8TH EDITION

INTERMEDIATE MICROECONOMICS

HAL R. VARIAN

14 Consumer’s Surplus

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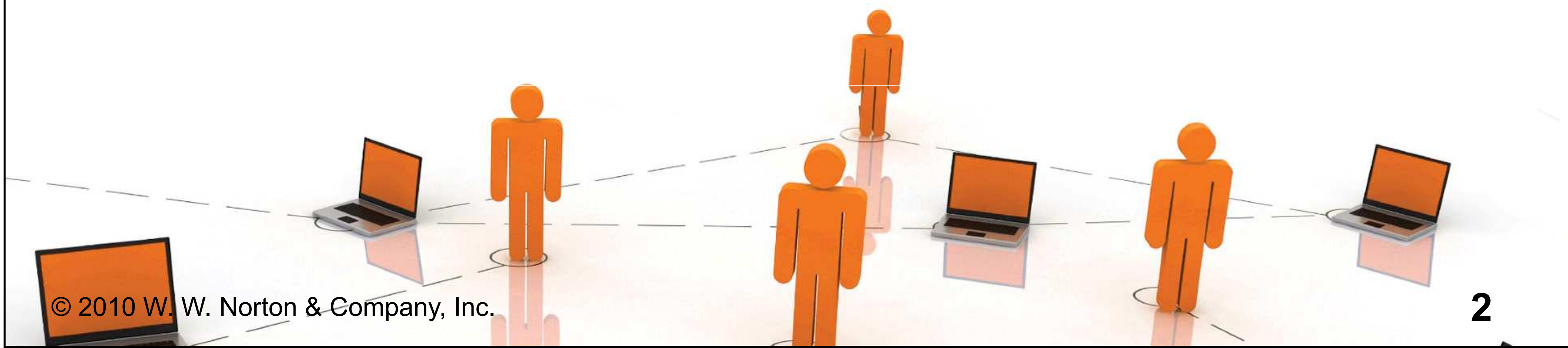
Monetary Measures of Gains-to-Trade

- You can buy as much gasoline as you wish at $1 per gallon once you enter the gasoline market.
- Q: What is the most you would pay to enter the market?



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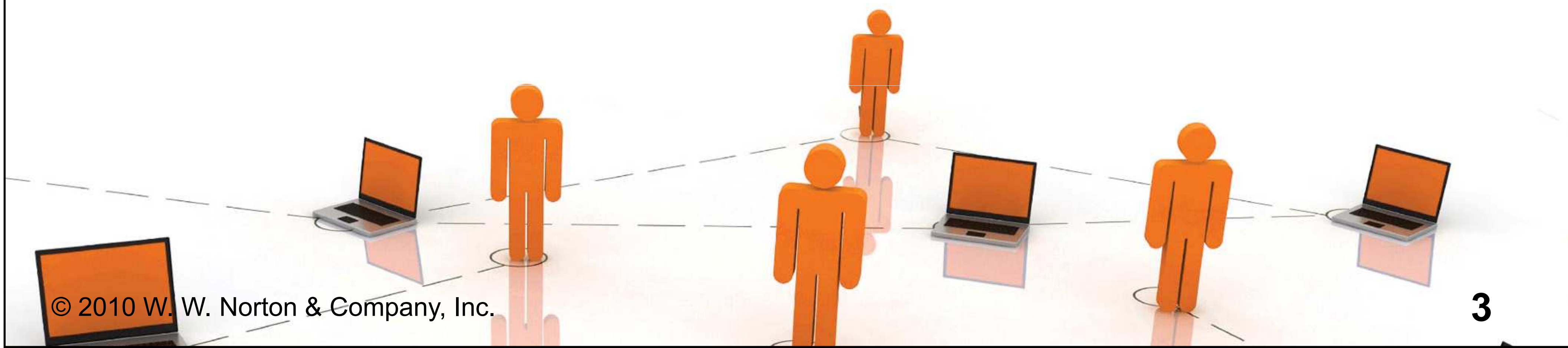
Monetary Measures of Gains-to-Trade

- **A: You would pay up to the dollar value of the gains-to-trade you would enjoy once in the market.**
- **How can such gains-to-trade be measured?**



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Monetary Measures of Gains-to-Trade

- Three such measures are:
  - Consumer’s Surplus
  - Equivalent Variation, and
  - Compensating Variation.
- Only in one special circumstance do these three measures coincide.

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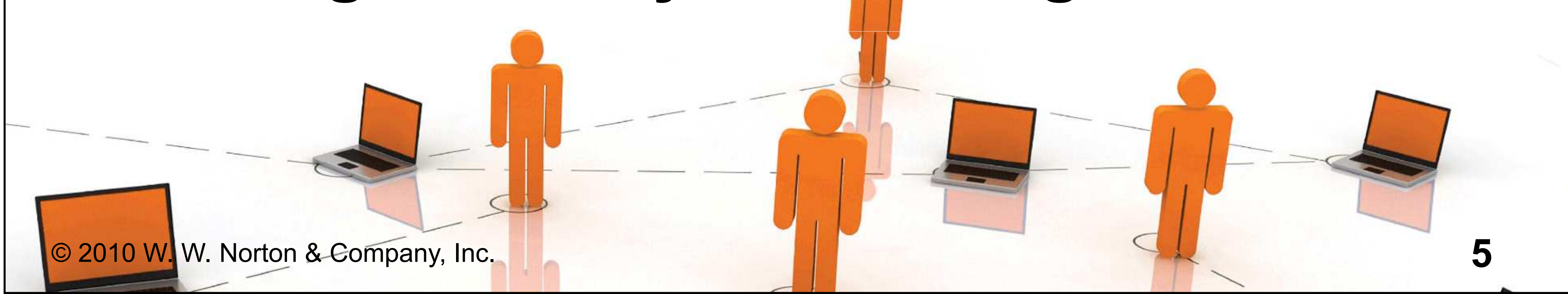
$ Equivalent Utility Gains

- Suppose gasoline can be bought only in lumps of one gallon.
- Use r_1 to denote the most a single consumer would pay for a 1st gallon -- call this her reservation price for the 1st gallon.
- r_1 is the dollar equivalent of the marginal utility of the 1st gallon.



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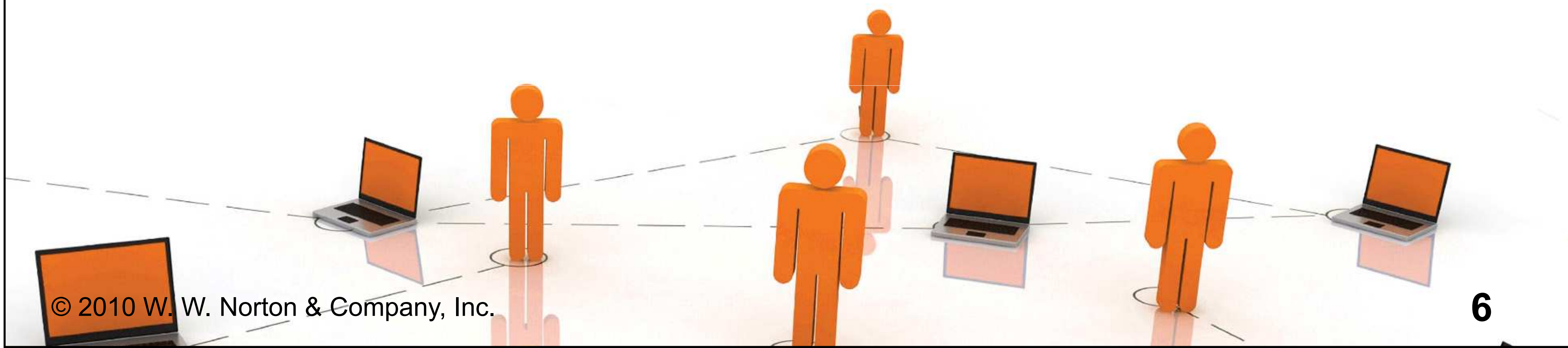
$ Equivalent Utility Gains

- Now that she has one gallon, use r_2 to denote the most she would pay for a 2nd gallon -- this is her reservation price for the 2nd gallon.
- r_2 is the dollar equivalent of the marginal utility of the 2nd gallon.



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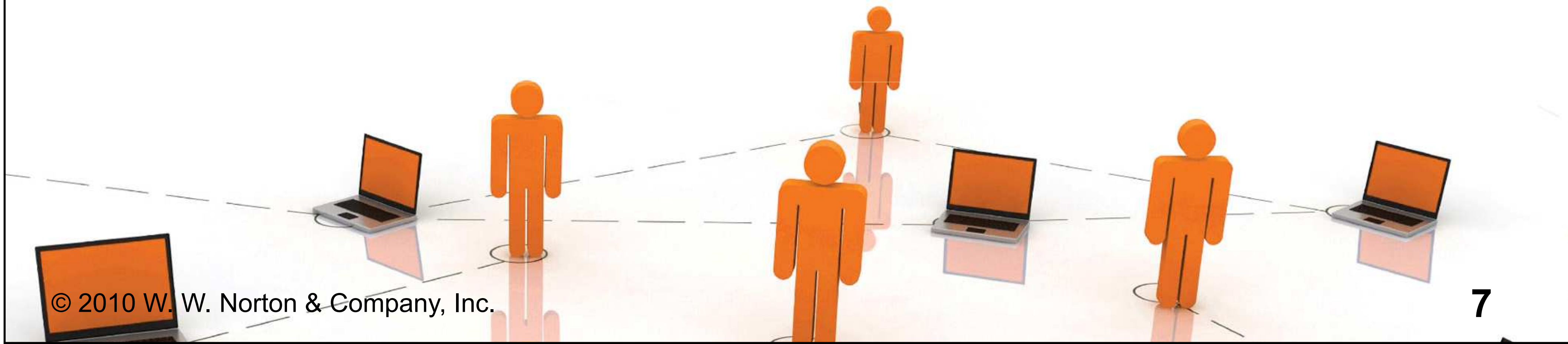
$ Equivalent Utility Gains

- Generally, if she already has n-1 gallons of gasoline then r_n denotes the most she will pay for an nth gallon.
- r_n is the dollar equivalent of the marginal utility of the nth gallon.



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$ Equivalent Utility Gains

- $r_1 + \ldots + r_n$ will therefore be the dollar equivalent of the total change to utility from acquiring n gallons of gasoline at a price of $0.
- So $r_1 + \ldots + r_n - p_G n$ will be the dollar equivalent of the total change to utility from acquiring n gallons of gasoline at a price of $$p_G$ each.

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$ Equivalent Utility Gains

- A plot of r_1, r_2, … , r_n, … against n is a reservation-price curve. This is not quite the same as the consumer's demand curve for gasoline.

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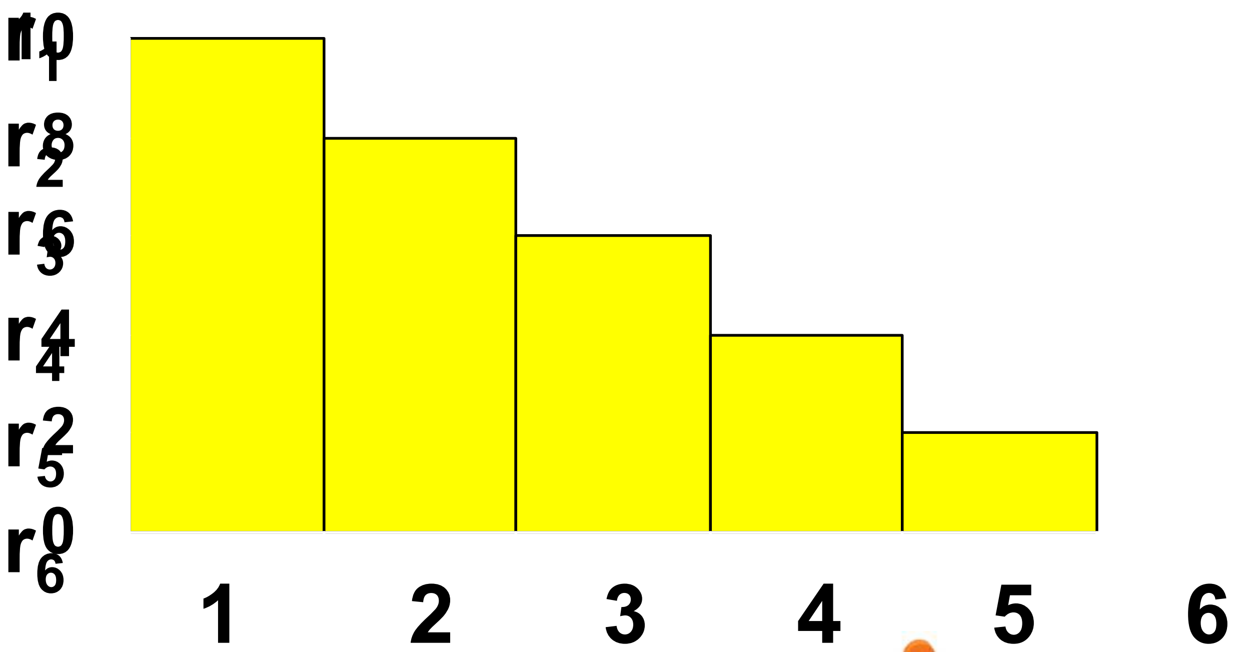
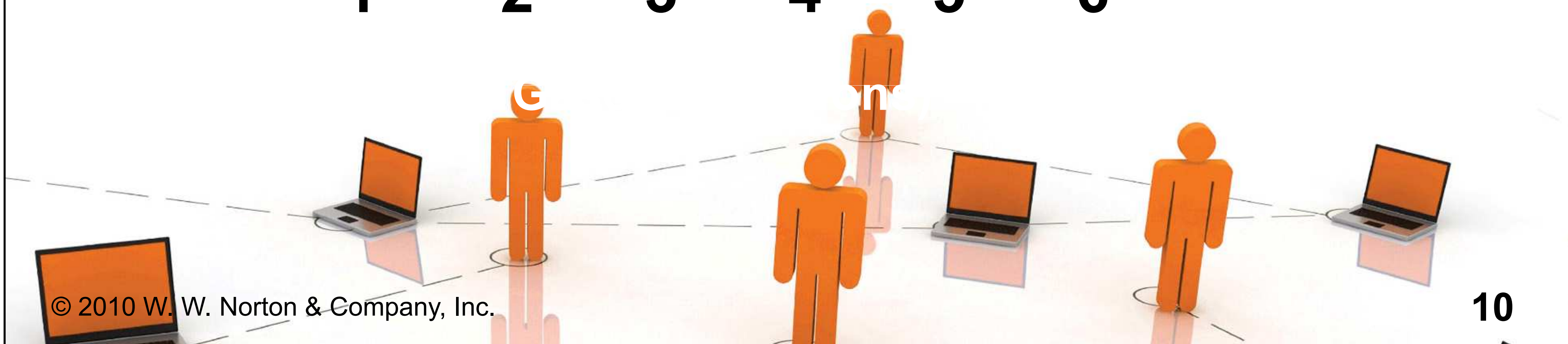
$ Equivalent Utility Gains





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$ Equivalent Utility Gains

- **What is the monetary value of our consumer's gain-to-trading in the gasoline market at a price of $\$p_G$?**

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$ Equivalent Utility Gains

- The dollar equivalent net utility gain for the 1st gallon is $\$(r_1 - p_G)$
- and is $\$(r_2 - p_G)$ for the 2nd gallon,
- and so on, so the dollar value of the gain-to-trade is

  $$\$(r_1 - p_G) + \$(r_2 - p_G) + \ldots$$

  for as long as $r_n - p_G > 0$.

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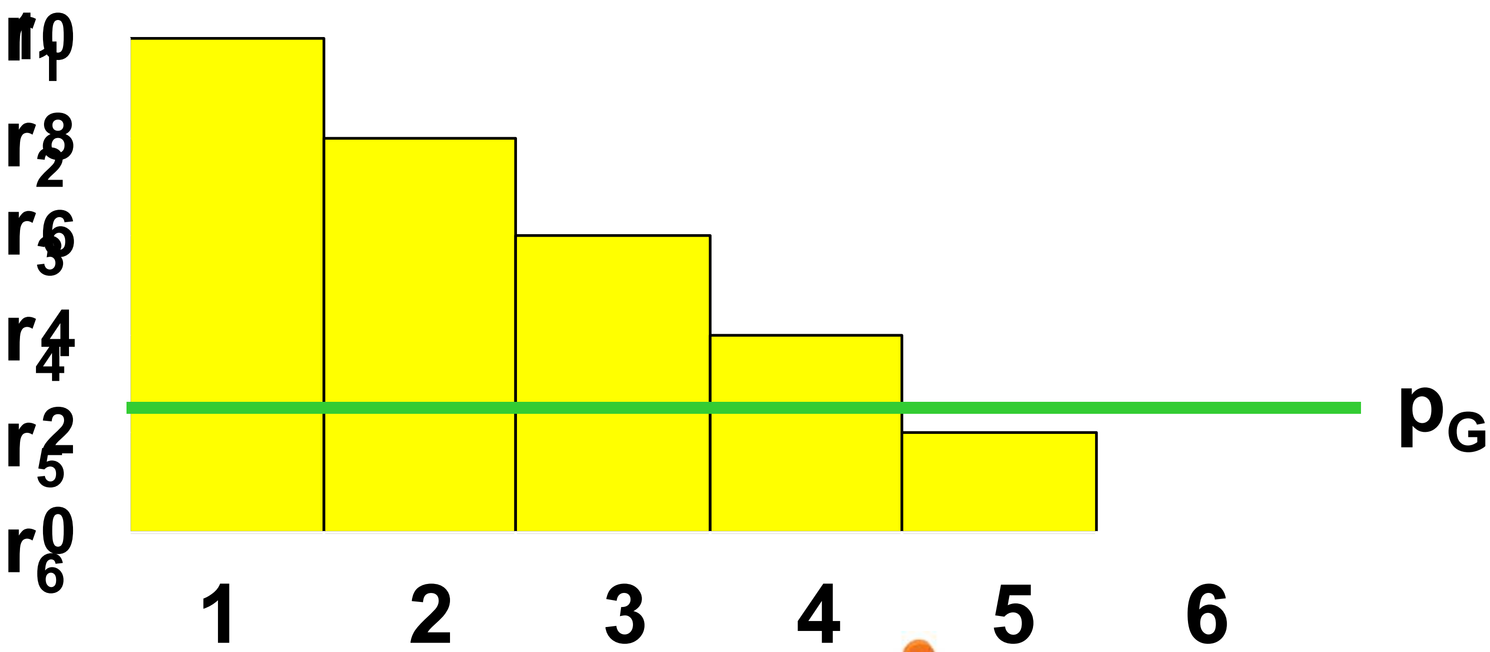
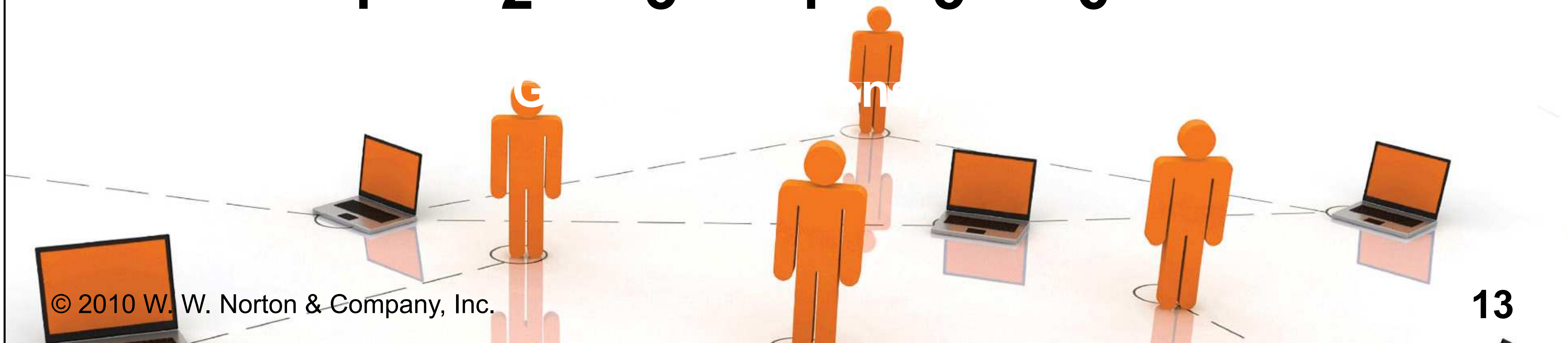
$ Equivalent Utility Gains





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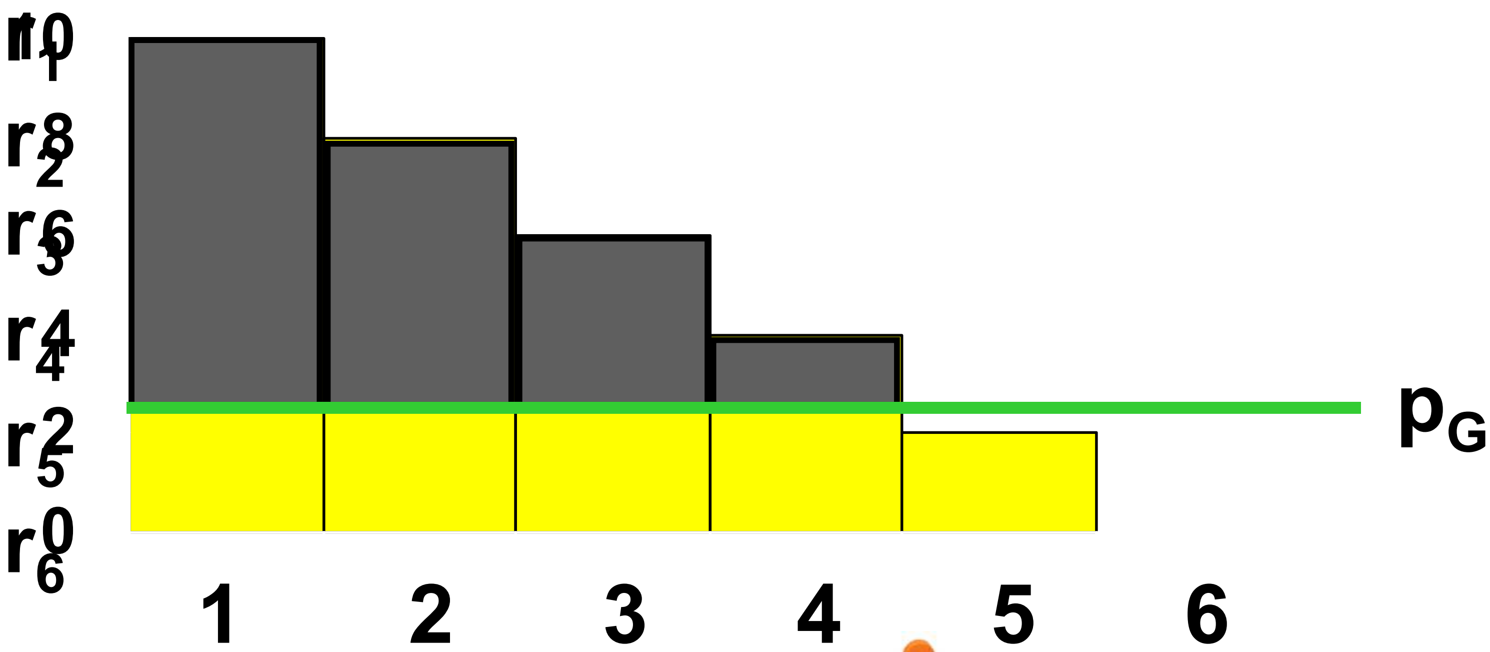
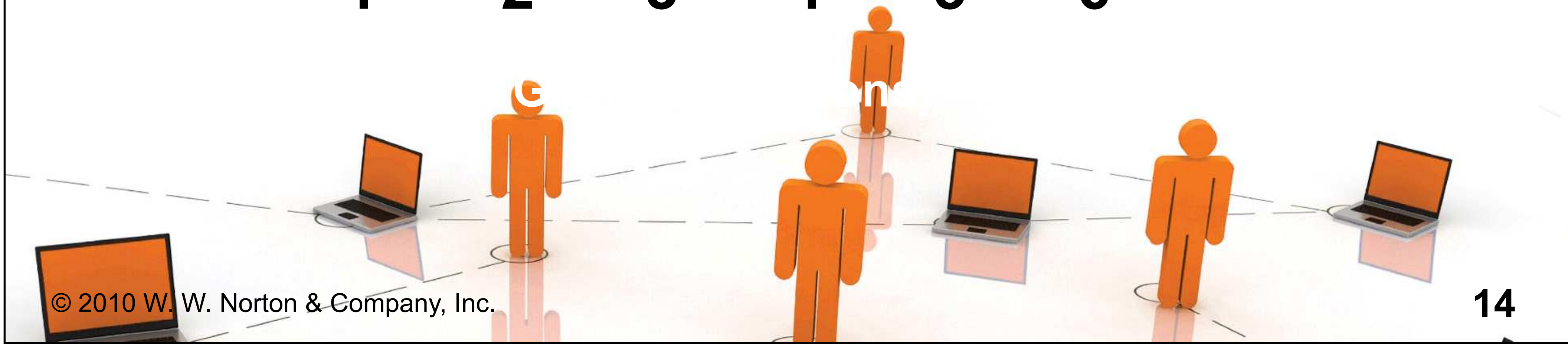
$ Equivalent Utility Gains





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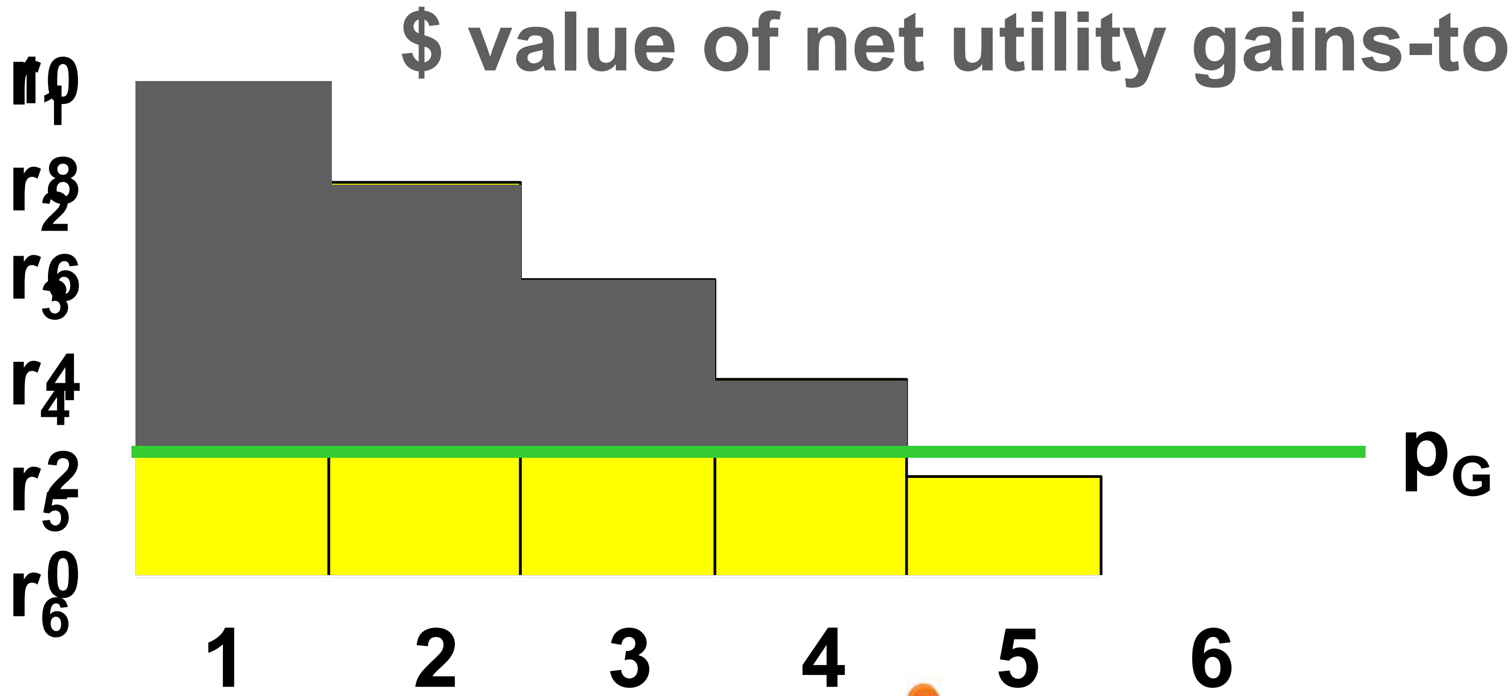
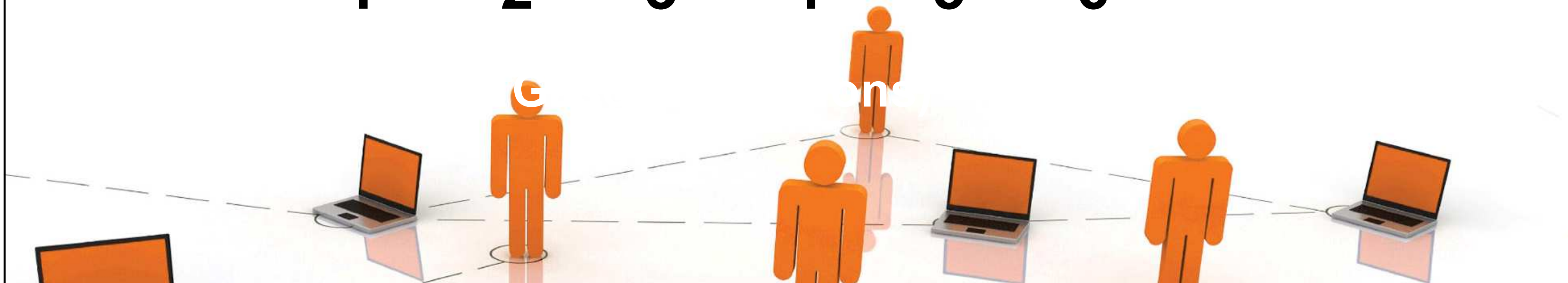
$ Equivalent Utility Gains





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$ Equivalent Utility Gains

- **Now suppose that gasoline is sold in half-gallon units.**
- **r_1, r_2, … , r_n, … denote the consumer's reservation prices for successive half-gallons of gasoline.**
- **Our consumer's new reservation price curve is**

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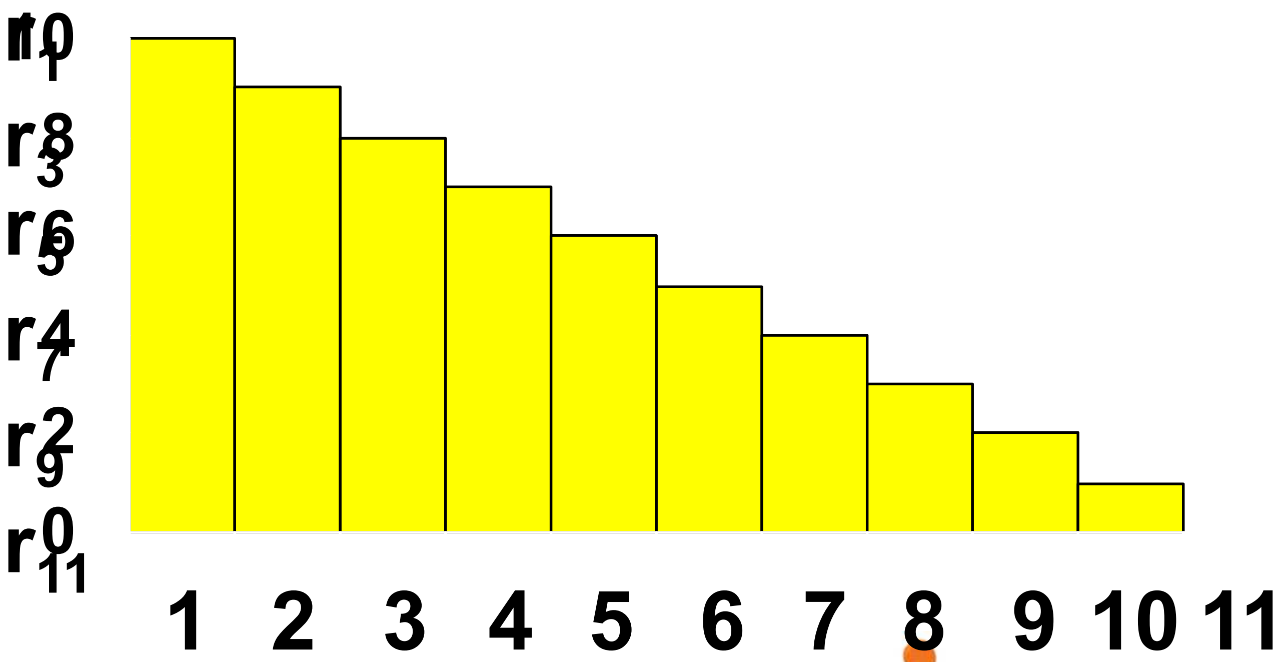
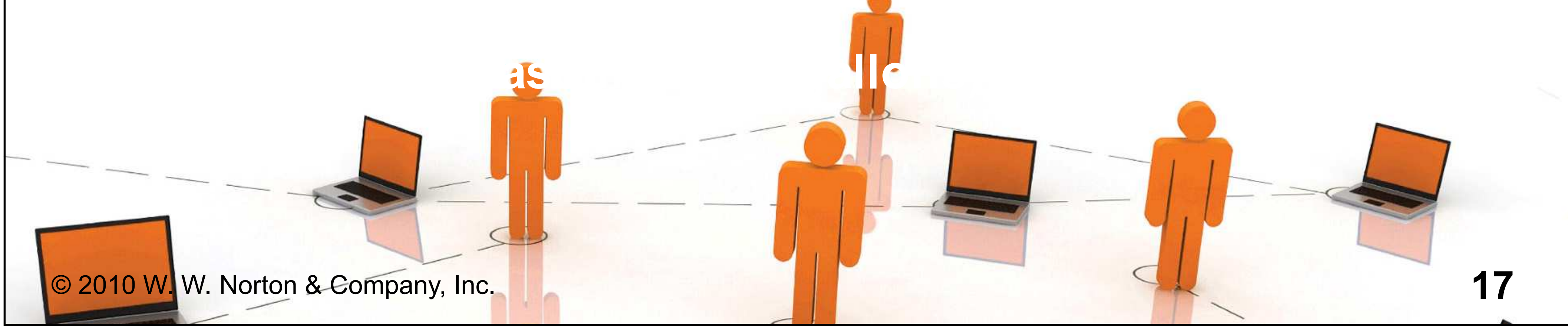
$ Equivalent Utility Gains





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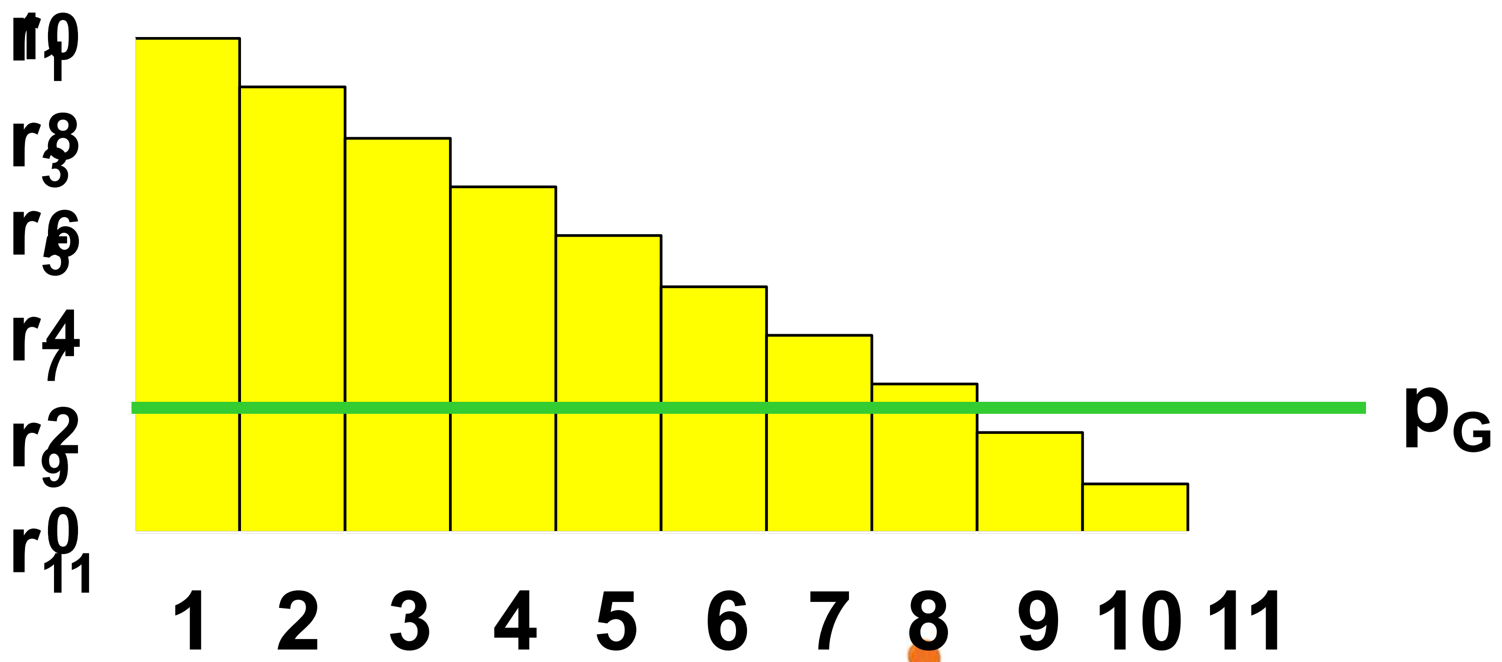
$ Equivalent Utility Gains



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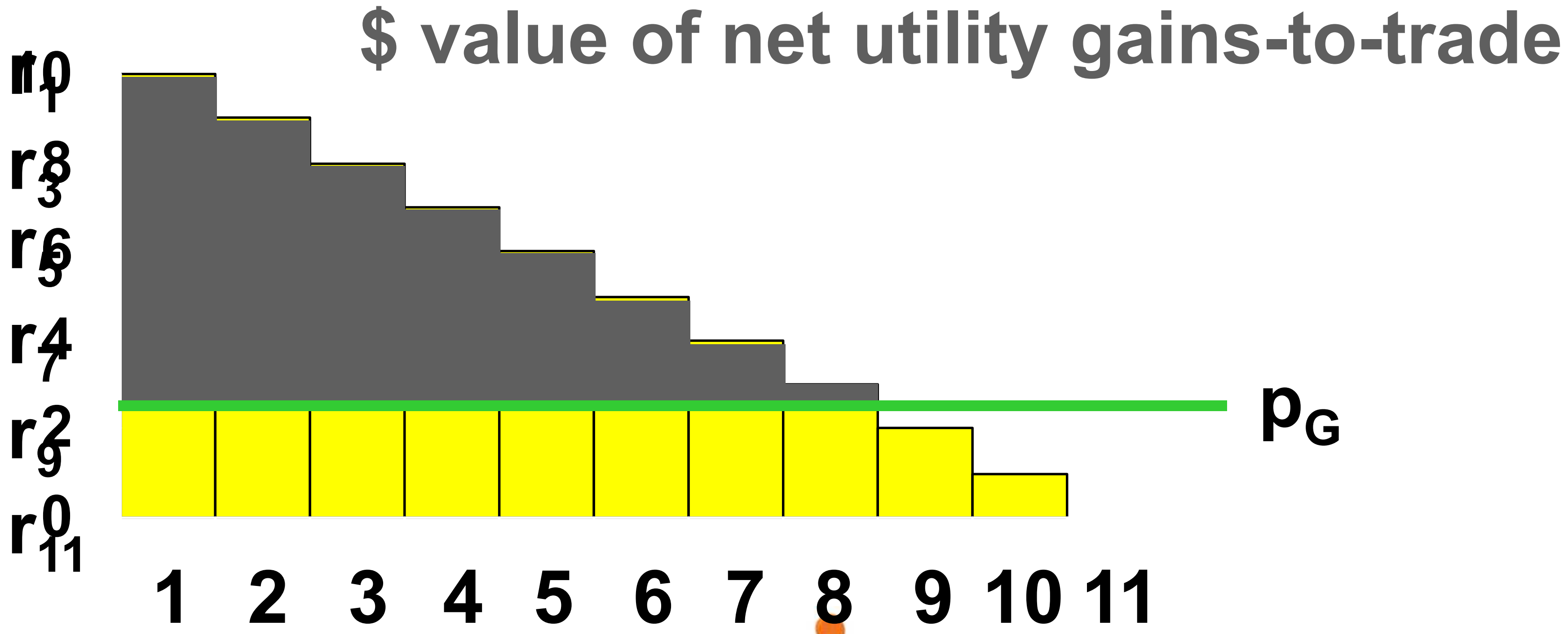
$ Equivalent Utility Gains



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$ Equivalent Utility Gains

- **And if gasoline is available in one-quarter gallon units ...**

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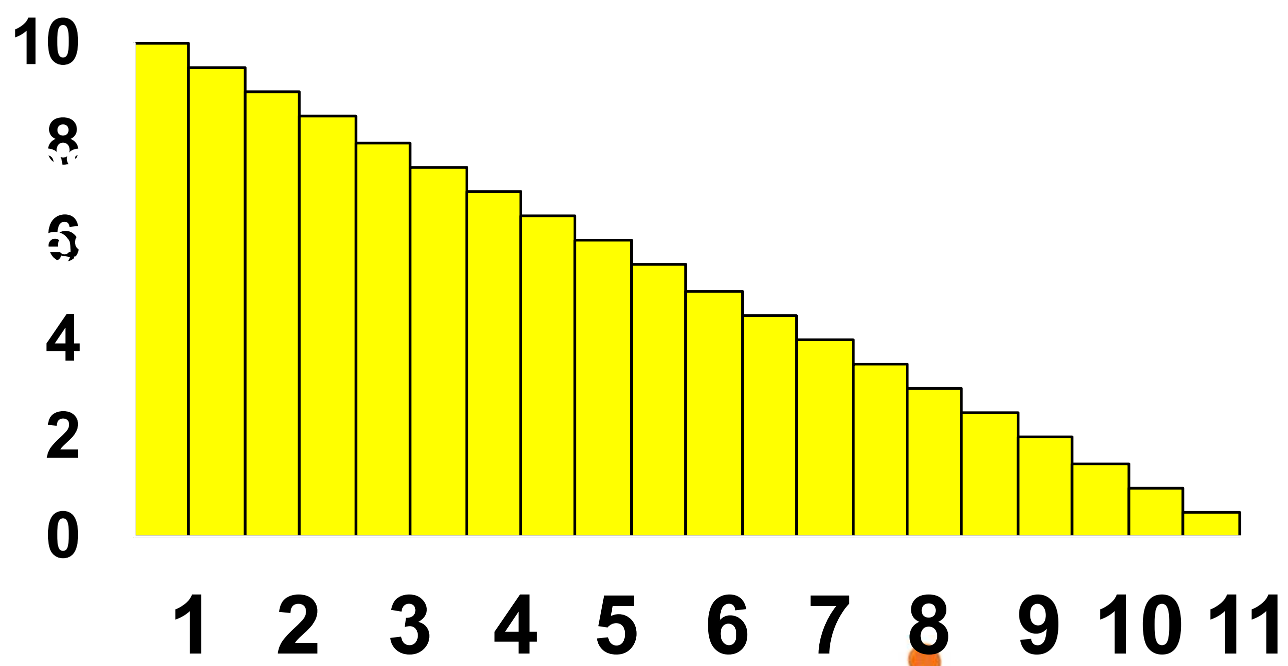
$ Equivalent Utility Gains



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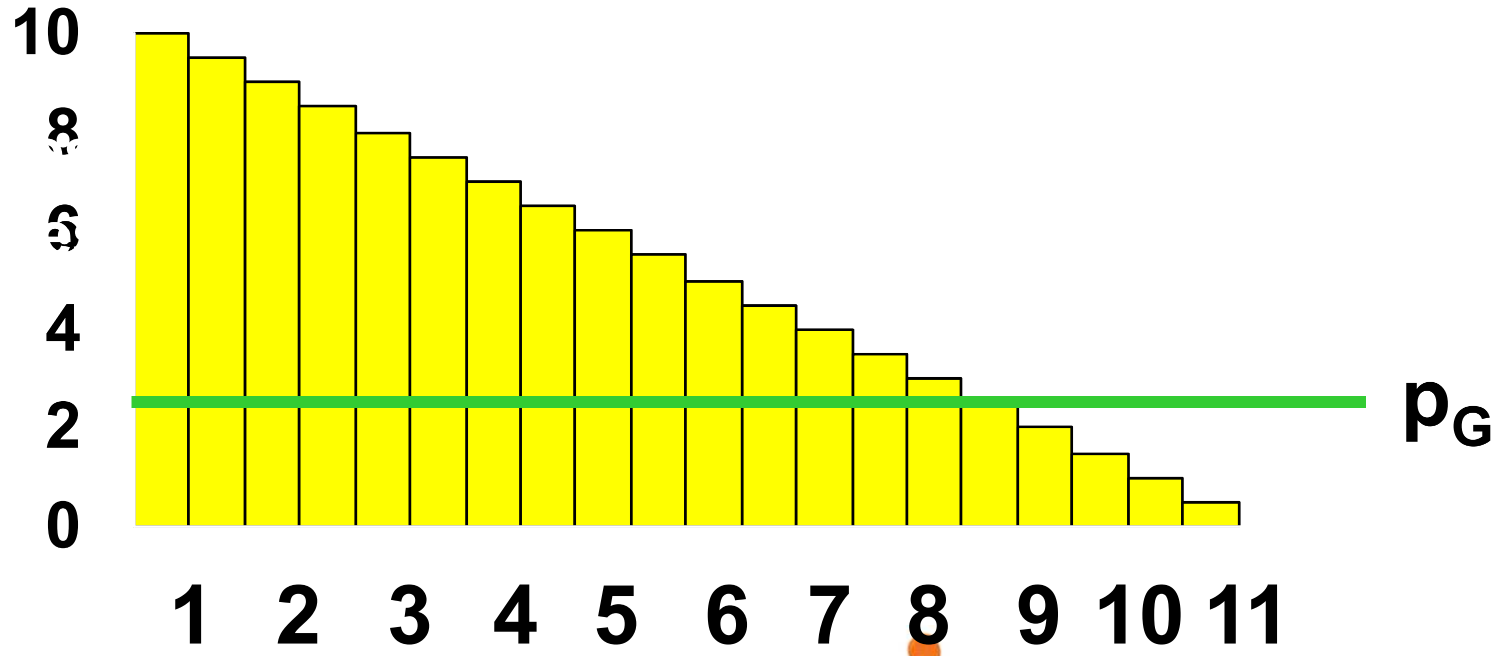
$ Equivalent Utility Gains



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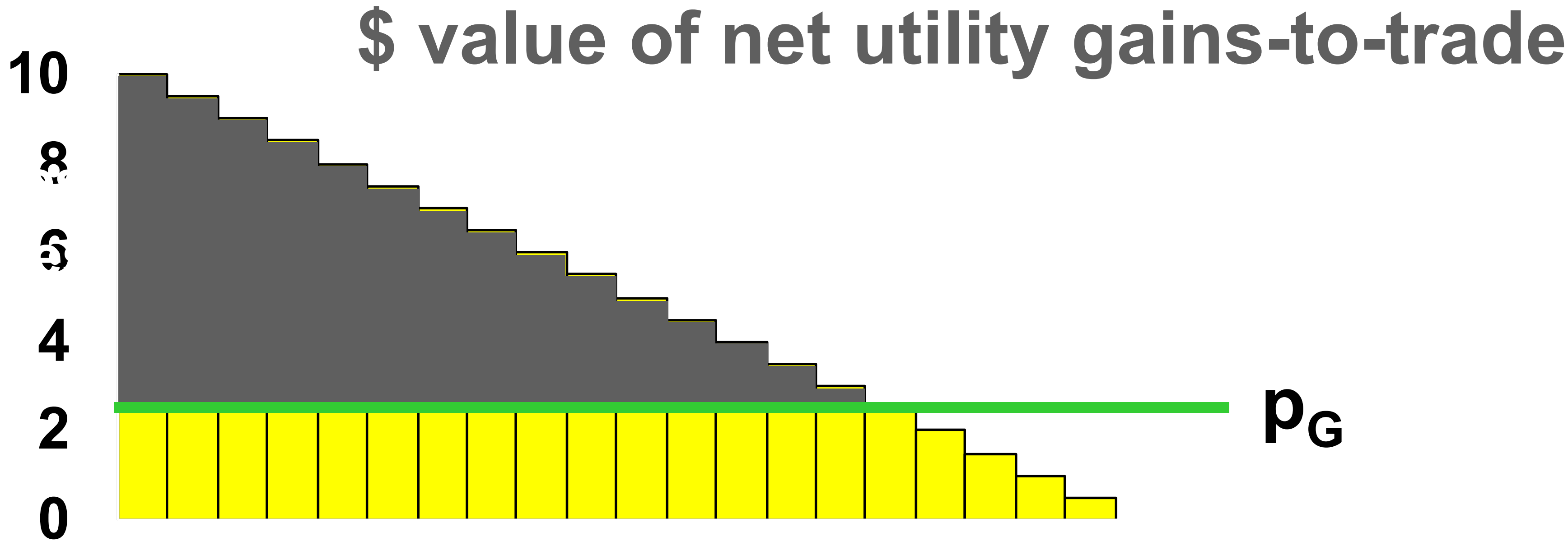
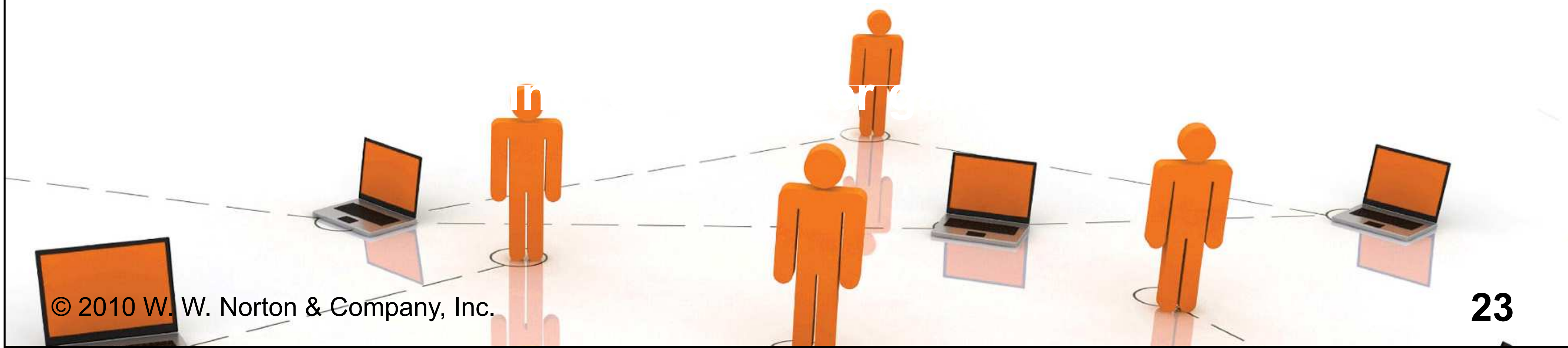
$ Equivalent Utility Gains





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$ Equivalent Utility Gains

- **Finally, if gasoline can be purchased in any quantity then ...**

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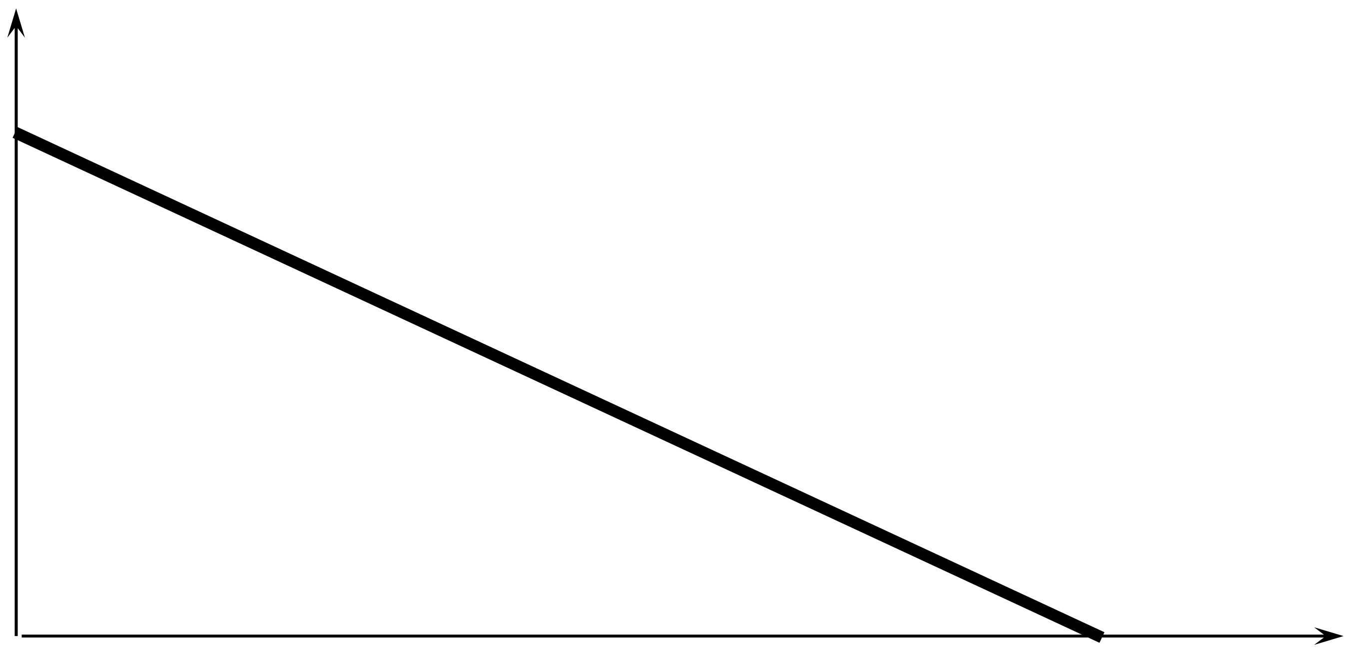
$ Equivalent Utility Gains



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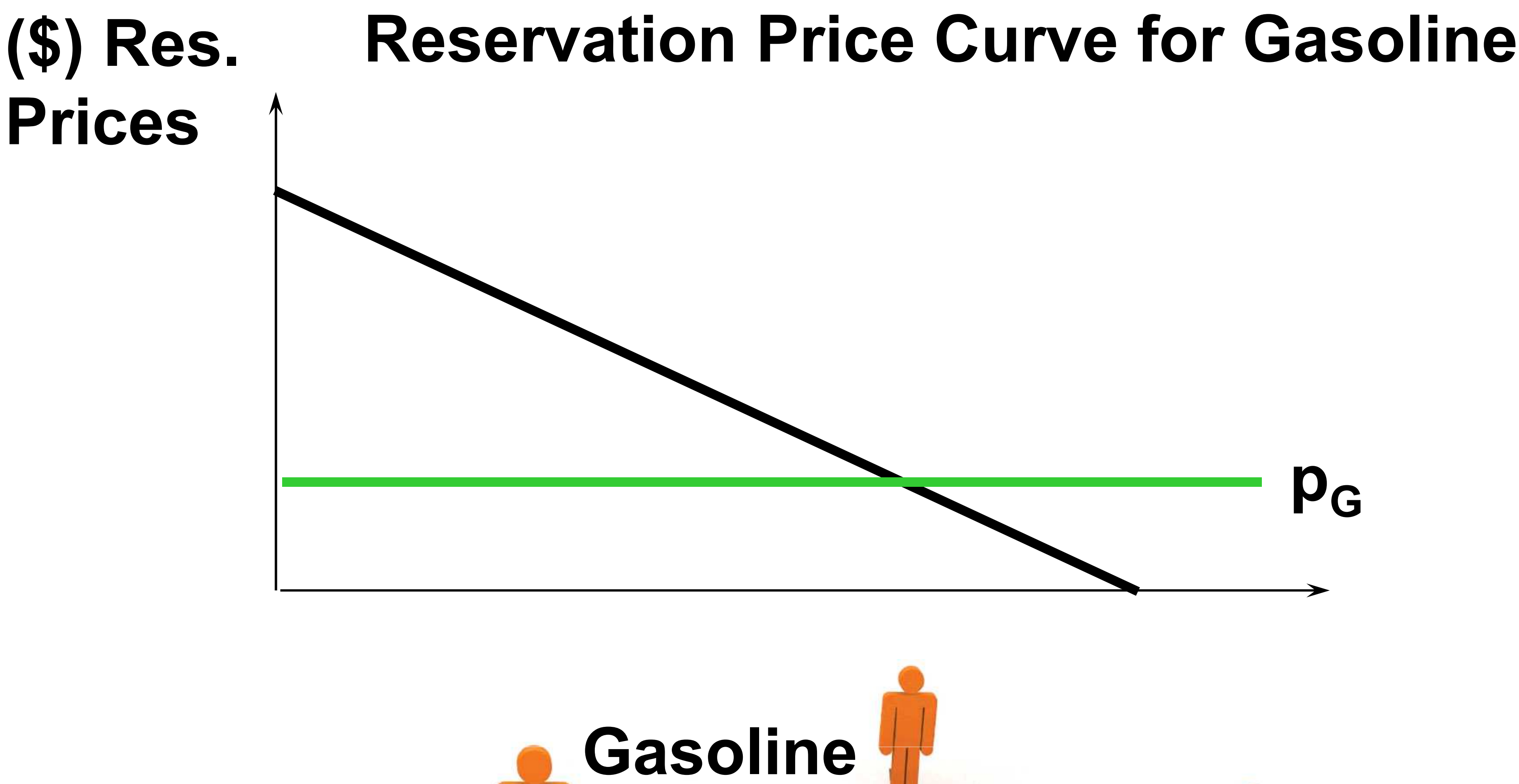
$ Equivalent Utility Gains



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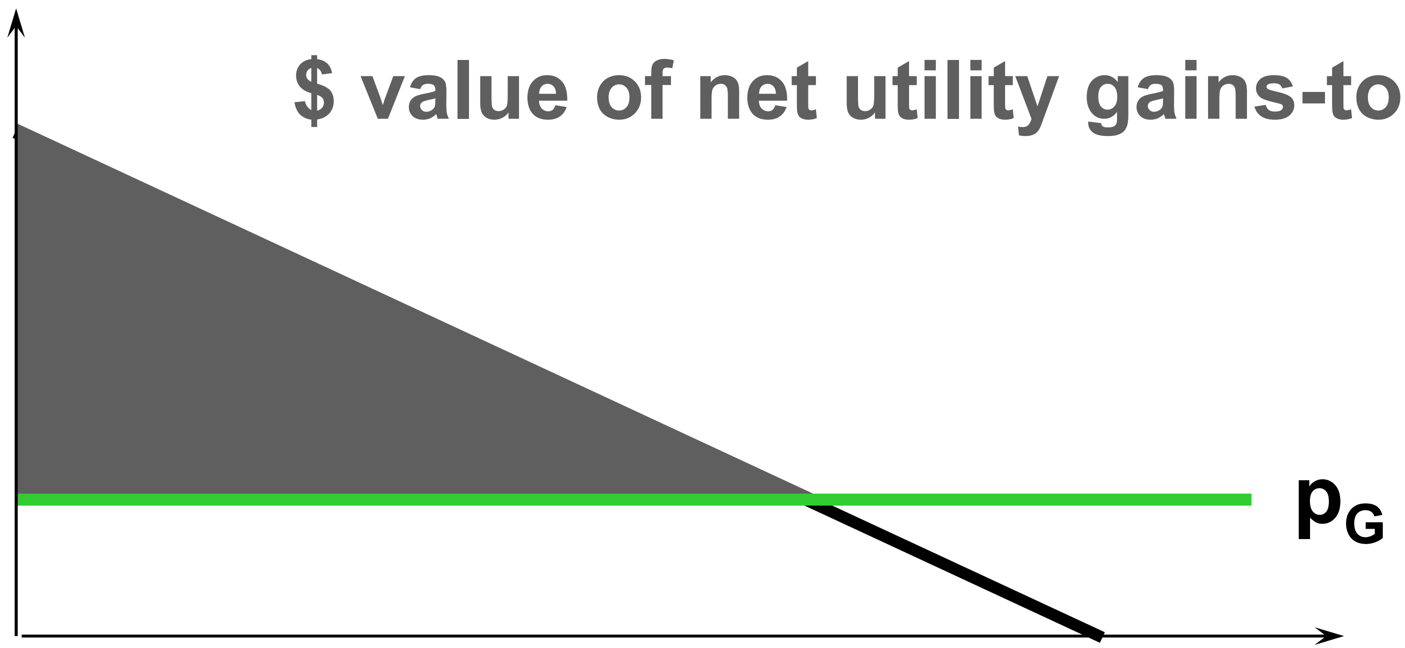
$ Equivalent Utility Gains

($) Res. Prices

Reservation Price Curve for Gasoline



Gasoline

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$ Equivalent Utility Gains

- **Unfortunately, estimating a consumer's reservation-price curve is difficult,**
- **so, as an approximation, the reservation-price curve is replaced with the consumer's ordinary demand curve.**

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Consumer's Surplus

- A consumer's reservation-price curve is not quite the same as her ordinary demand curve. Why not?
- A reservation-price curve describes sequentially the values of successive single units of a commodity.
- An ordinary demand curve describes the most that would be paid for q units of a commodity purchased simultaneously.

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Consumer's Surplus

- **Approximating the net utility gain area under the reservation-price curve by the corresponding area under the ordinary demand curve gives the Consumer's Surplus measure of net utility gain.**

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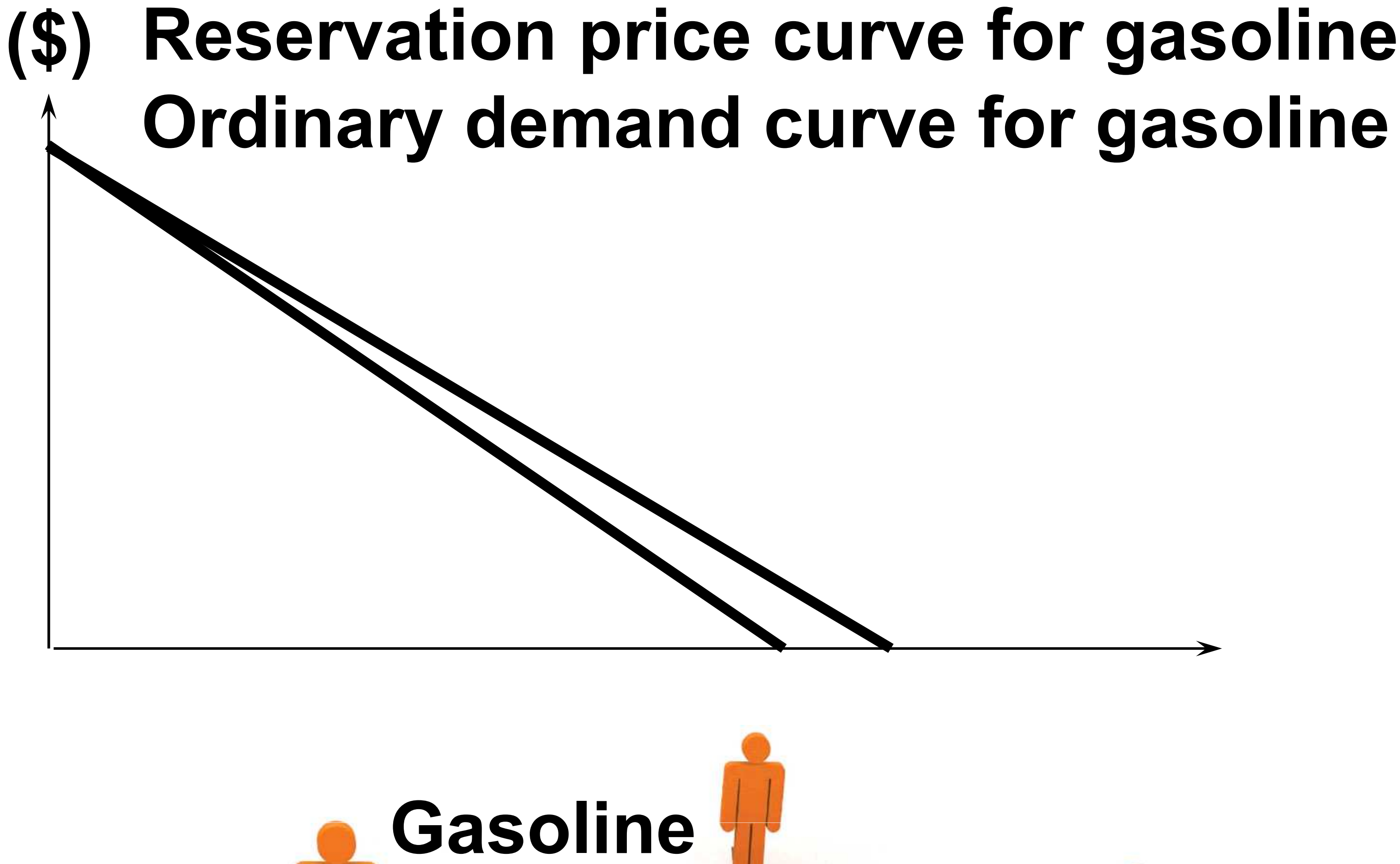
Consumer's Surplus



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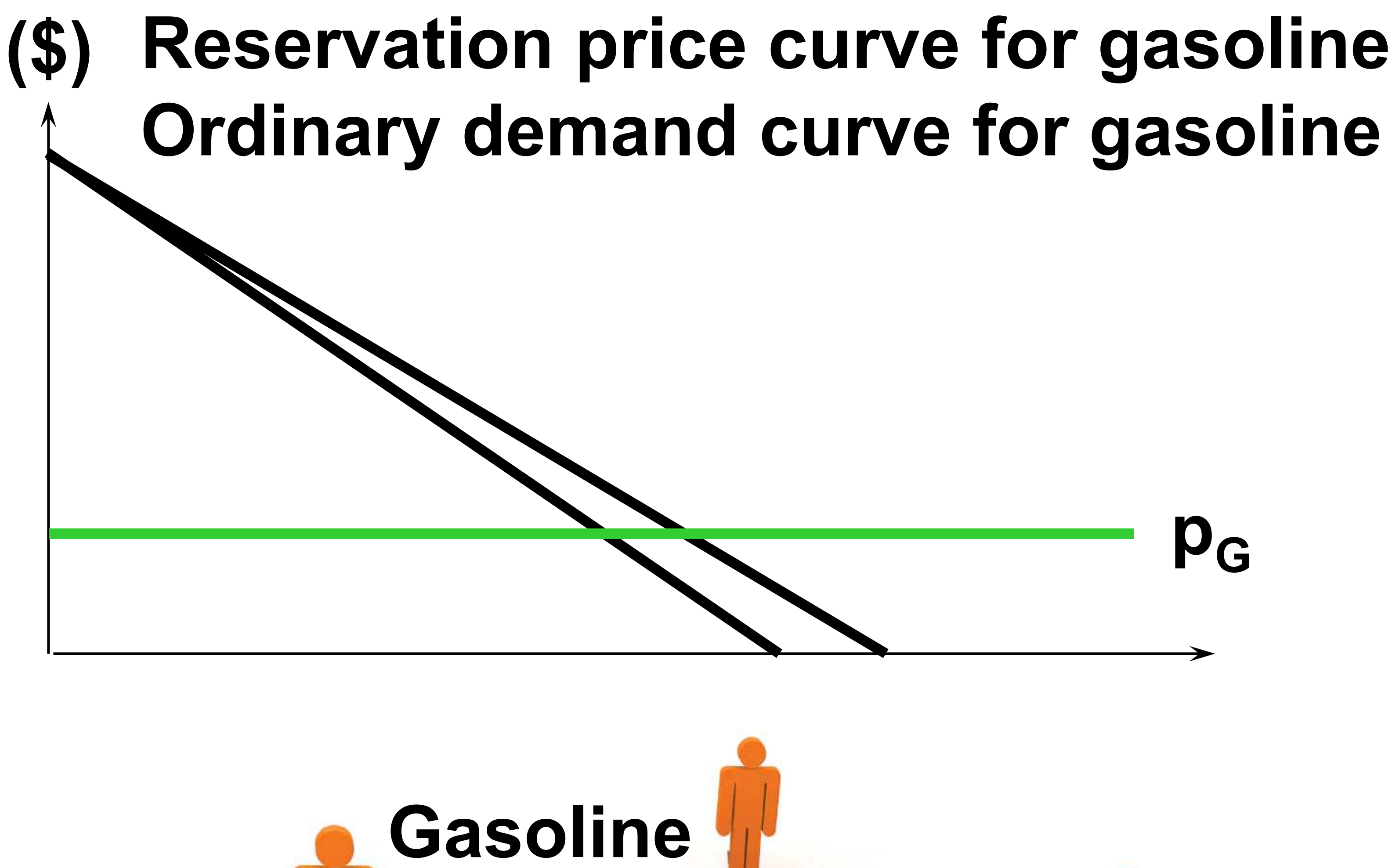
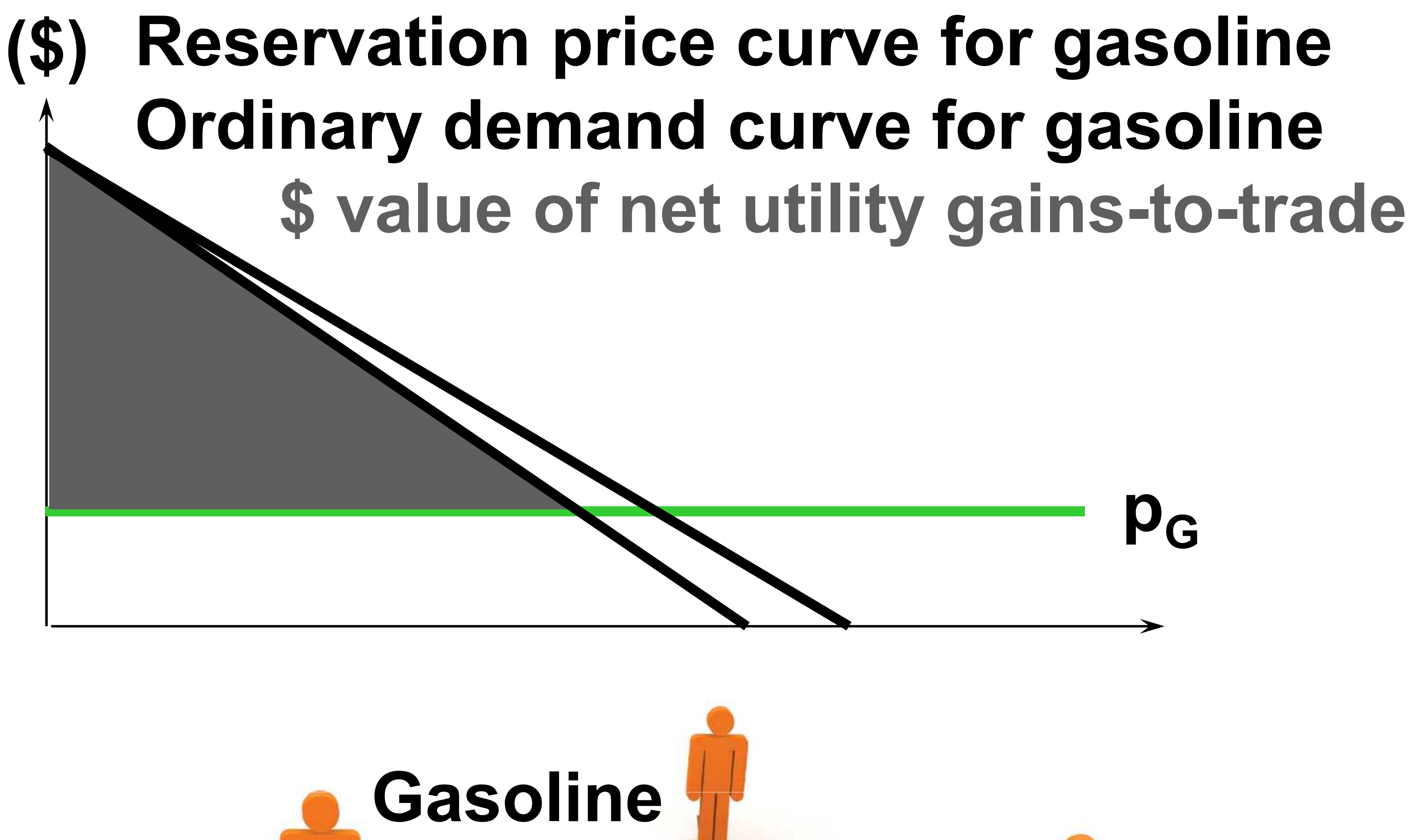
Consumer's Surplus



Consumer's Surplus



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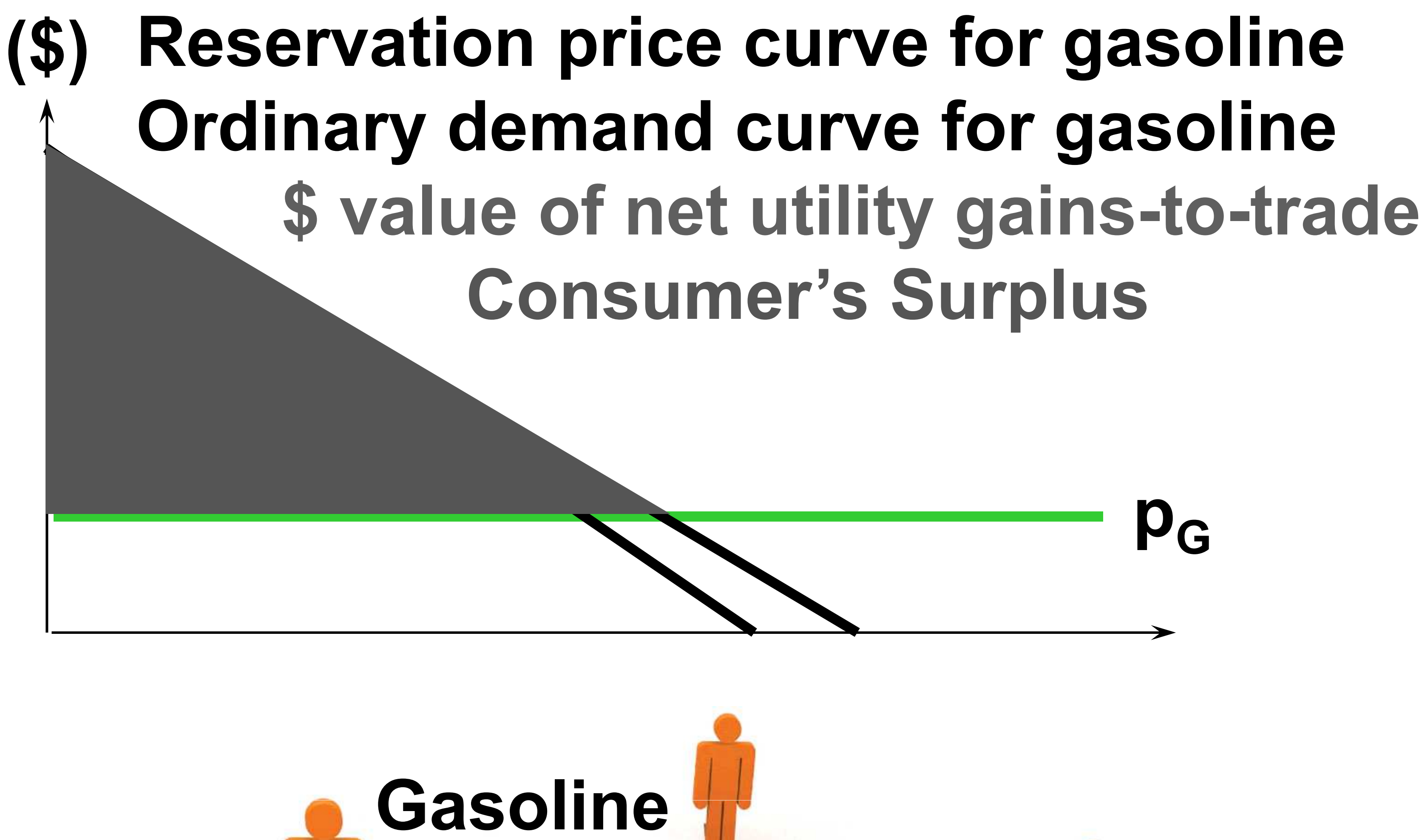
Consumer's Surplus



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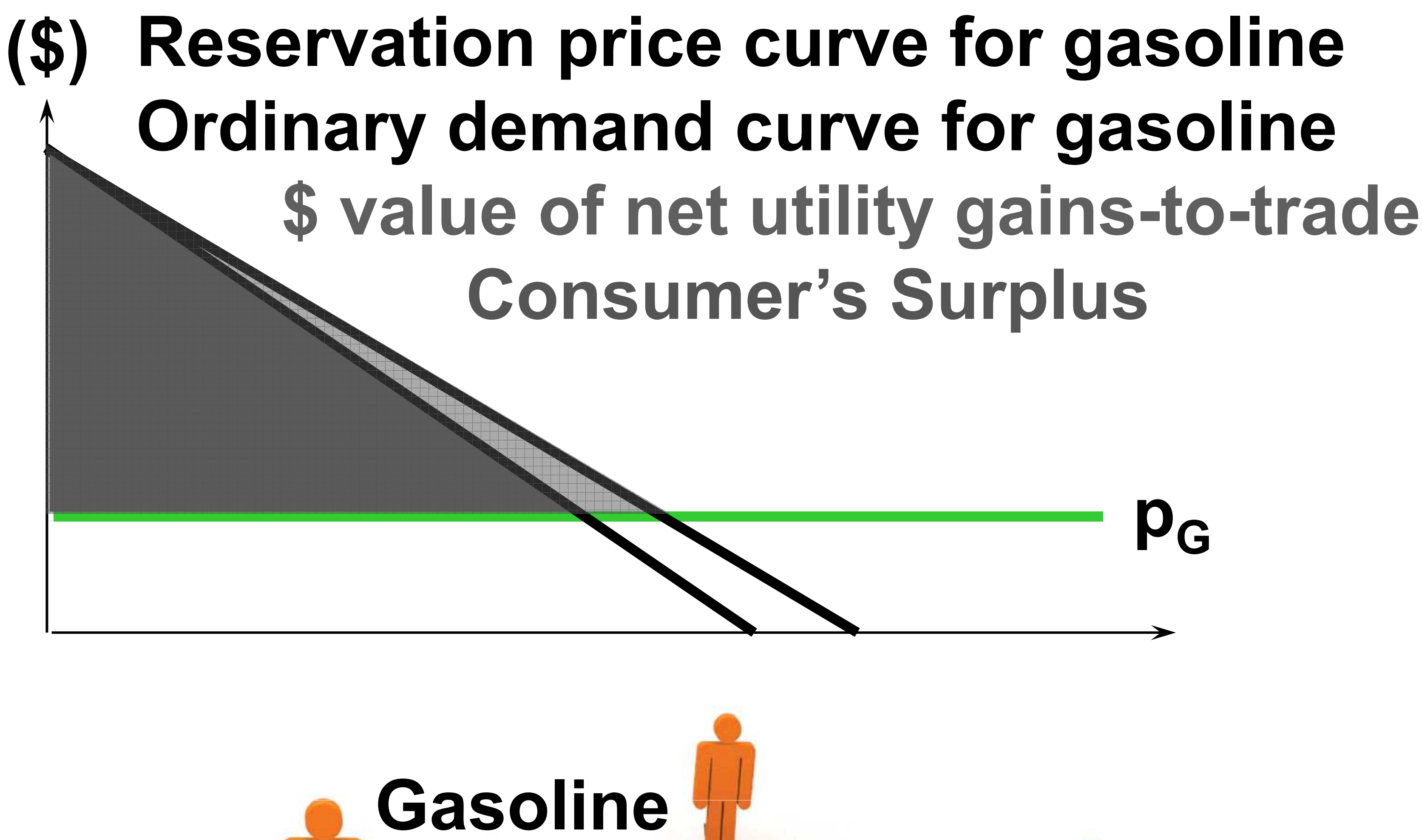
Consumer's Surplus



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Consumer's Surplus

- **The difference between the consumer's reservation-price and ordinary demand curves is due to income effects.**
- **But, if the consumer's utility function is quasilinear in income then there are no income effects and Consumer's Surplus is an exact $ measure of gains-to-trade.**

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Consumer's Surplus

The consumer's utility function is quasilinear in x_2.

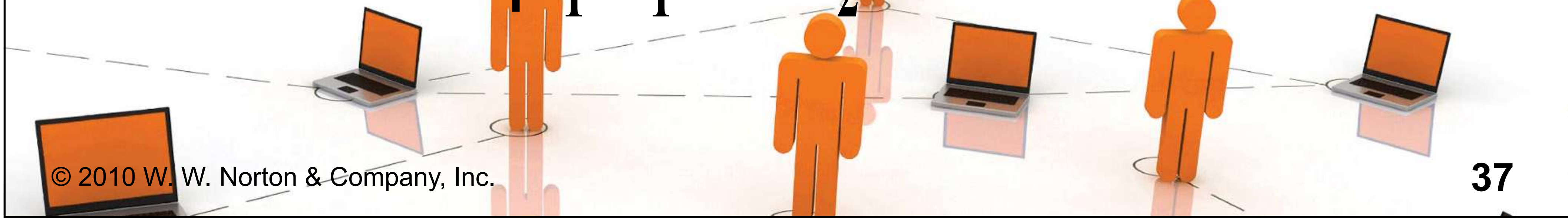
$$U(x_1, x_2) = v(x_1) + x_2$$

Take $p_2 = 1$. Then the consumer's choice problem is to maximize

$$U(x_1, x_2) = v(x_1) + x_2$$

subject to

$$p_1 x_1 + x_2 = m.$$



Consumer's Surplus

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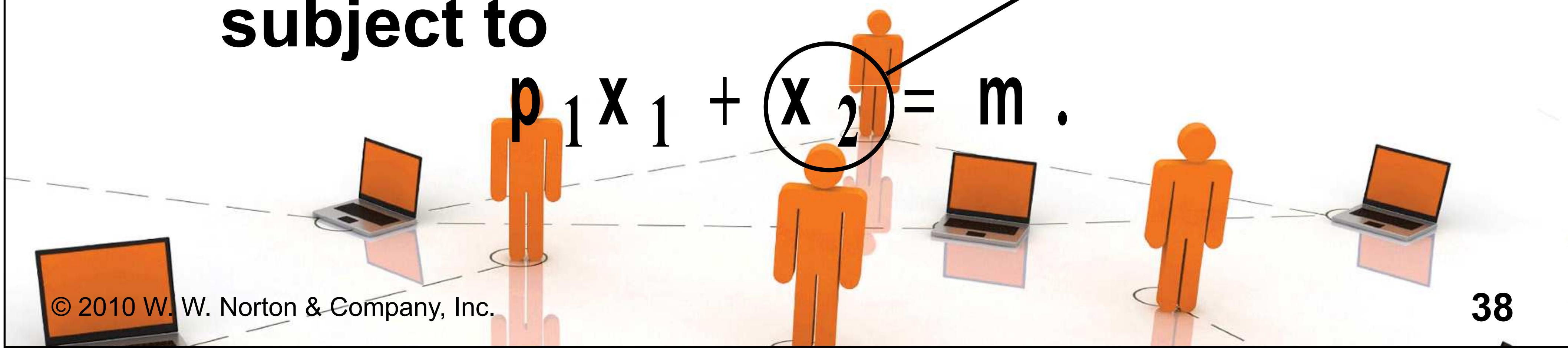

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Consumer's Surplus

That is, choose x_1 to maximize

$$v(x_1) + m - p_1 x_1.$$

The first-order condition is

$$v'(x_1) - p_1 = 0$$

That is, $$p_1 = v'(x_1).$$

This is the equation of the consumer's ordinary demand for commodity 1.

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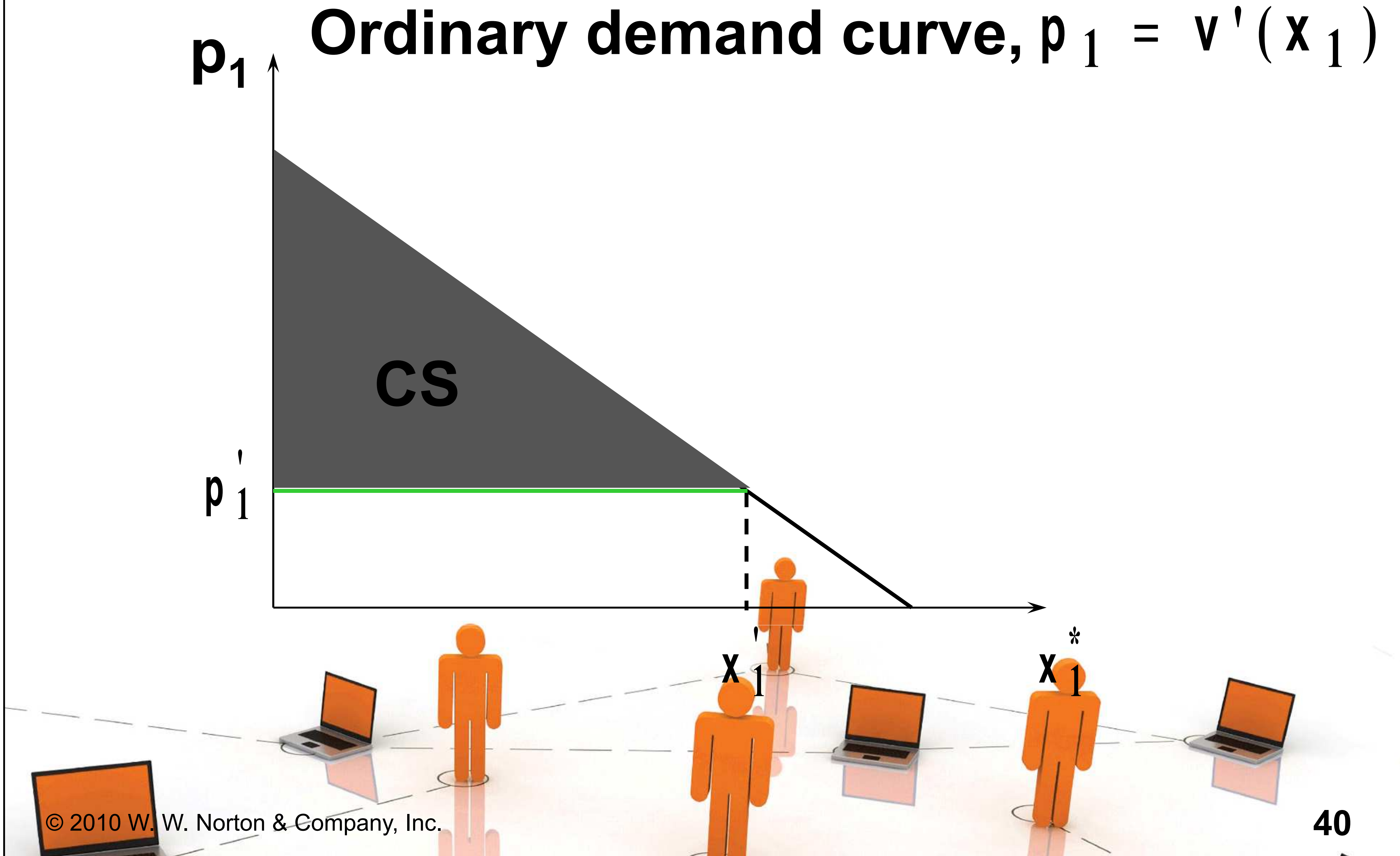

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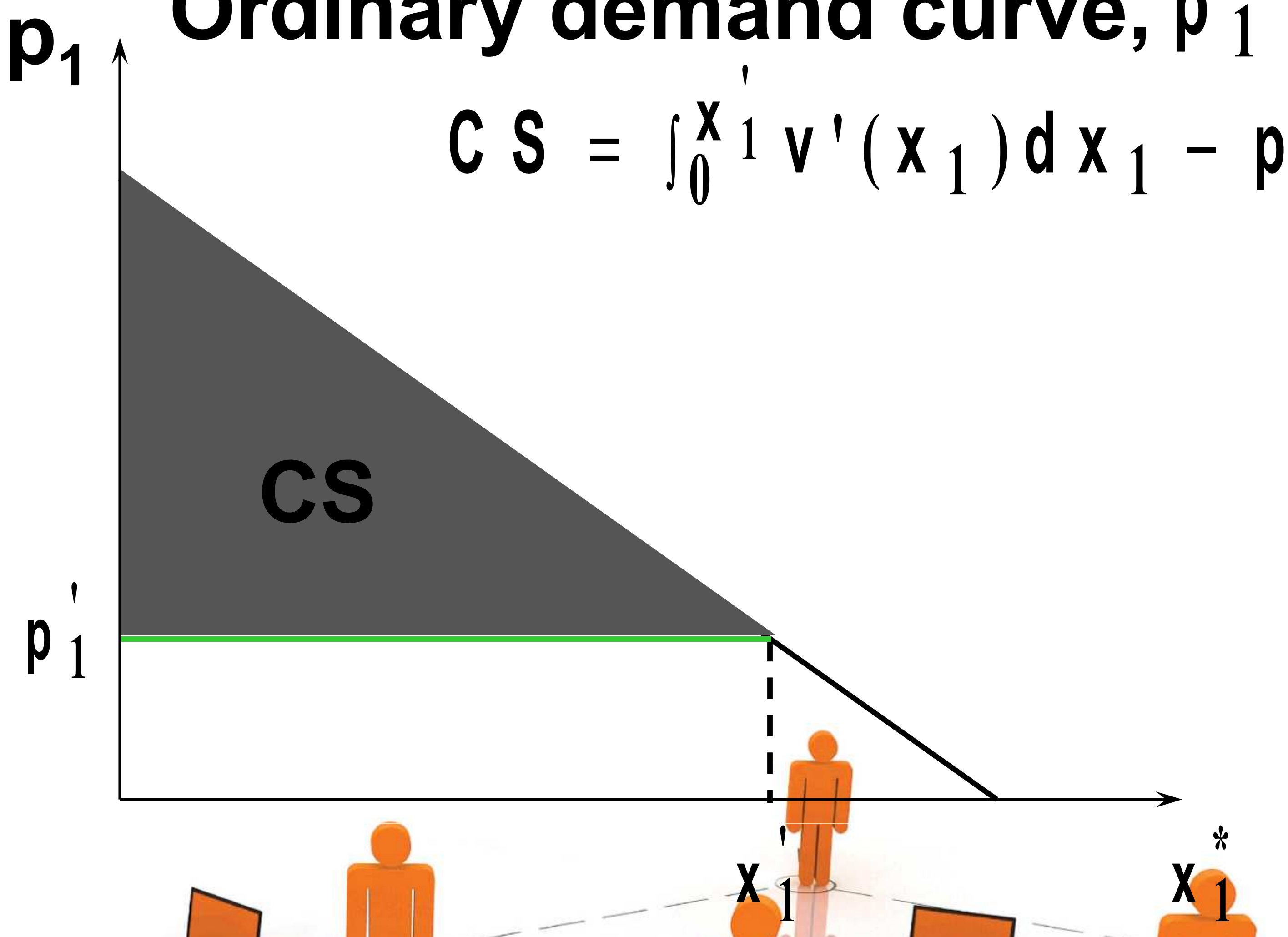
Consumer's Surplus

Ordinary demand curve, $p_1 = v'(x_1)$

$$CS = \int_0^{x_1'} v'(x_1)\,dx_1 - p_1' x_1'$$



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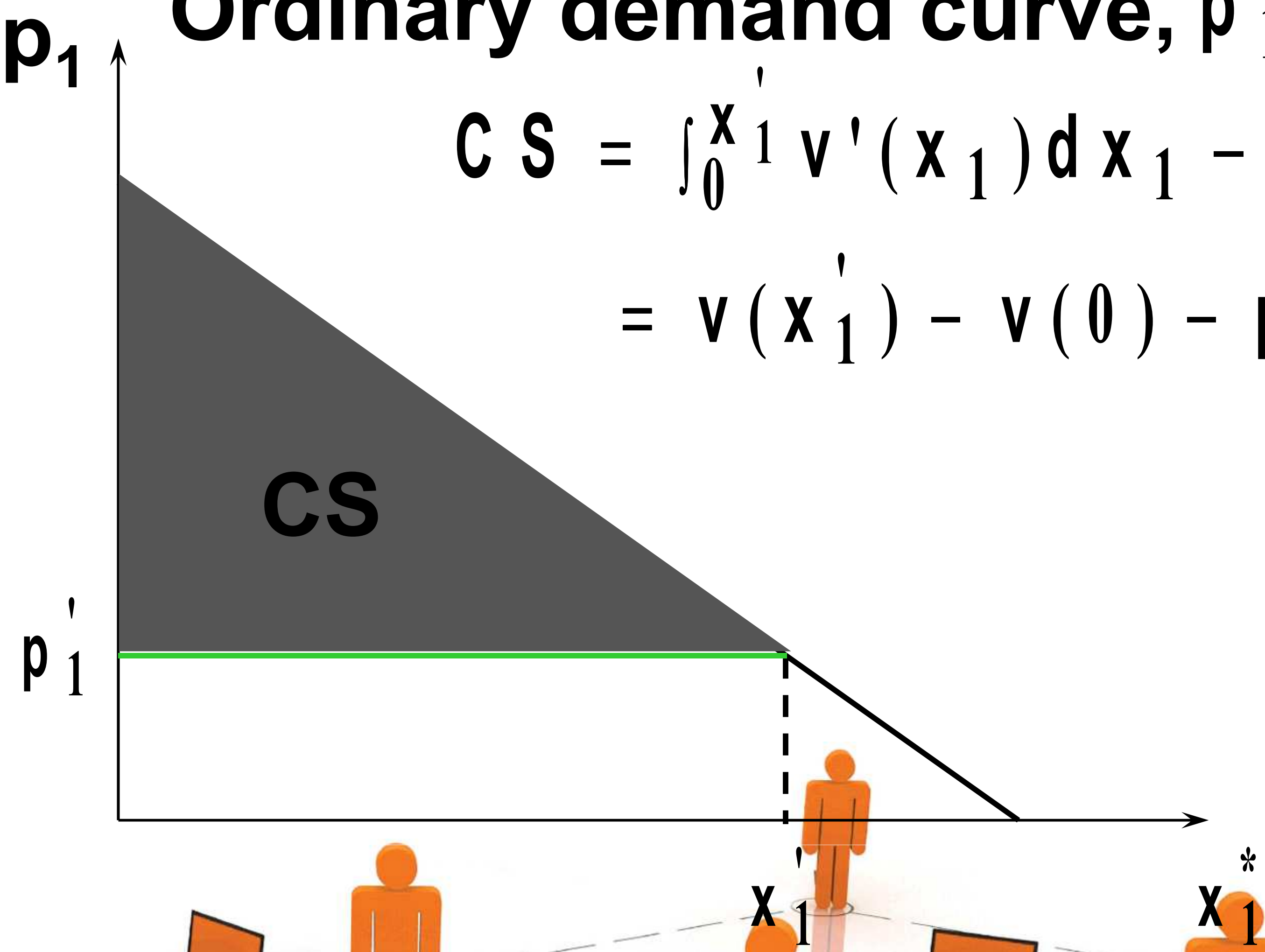
Consumer's Surplus

Ordinary demand curve, $p_1 = v'(x_1)$

$$CS = \int_0^{x_1'} v'(x_1)\,dx_1 - p_1'x_1'$$

$$= v(x_1') - v(0) - p_1'x_1'$$



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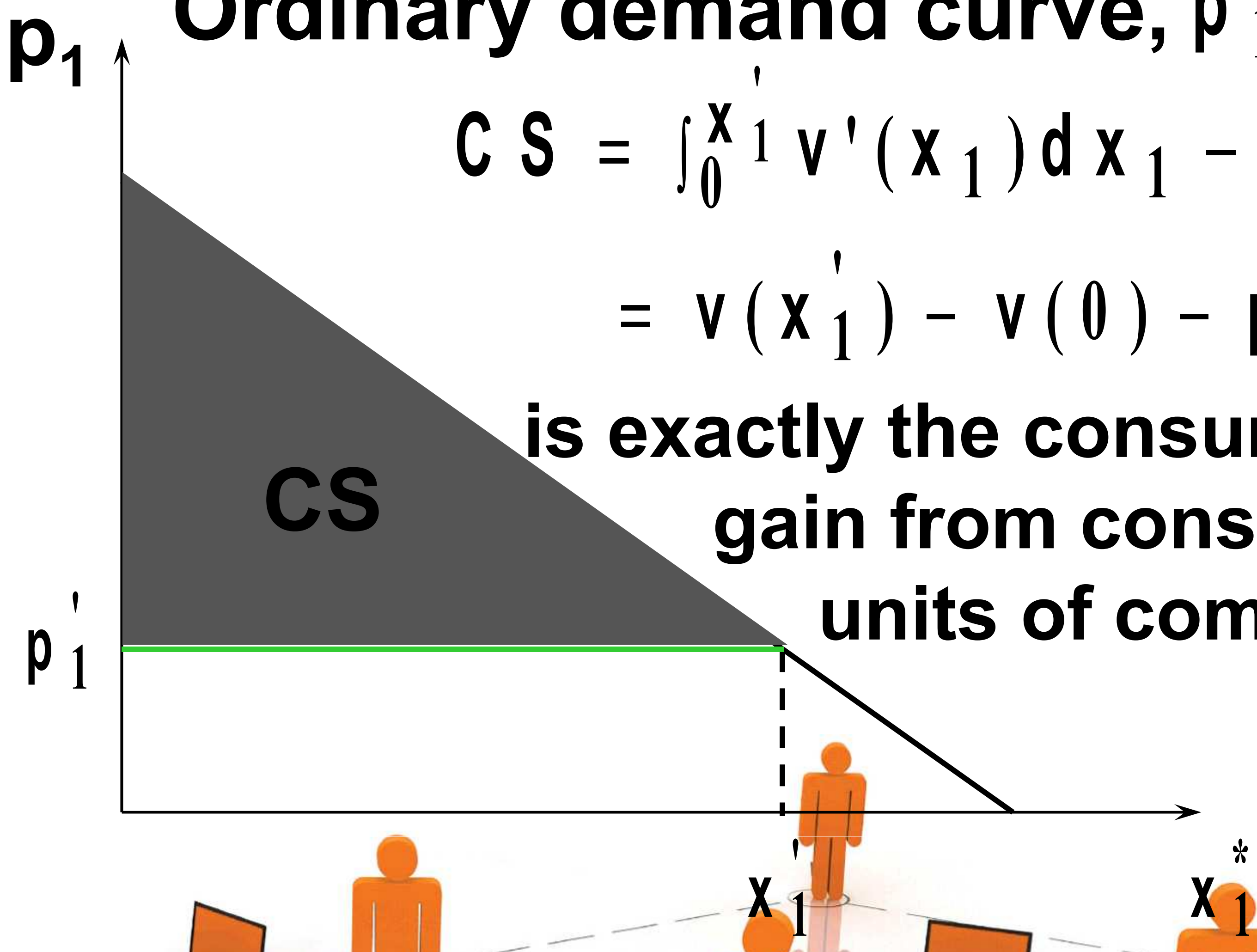
Consumer’s Surplus

Ordinary demand curve, $p_1 = v'(x_1)$

$$CS = \int_0^{x_1'} v'(x_1)\,dx_1 - p_1'x_1'$$

$$= v(x_1') - v(0) - p_1'x_1'$$

is exactly the consumer’s utility gain from consuming x_1’ units of commodity 1.



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Consumer's Surplus

- **Consumer's Surplus is an exact dollar measure of utility gained from consuming commodity 1 when the consumer's utility function is quasilinear in commodity 2.**
- **Otherwise Consumer's Surplus is an approximation.**

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Consumer's Surplus

- **The change to a consumer's total utility due to a change to p_1 is approximately the change in her Consumer's Surplus.**

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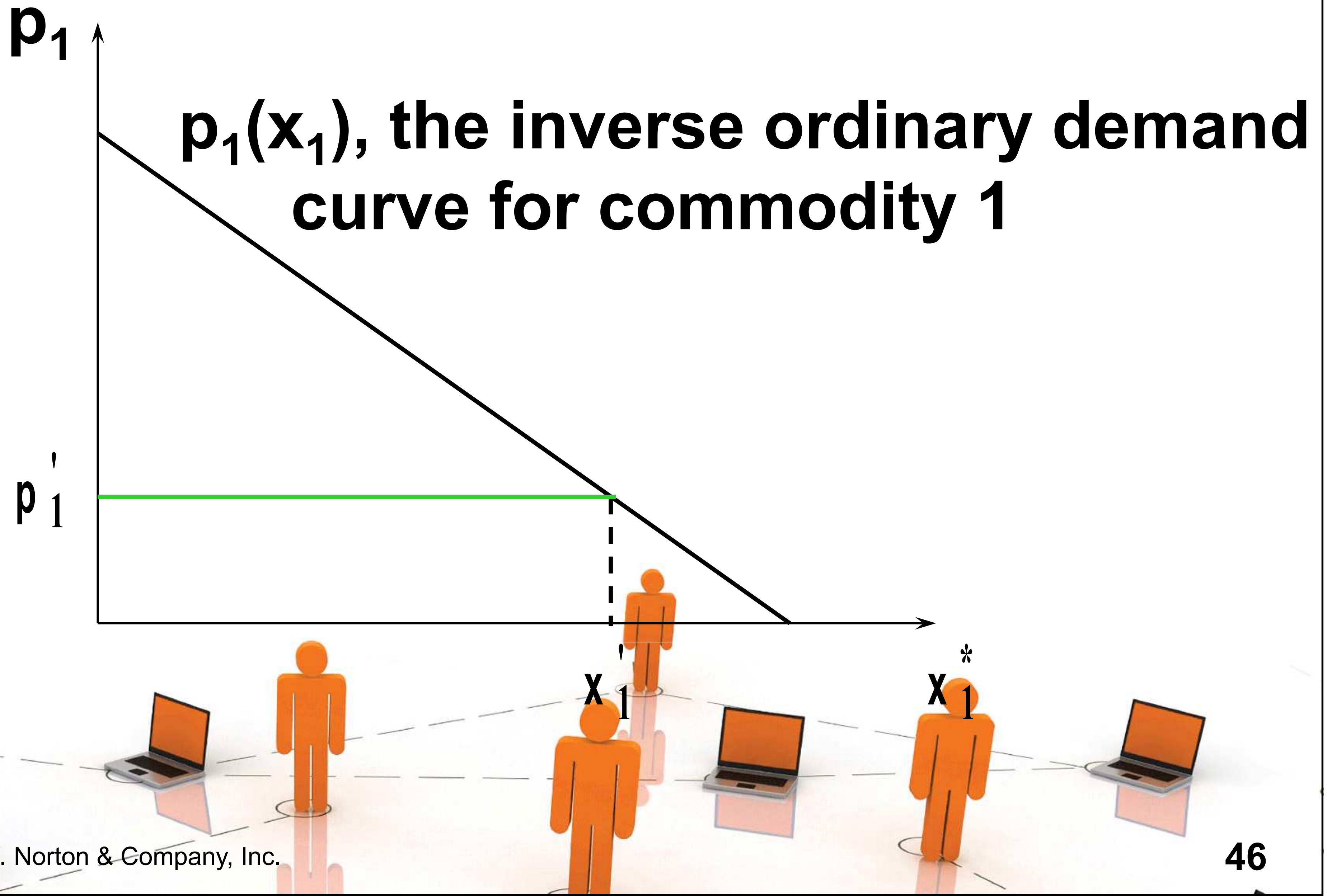
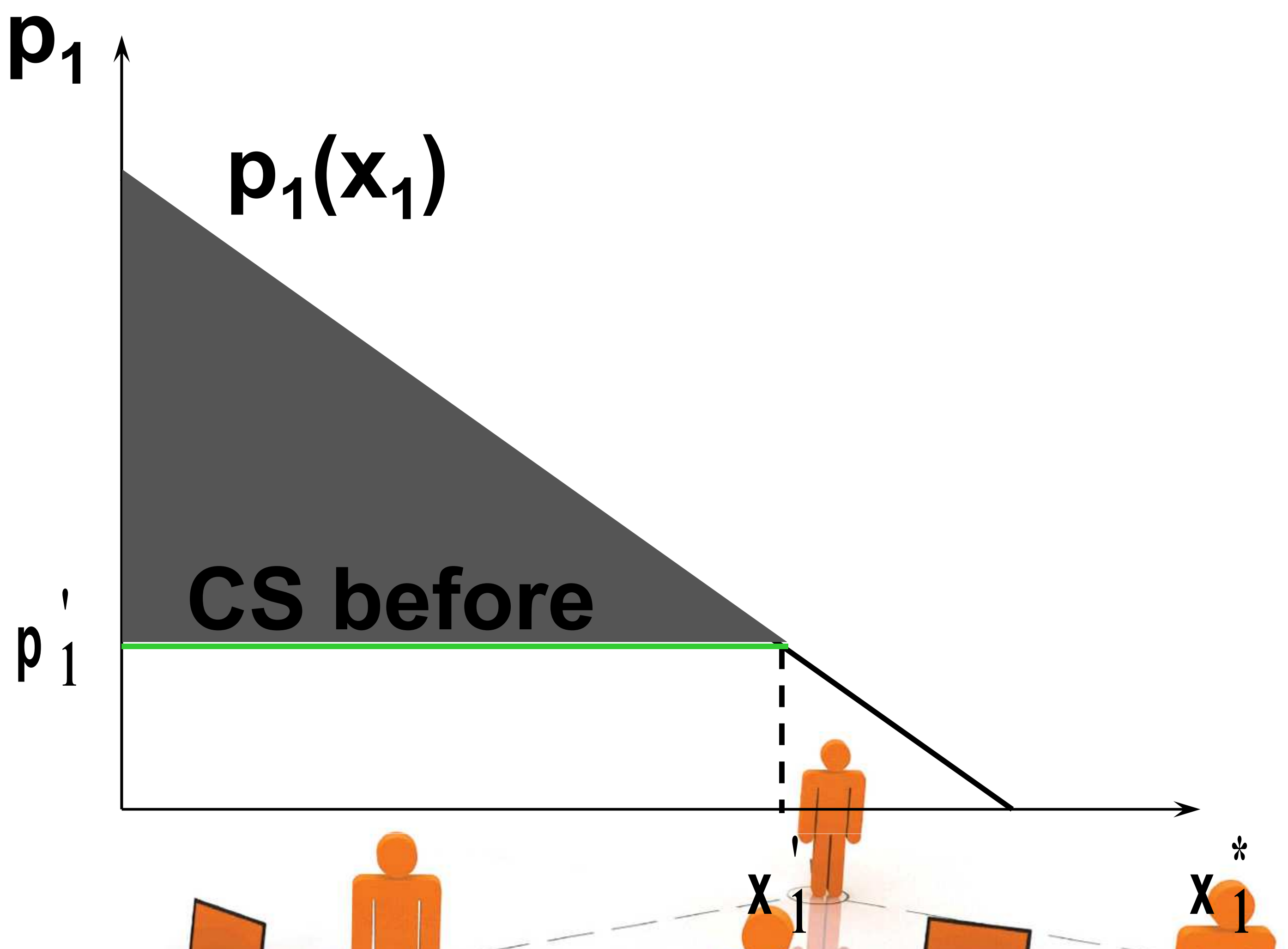
Consumer's Surplus



Consumer's Surplus



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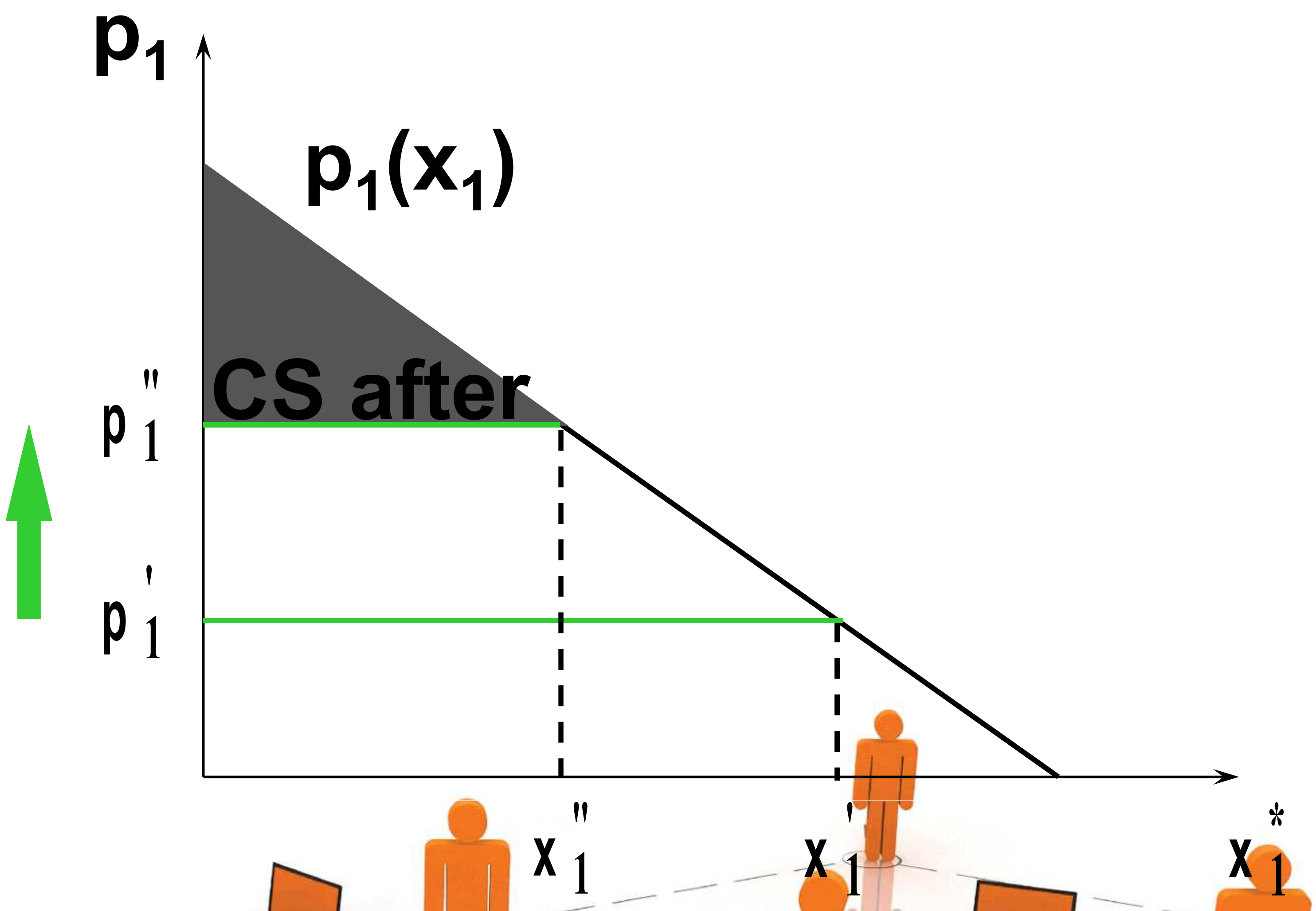

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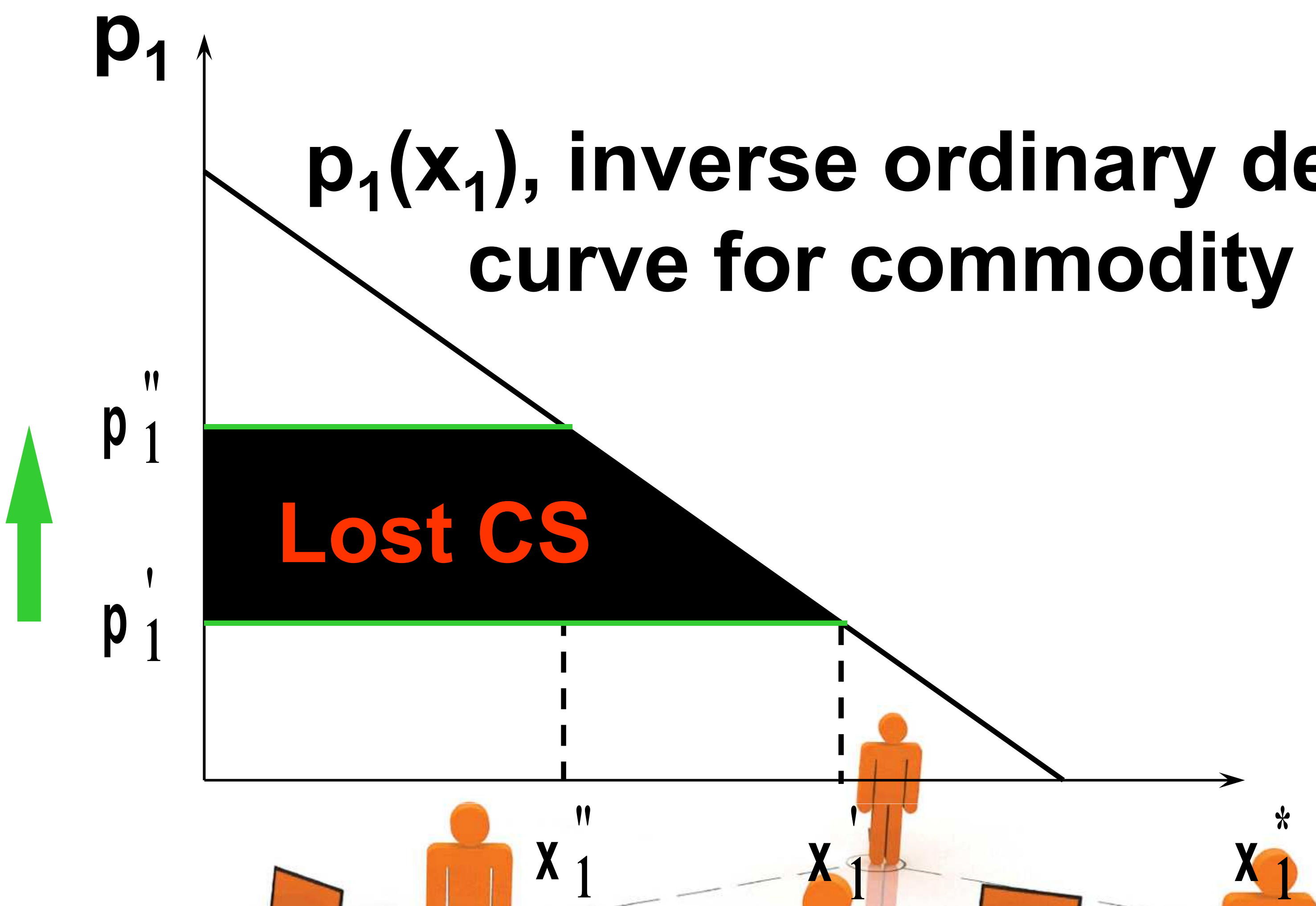

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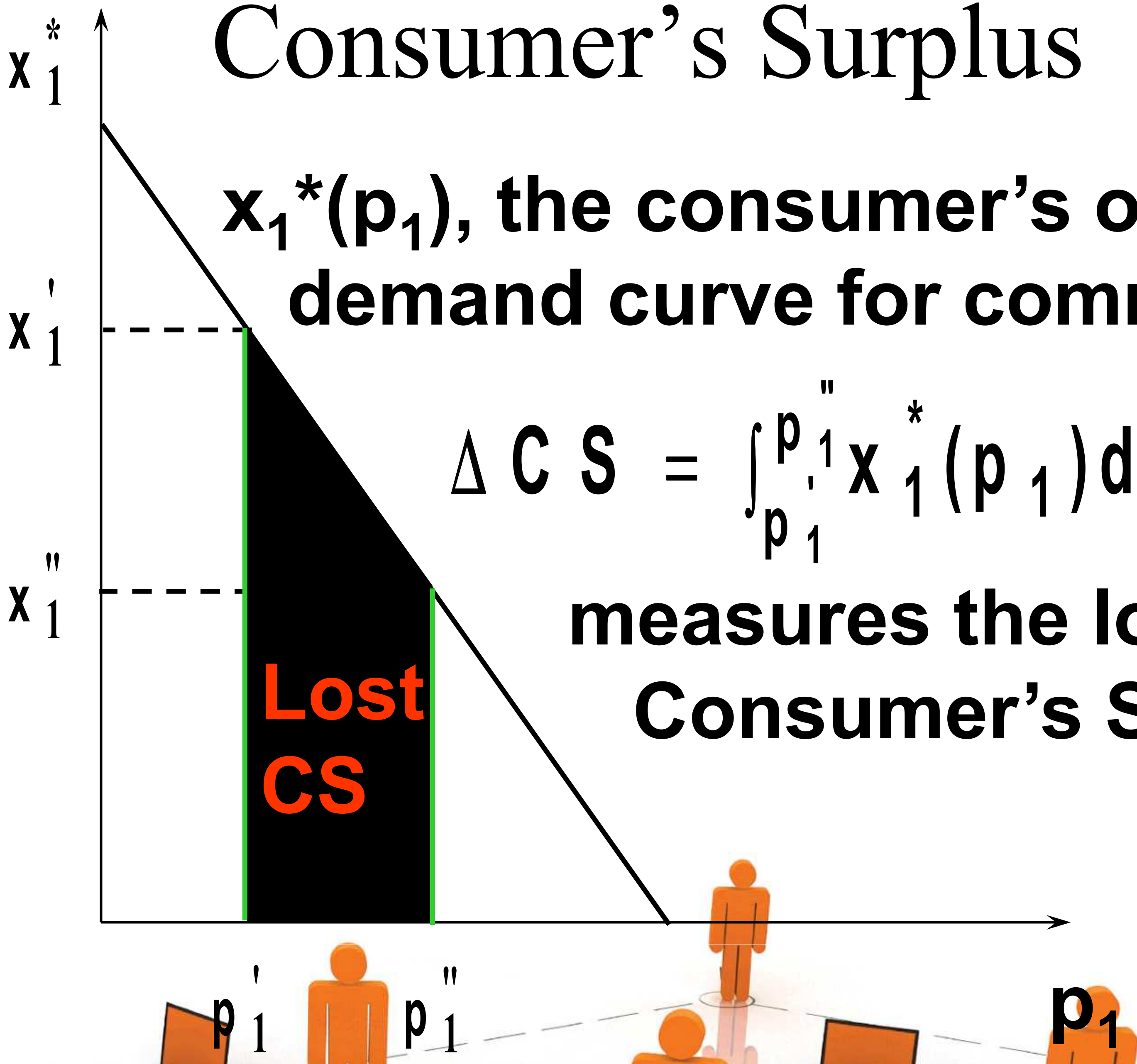

$x_1^*(p_1)$, the consumer's ordinary demand curve for commodity 1.

$$\Delta CS = \int_{p_1'}^{p_1''} x_1^*(p_1)\,dp_1$$

measures the loss in Consumer's Surplus.

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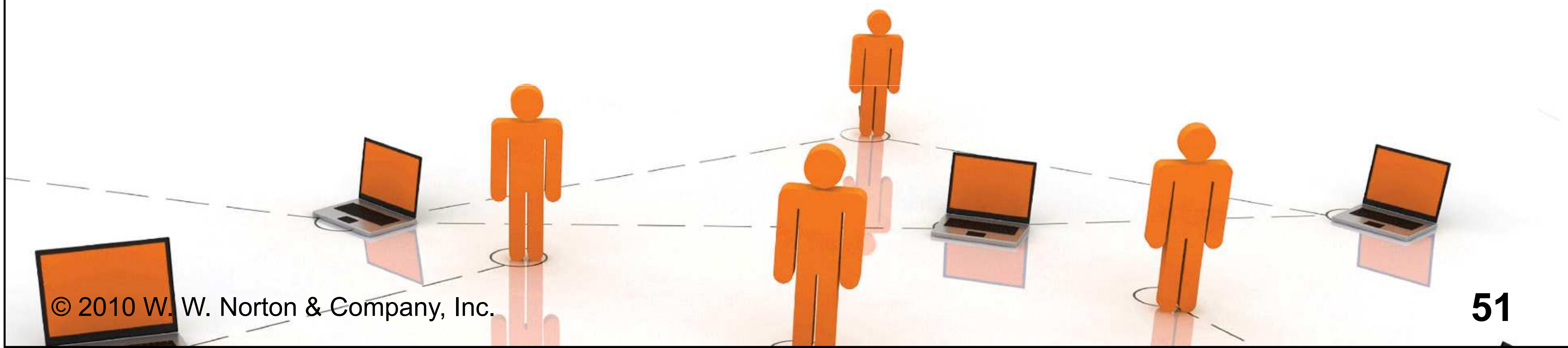
Compensating Variation and Equivalent Variation

- **Two additional dollar measures of the total utility change caused by a price change are Compensating Variation and Equivalent Variation.**



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Compensating Variation

- p_1 rises.
- Q: What is the least extra income that, at the new prices, just restores the consumer's original utility level?

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Compensating Variation

- p_1 rises.
- Q: What is the least extra income that, at the new prices, just restores the consumer's original utility level?
- A: The Compensating Variation.

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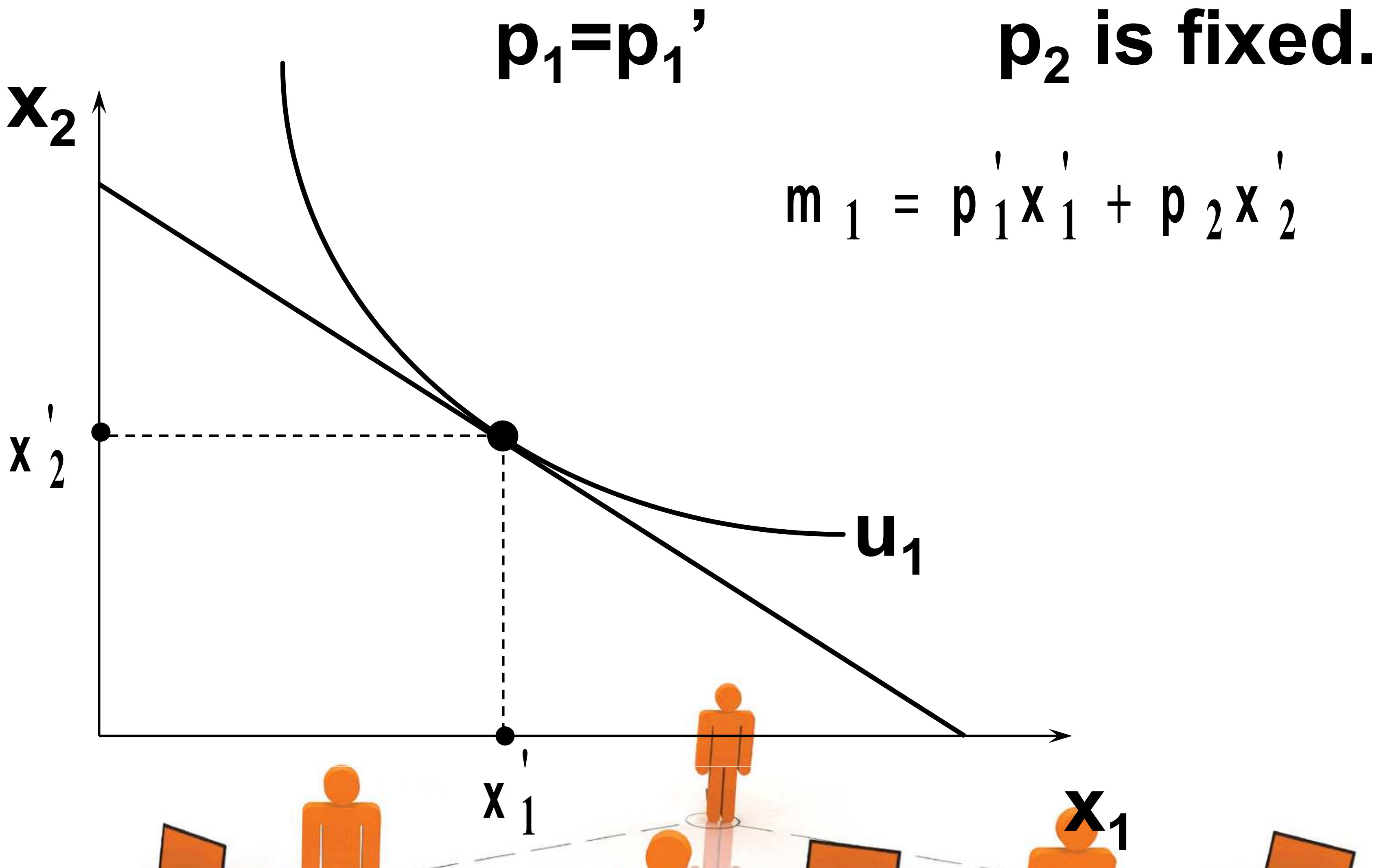
Compensating Variation



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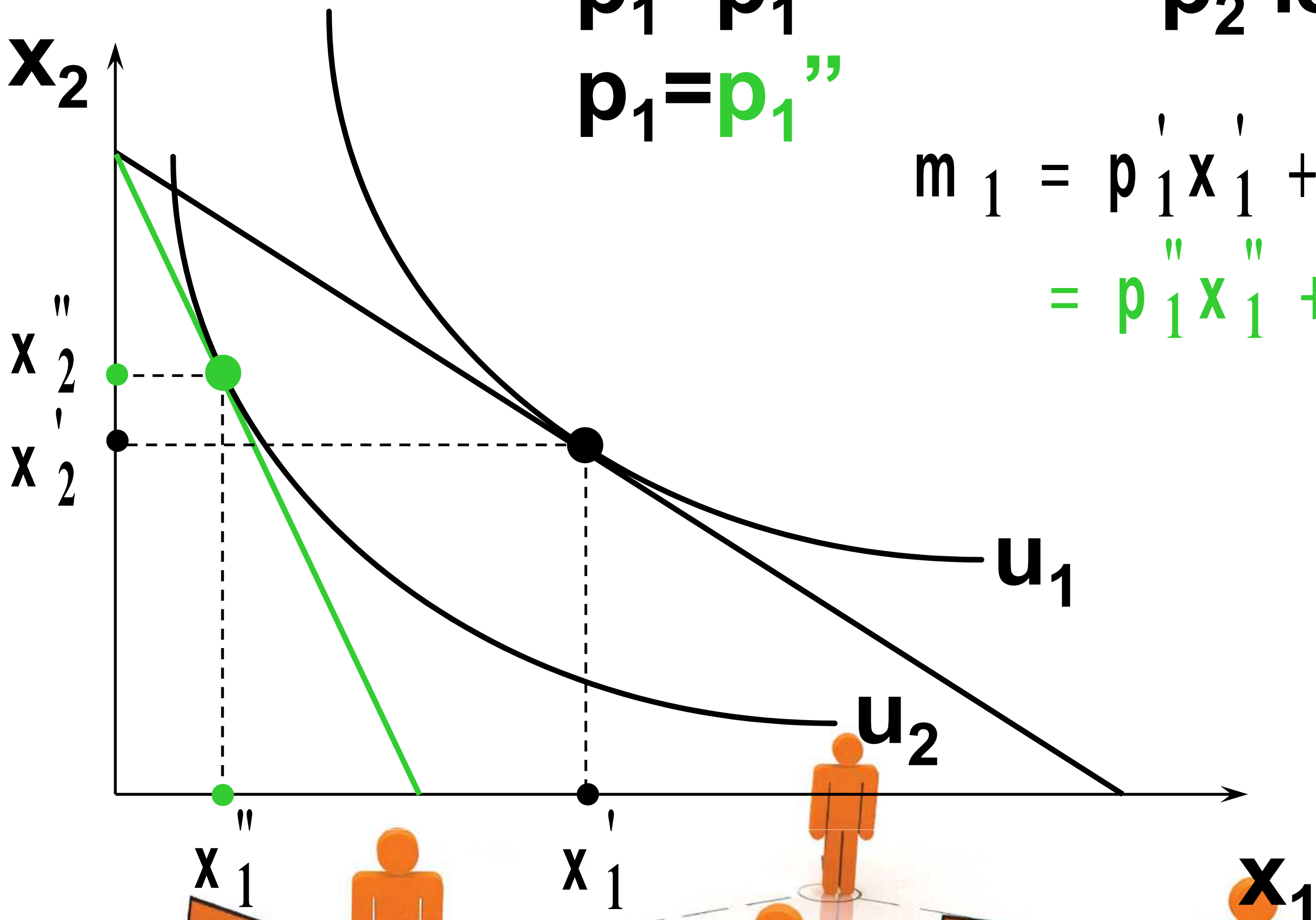
Compensating Variation

$p_1 = p_1'$
$p_1 = p_1''$

p_2 is fixed.

$$m_1 = p_1' x_1' + p_2 x_2'$$
$$= p_1'' x_1'' + p_2 x_2''$$



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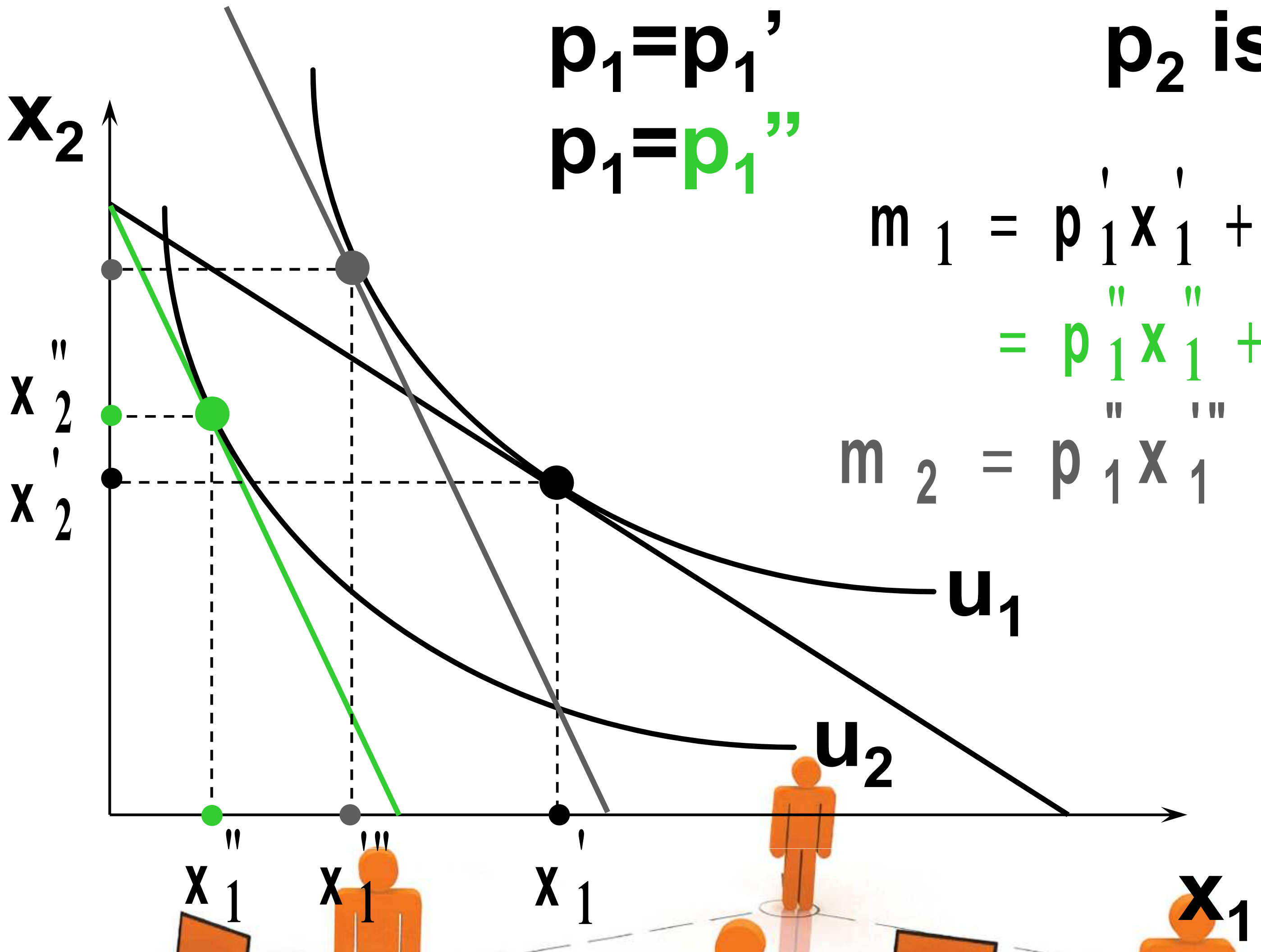
Compensating Variation

$p_1 = p_1'$

$p_1 = p_1''$

p_2 is fixed.

$$m_1 = p_1' x_1' + p_2 x_2'$$
$$= p_1'' x_1'' + p_2 x_2''$$
$$m_2 = p_1'' x_1''' + p_2 x_2'''$$



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Compensating Variation

$p_1 = p_1'$

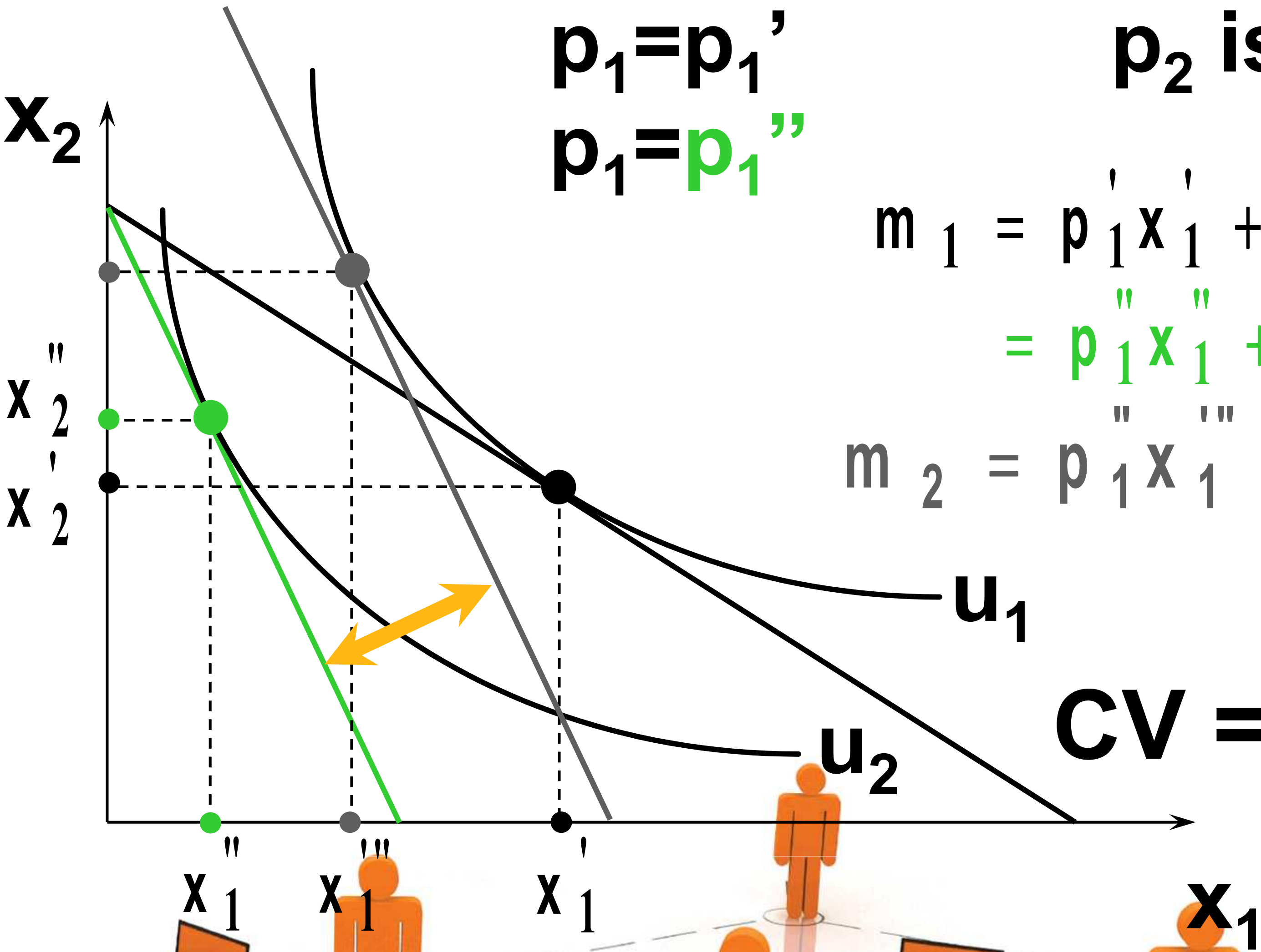
$p_1 = p_1''$

p_2 is fixed.

$$m_1 = p_1' x_1' + p_2 x_2'$$

$$= p_1'' x_1'' + p_2 x_2''$$

$$m_2 = p_1'' x_1''' + p_2 x_2'''$$



$CV = m_2 - m_1$.

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Equivalent Variation

- p_1 **rises.**
- **Q: What is the least extra income that, at the original prices, just restores the consumer's original utility level?**
- **A: The Equivalent Variation.**

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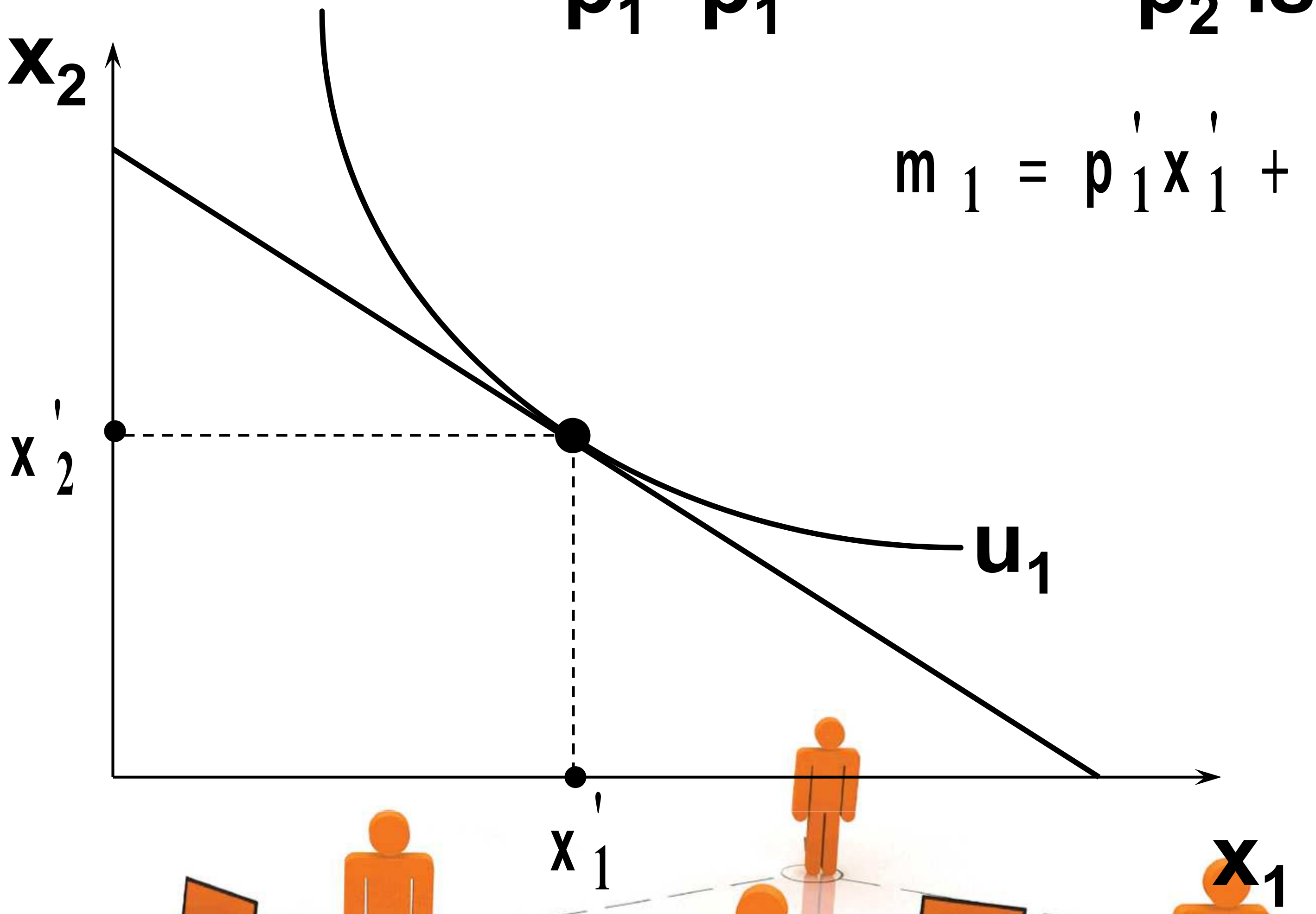
Equivalent Variation

$p_1 = p_1'$

p_2 is fixed.

$$m_1 = p_1' x_1' + p_2 x_2'$$



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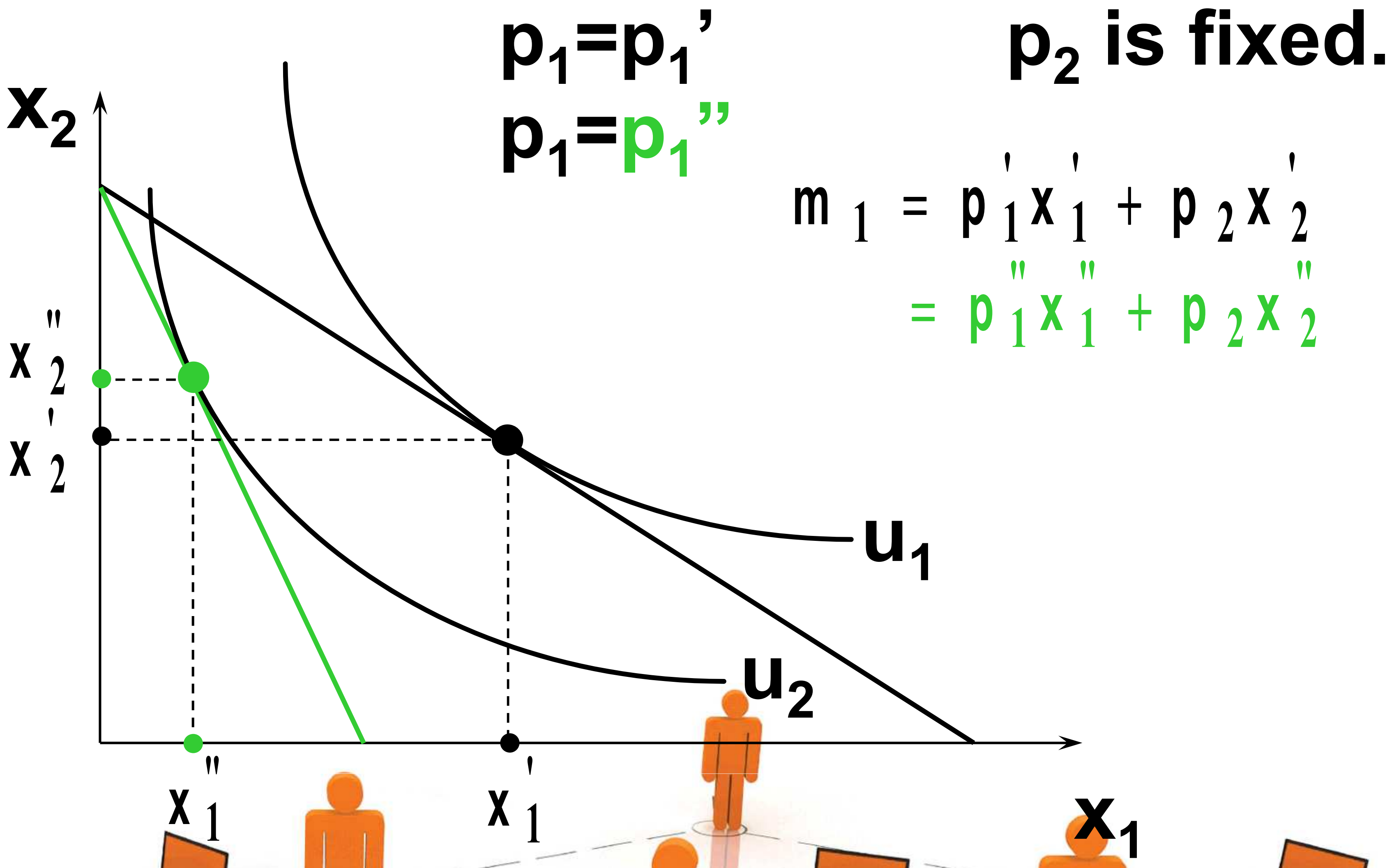
Equivalent Variation



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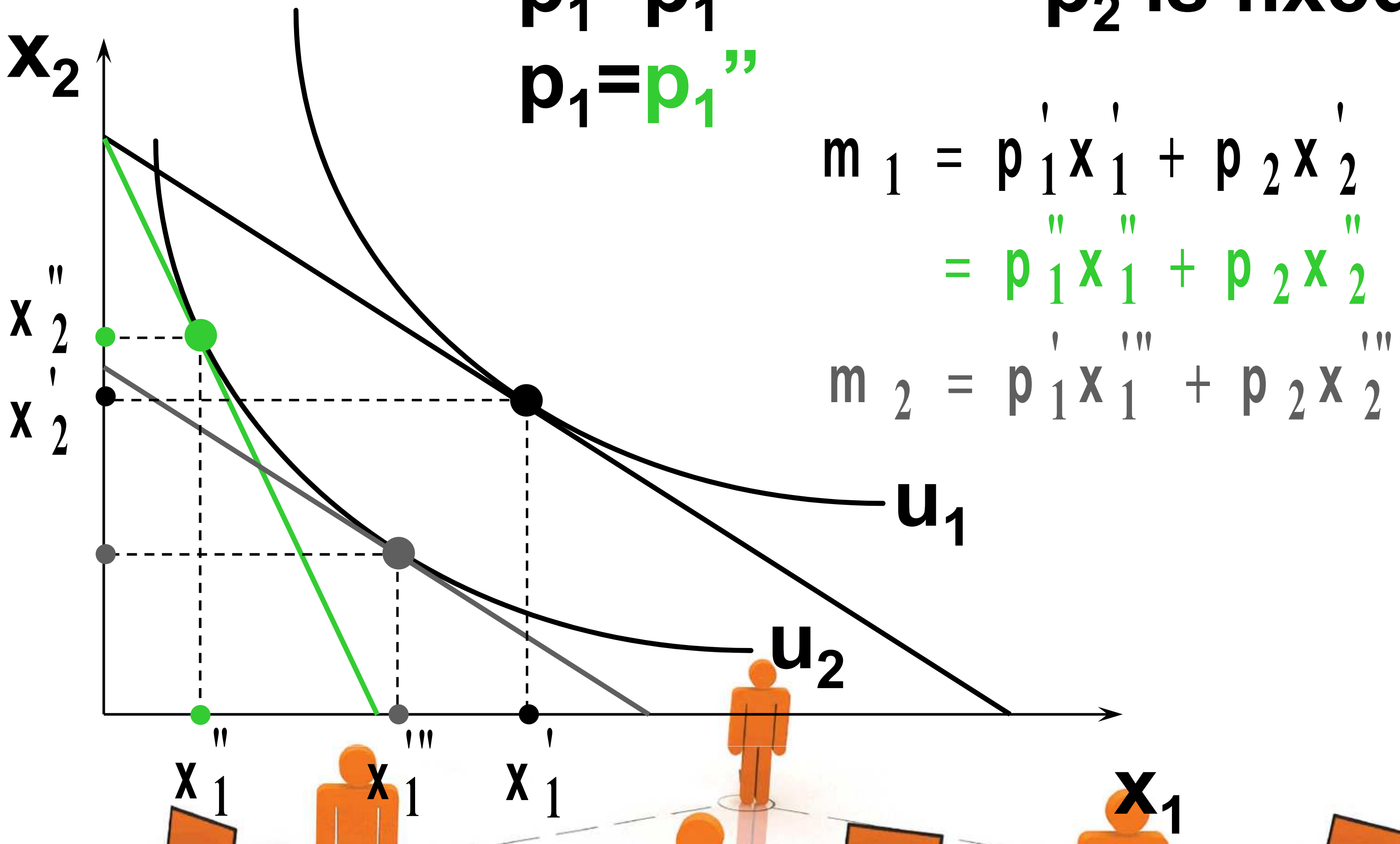
Equivalent Variation



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Equivalent Variation

$p_1 = p_1'$

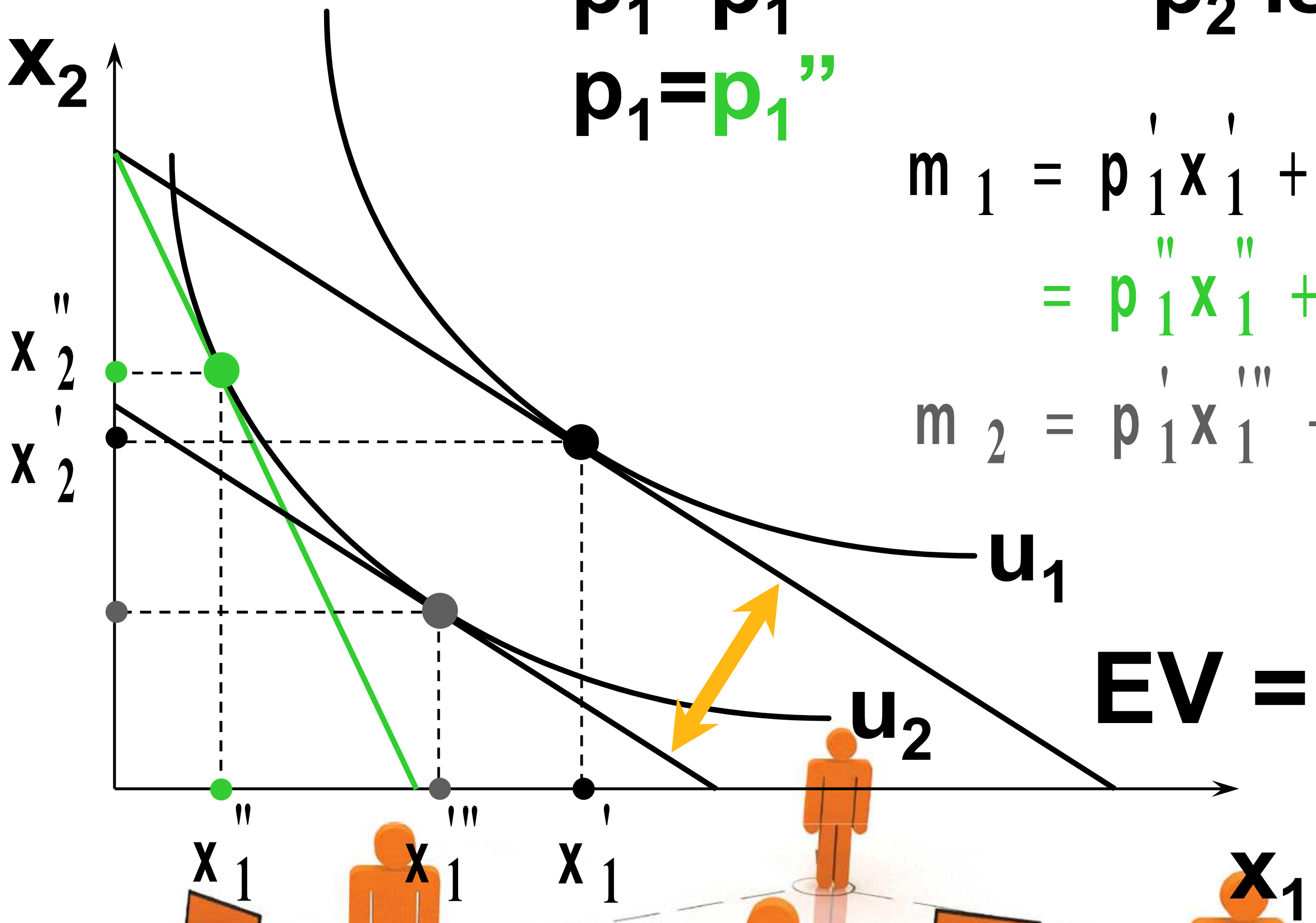
$p_1 = p_1''$

p_2 is fixed.

$$m_1 = p_1' x_1' + p_2 x_2'$$

$$= p_1'' x_1'' + p_2 x_2''$$

$$m_2 = p_1' x_1''' + p_2 x_2'''$$



EV = m_1 - m_2.

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Consumer’s Surplus, Compensating Variation and Equivalent Variation

- **Relationship 1: When the consumer’s preferences are quasilinear, all three measures are the same.**

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Consumer’s Surplus, Compensating Variation and Equivalent Variation

- **Consider first the change in Consumer’s Surplus when p_1 rises from p_1' to p_1''.**

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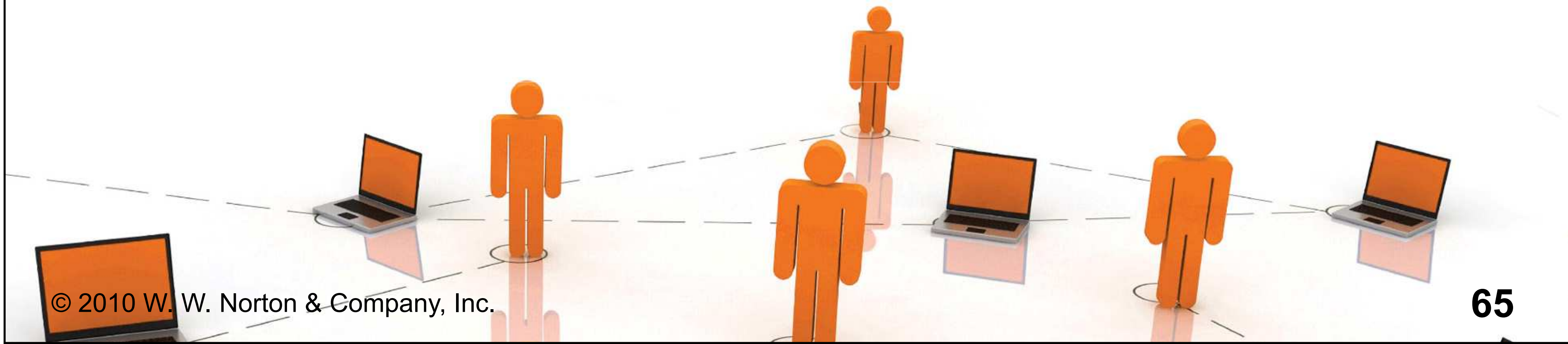
Consumer’s Surplus, Compensating Variation and Equivalent Variation

If $$U(x_1, x_2) = v(x_1) + x_2$$ **then**

$$CS(p_1') = v(x_1') - v(0) - p_1' x_1'$$



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Consumer's Surplus, Compensating Variation and Equivalent Variation

If $U(x_1, x_2) = v(x_1) + x_2$ **then**

$$CS(p_1') = v(x_1') - v(0) - p_1' x_1'$$

and so the change in CS when p_1 rises from p_1' to p_1" is

$$\Delta CS = CS(p_1') - CS(p_1'')$$

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Consumer's Surplus, Compensating Variation and Equivalent Variation

If $U(x_1, x_2) = v(x_1) + x_2$ **then**

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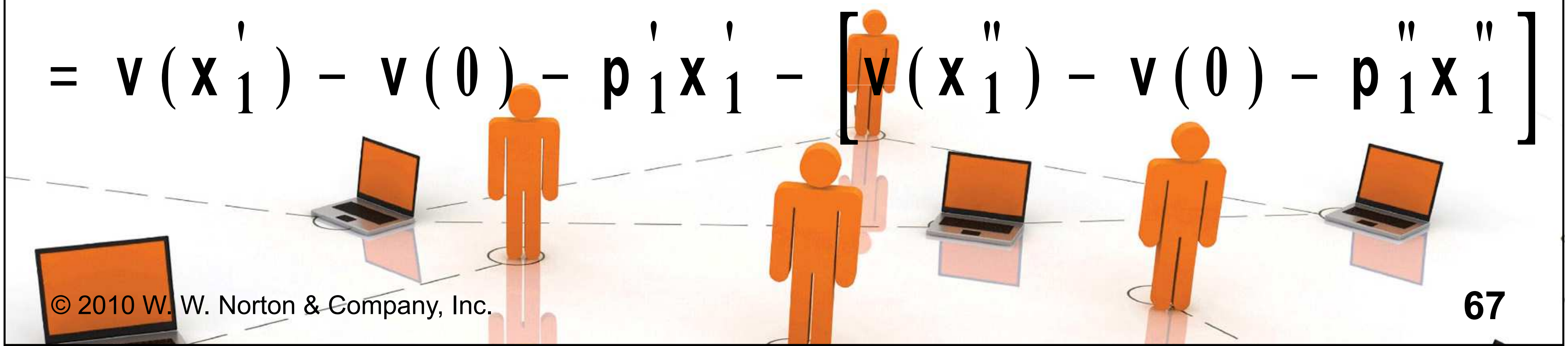
$$\Delta CS = CS(p_1') - CS(p_1'')$$

$$= v(x_1') - v(0) - p_1' x_1' - \left[v(x_1'') - v(0) - p_1'' x_1'' \right]$$



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Consumer's Surplus, Compensating Variation and Equivalent Variation

If $U(x_1, x_2) = v(x_1) + x_2$ **then**

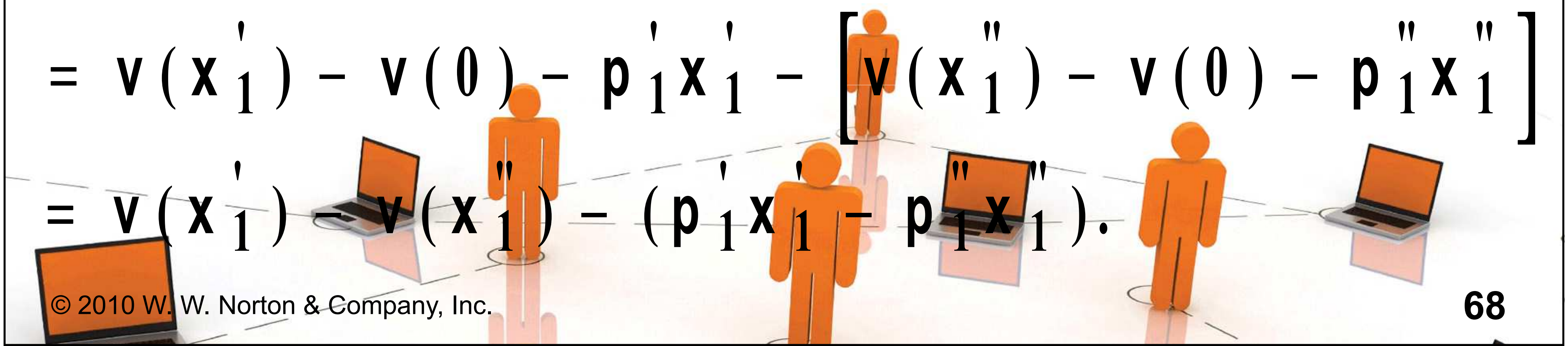
$$CS(p_1') = v(x_1') - v(0) - p_1'x_1'$$

and so the change in CS when p_1 rises from p_1' to p_1" is

$$\Delta CS = CS(p_1') - CS(p_1'')$$

$$= v(x_1') - v(0) - p_1'x_1' - \left[v(x_1'') - v(0) - p_1''x_1''\right]$$

$$= v(x_1') - v(x_1'') - (p_1'x_1' - p_1''x_1'').$$



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Consumer's Surplus, Compensating Variation and Equivalent Variation

- Now consider the change in CV when p_1 rises from p_1' to p_1''.
- The consumer's utility for given p_1 is

$$v(x_1^*(p_1)) + m - p_1 x_1^*(p_1)$$

and CV is the extra income which, at the new prices, makes the consumer's utility the same as at the old prices. That is, ...

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Consumer's Surplus, Compensating Variation and Equivalent Variation

$$v(x_1') + m - p_1' x_1'$$
$$= v(x_1'') + m + CV - p_1'' x_1''.$$

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Consumer's Surplus, Compensating Variation and Equivalent Variation

$$v(x_1') + m - p_1' x_1'$$
$$= v(x_1'') + m + CV - p_1'' x_1''.$$

So

$$CV = v(x_1') - v(x_1'') - (p_1' x_1' - p_1'' x_1'')$$
$$= \Delta CS.$$

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Consumer's Surplus, Compensating Variation and Equivalent Variation

- **Now consider the change in EV when p_1 rises from p_1' to p_1''.**
- **The consumer's utility for given p_1 is**

$$v(x_1^*(p_1)) + m - p_1 x_1^*(p_1)$$

and EV is the extra income which, at the old prices, makes the consumer's utility the same as at the new prices. That is, ...

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Consumer's Surplus, Compensating Variation and Equivalent Variation

$$v(x_1') + m - p_1' x_1'$$
$$= v(x_1'') + m + EV - p_1'' x_1''.$$

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Consumer's Surplus, Compensating Variation and Equivalent Variation

$$v(x_1') + m - p_1' x_1'$$
$$= v(x_1'') + m + EV - p_1'' x_1''.$$

That is,

$$EV = v(x_1') - v(x_1'') - (p_1' x_1' - p_1'' x_1'')$$
$$= \Delta CS.$$

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Consumer's Surplus, Compensating Variation and Equivalent Variation

So when the consumer has quasilinear utility,

$$\mathbf{CV = EV = \Delta CS.}$$

But, otherwise, we have:

Relationship 2: In size, $EV < \Delta CS < CV$.

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Producer's Surplus

- **Changes in a firm's welfare can be measured in dollars much as for a consumer.**

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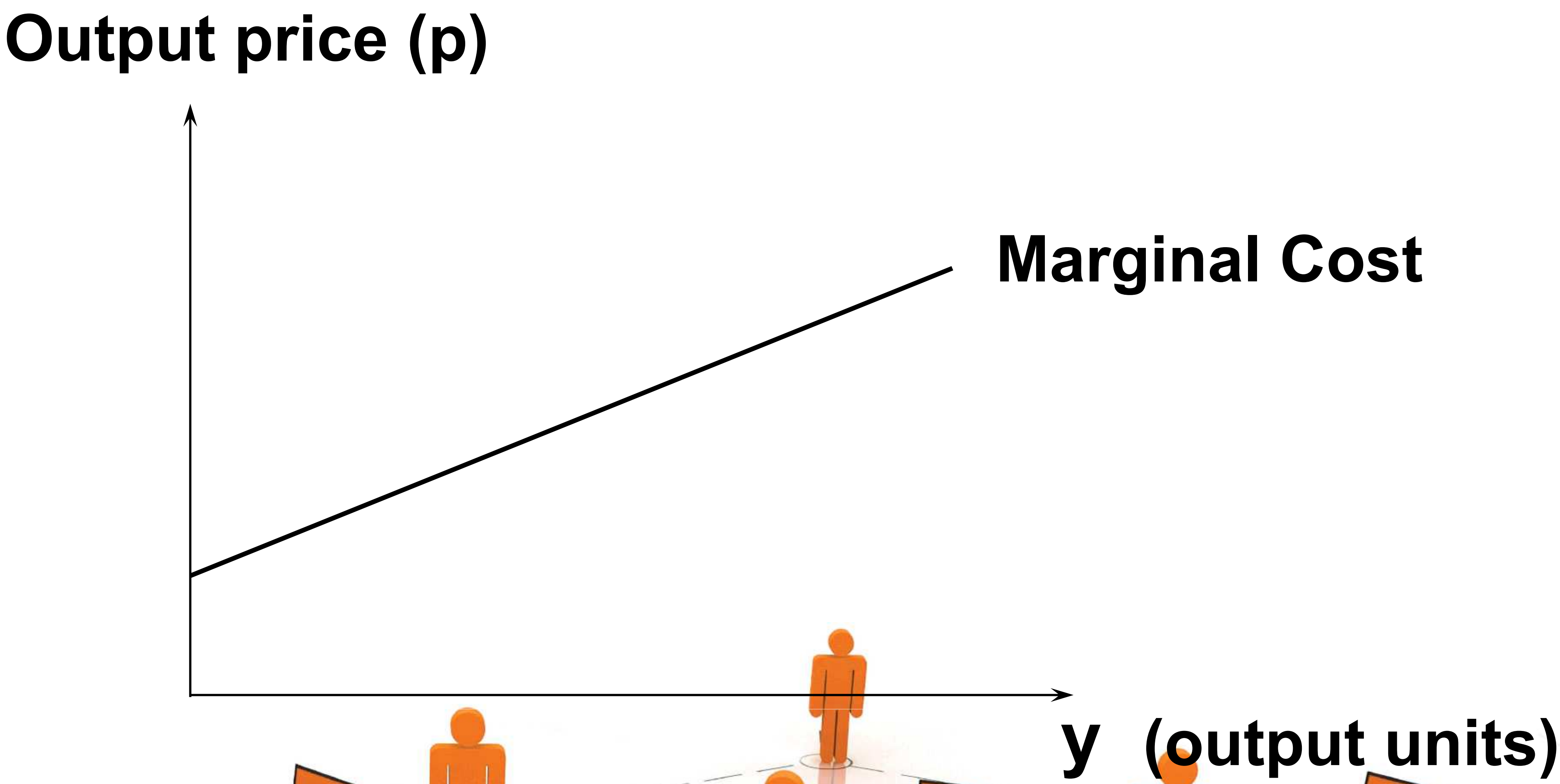
Producer's Surplus



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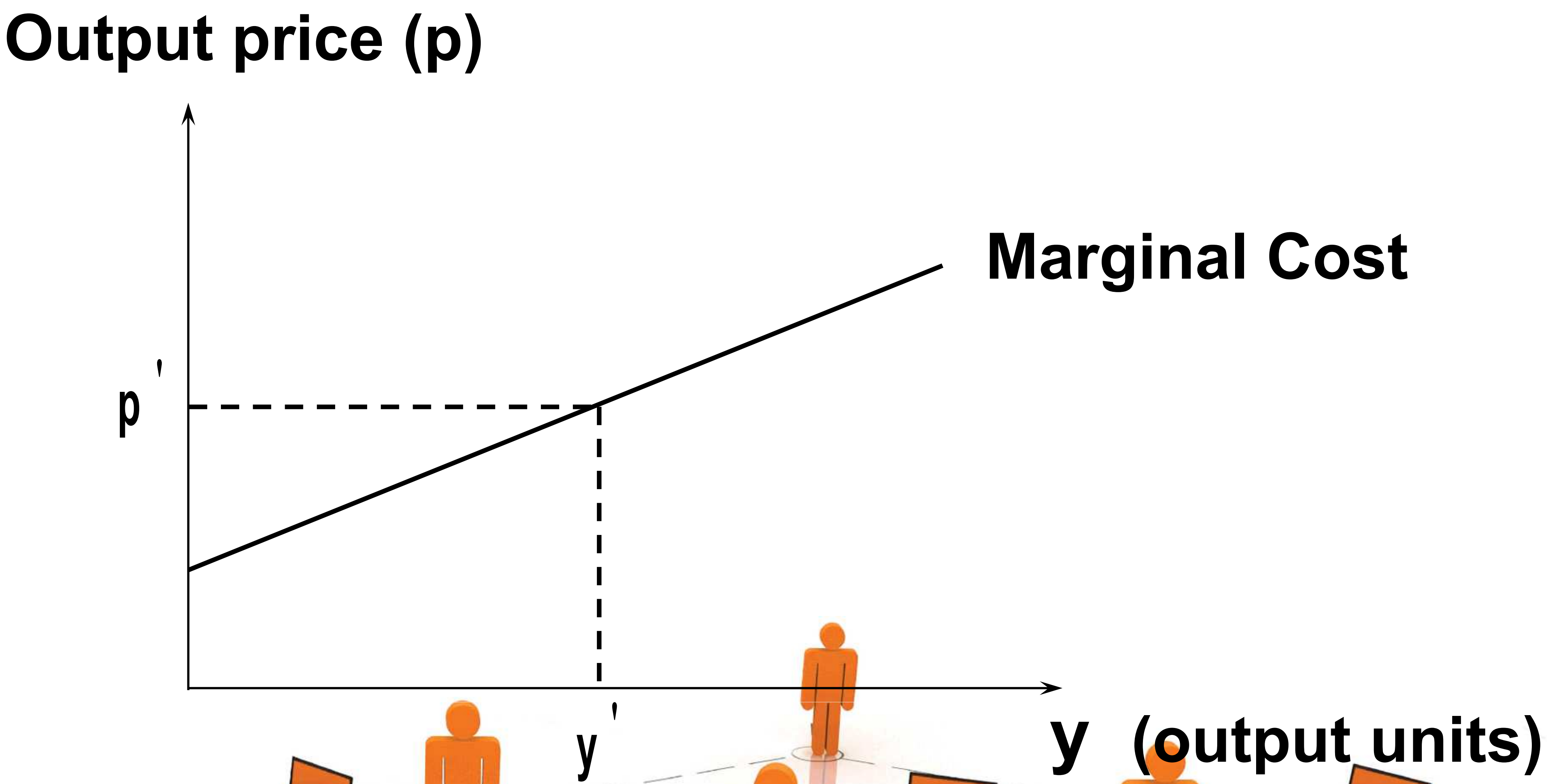
Producer's Surplus



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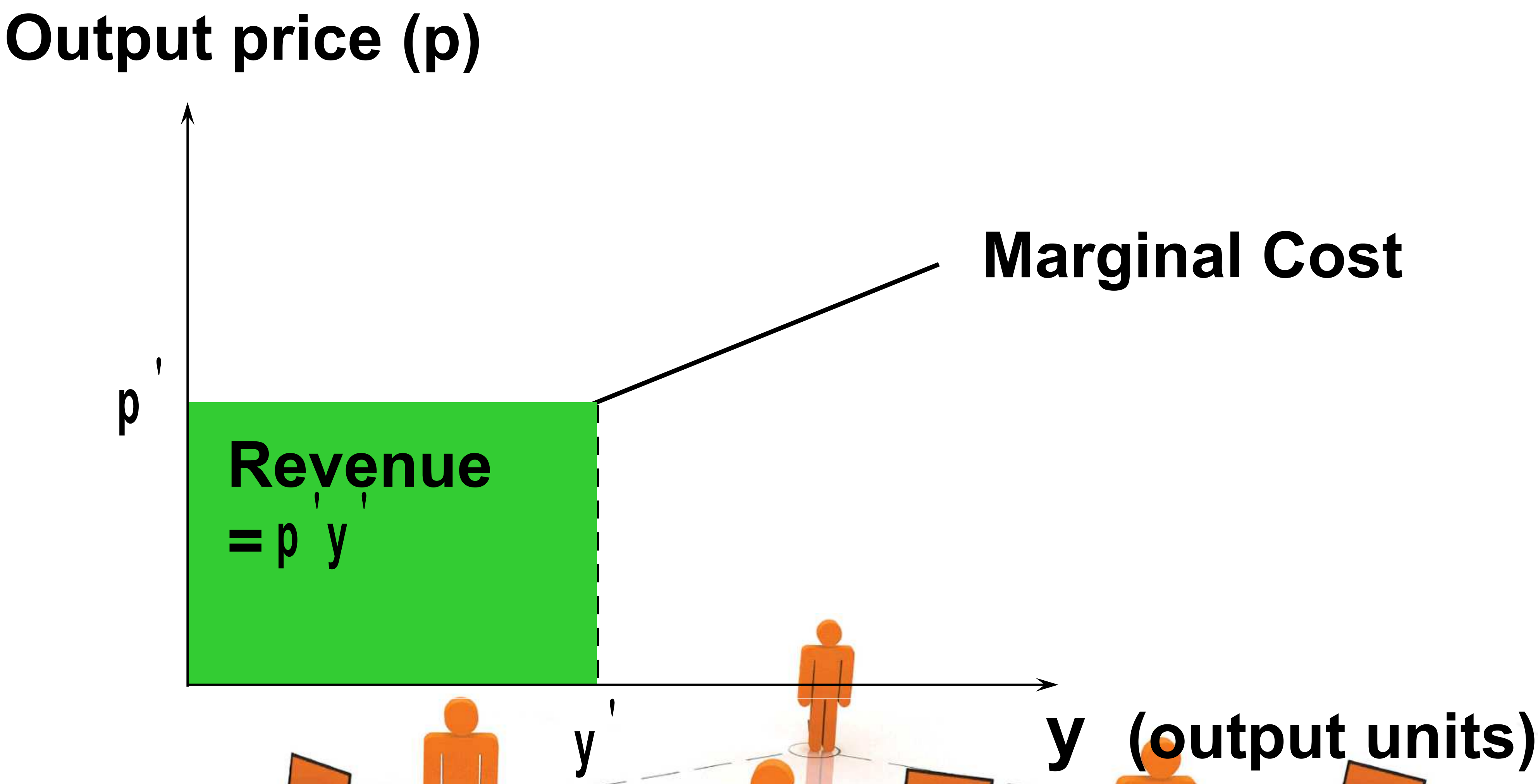
Producer's Surplus



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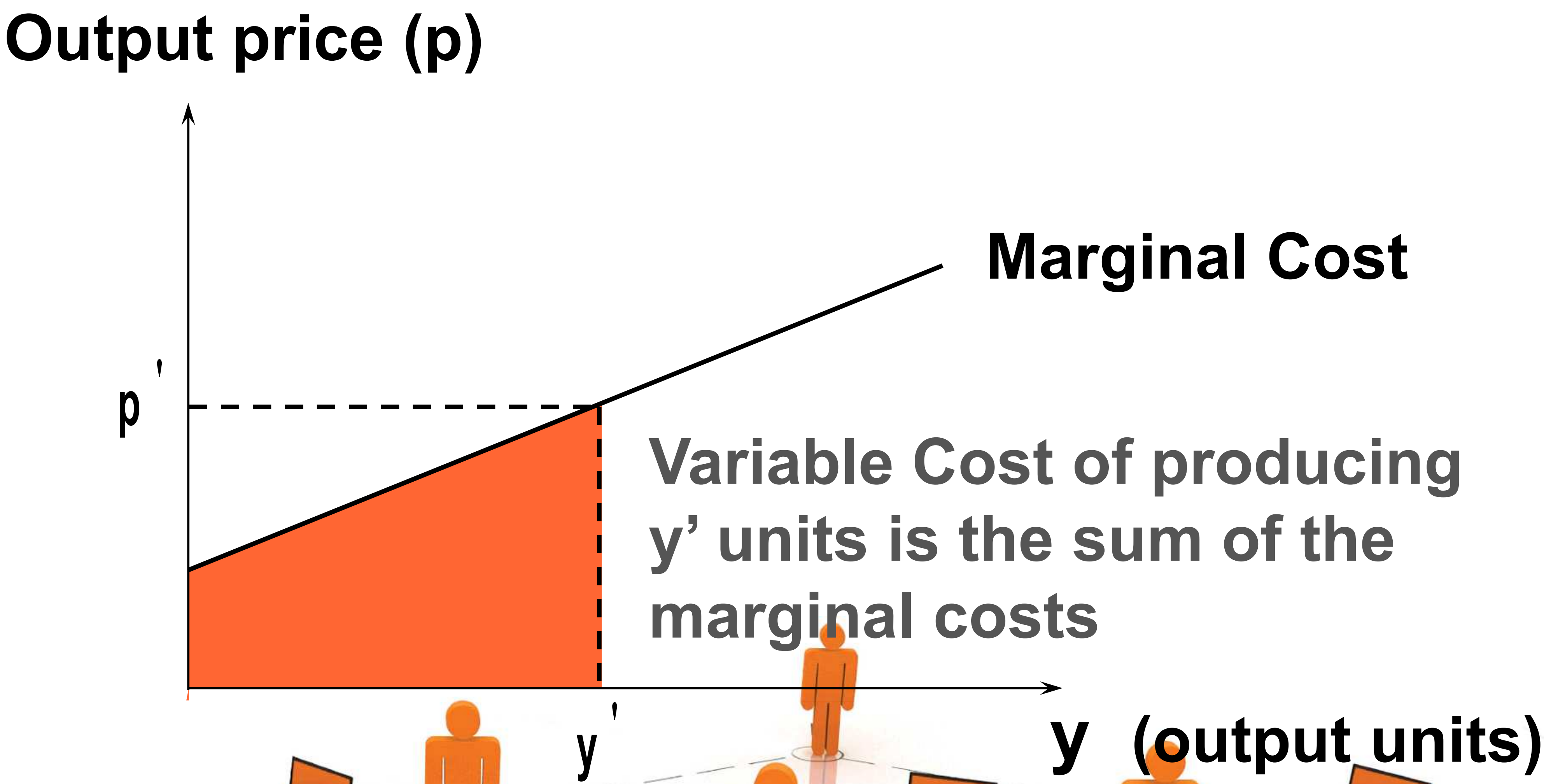
Producer's Surplus



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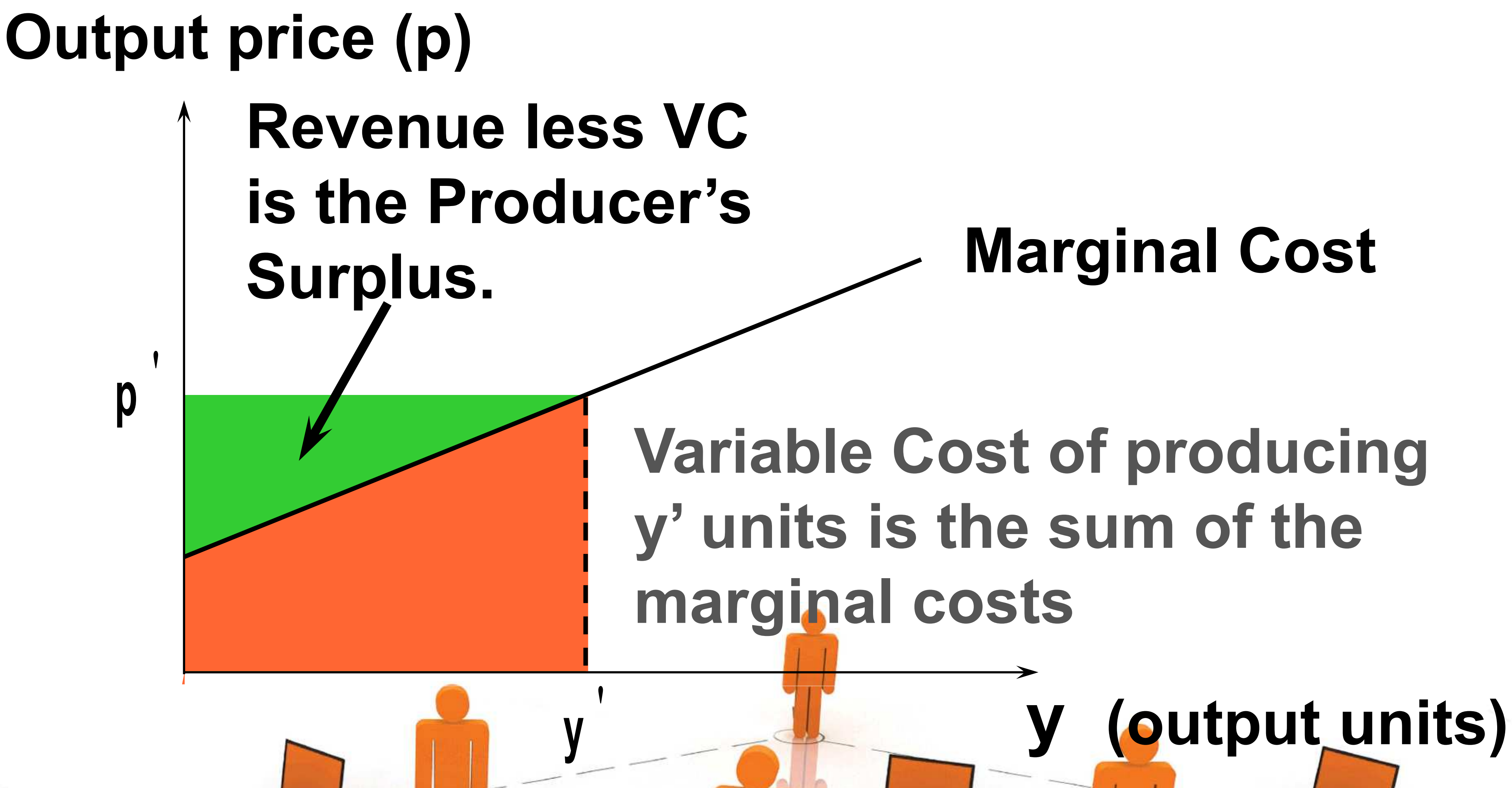
Producer's Surplus



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Benefit-Cost Analysis

- **Can we measure in money units the net gain, or loss, caused by a market intervention; *e.g.,* the imposition or the removal of a market regulation?**
- **Yes, by using measures such as the Consumer's Surplus and the Producer's Surplus.**

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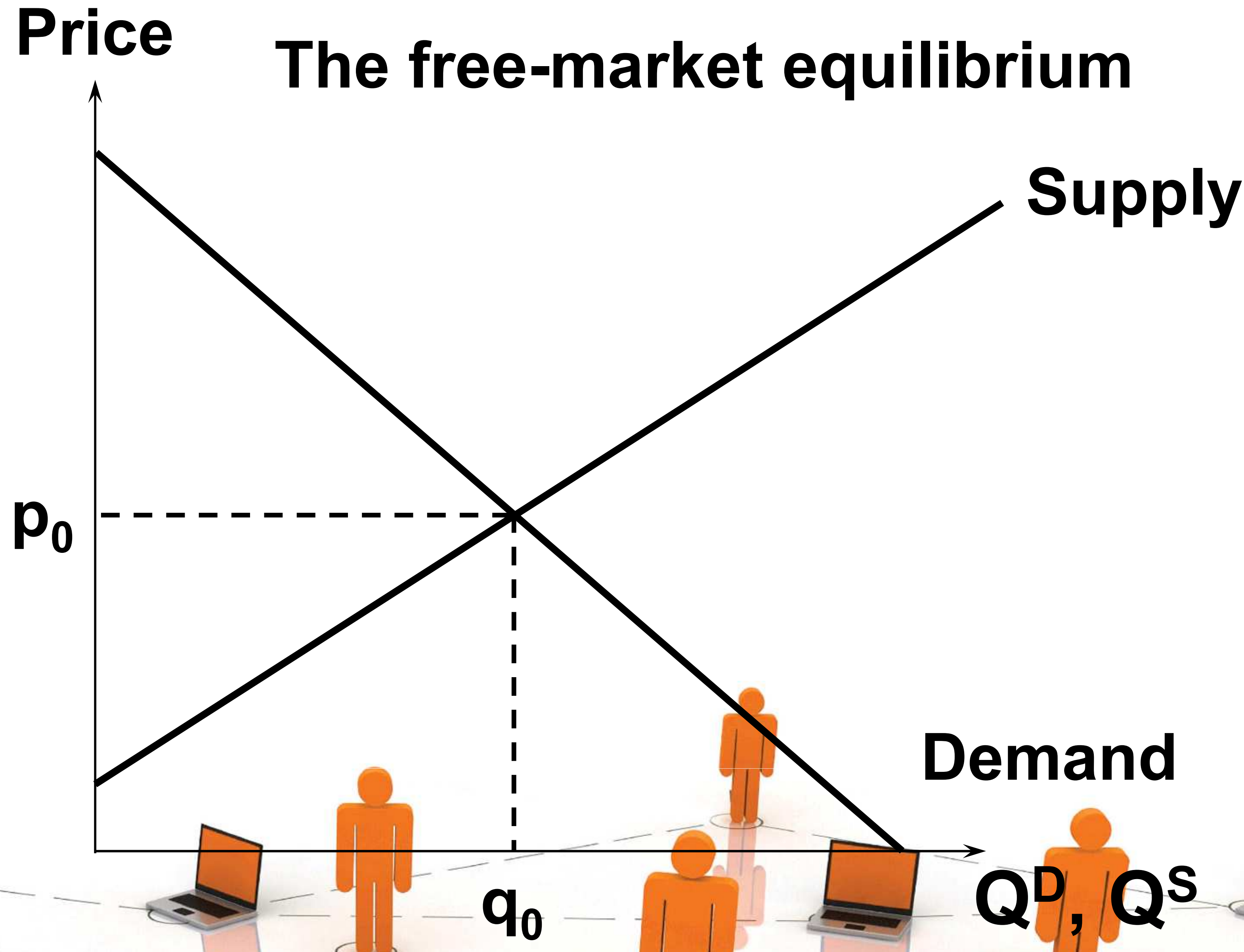
Benefit-Cost Analysis



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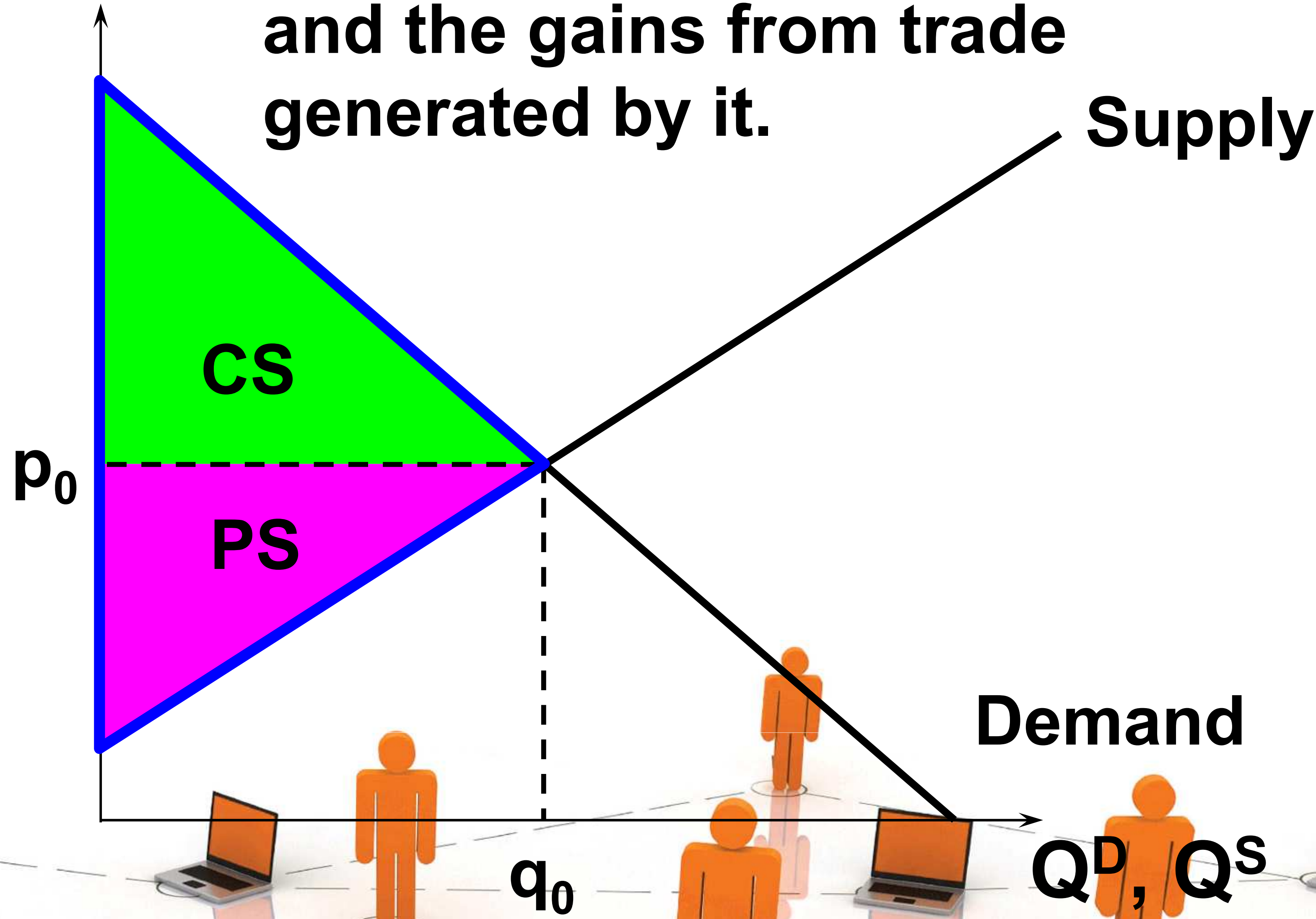
Benefit-Cost Analysis

The free-market equilibrium and the gains from trade generated by it.



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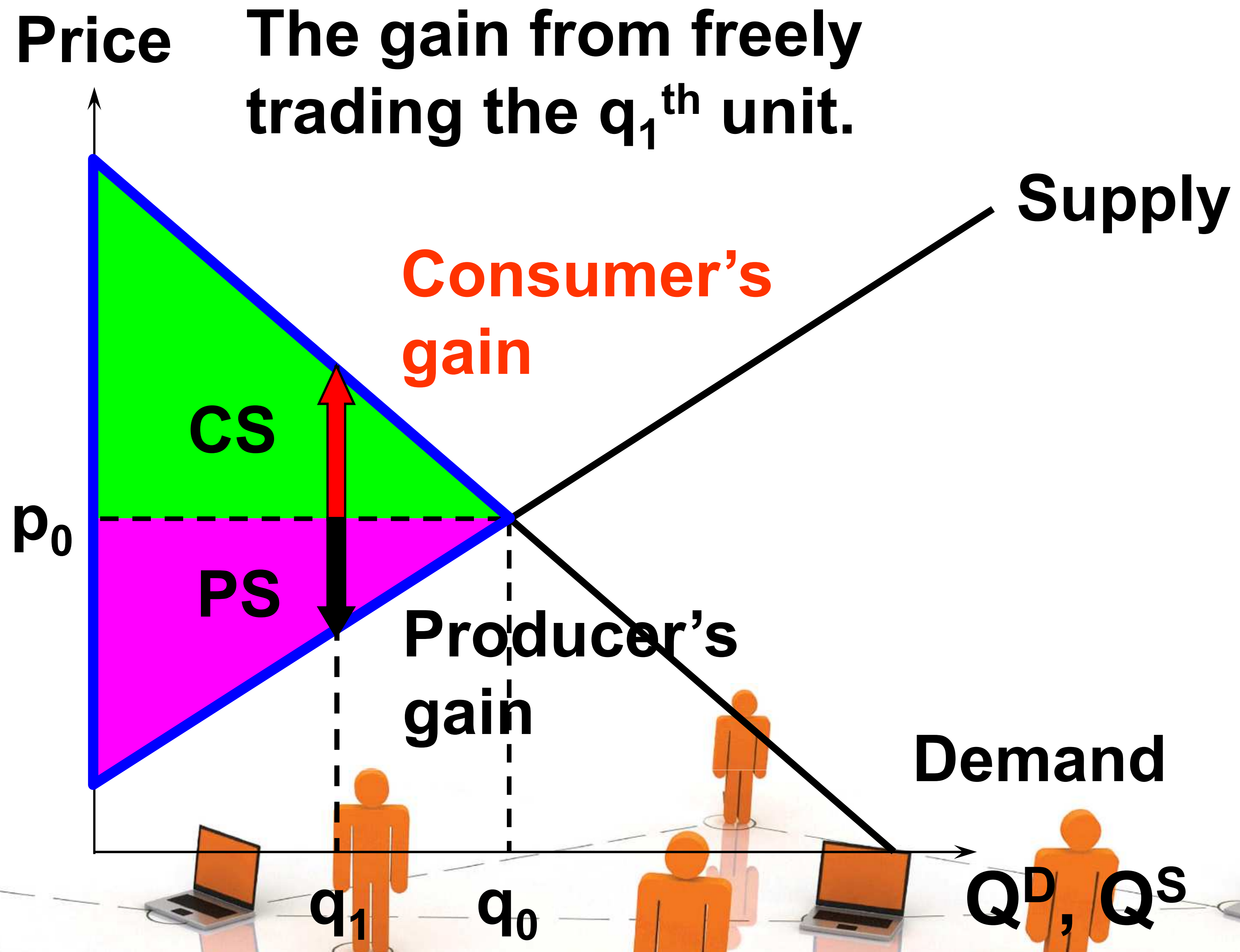
Benefit-Cost Analysis



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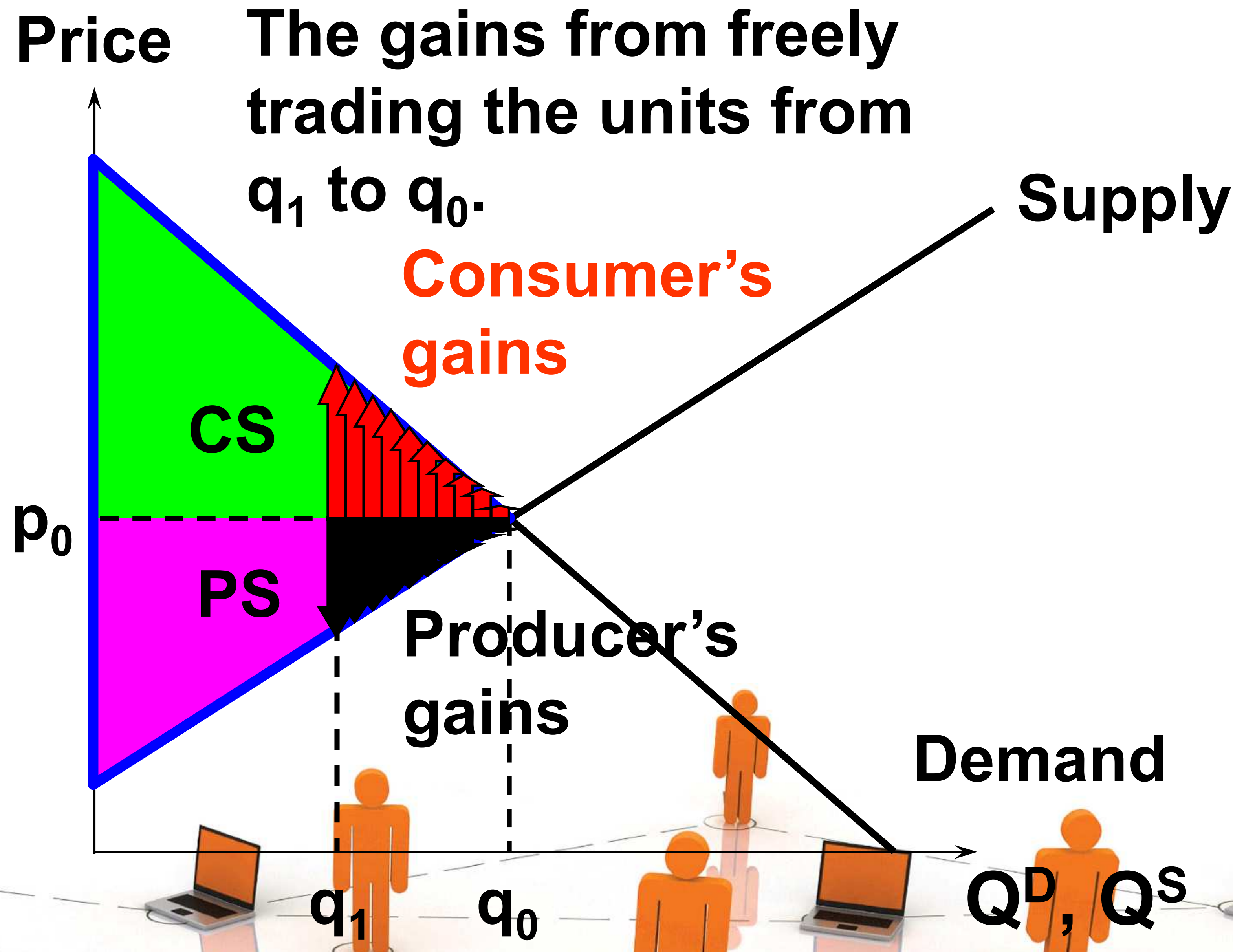
Benefit-Cost Analysis



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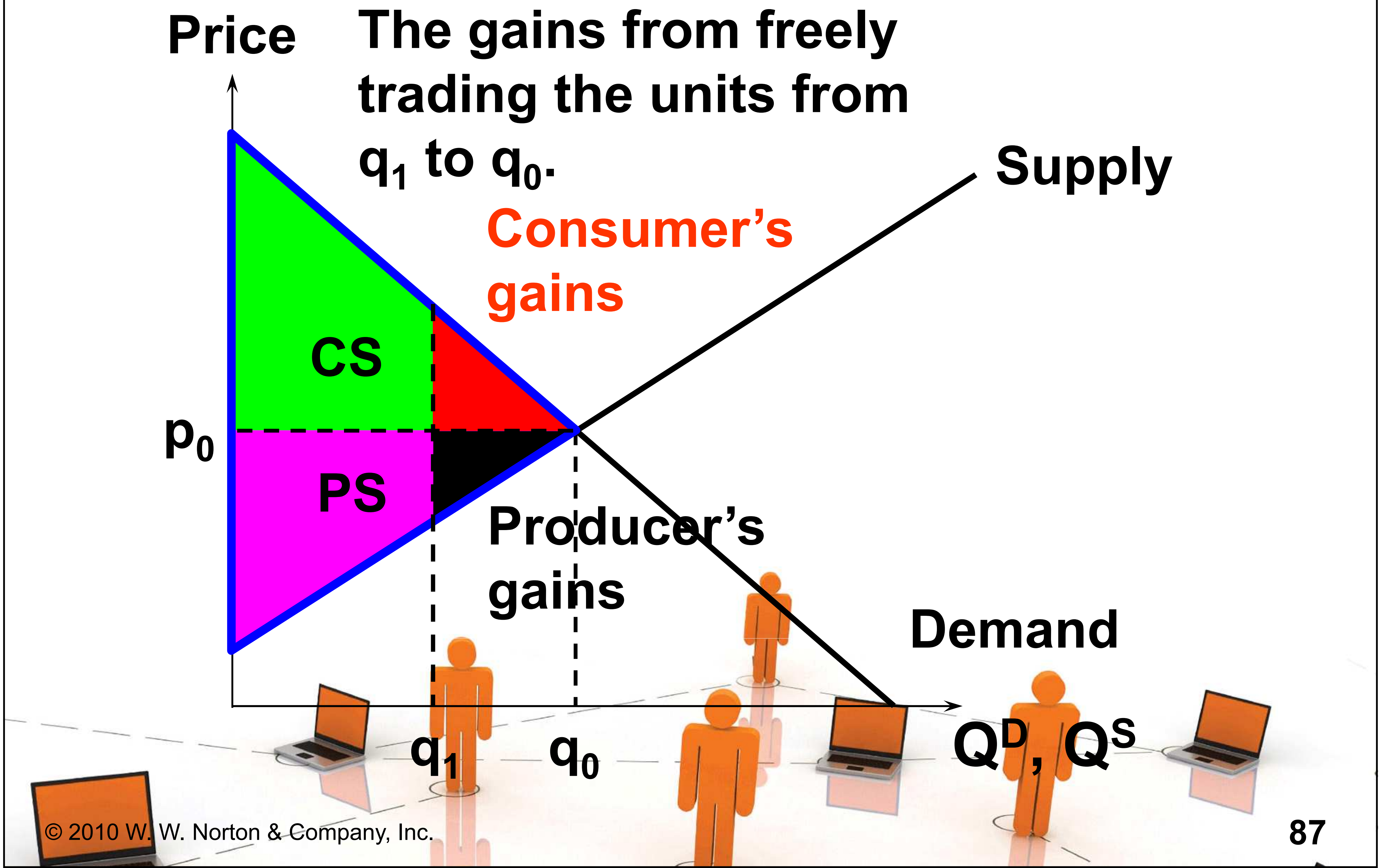
Benefit-Cost Analysis



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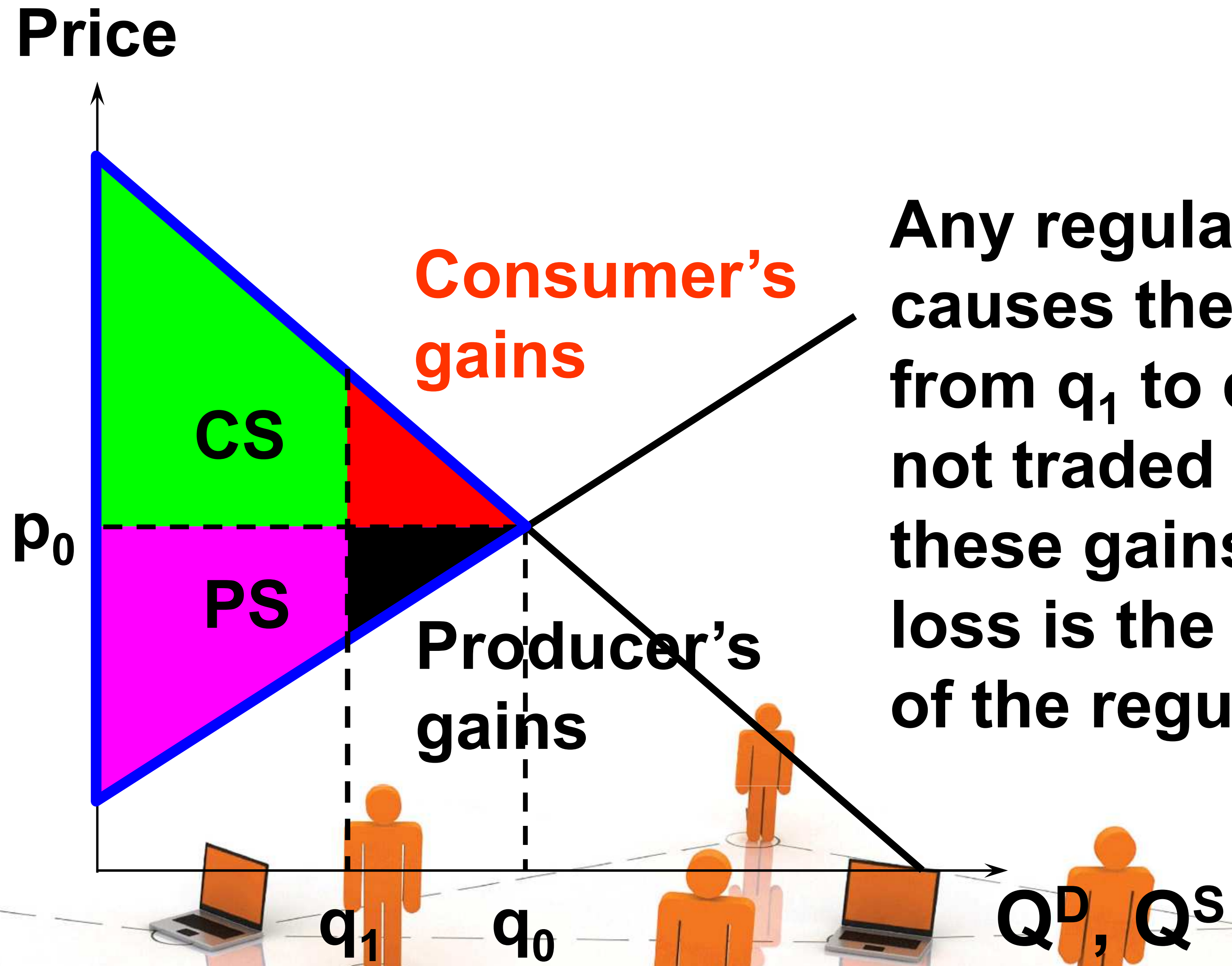
Benefit-Cost Analysis



Any regulation that causes the units from q_1 to q_0 to be not traded destroys these gains. This loss is the net cost of the regulation.

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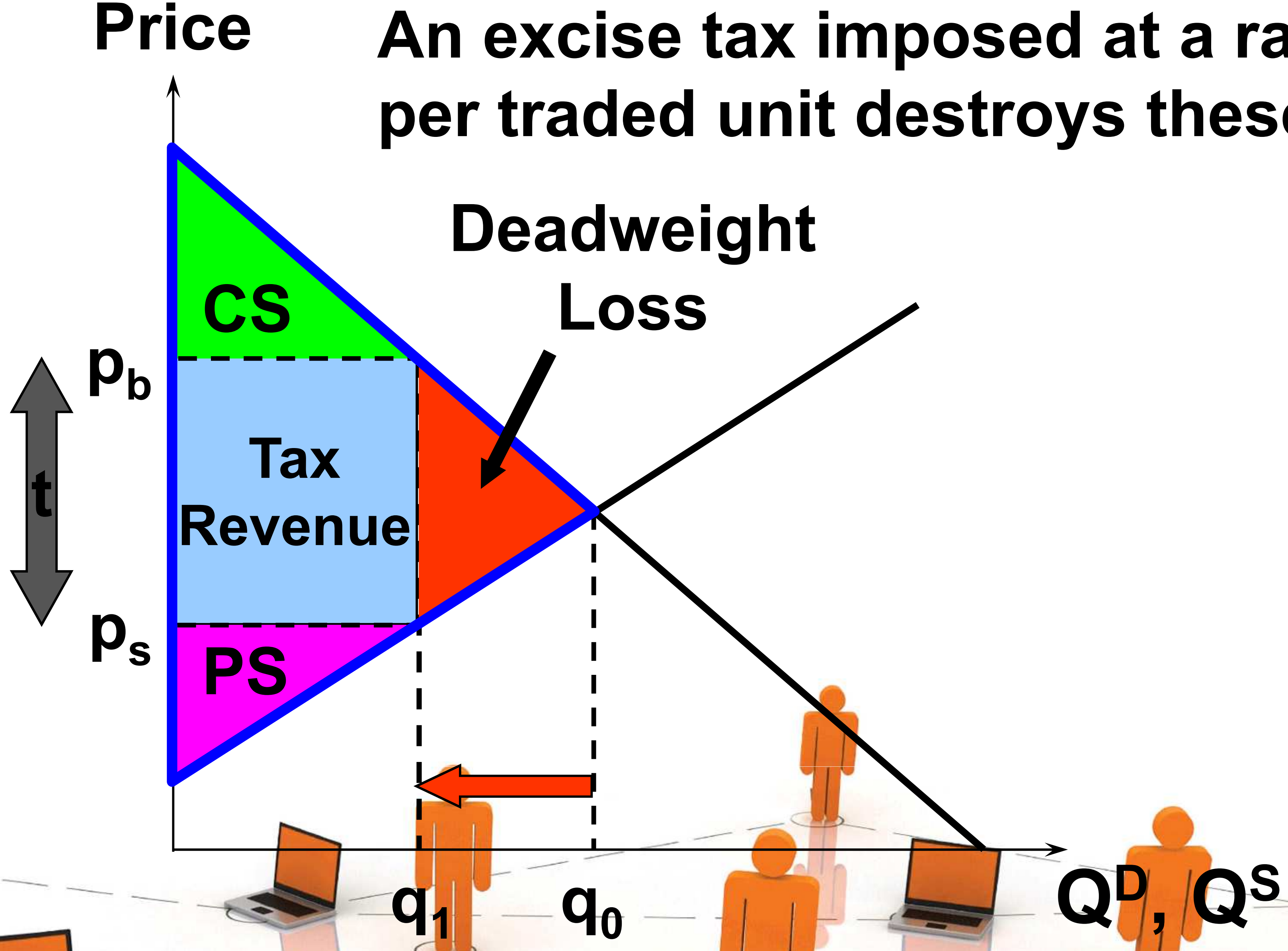
Benefit-Cost Analysis

An excise tax imposed at a rate of $t per traded unit destroys these gains.



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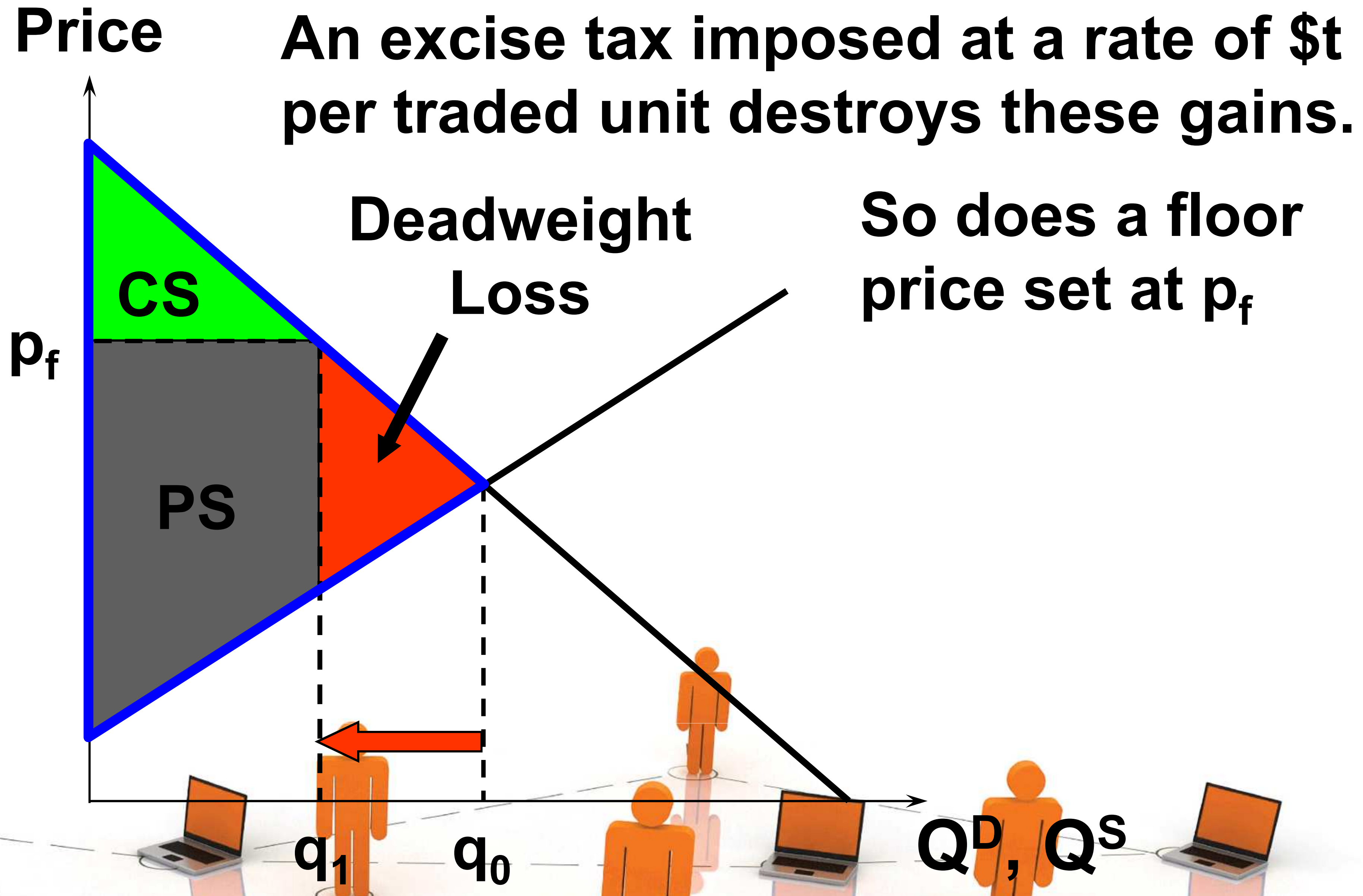
Benefit-Cost Analysis



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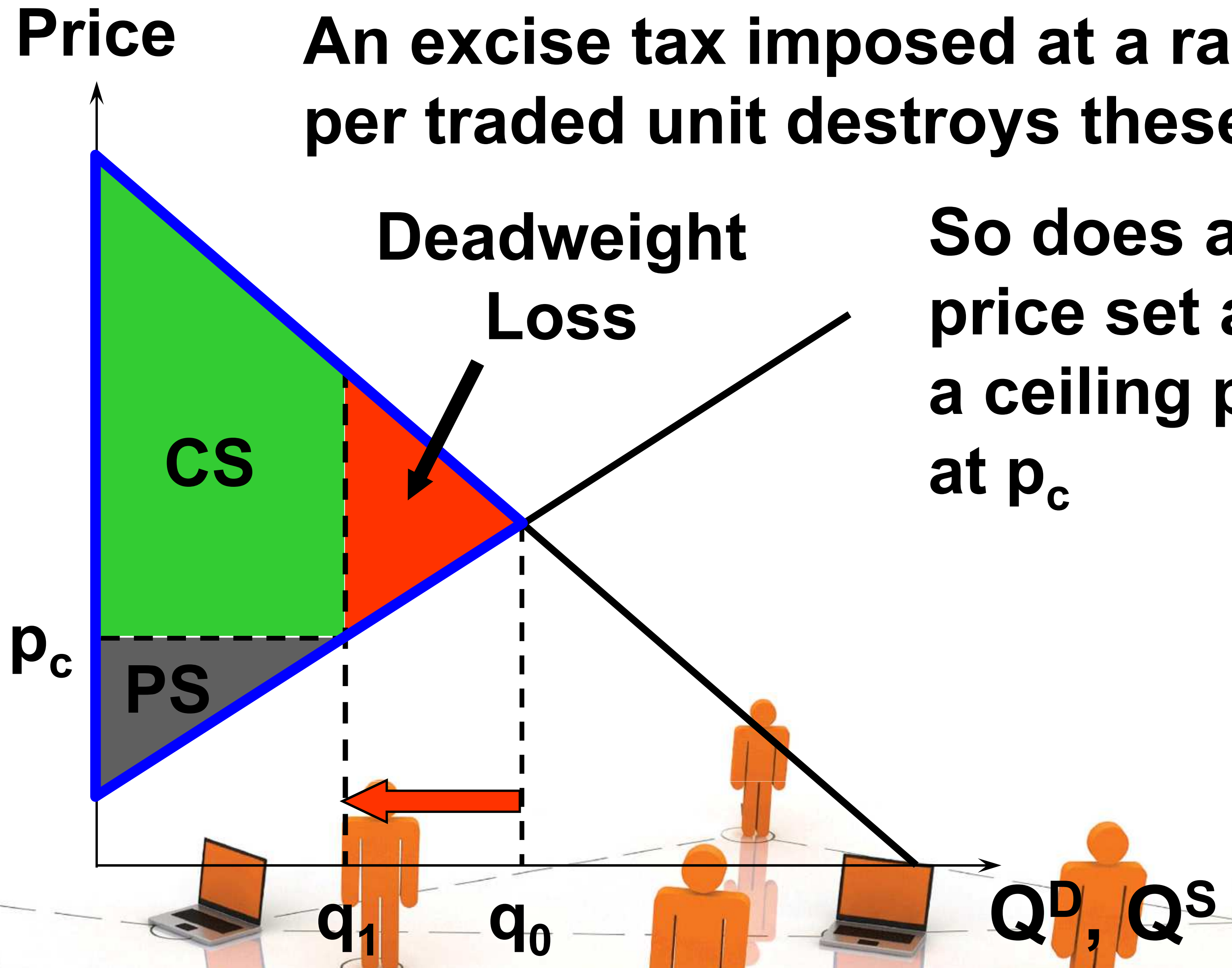
Benefit-Cost Analysis

An excise tax imposed at a rate of $t per traded unit destroys these gains.

So does a floor price set at p_f, a ceiling price set at p_c



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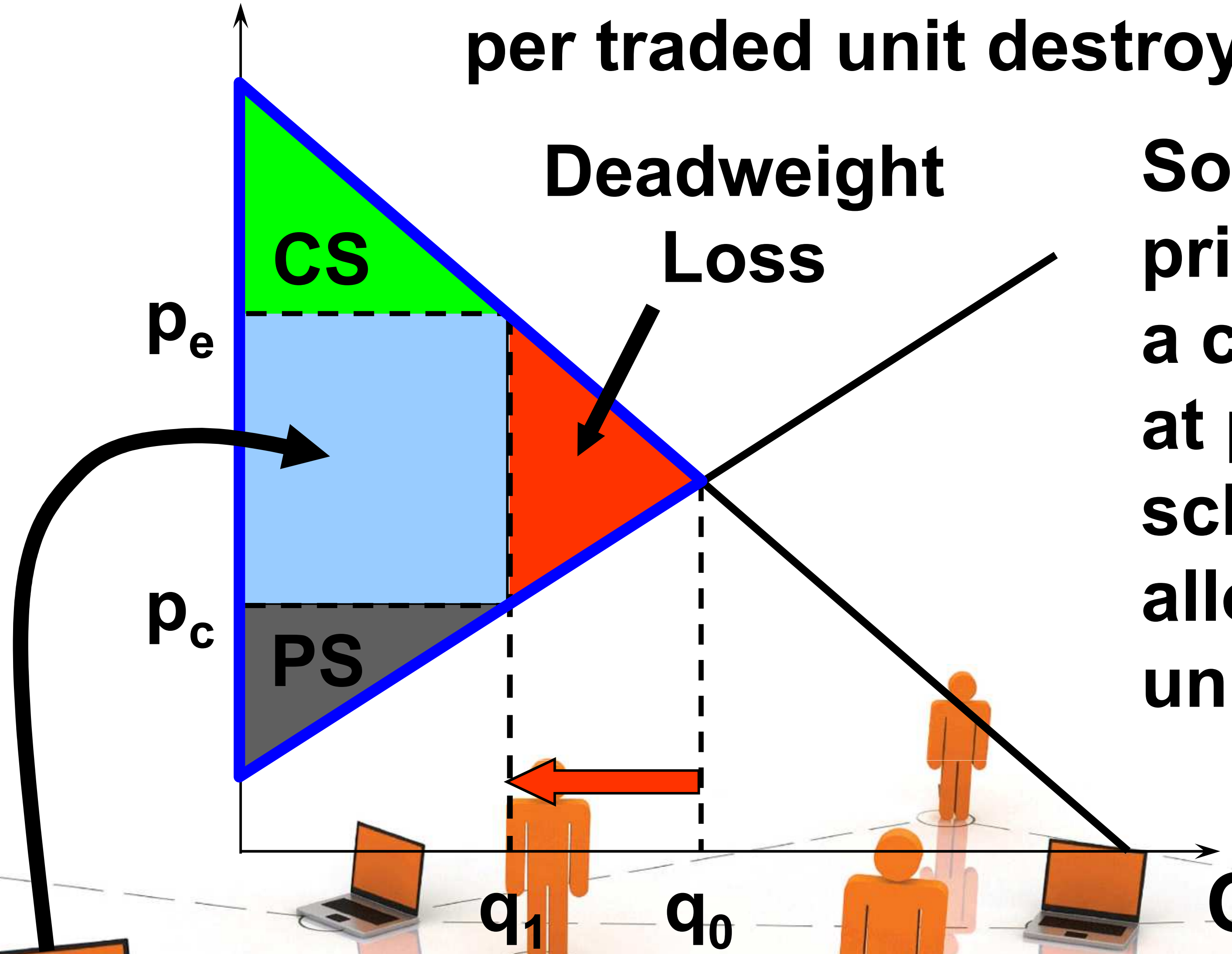
Benefit-Cost Analysis

An excise tax imposed at a rate of $t per traded unit destroys these gains.



So does a floor price set at p_f, a ceiling price set at p_c, and a ration scheme that allows only q_1 units to be traded.

Revenue received by holders of ration coupons.

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