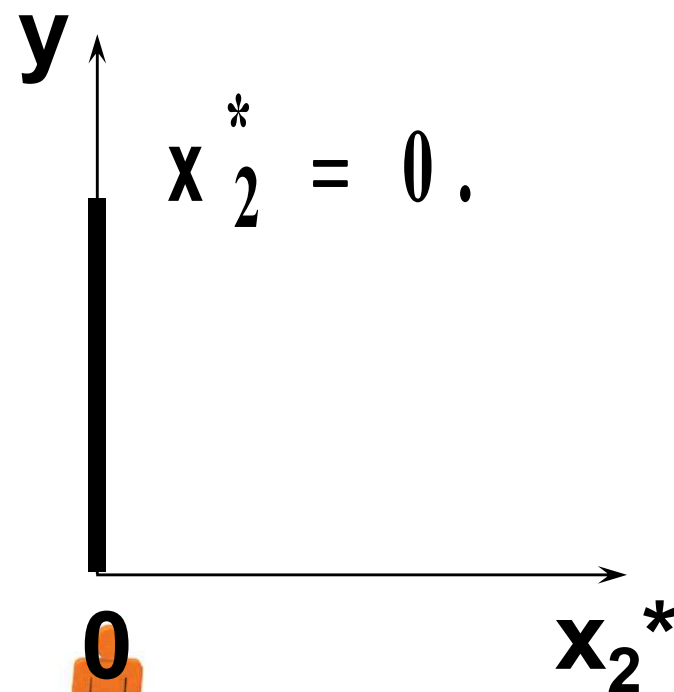
Suppose $p_1 < p_2$. Then $x_1^* = \frac{y}{p_1}$ and $x_2^* = 0$



Income Changes and Perfectly-Substitutable Preferences



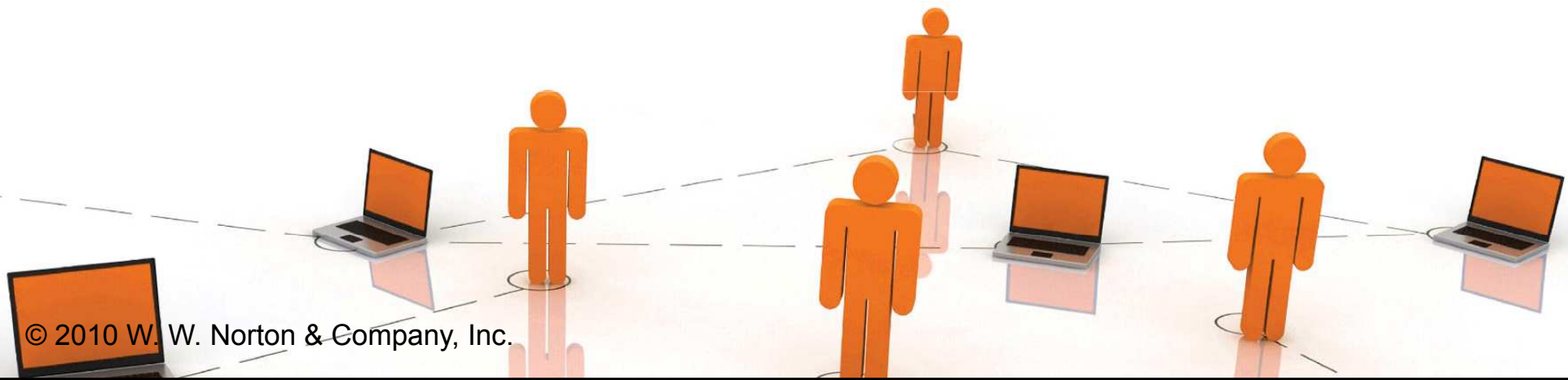
**Engel curve
for good 1**



**Engel curve
for good 2**

Income Changes

- ◆ In every example so far the Engel curves have all been straight lines?
Q: Is this true in general?
- ◆ A: No. Engel curves are straight lines if the consumer's preferences are homothetic.



Homotheticity

- ◆ A consumer's preferences are homothetic if and only if

$$(x_1, x_2) \prec (y_1, y_2) \iff (kx_1, kx_2) \prec (ky_1, ky_2)$$

for every $k > 0$.

- ◆ That is, the consumer's MRS is the same anywhere on a straight line drawn from the origin.

Income Effects -- A Nonhomothetic Example

- ◆ **Quasilinear preferences are not homothetic.**

$$U(x_1, x_2) = f(x_1) + x_2.$$

- ◆ **For example,**

$$U(x_1, x_2) = \sqrt{x_1} + x_2.$$

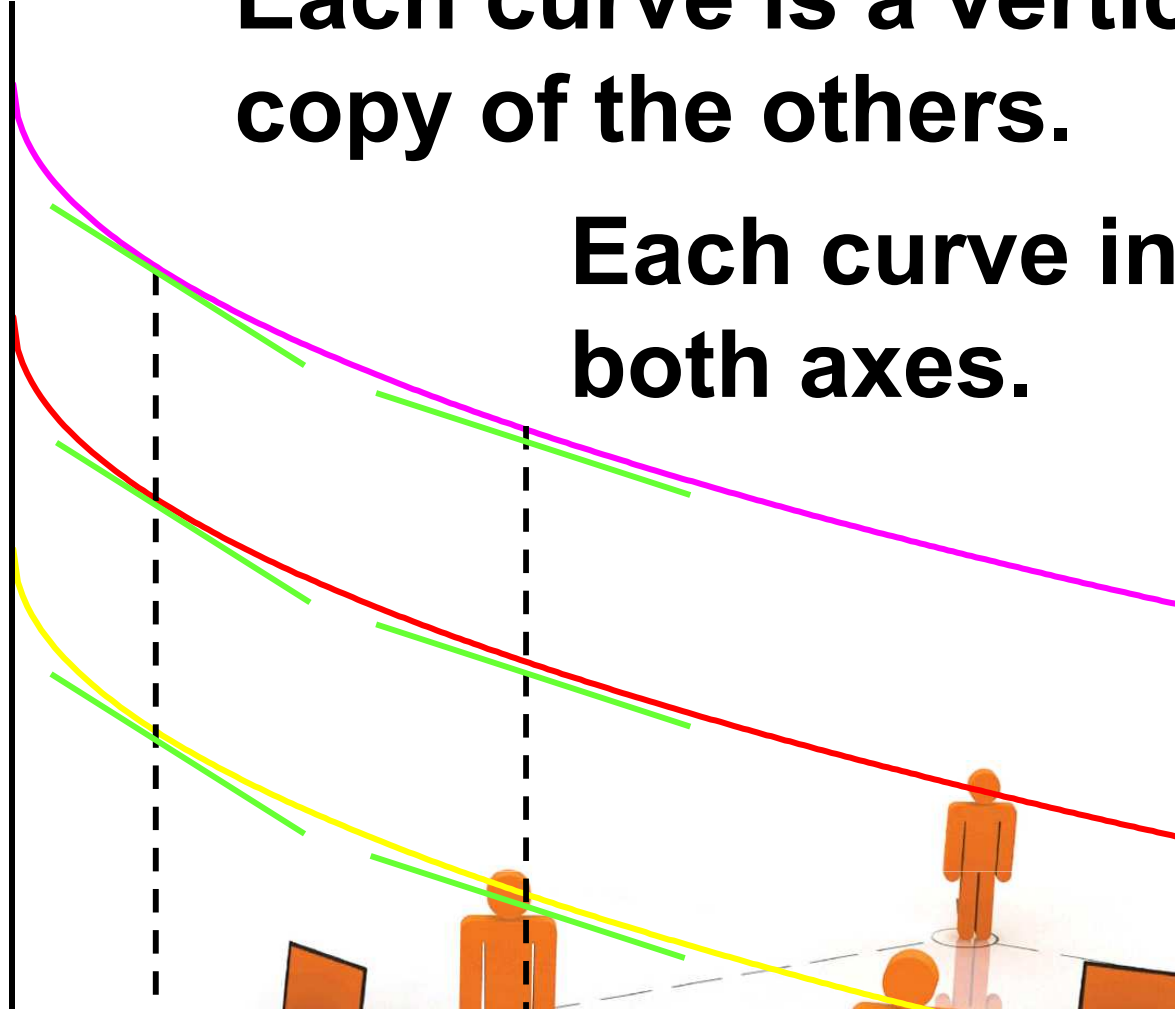


Quasi-linear Indifference Curves

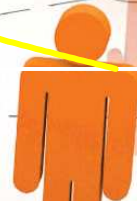
Each curve is a vertically shifted copy of the others.

Each curve intersects both axes.

X_2

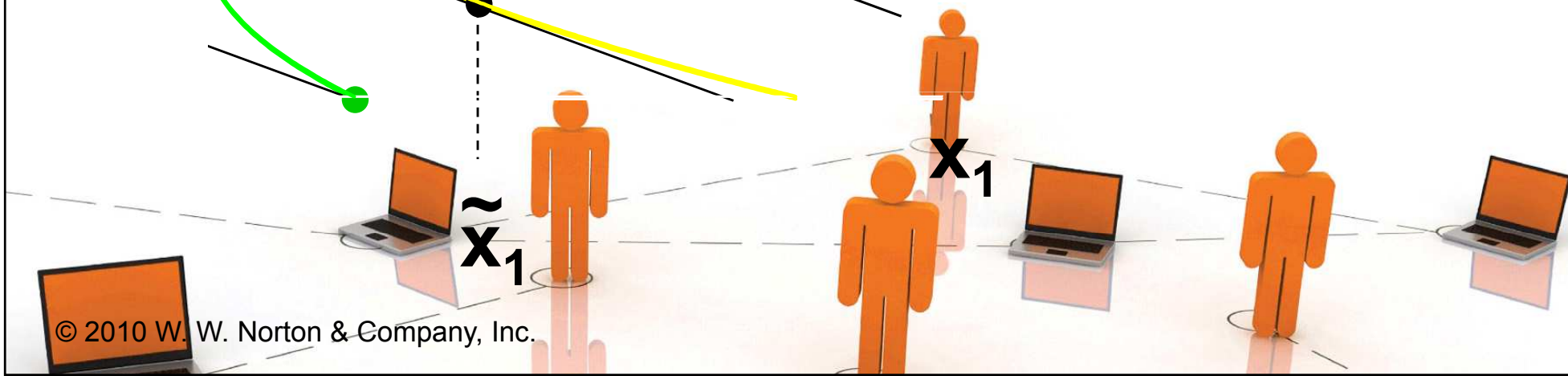
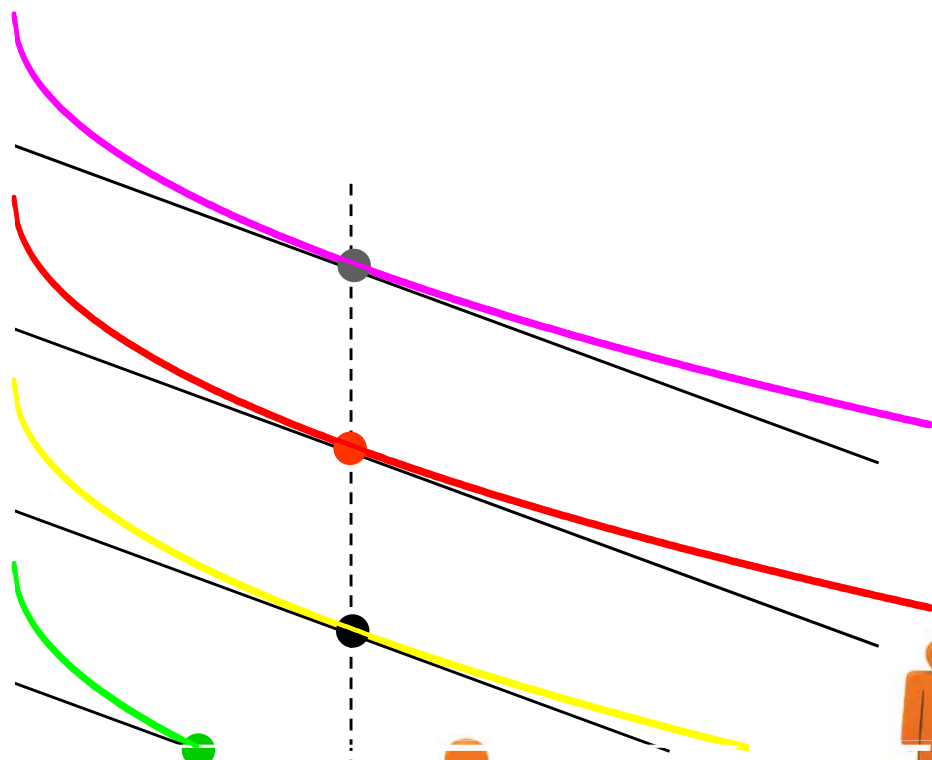


X_1



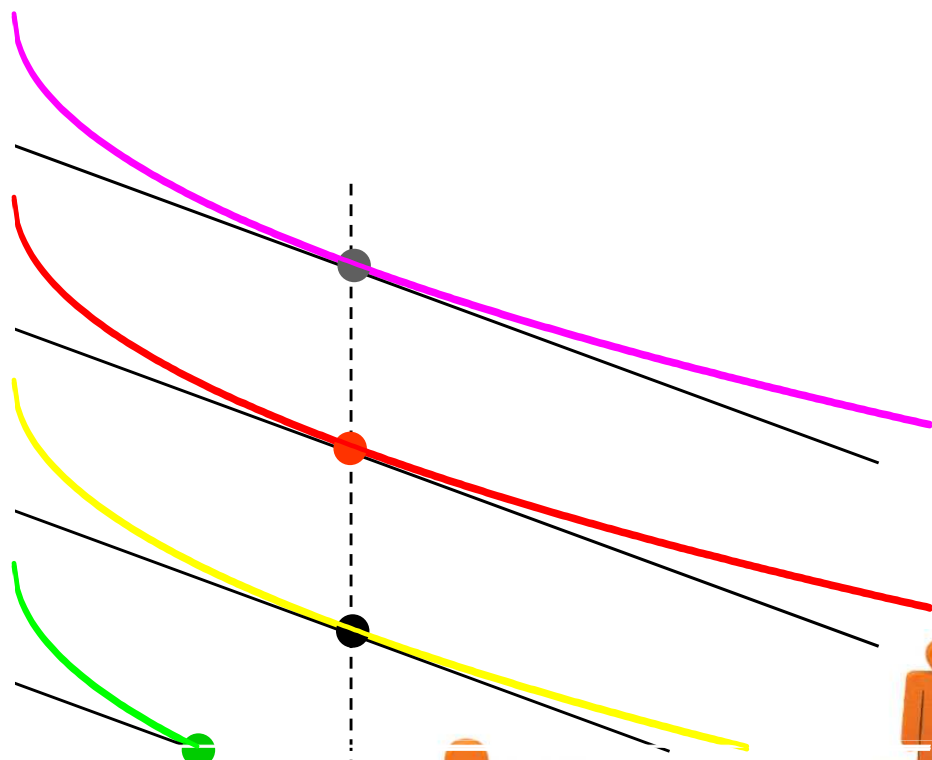
Income Changes; Quasilinear Utility

x_2



Income Changes; Quasilinear Utility

x_2



y

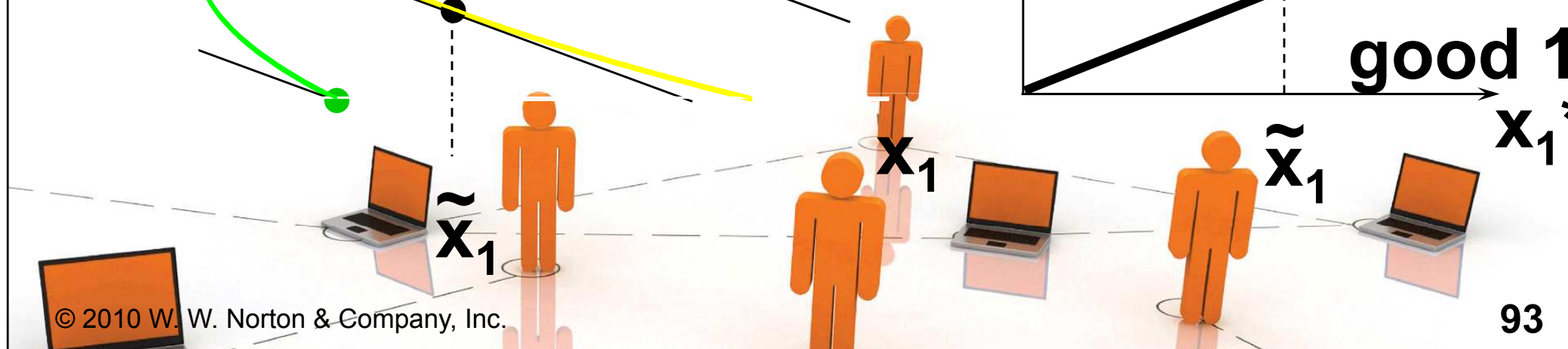
Engel curve
for
good 1

x_1^*

\tilde{x}_1

x_1

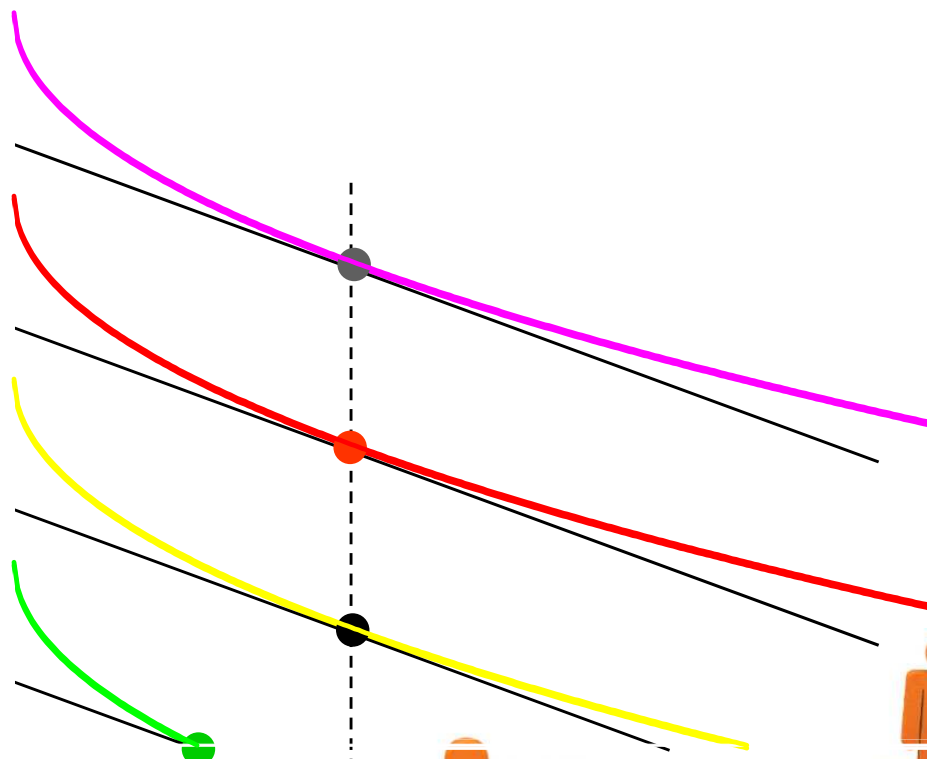
\tilde{x}_1



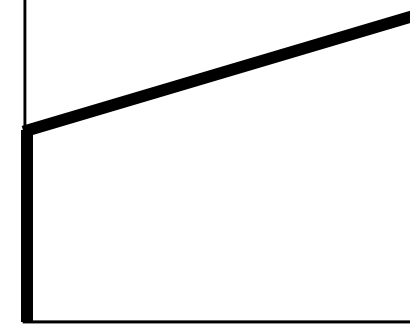
Income Changes; Quasilinear

Utility

x_2



y



Engel
curve
for
good 2
 x_2^*

\tilde{x}_1

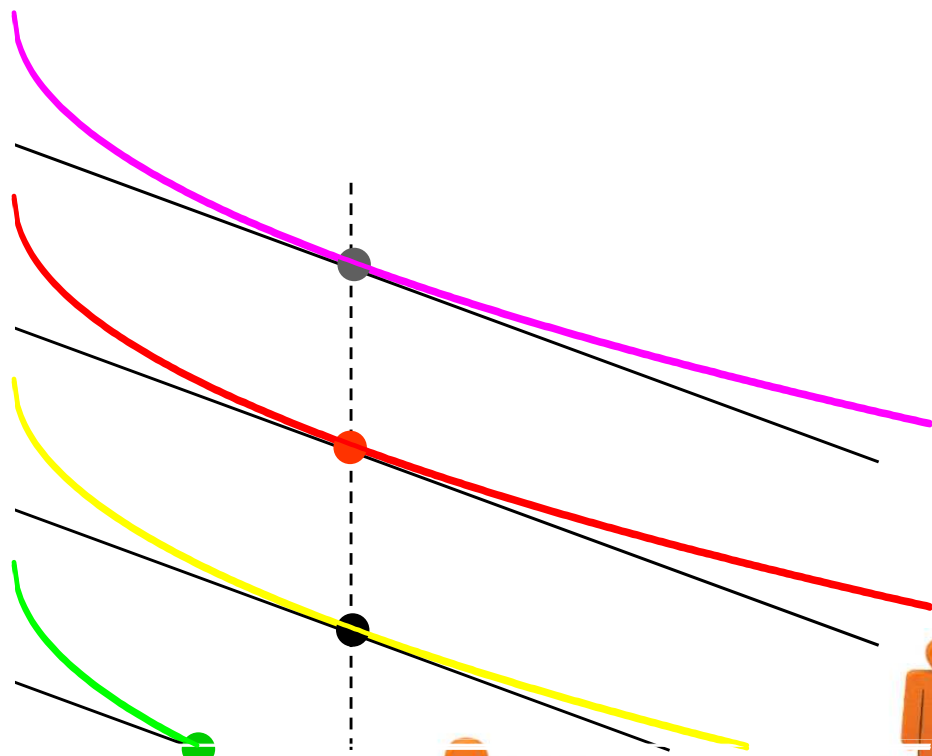
x_1



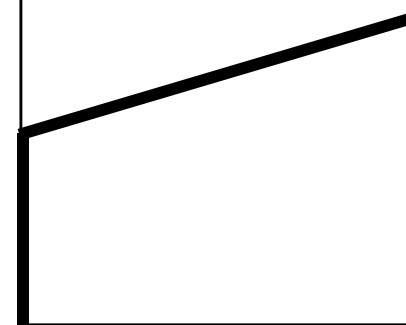
Income Changes; Quasilinear

Utility

x_2



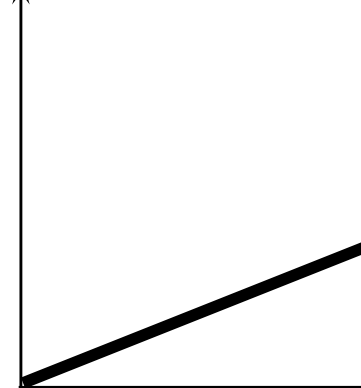
y



Engel curve for good 2

x_2^*

y



Engel curve for good 1

x_1^*

\tilde{x}_1

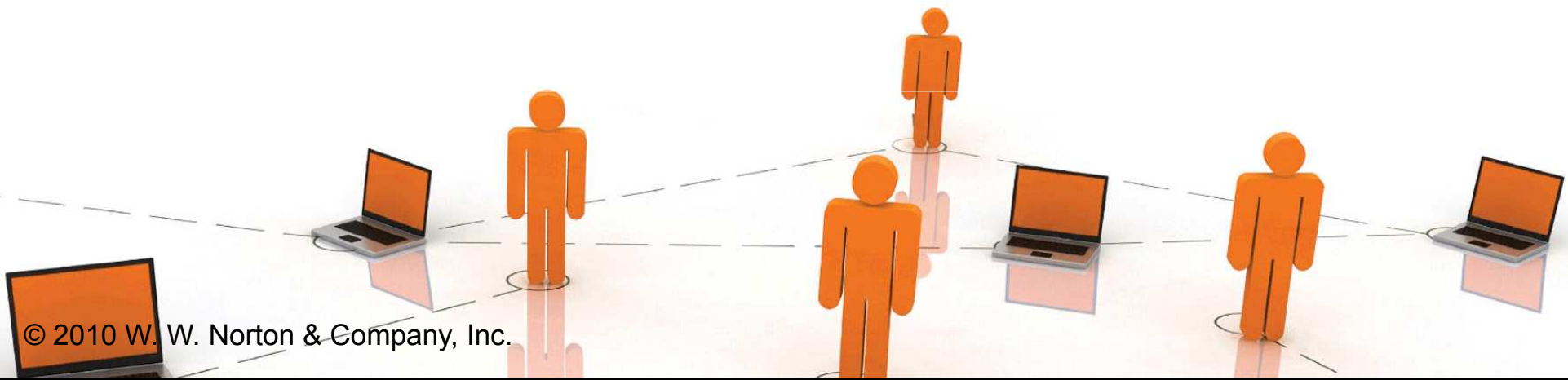
x_1

\tilde{x}_1



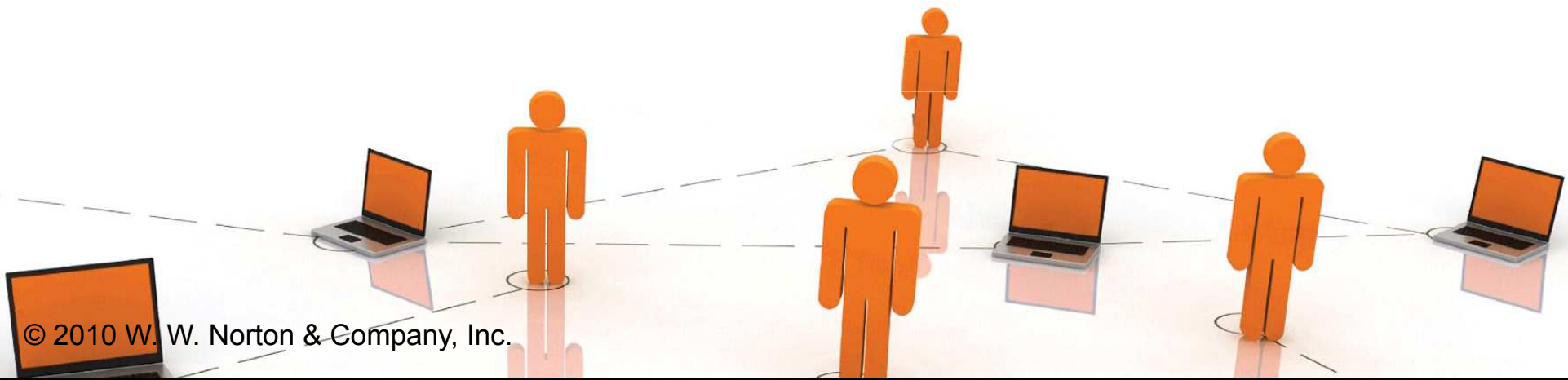
Income Effects

- ◆ **A good for which quantity demanded rises with income is called normal.**
- ◆ **Therefore a normal good's Engel curve is positively sloped.**



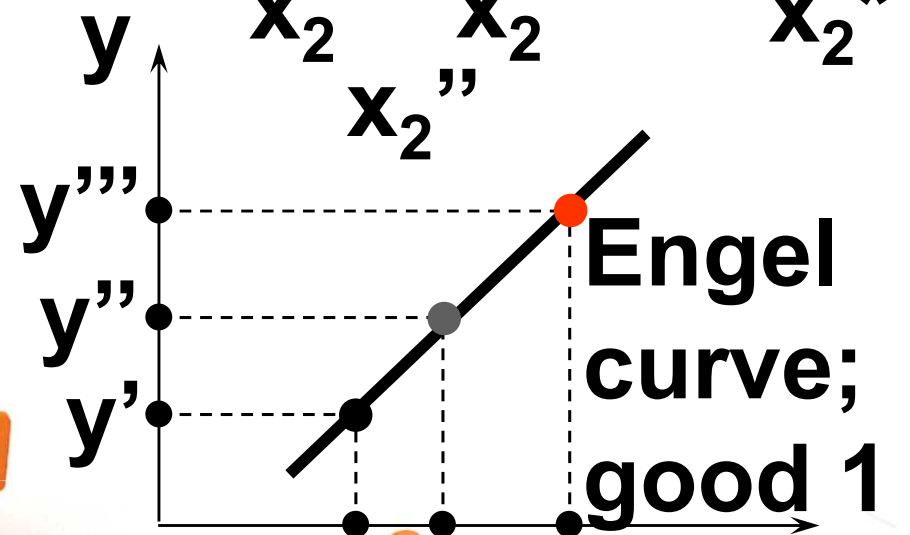
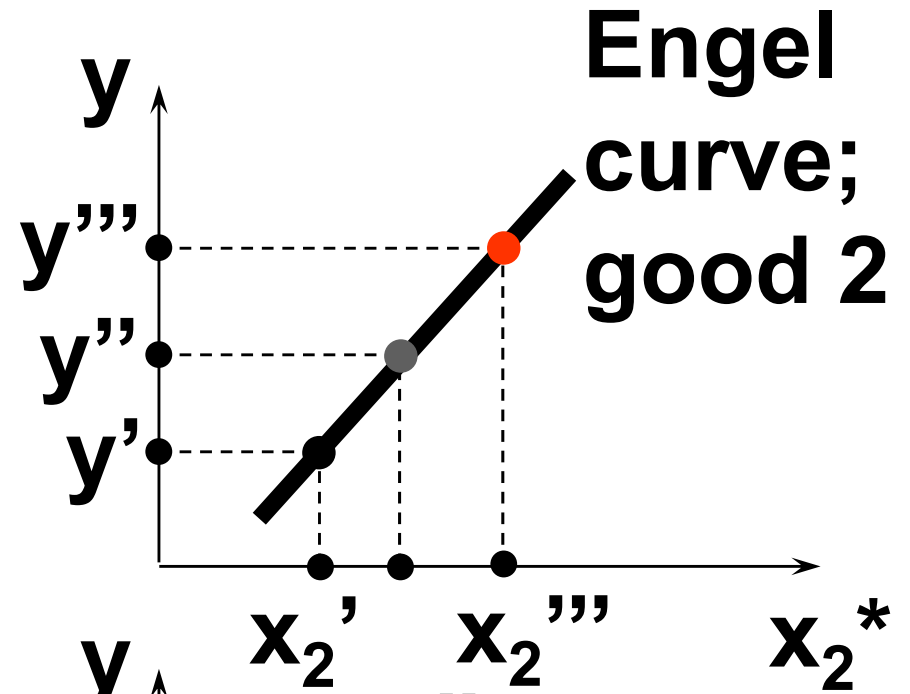
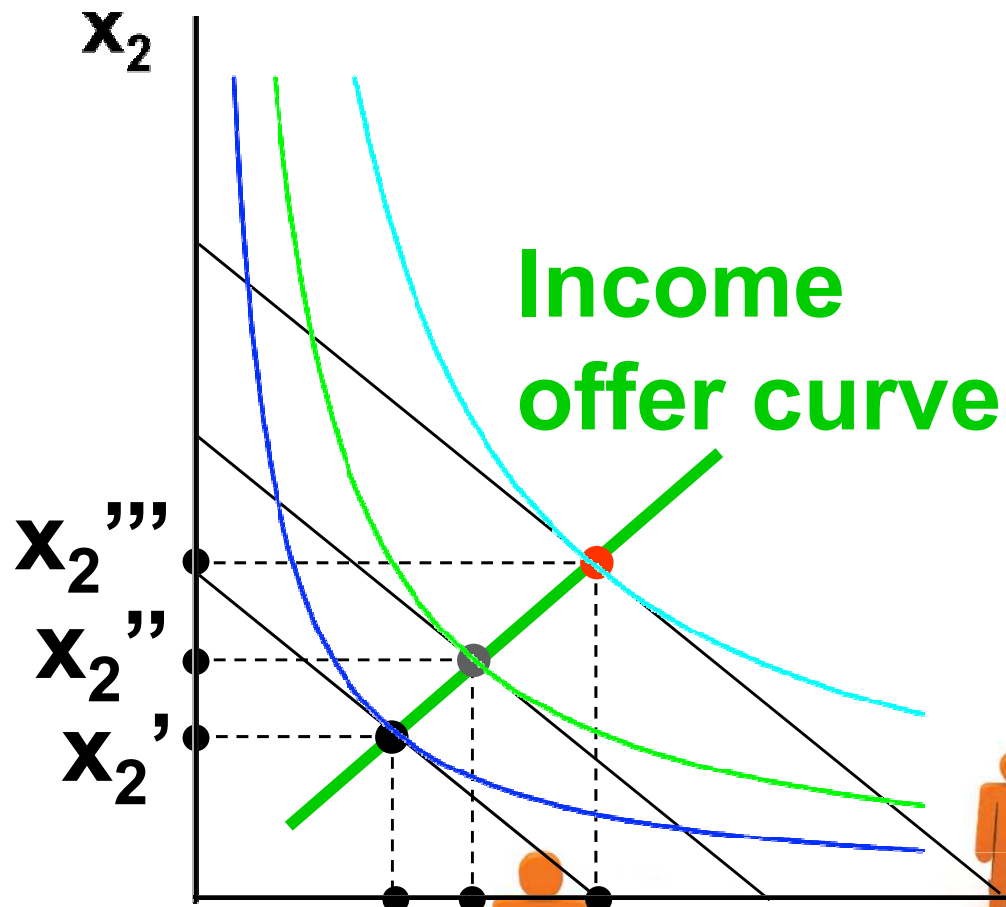
Income Effects

- ◆ **A good for which quantity demanded falls as income increases is called income inferior.**
- ◆ **Therefore an income inferior good's Engel curve is negatively sloped.**

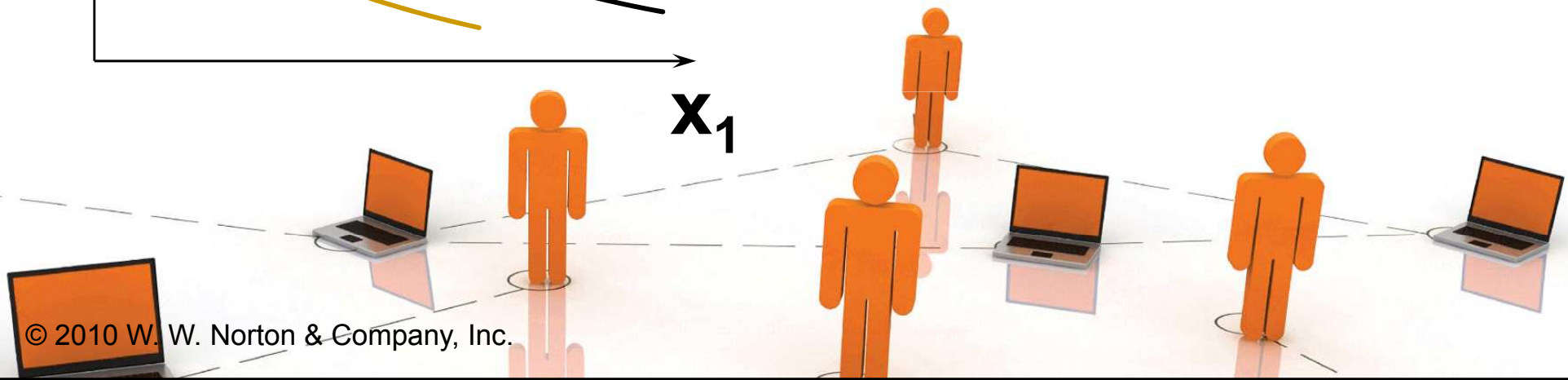
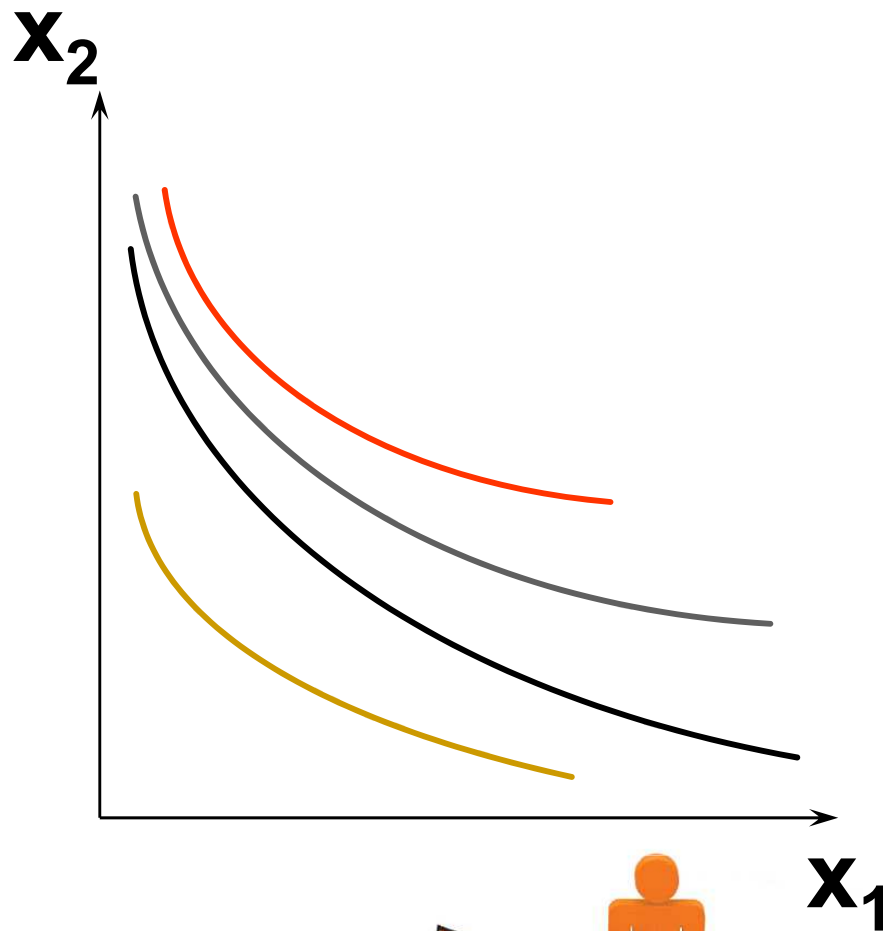


Income Changes; Goods

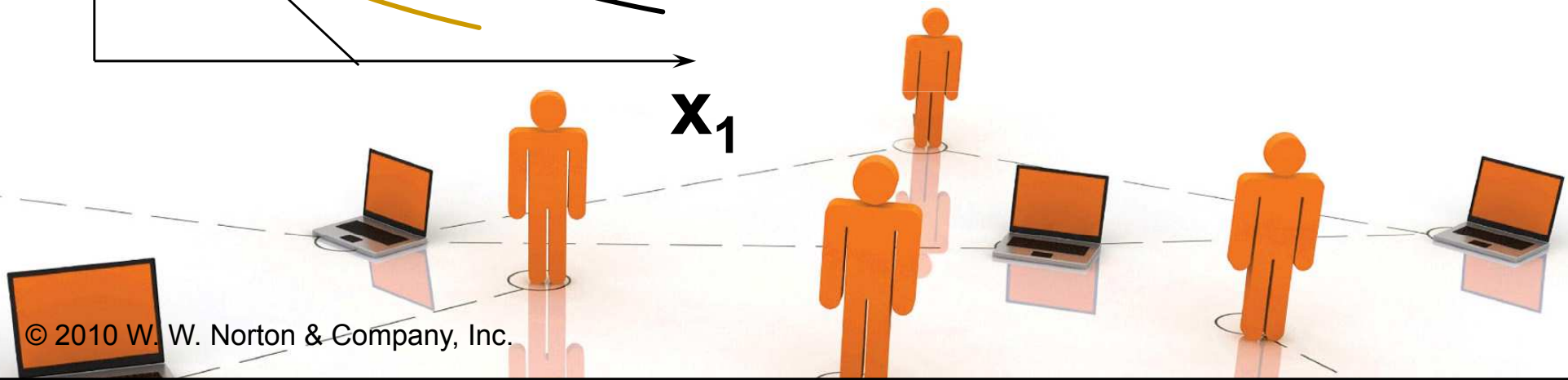
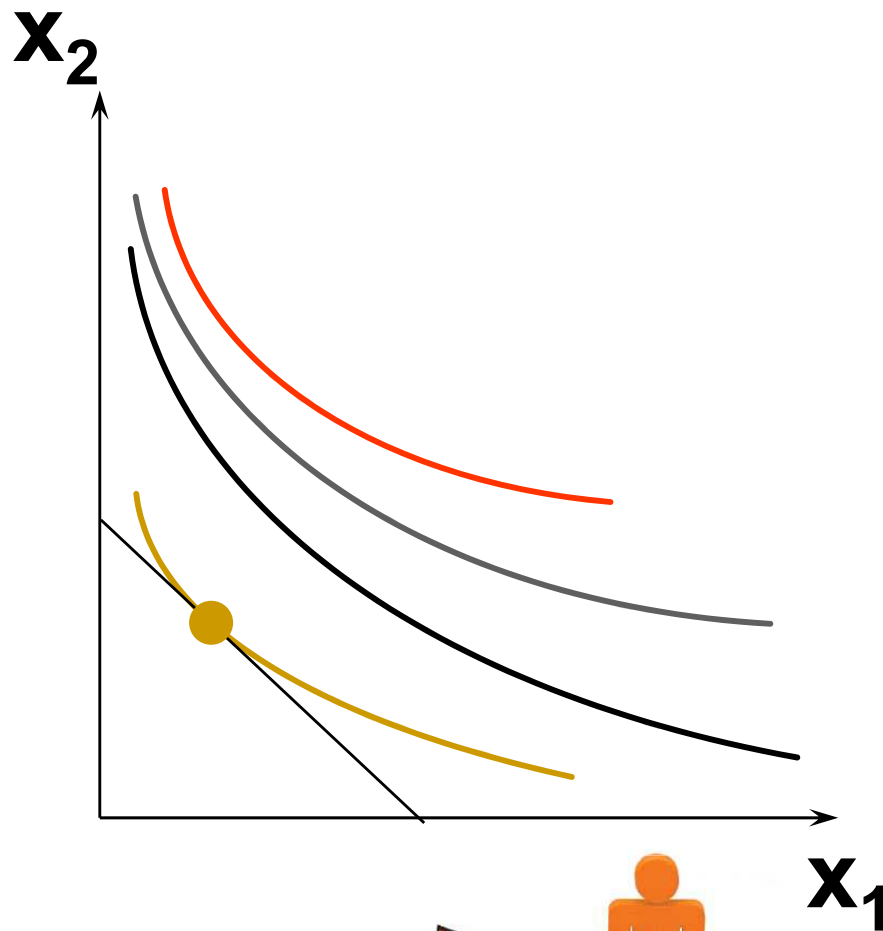
1 & 2 Normal



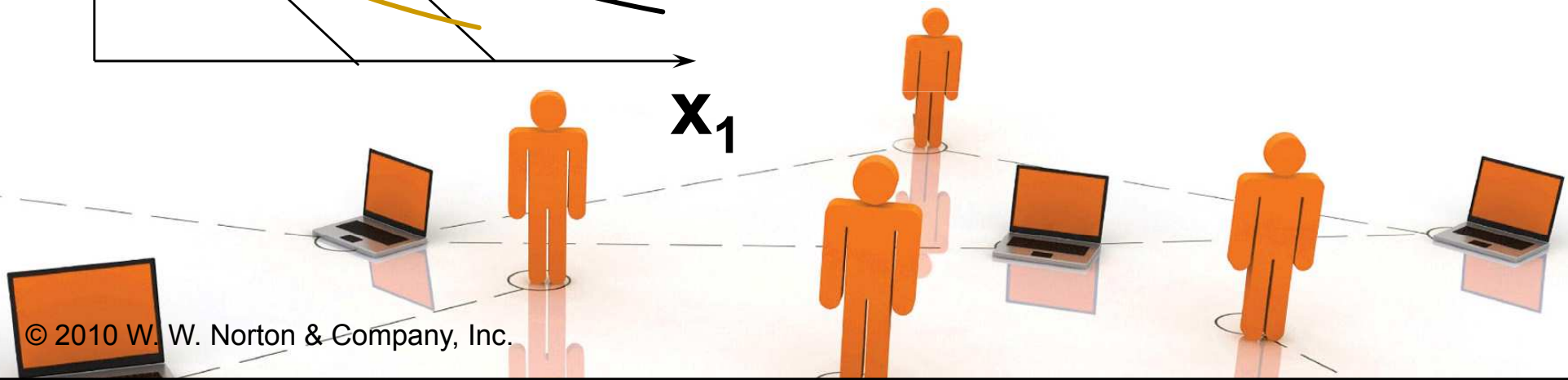
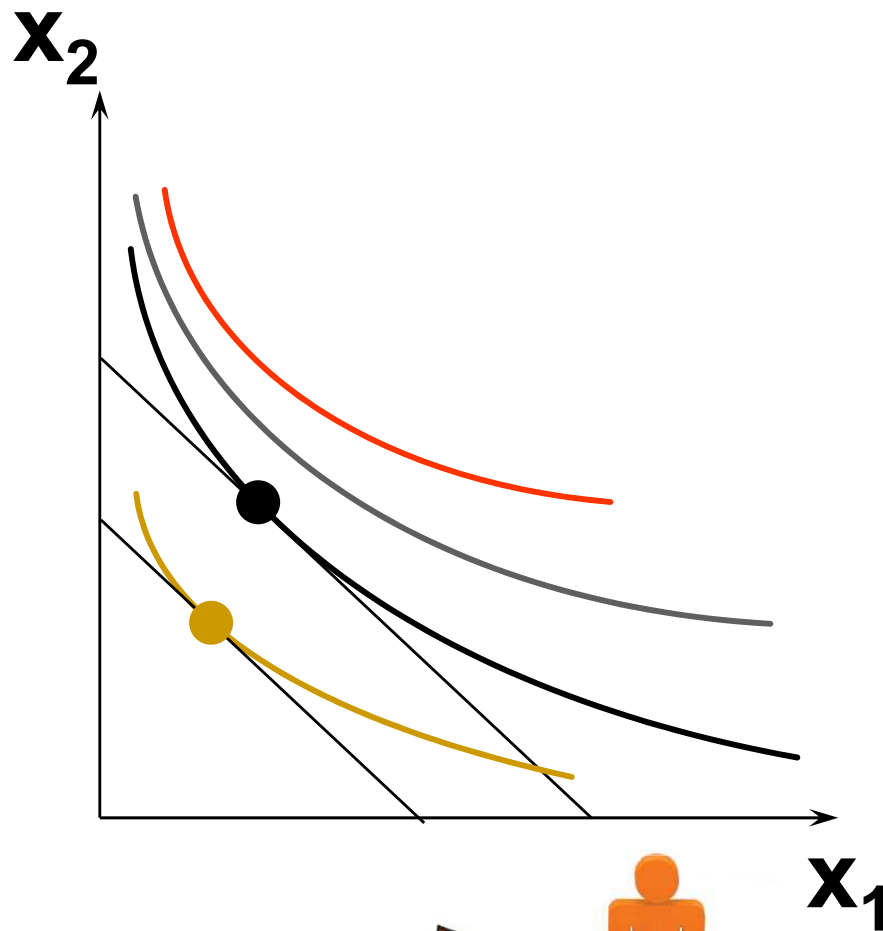
Income Changes; Good 2 Is Normal, Good 1 Becomes Income Inferior



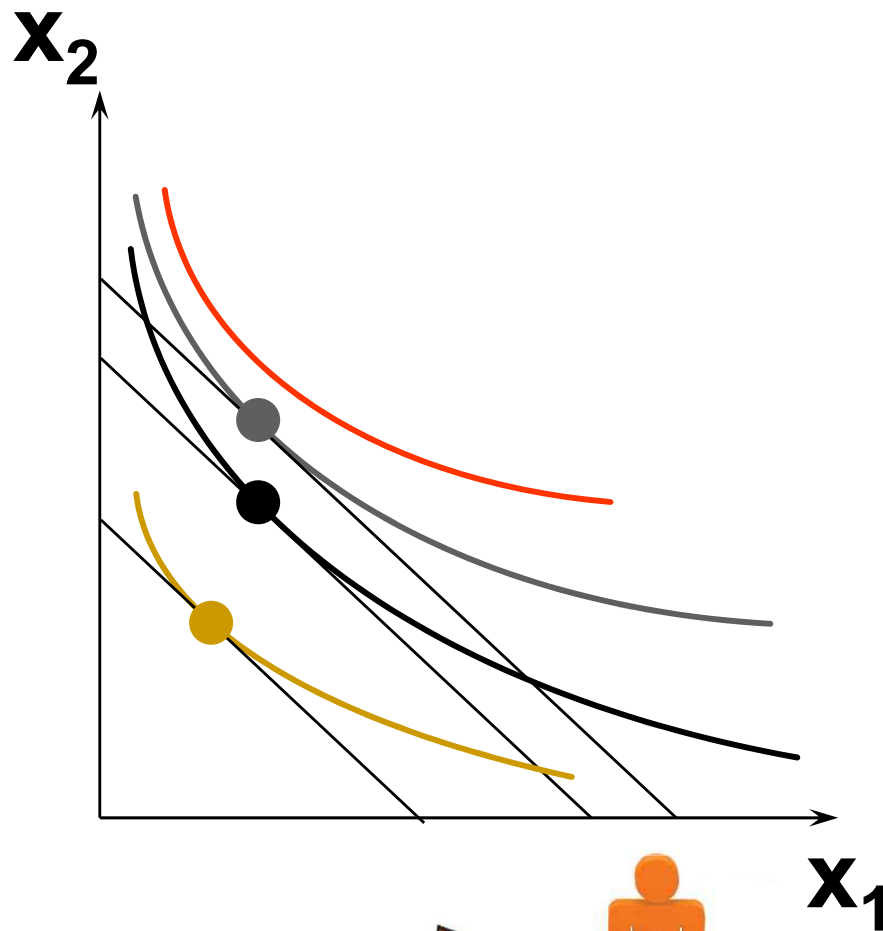
Income Changes; Good 2 Is Normal, Good 1 Becomes Income Inferior



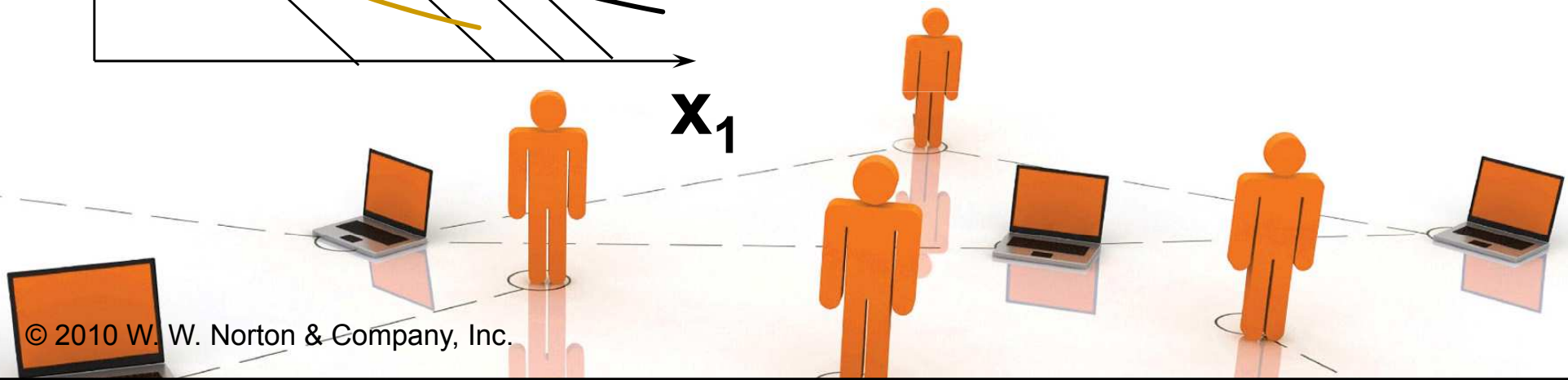
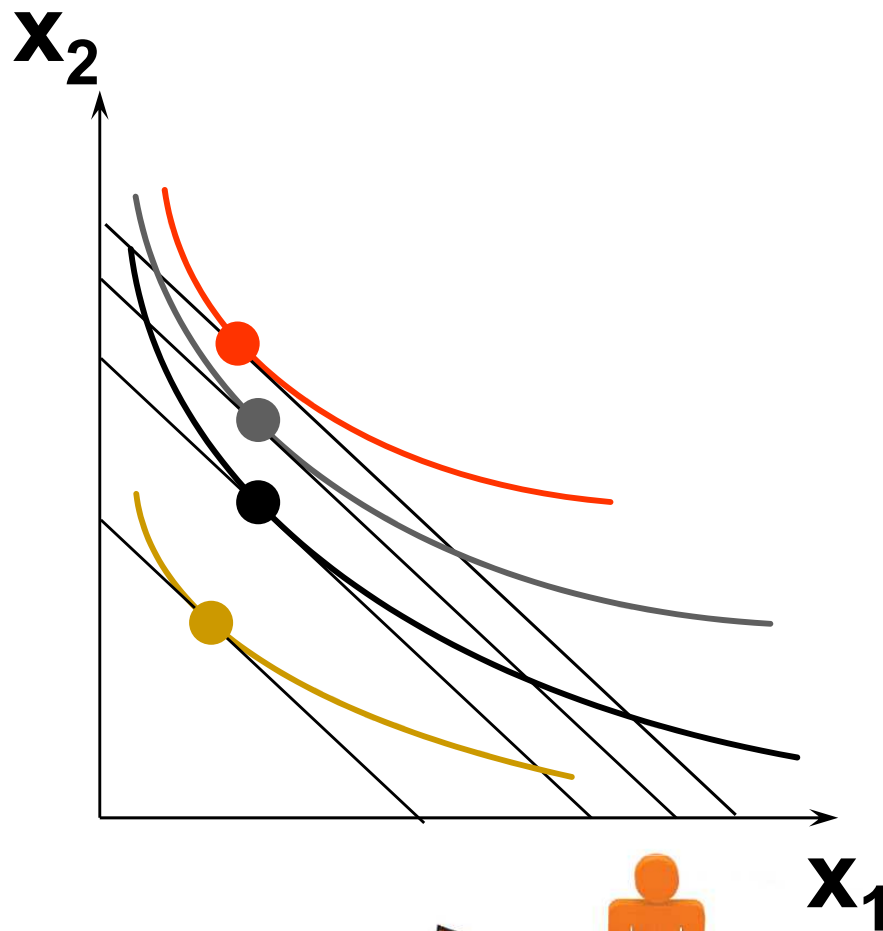
Income Changes; Good 2 Is Normal, Good 1 Becomes Income Inferior



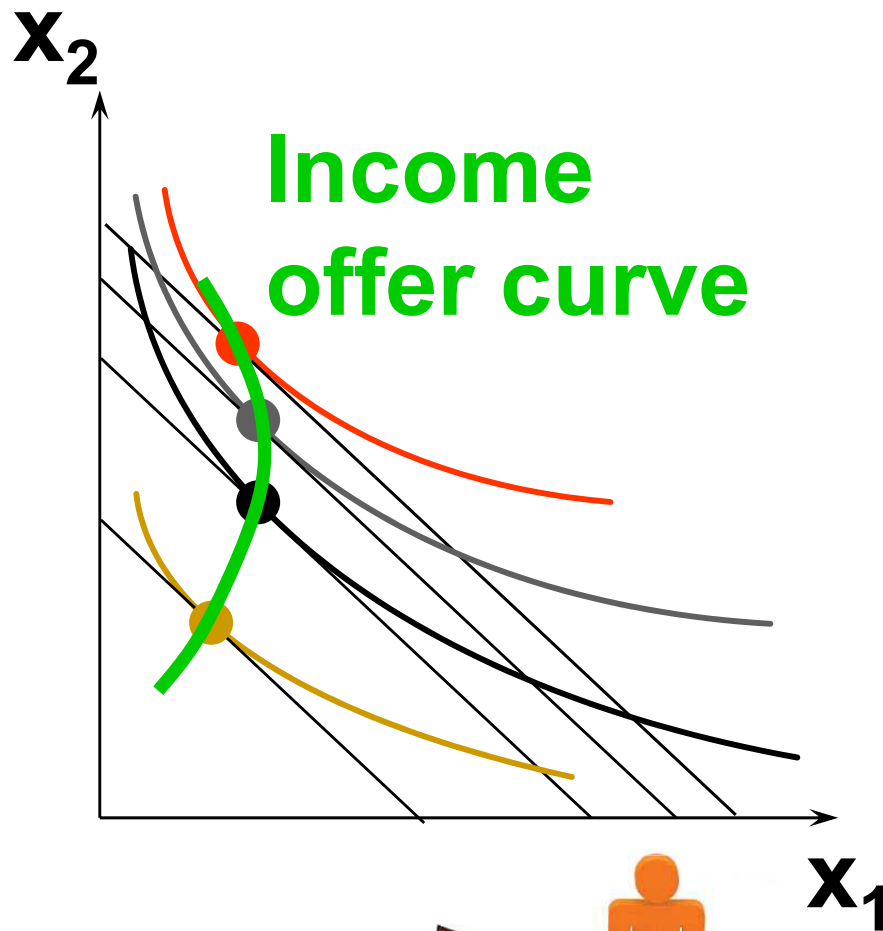
Income Changes; Good 2 Is Normal, Good 1 Becomes Income Inferior



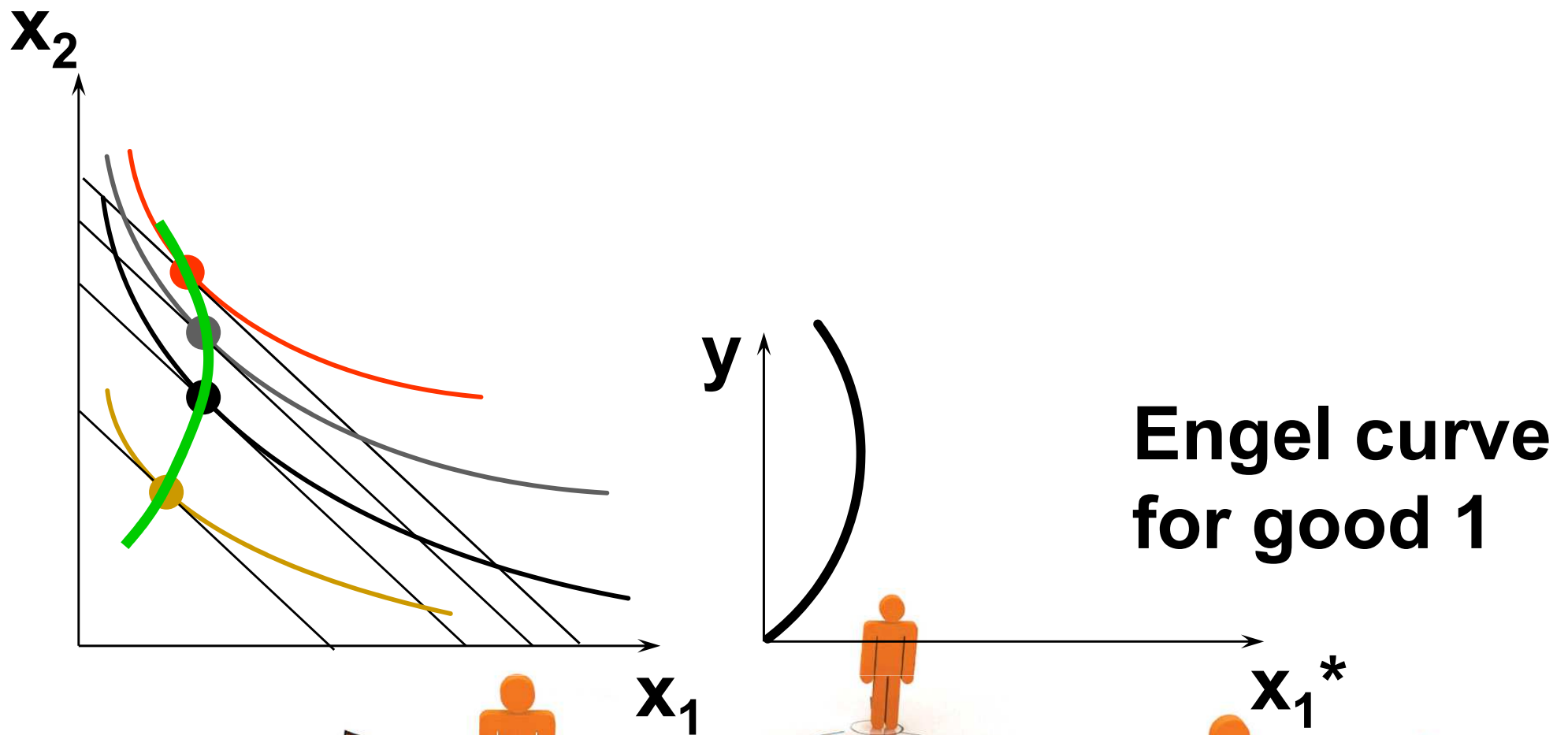
Income Changes; Good 2 Is Normal, Good 1 Becomes Income Inferior



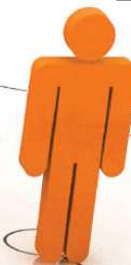
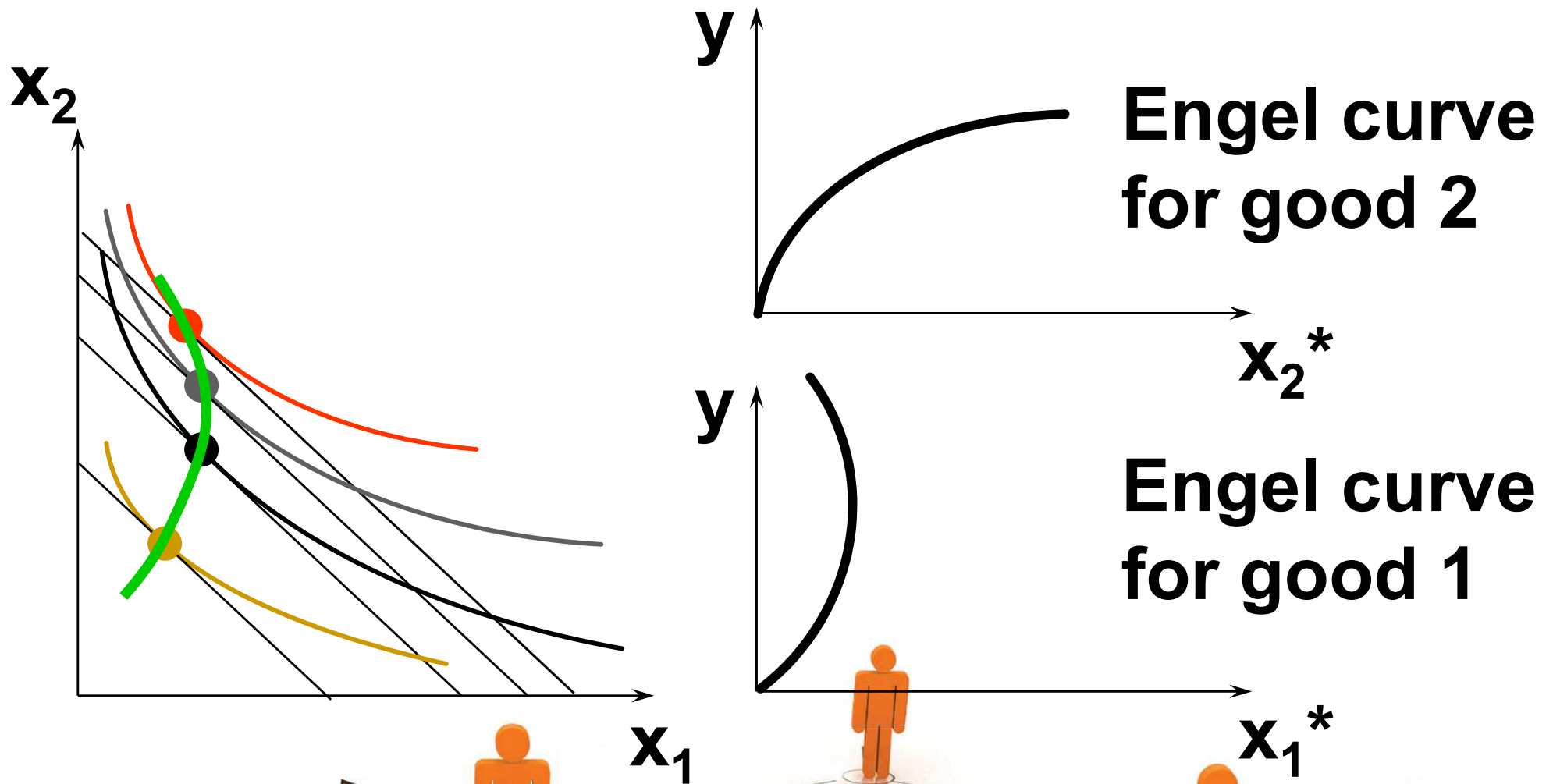
Income Changes; Good 2 Is Normal, Good 1 Becomes Income Inferior



Income Changes; Good 2 Is Normal, Good 1 Becomes Income Inferior

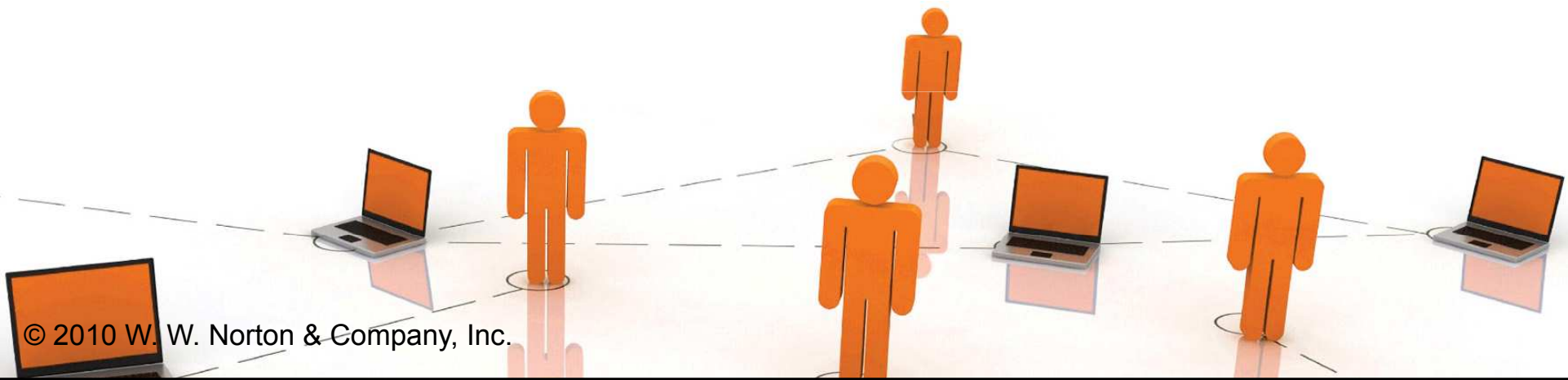


Income Changes; Good 2 Is Normal, Good 1 Becomes Income Inferior



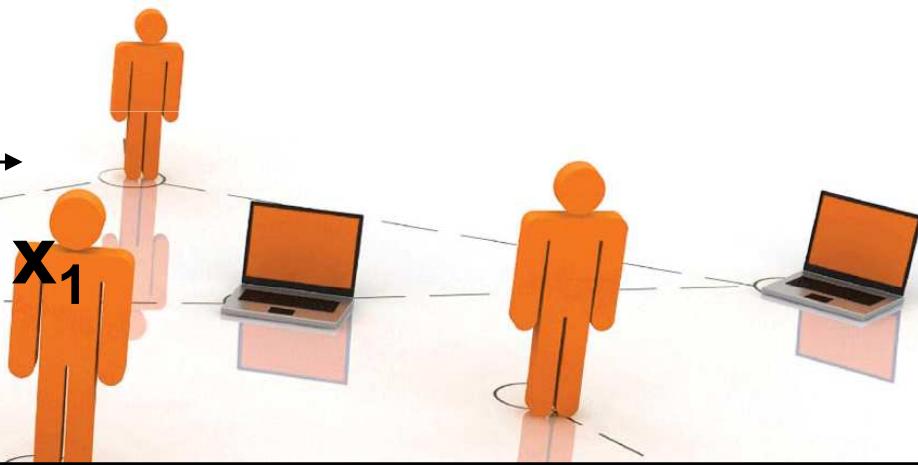
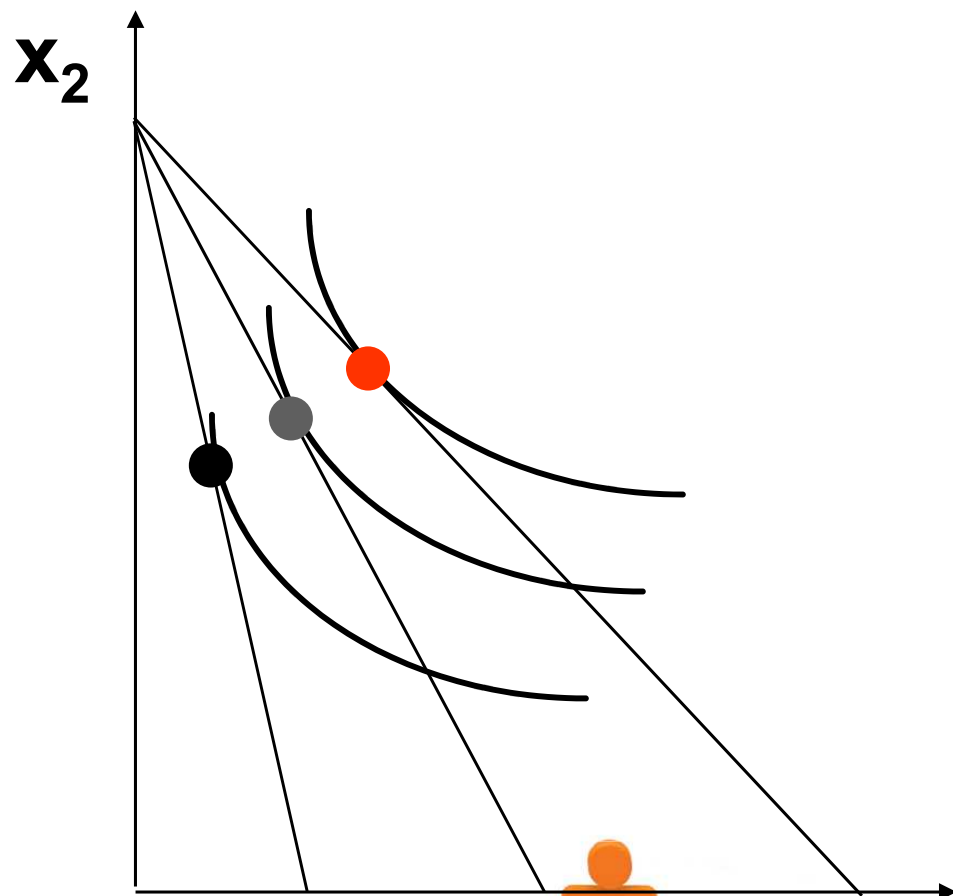
Ordinary Goods

- ◆ **A good is called ordinary if the quantity demanded of it always increases as its own price decreases.**



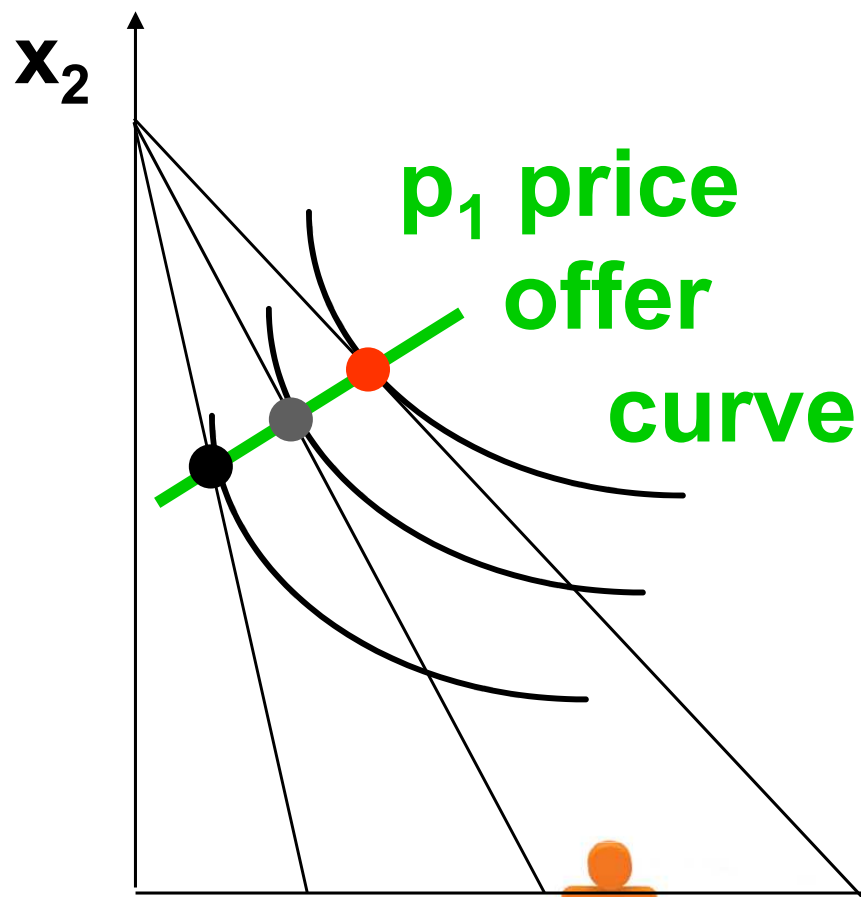
Ordinary Goods

Fixed p_2 and y .



Ordinary Goods

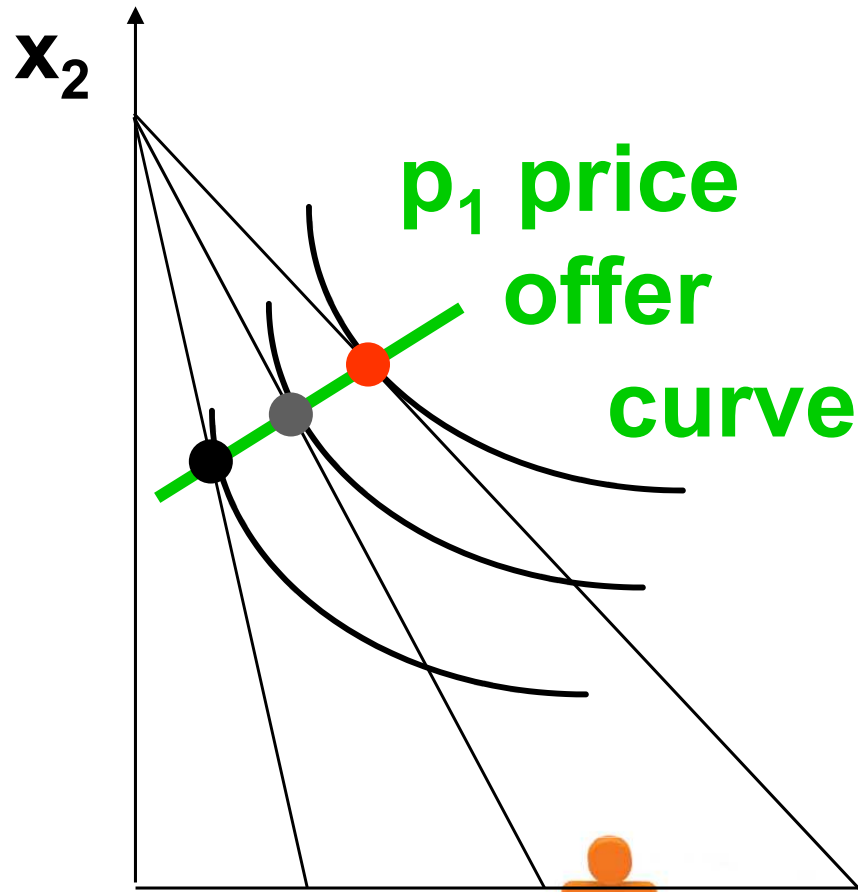
Fixed p_2 and y .



x_1

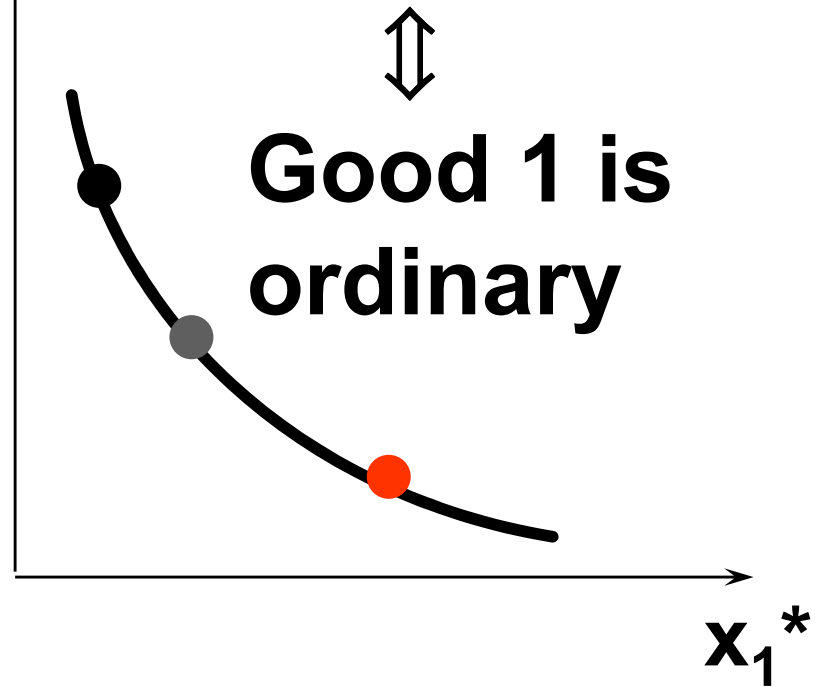
Ordinary Goods

Fixed p_2 and y .



Downward-sloping demand curve

p_1



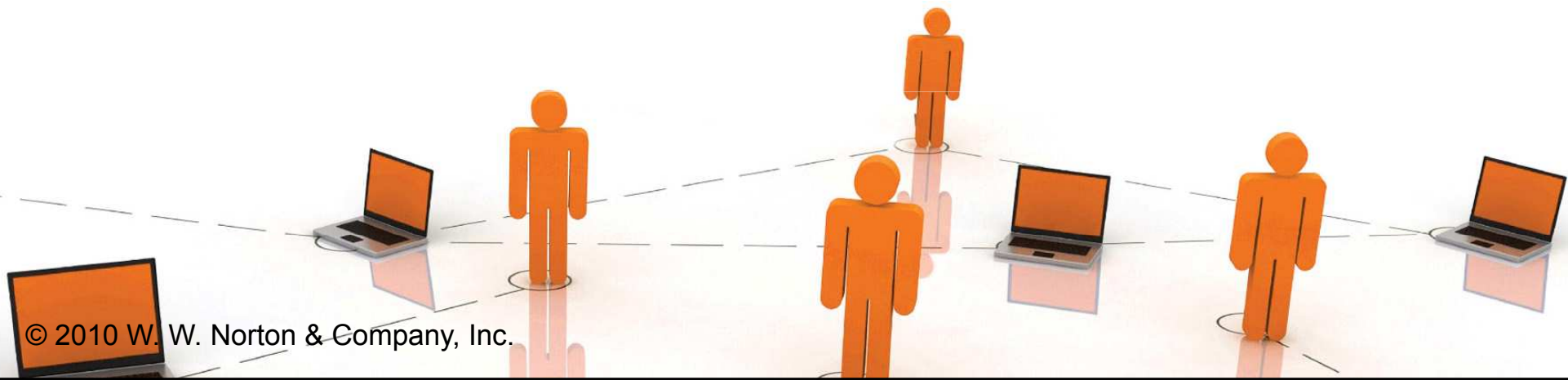
Good 1 is ordinary

x_1



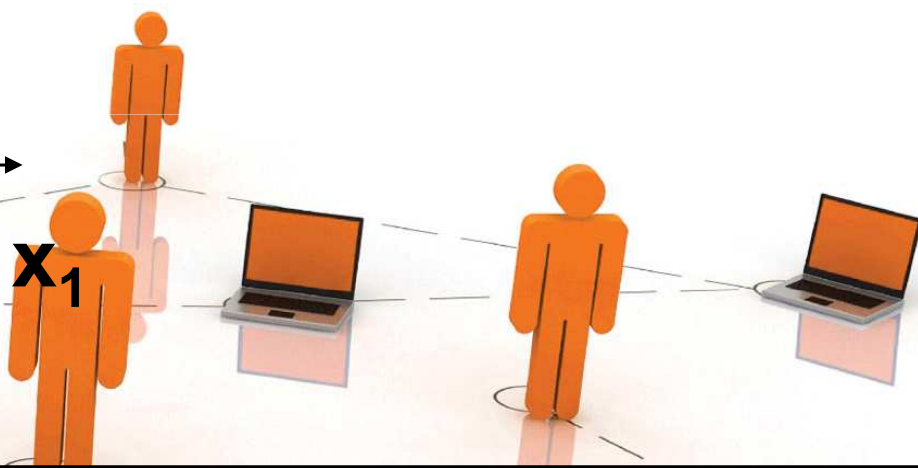
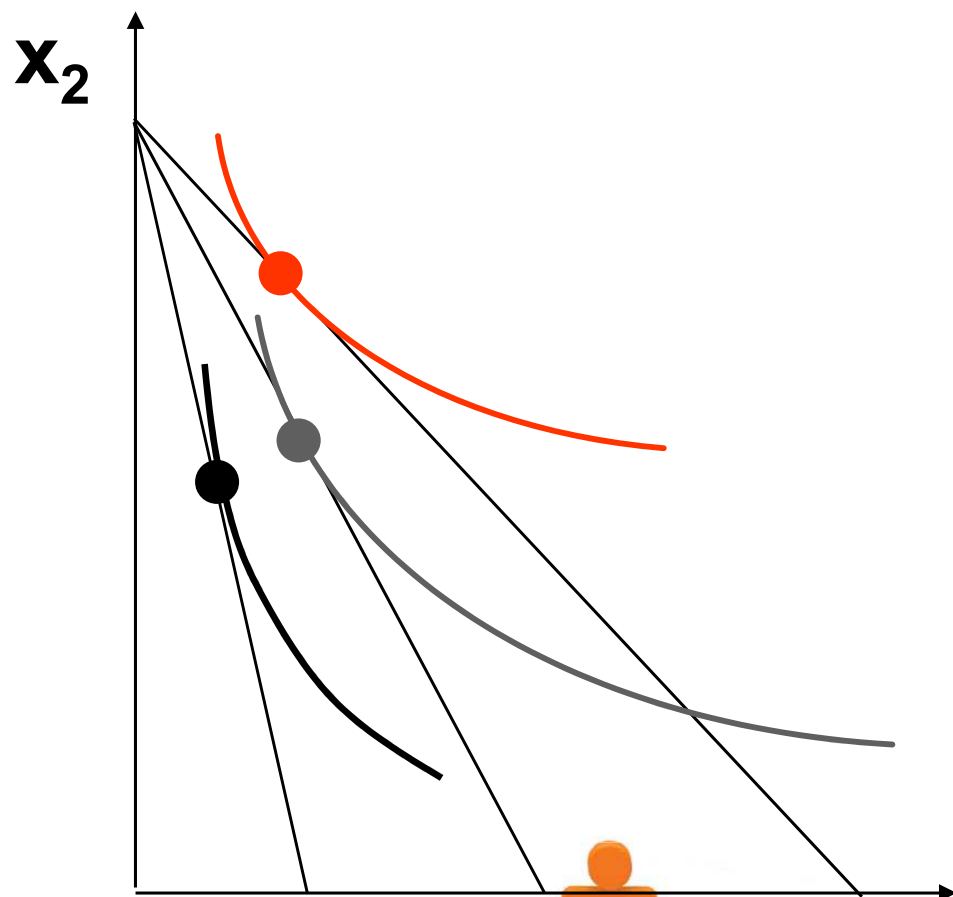
Giffen Goods

- ◆ **If, for some values of its own price, the quantity demanded of a good rises as its own-price increases then the good is called Giffen.**



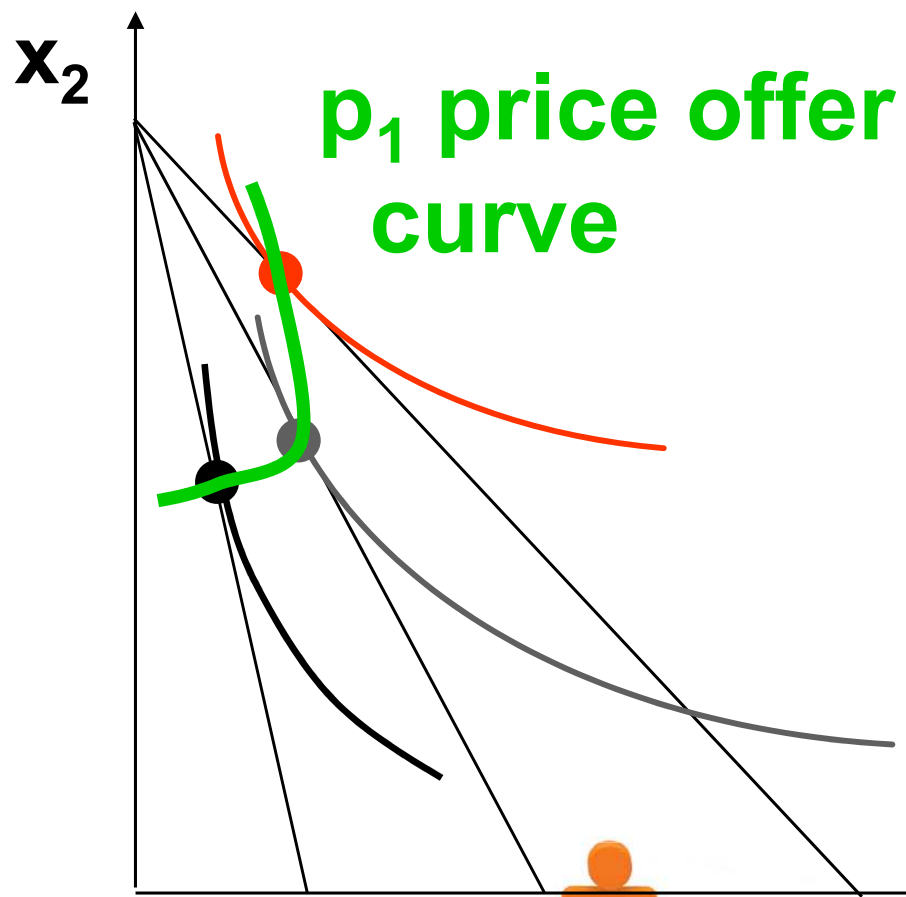
Ordinary Goods

Fixed p_2 and y .



Ordinary Goods

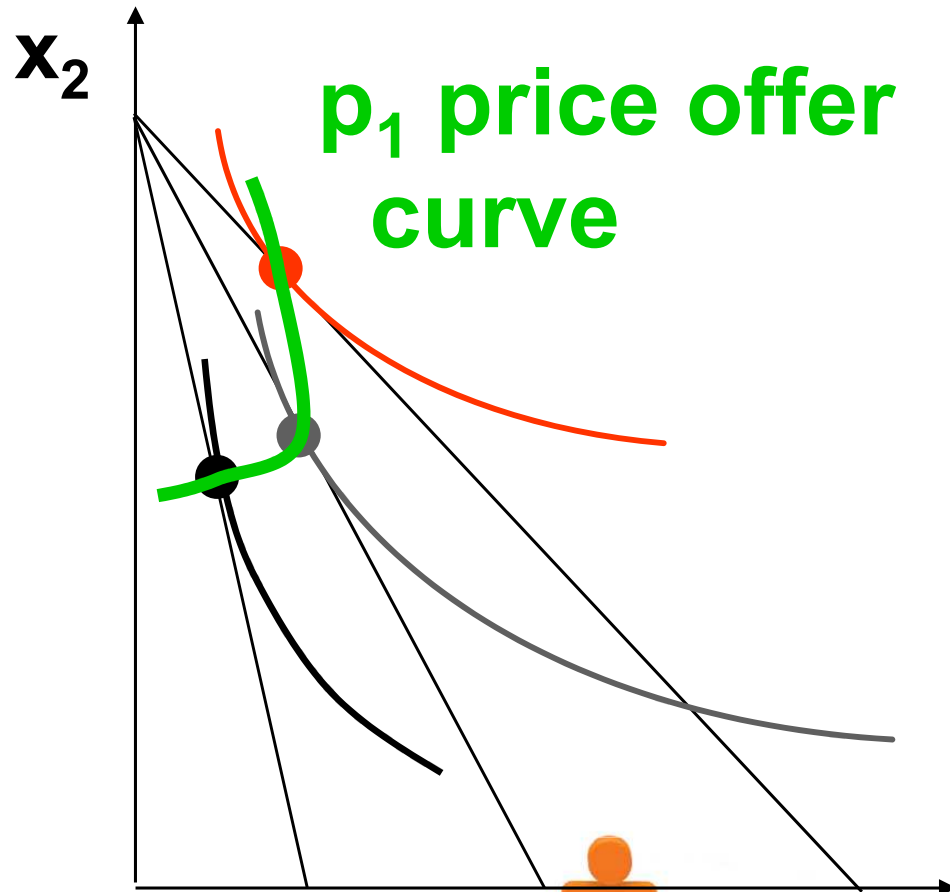
Fixed p_2 and y .



x_1

Ordinary Goods

Fixed p_2 and y .



Demand curve has

a positively sloped part

p_1



Good 1 is Giffen

x_1^*

x_1

Cross-Price Effects

- ◆ **If an increase in p_2**
 - **increases demand for commodity 1 then commodity 1 is a gross substitute for commodity 2.**
 - **reduces demand for commodity 1 then commodity 1 is a gross complement for commodity 2.**



Cross-Price Effects

A perfect-complements example:

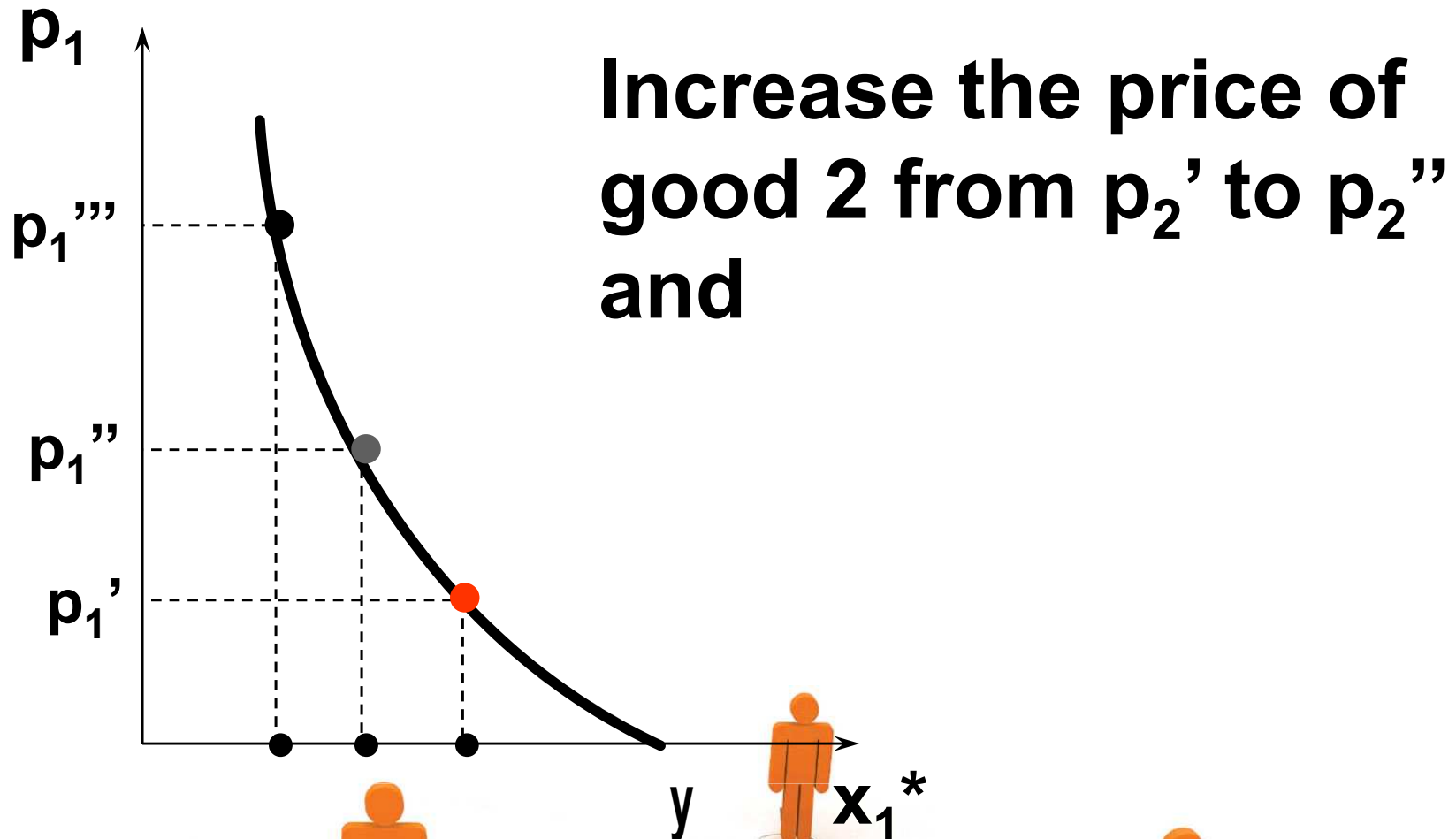
$$x_1^* = \frac{y}{p_1 + p_2}$$

so

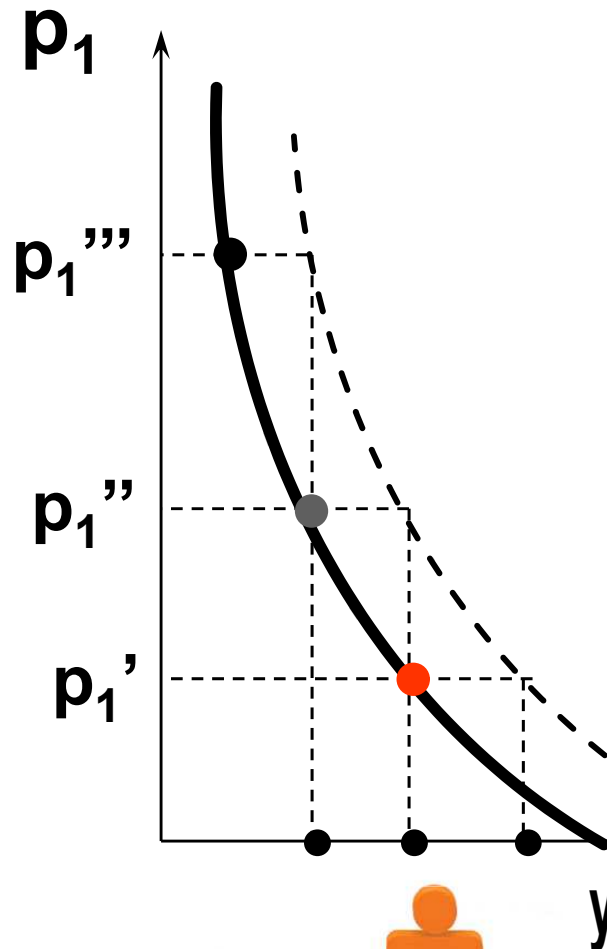
$$\frac{\partial x_1^*}{\partial p_2} = - \frac{y}{(p_1 + p_2)^2} < 0.$$

Therefore commodity 2 is a gross complement for commodity 1.

Cross-Price Effects



Cross-Price Effects



Increase the price of good 2 from p_2' to p_2'' and the demand curve for good 1 shifts inwards -- good 2 is a complement for good 1.



p_2''

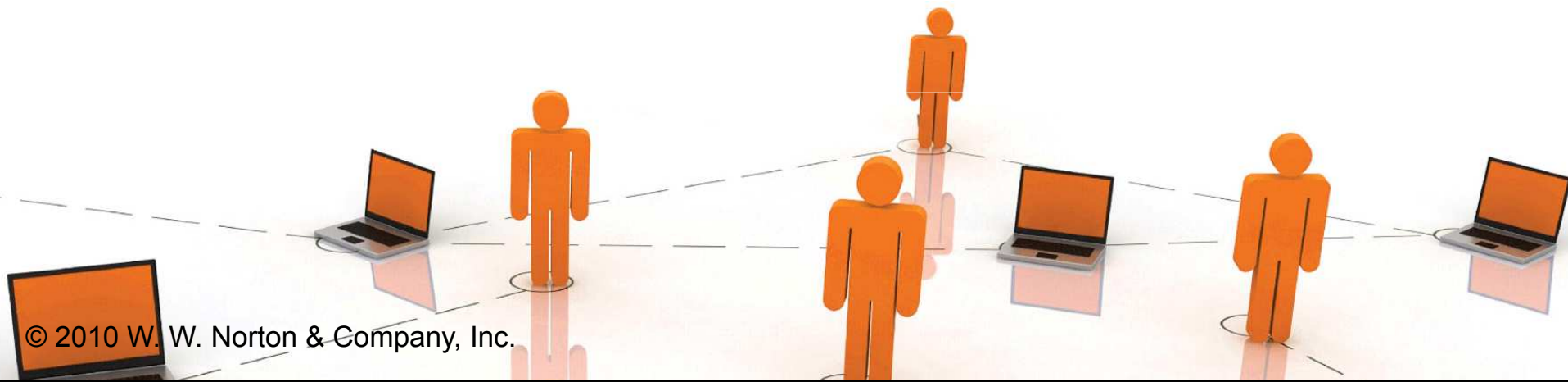


Cross-Price Effects

A Cobb- Douglas example:

$$x_2^* = \frac{b y}{(a + b) p_2}$$

so



Cross-Price Effects

A Cobb- Douglas example:

$$x_2^* = \frac{b y}{(a + b) p_2}$$

so

$$\frac{\partial x_2^*}{\partial p_1} = 0.$$

Therefore commodity 1 is neither a gross complement nor a gross substitute for commodity 2.