8TH EDITION

INTERMEDIATE MICROECONOMICS

HAL R. VARIAN

21 Cost Curves

Types of Cost Curves

- **A total cost curve is the graph of a firm's total cost function.**
- **A variable cost curve is the graph of a firm's variable cost function.**
- **An average total cost curve is the graph of a firm's average total cost function.**

Types of Cost Curves

- **An average variable cost curve is the graph of a firm's average variable cost function.**
- **An average fixed cost curve is the graph of a firm's average fixed cost function.**
- **A marginal cost curve is the graph of a firm's marginal cost function.**

Types of Cost Curves

- **How are these cost curves related to each other?**
- **How are a firm's long-run and short-run cost curves related?**

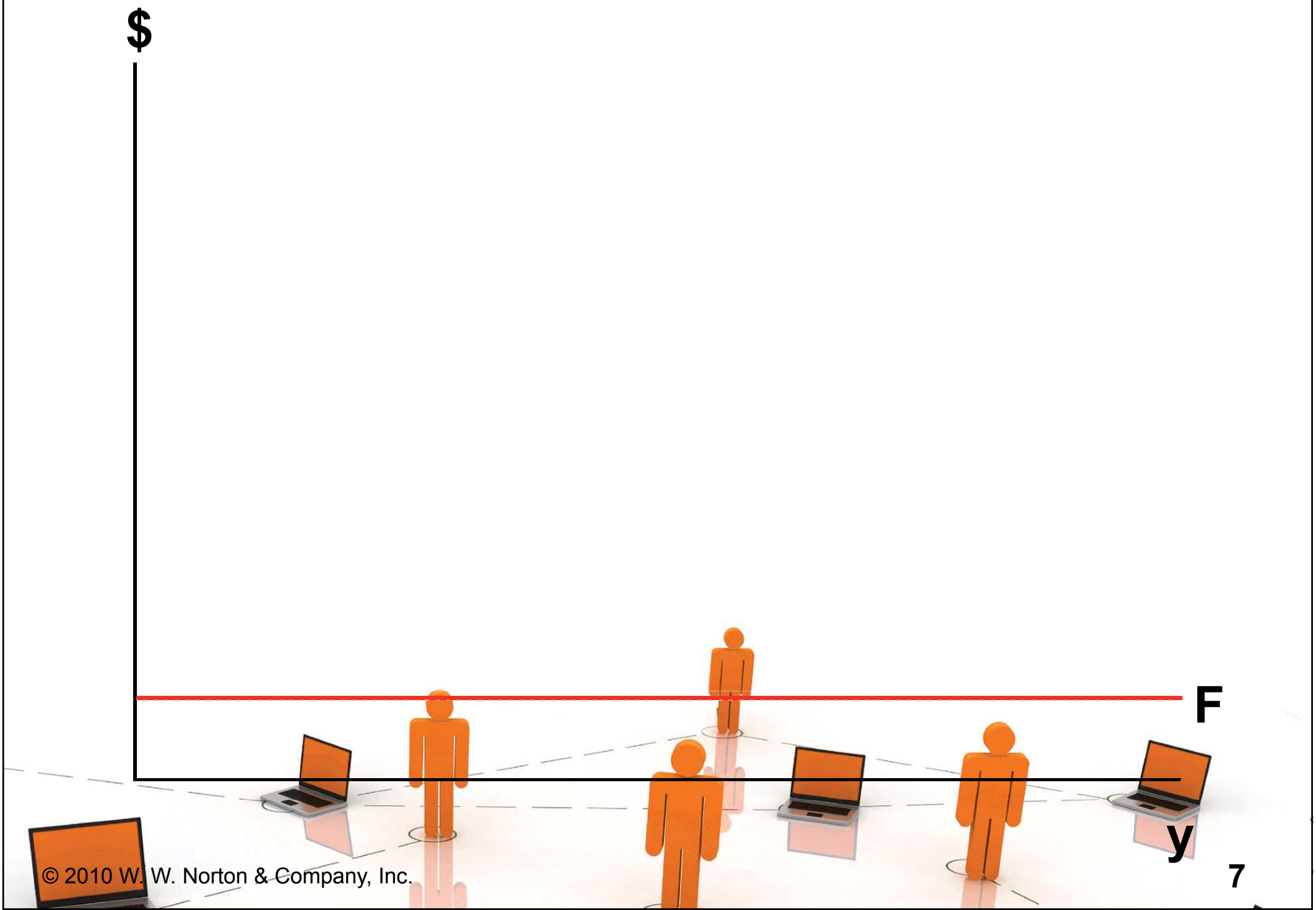
Fixed, Variable & Total Cost Functions

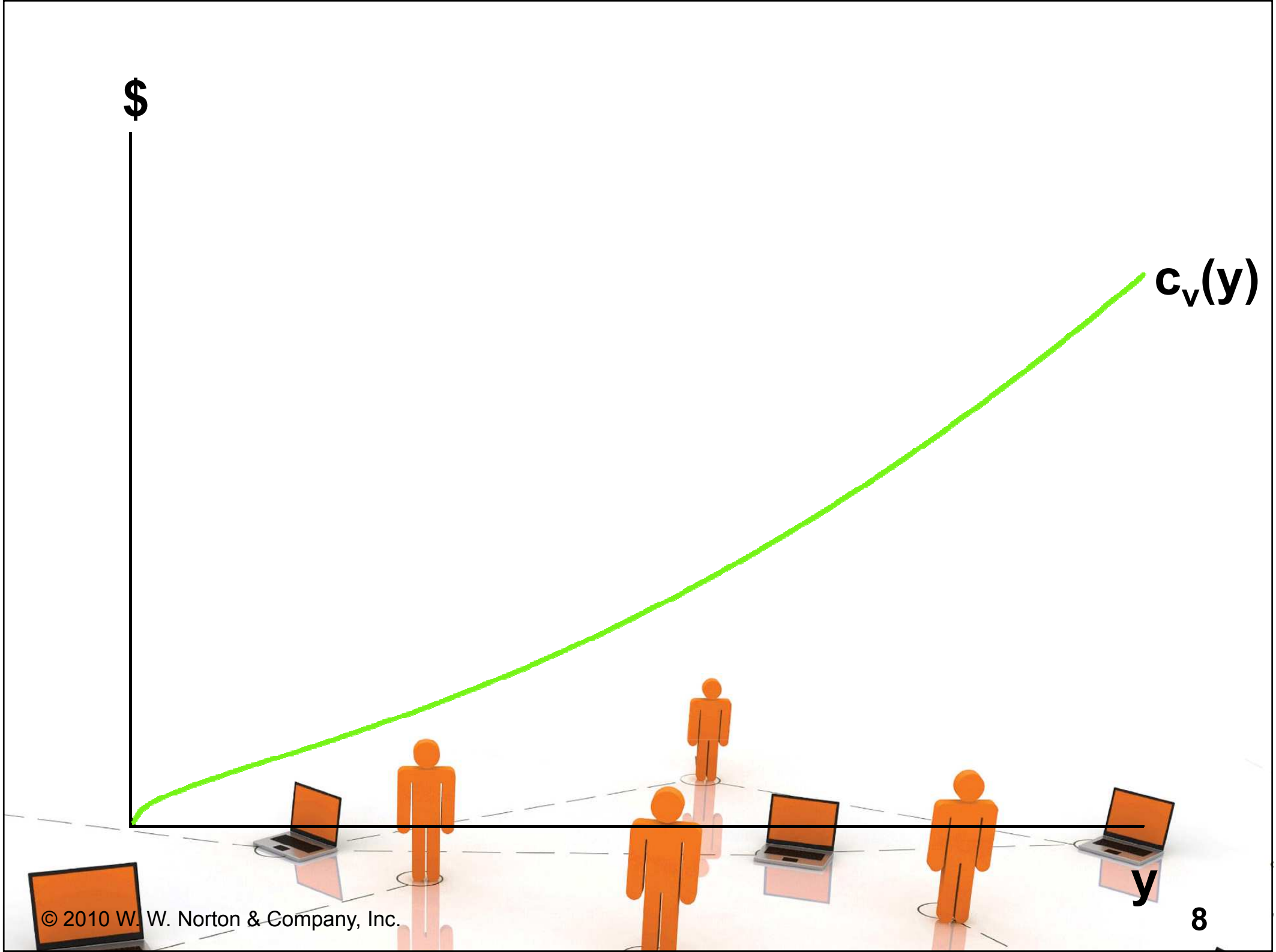
- **F is the total cost to a firm of its short-run fixed inputs. F, the firm's fixed cost, does not vary with the firm's output level.**
- **$c_v(y)$ is the total cost to a firm of its variable inputs when producing y output units. $c_v(y)$ is the firm's variable cost function.**
- **$c_v(y)$ depends upon the levels of the fixed inputs.**

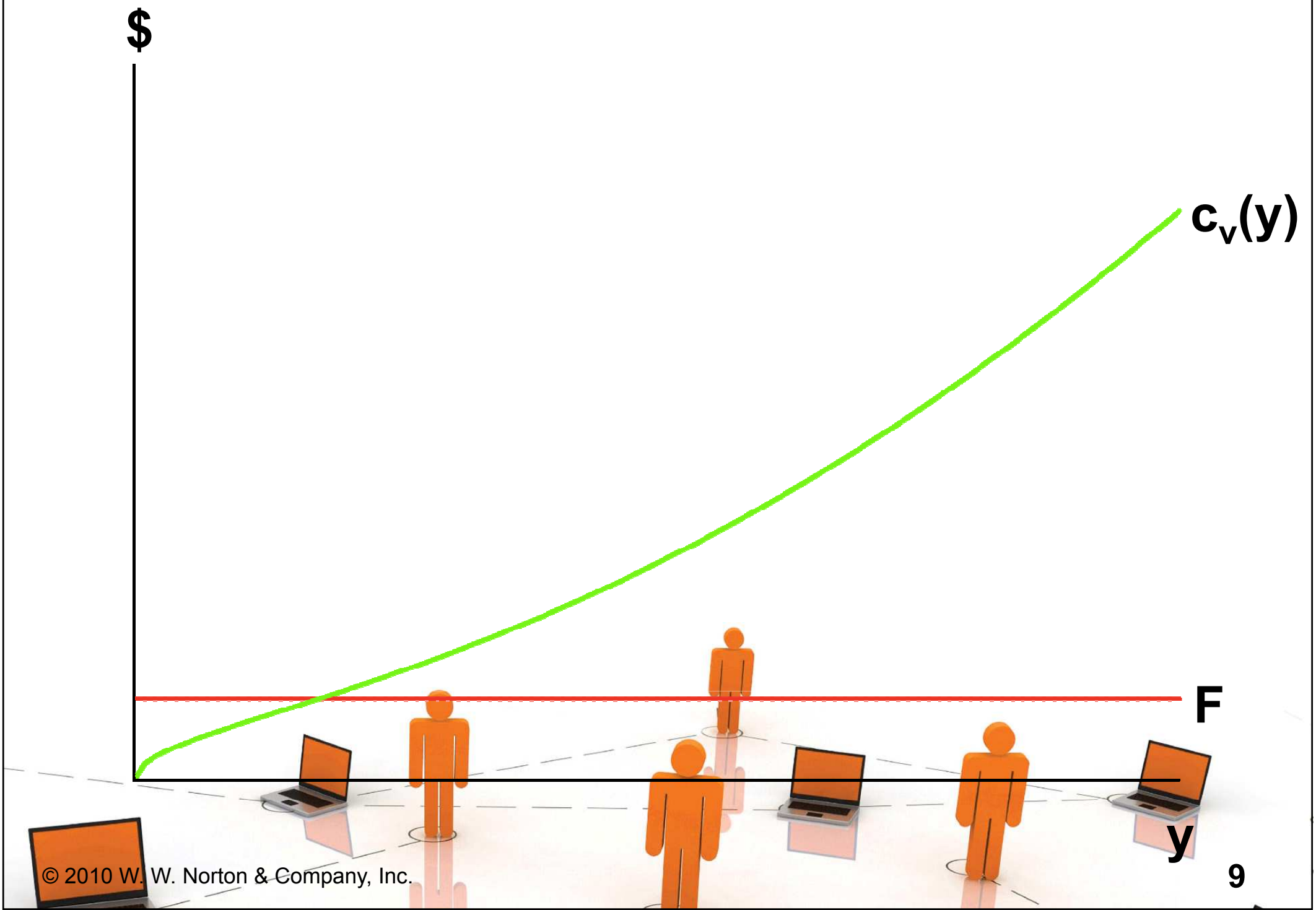
Fixed, Variable & Total Cost Functions

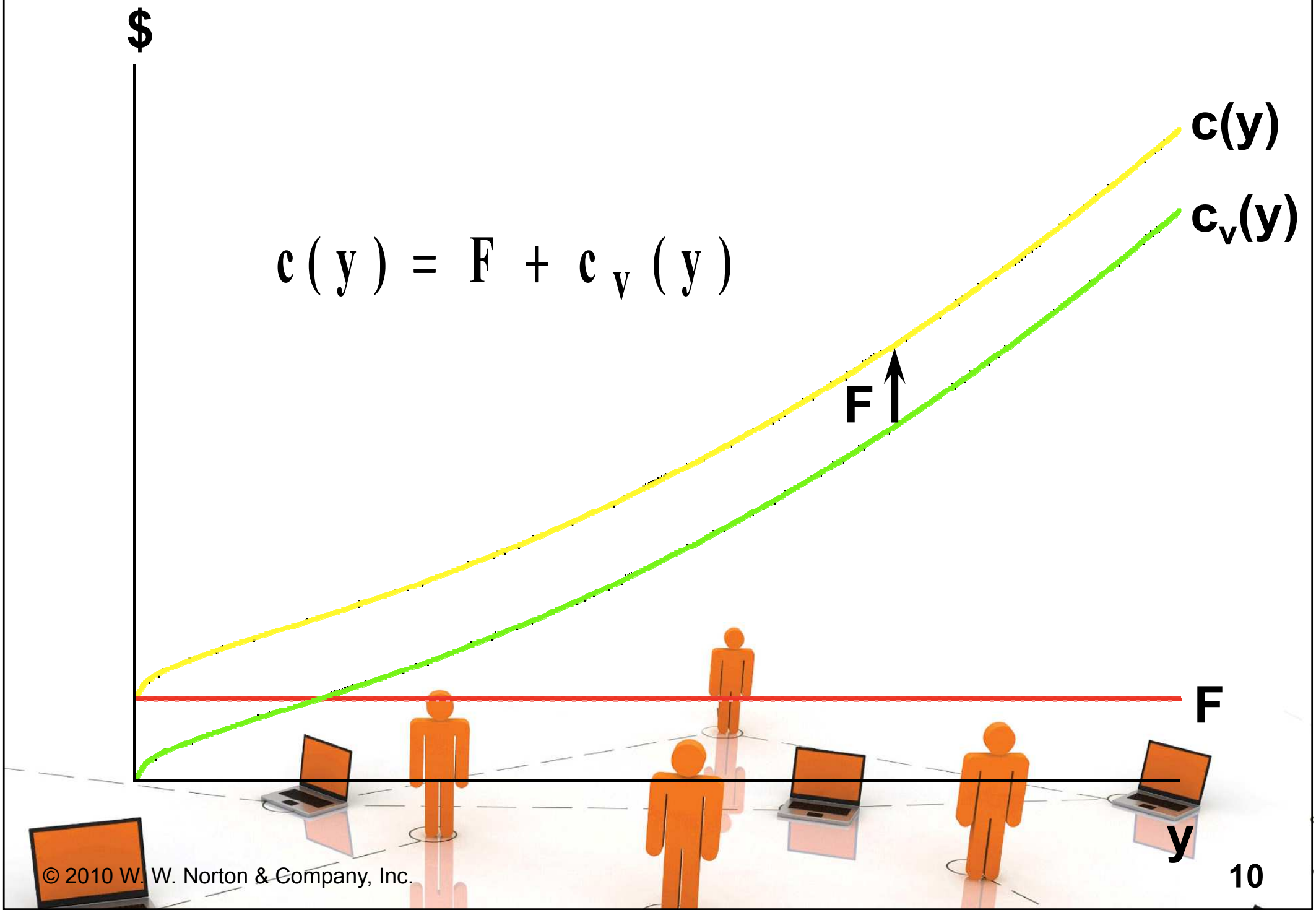
- **c(y) is the total cost of all inputs, fixed and variable, when producing y output units. c(y) is the firm's total cost function;**

$$c(y) = F + c_V(y).$$









Av. Fixed, Av. Variable & Av. Total Cost Curves

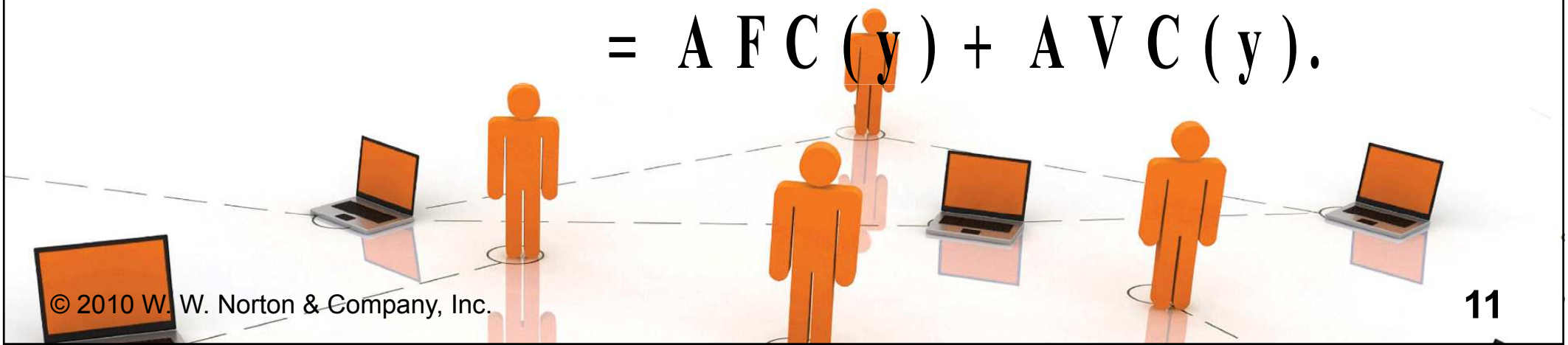
- **The firm's total cost function is**

$$c(y) = F + c_V(y).$$

For y > 0, the firm's average total cost function is

$$AC(y) = \frac{F}{y} + \frac{c_V(y)}{y}$$

$$= AFC(y) + AVC(y).$$

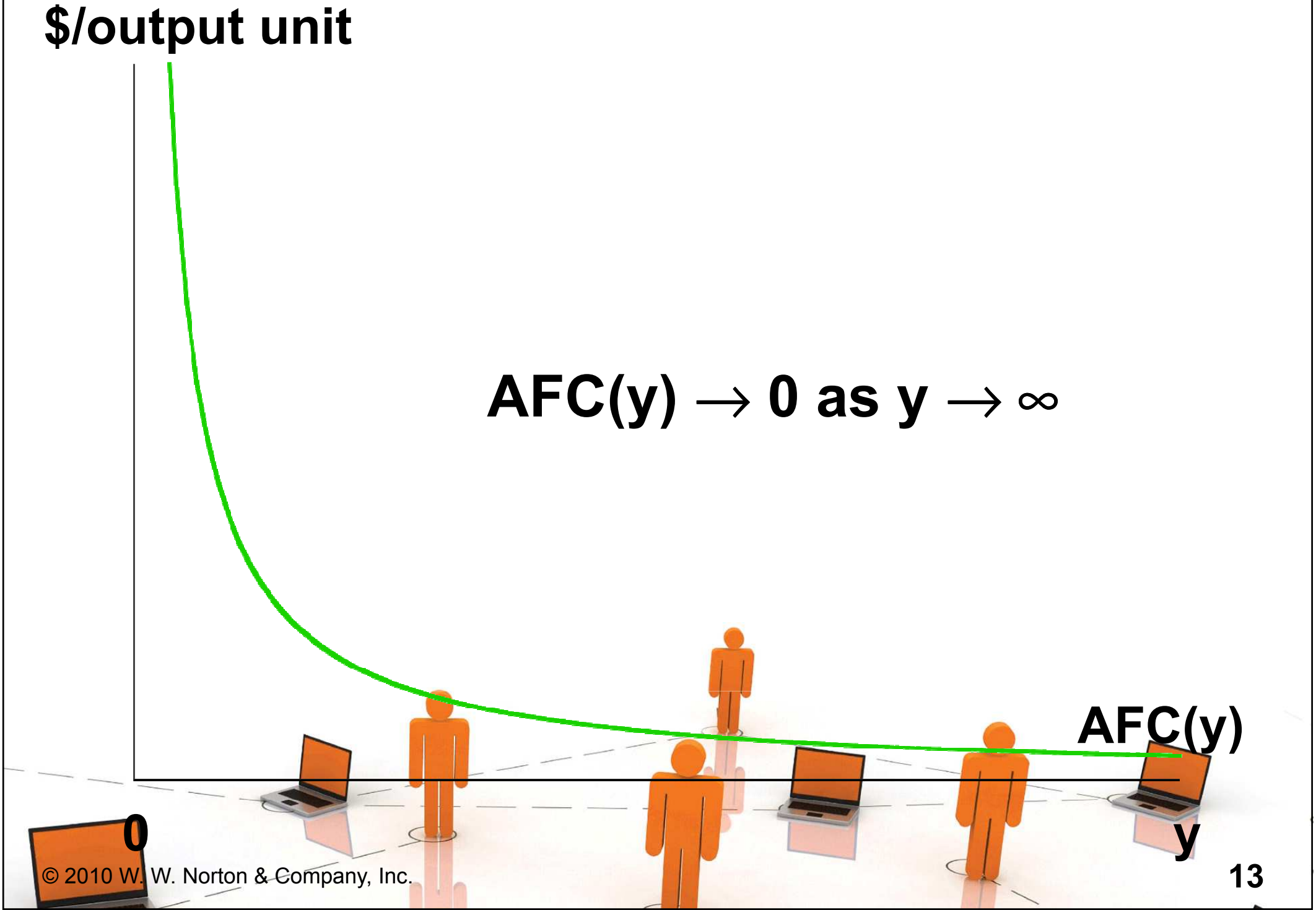


Av. Fixed, Av. Variable & Av. Total Cost Curves

- **What does an average fixed cost curve look like?**

$$AFC(y) = \frac{F}{y}$$

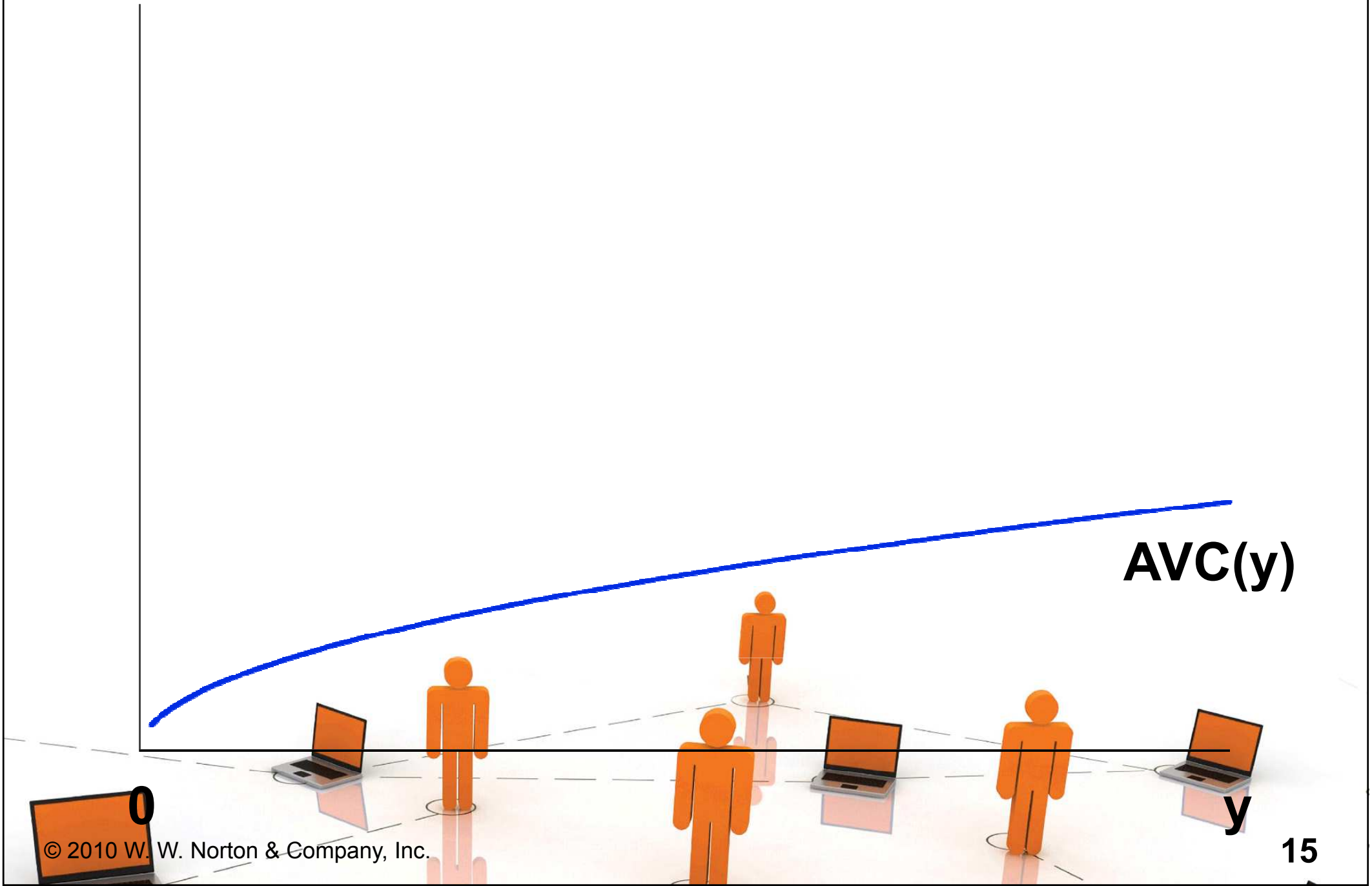
- **AFC(y) is a rectangular hyperbola so its graph looks like ...**

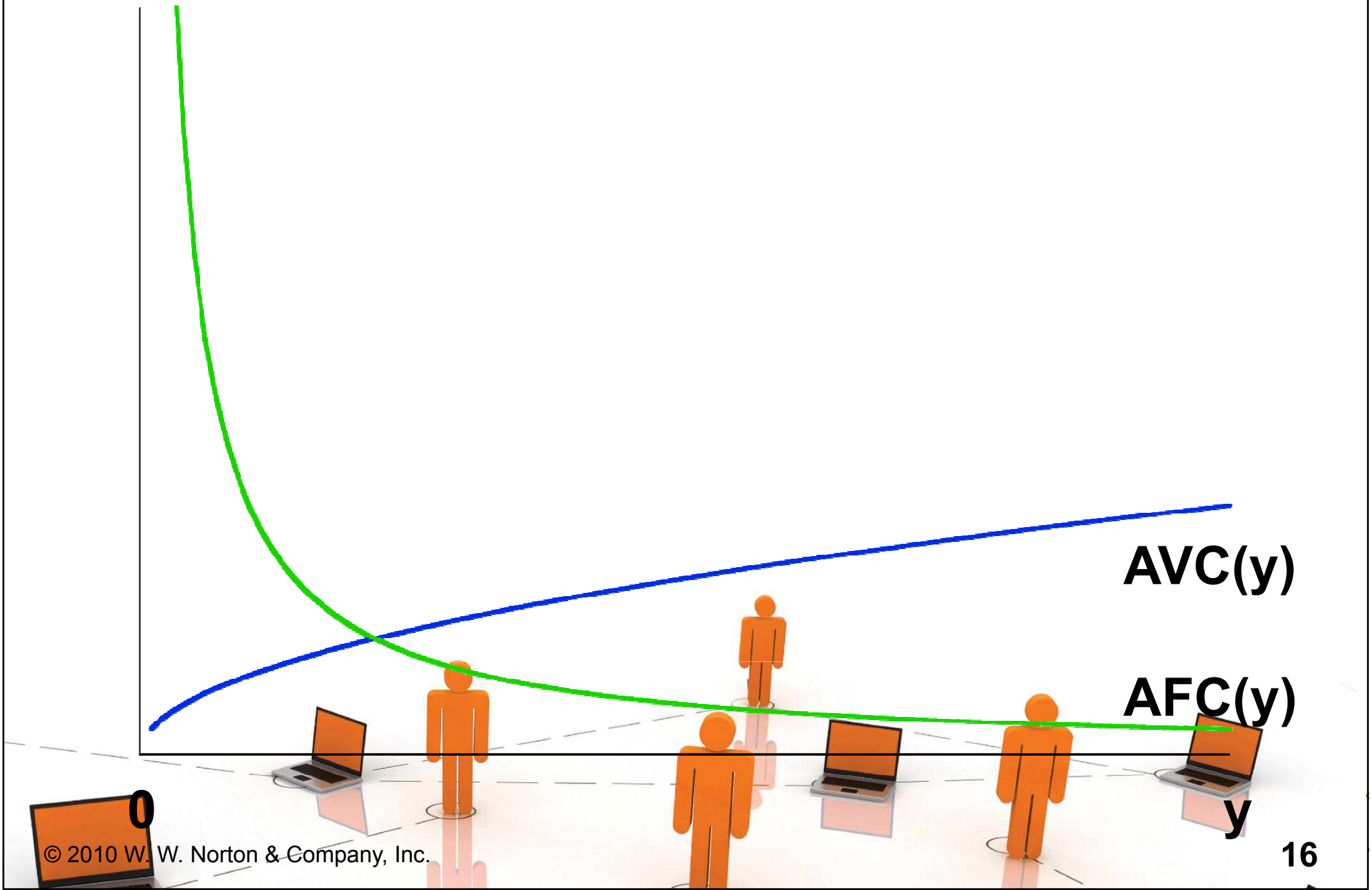


Av. Fixed, Av. Variable & Av. Total Cost Curves

- **In a short-run with a fixed amount of at least one input, the Law of Diminishing (Marginal) Returns must apply, causing the firm's average variable cost of production to increase eventually.**

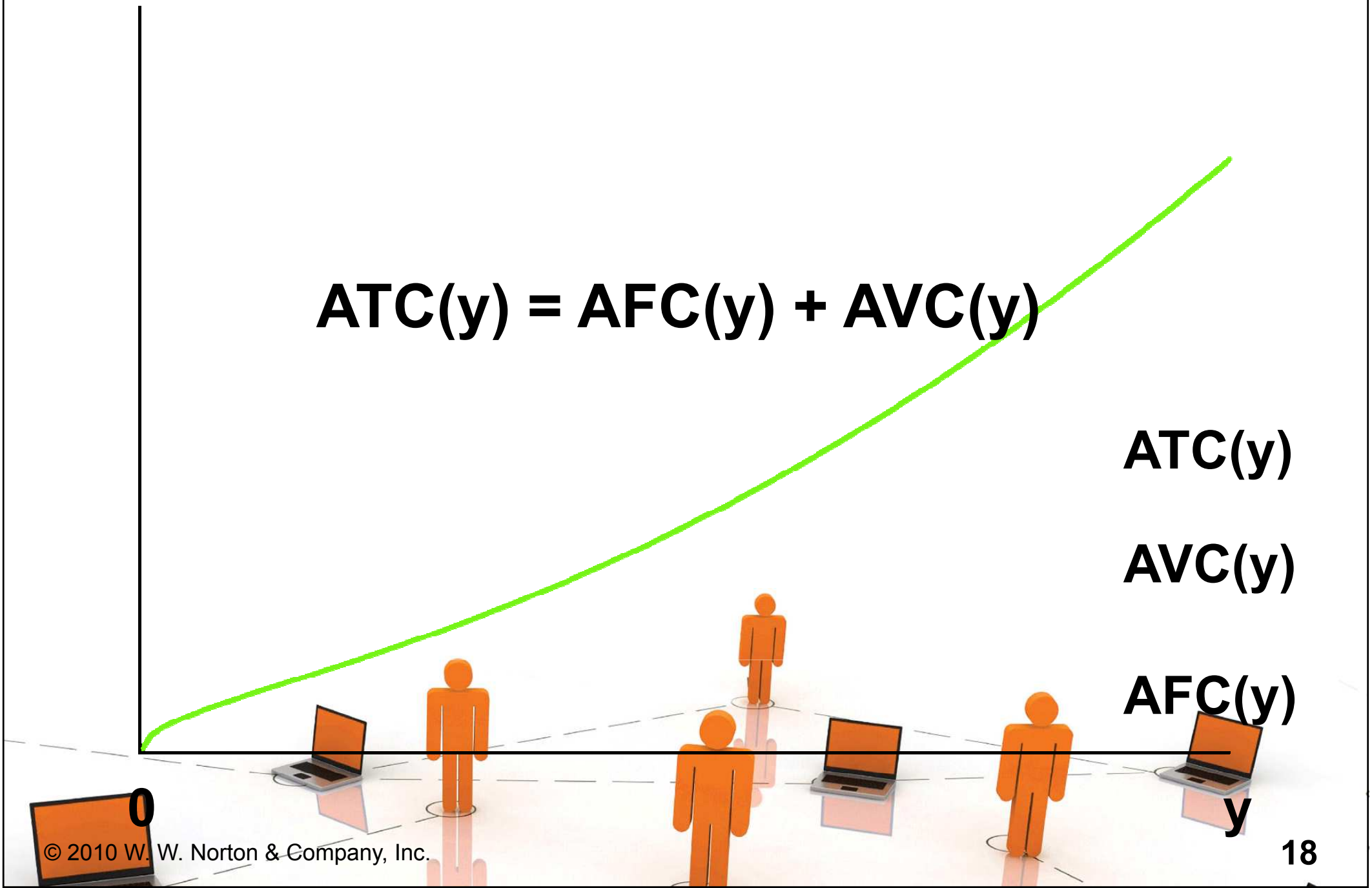
$/output unit

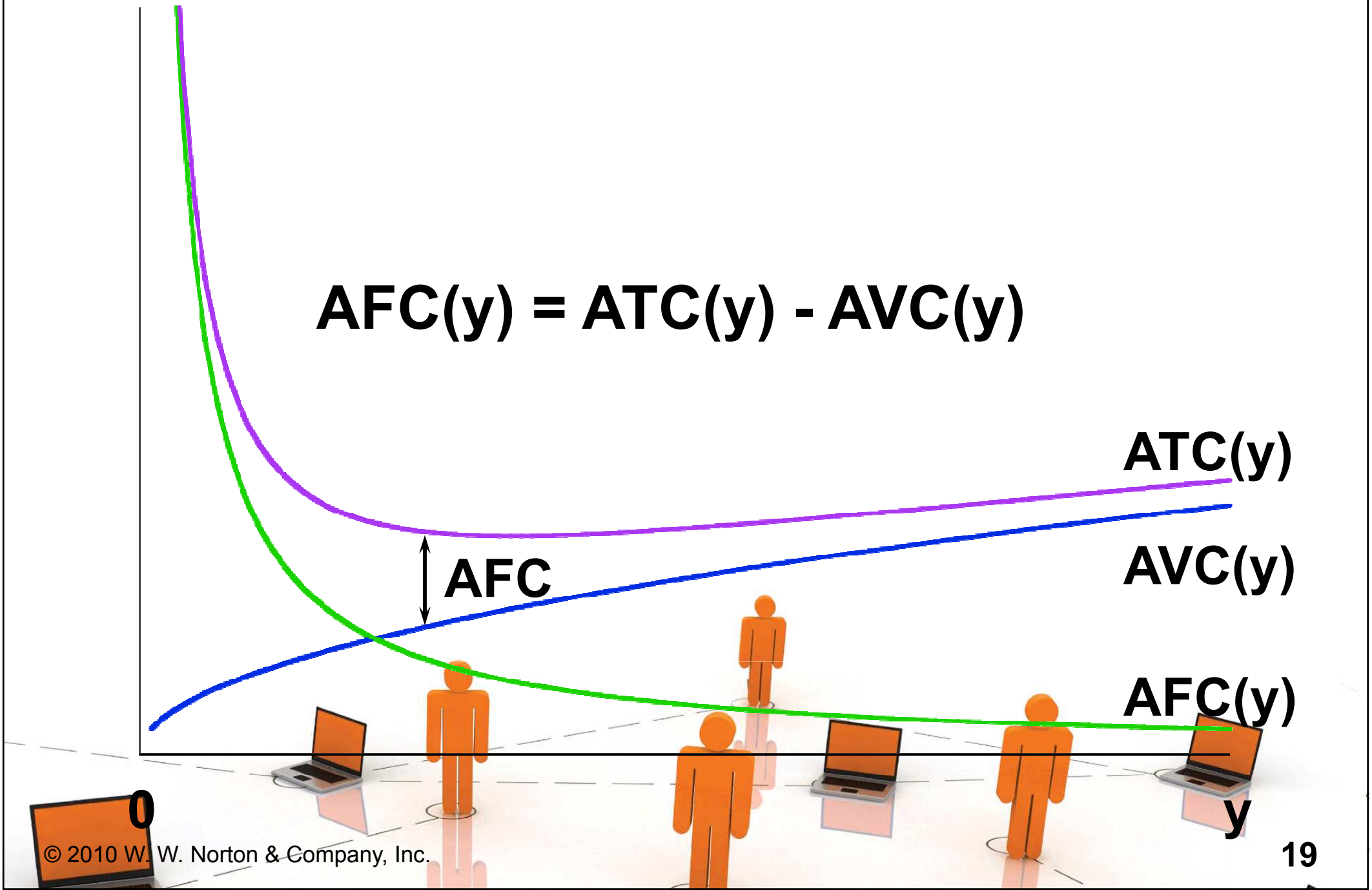




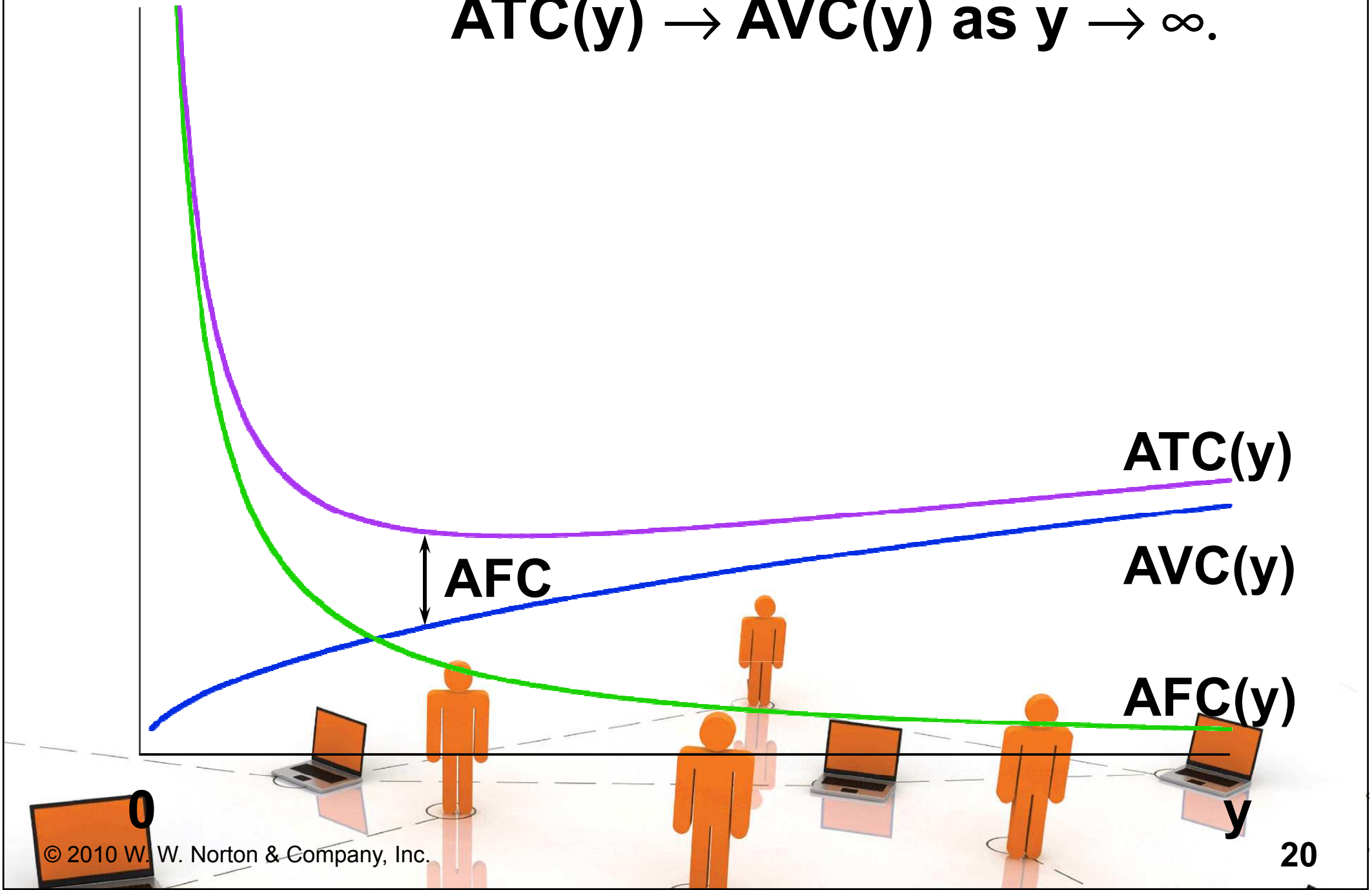
Av. Fixed, Av. Variable & Av. Total Cost Curves

- **And ATC(y) = AFC(y) + AVC(y)**



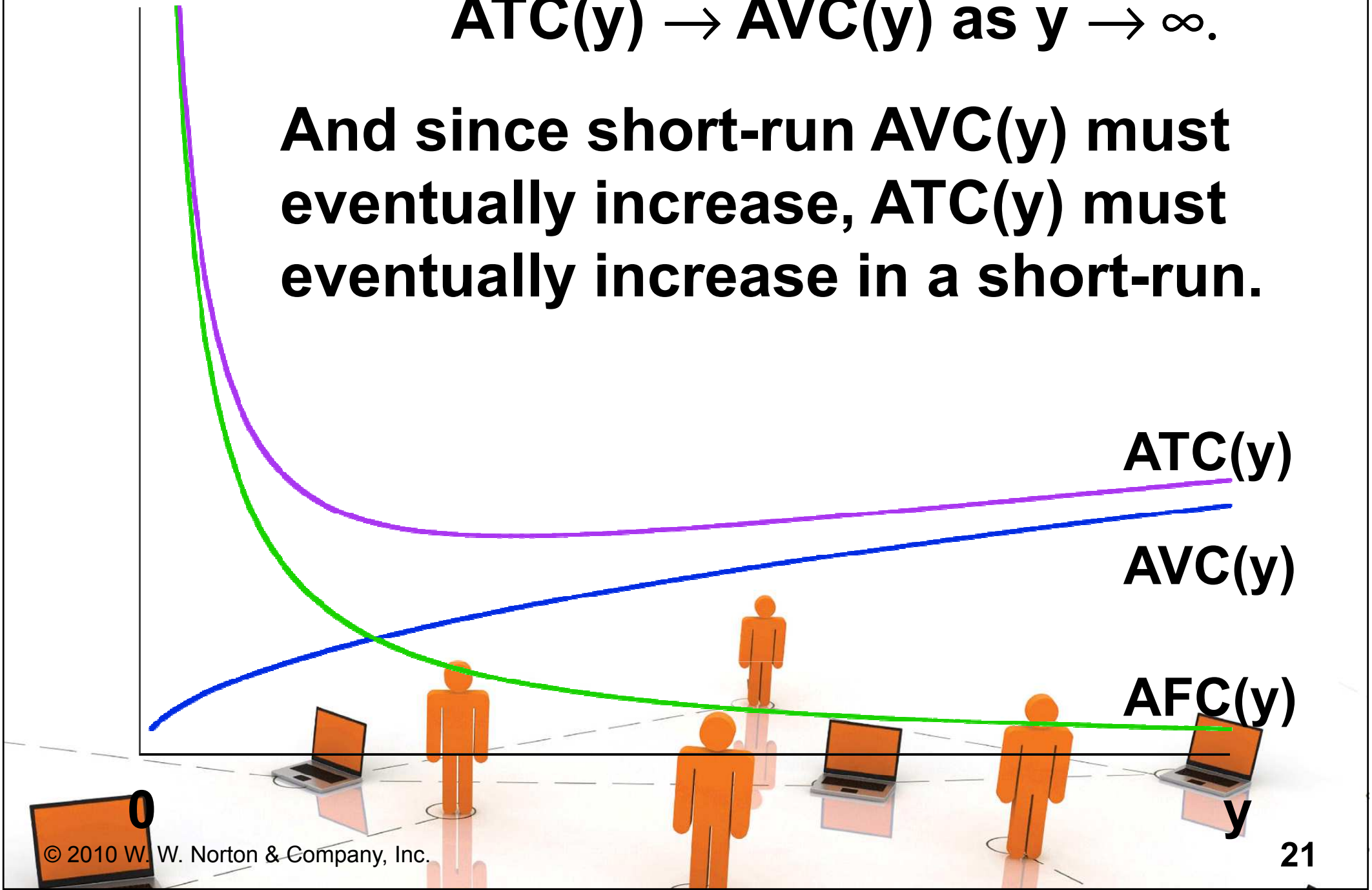


Since AFC(y) → 0 as y → ∞,
ATC(y) → AVC(y) as y → ∞.



Since AFC(y) $\rightarrow$ 0 as y $\rightarrow \infty$,
ATC(y) $\rightarrow$ AVC(y) as y $\rightarrow \infty$.

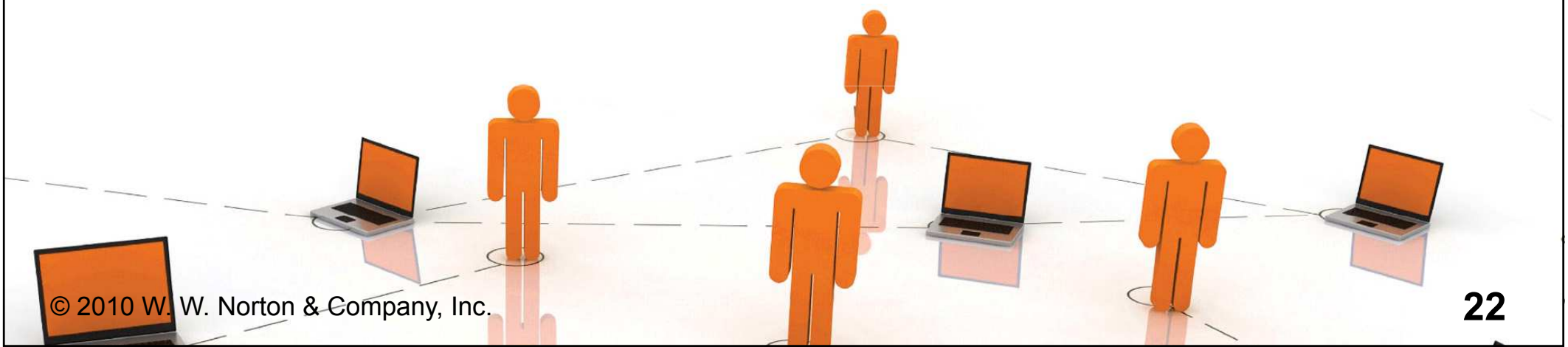
And since short-run AVC(y) must eventually increase, ATC(y) must eventually increase in a short-run.



Marginal Cost Function

- **Marginal cost is the rate-of-change of variable production cost as the output level changes. That is,**

$$MC(y) = \frac{\partial c_v(y)}{\partial y}.$$



Marginal Cost Function

- **The firm's total cost function is**

$$c(y) = F + c_v(y)$$

and the fixed cost F does not change with the output level y, so

$$MC(y) = \frac{\partial c_v(y)}{\partial y} = \frac{\partial c(y)}{\partial y}.$$

- **MC is the slope of both the variable cost and the total cost functions.**

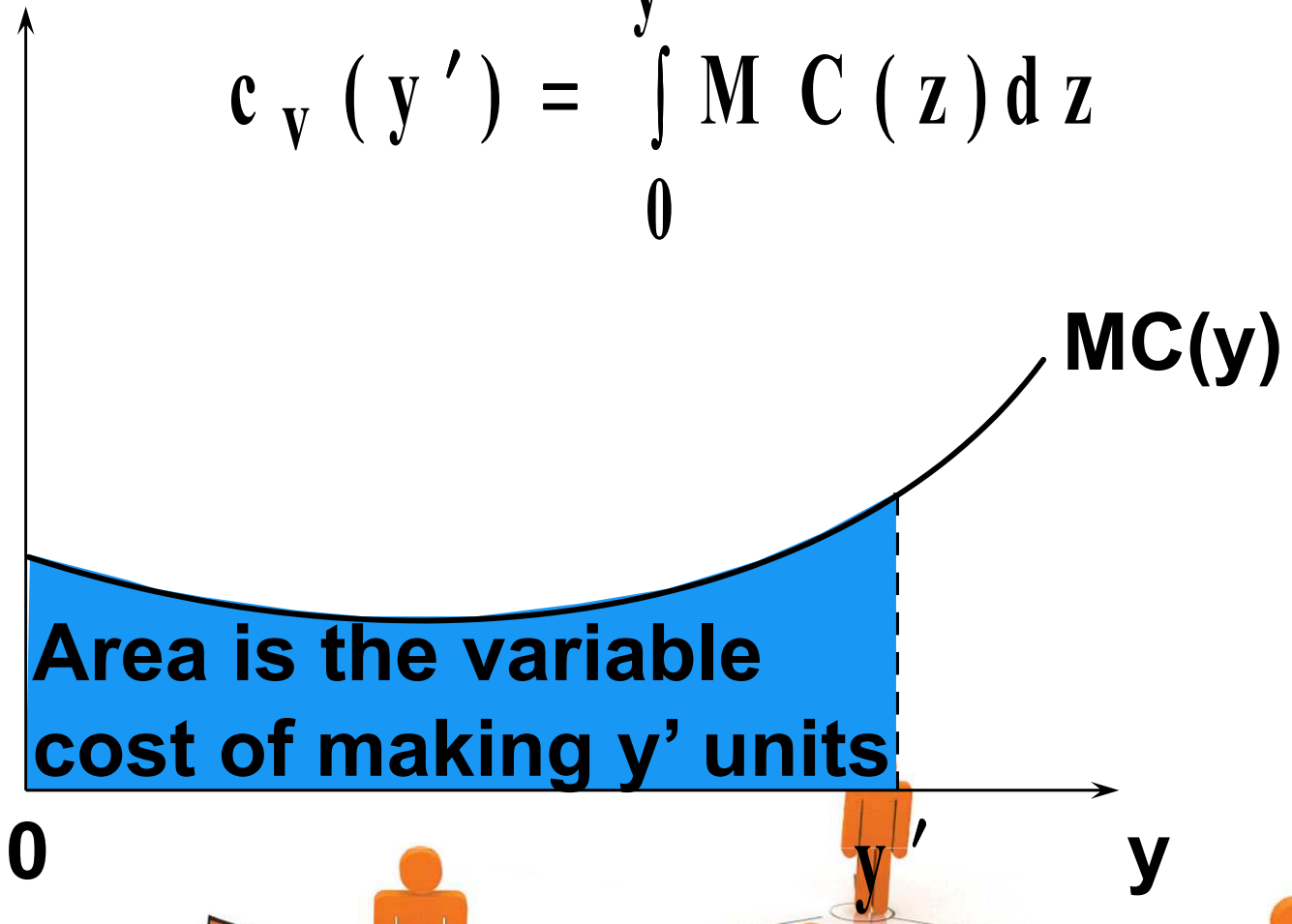
Marginal and Variable Cost Functions

- **Since MC(y) is the derivative of $c_v(y)$, $c_v(y)$ must be the integral of MC(y). That is,** $$MC(y) = \frac{\partial c_v(y)}{\partial y}$$

$$\Rightarrow \quad c_v(y) = \int_0^y MC(z)\,dz.$$

Marginal and Variable Cost Functions

$$c_V(y') = \int_0^{y'} MC(z)\,dz$$



Marginal & Average Cost Functions

- **How is marginal cost related to average variable cost?**

Marginal & Average Cost Functions

Since $$AVC(y) = \frac{c_V(y)}{y},$$

$$\frac{\partial AVC(y)}{\partial y} = \frac{y \times MC(y) - 1 \times c_V(y)}{y^2}.$$

Marginal & Average Cost Functions

Since $$AVC(y) = \frac{c_V(y)}{y},$$

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Therefore,

$$\frac{\partial AVC(y)}{\partial y} \begin{matrix} > \\ = \\ < \end{matrix} 0 \quad \textbf{as} \quad y \times MC(y) \begin{matrix} > \\ = \\ < \end{matrix} c_V(y).$$

Marginal & Average Cost Functions

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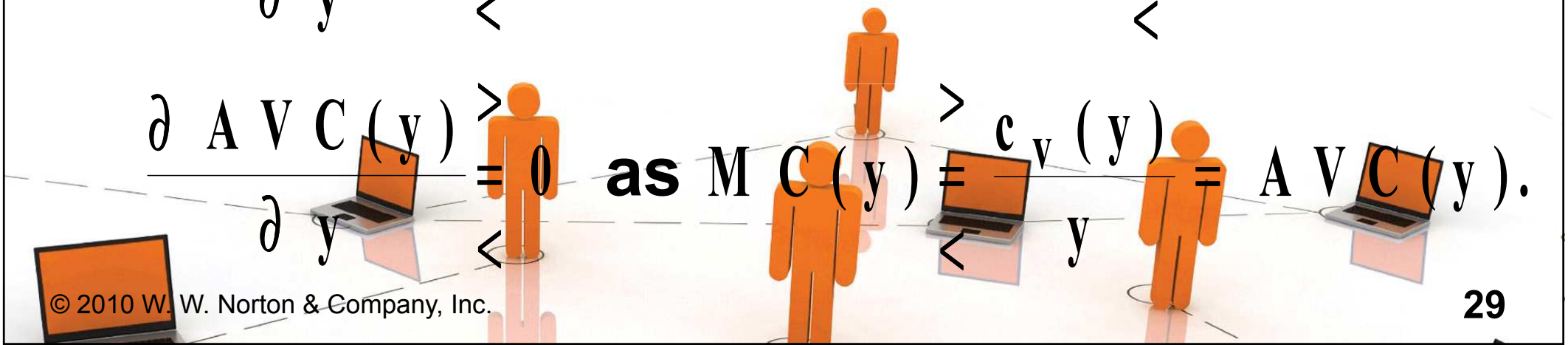
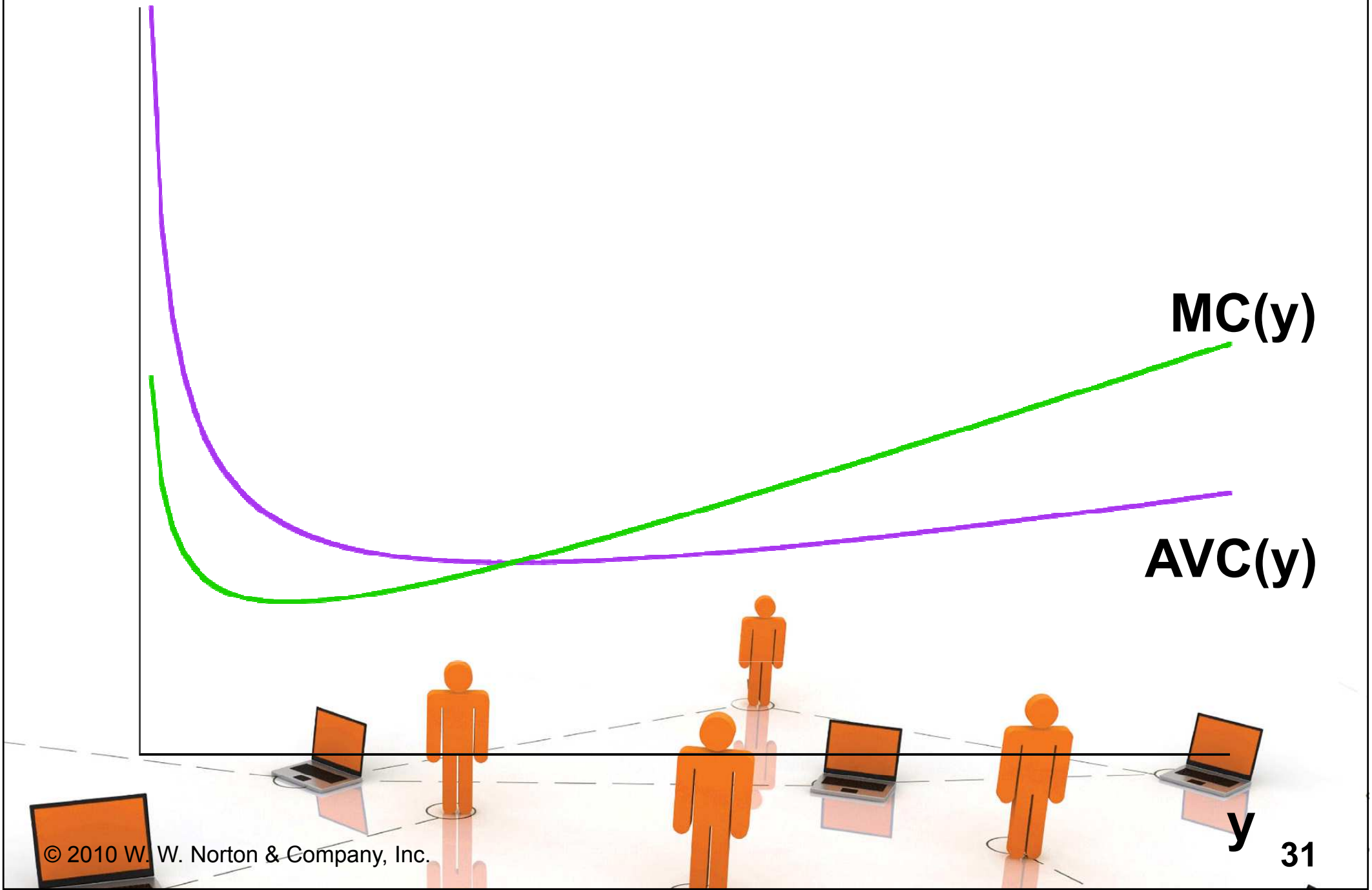
$$\frac{\partial AVC(y)}{\partial y} \begin{matrix} > \\ = \\ < \end{matrix} 0 \quad \textbf{as} \quad MC(y) \begin{matrix} > \\ = \\ < \end{matrix} \frac{c_V(y)}{y} = AVC(y).$$



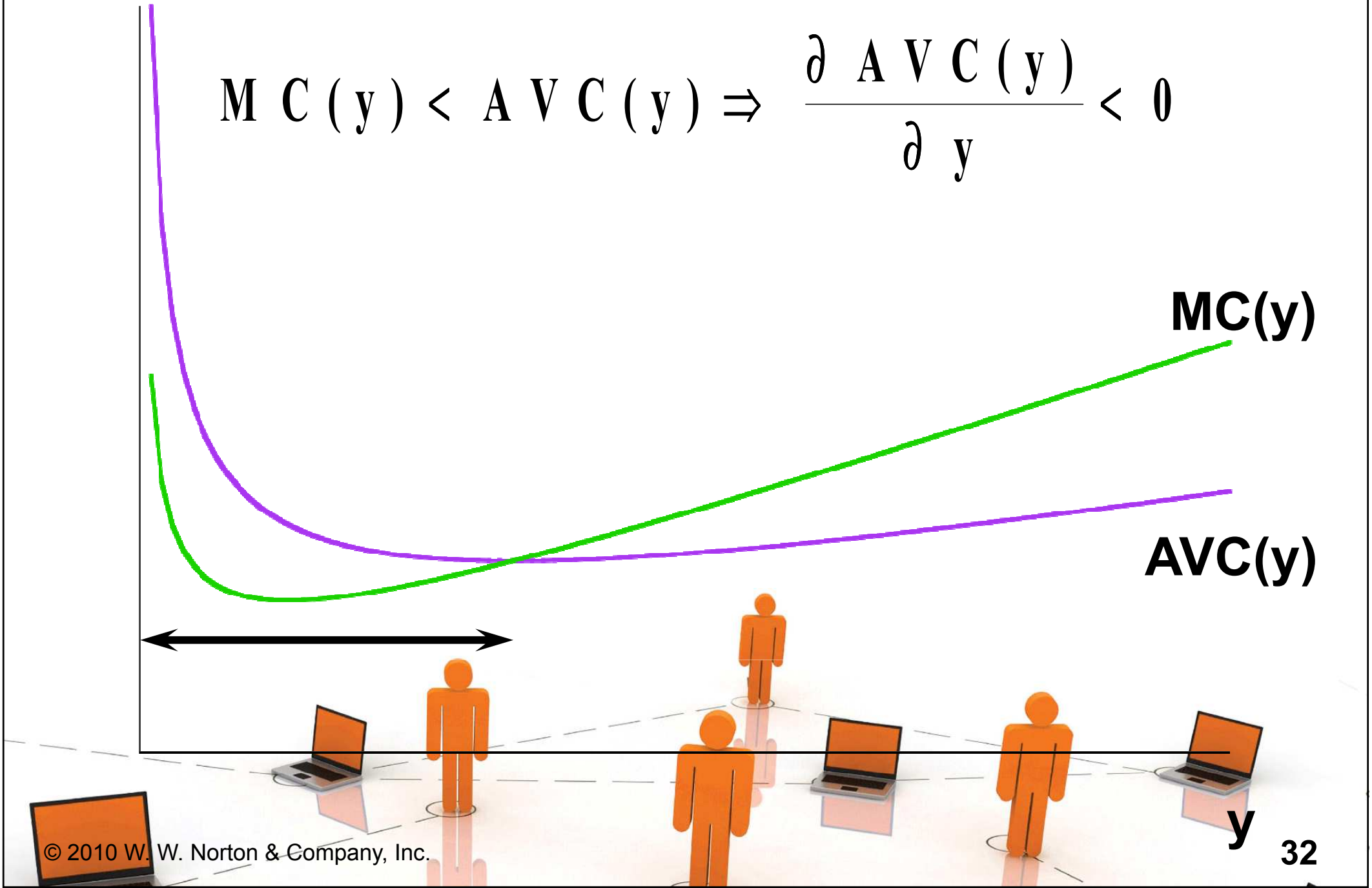
Marginal & Average Cost Functions

$$\frac{\partial AVC(y)}{\partial y} \begin{matrix} > \\ = \\ < \end{matrix} 0 \quad \textbf{as} \quad MC(y) \begin{matrix} > \\ = \\ < \end{matrix} AVC(y).$$



$/output unit

$$MC(y) < AVC(y) \Rightarrow \frac{\partial AVC(y)}{\partial y} < 0$$



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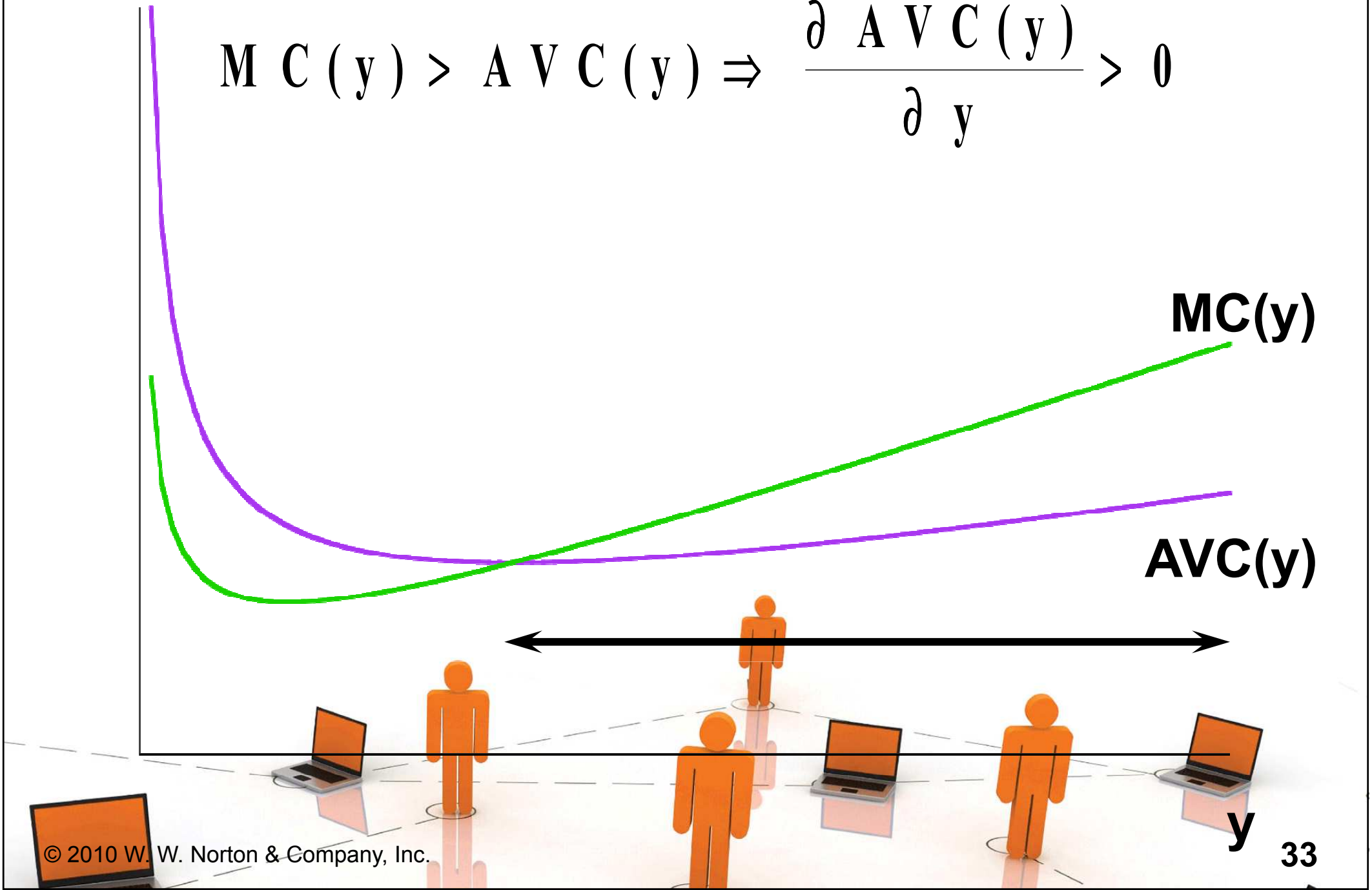
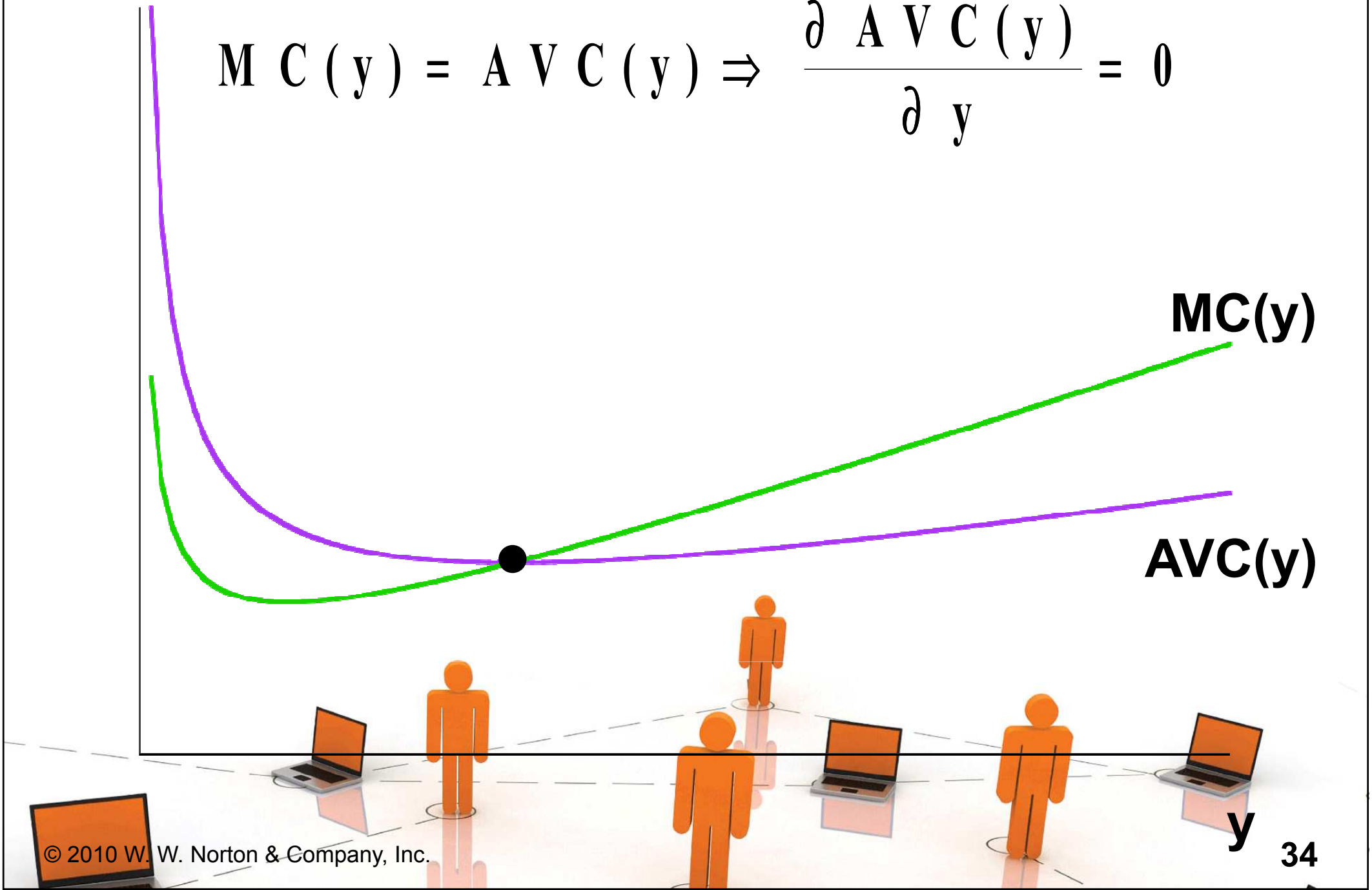


$/output unit

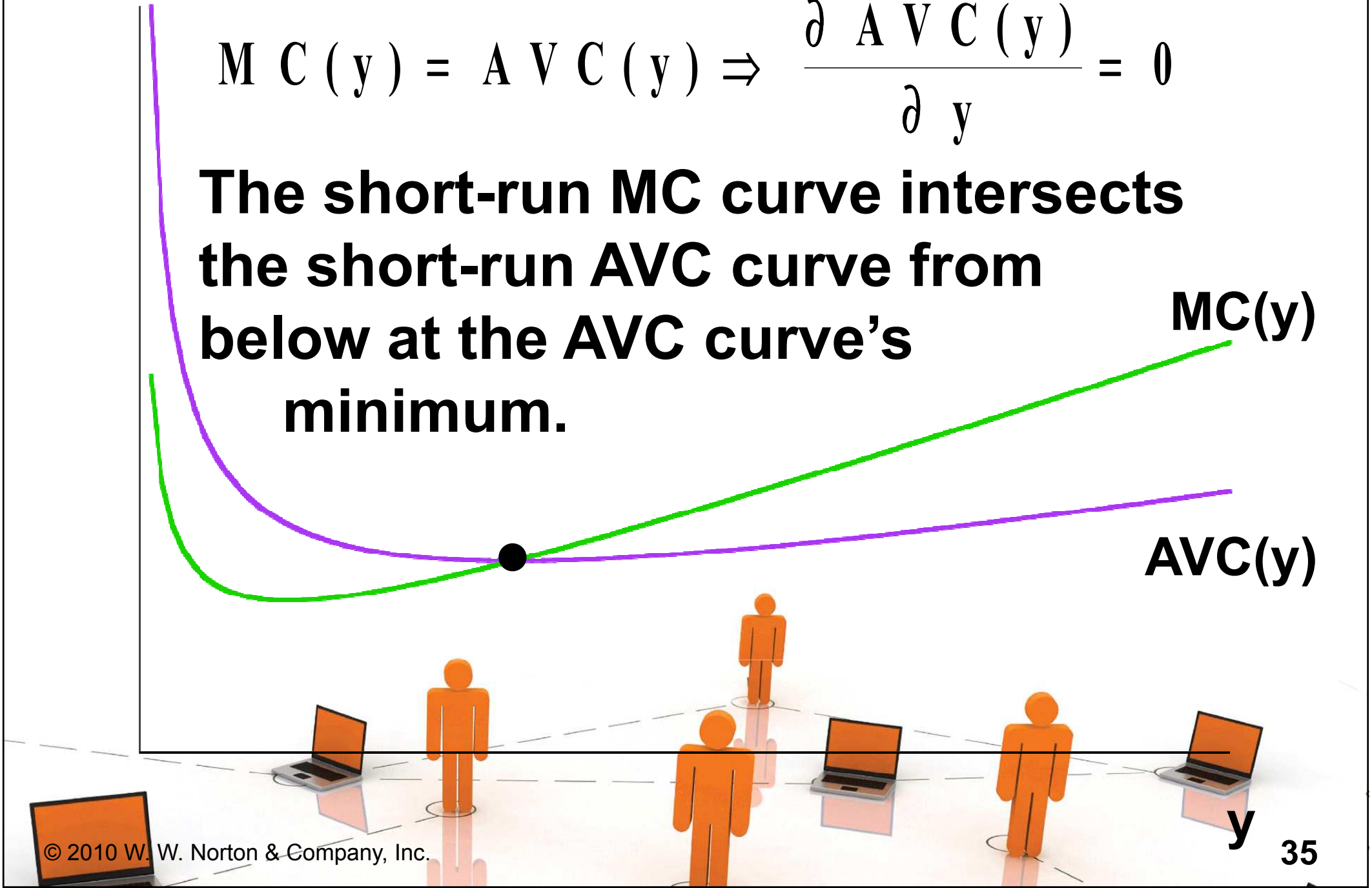
$$MC(y) = AVC(y) \Rightarrow \frac{\partial AVC(y)}{\partial y} = 0$$



$/output unit

$$MC(y) = AVC(y) \Rightarrow \frac{\partial AVC(y)}{\partial y} = 0$$

The short-run MC curve intersects the short-run AVC curve from below at the AVC curve's minimum.



Marginal & Average Cost Functions

Similarly, since $ATC(y) = \frac{c(y)}{y}$,

$$\frac{\partial ATC(y)}{\partial y} = \frac{y \times MC(y) - 1 \times c(y)}{y^2}.$$

Marginal & Average Cost Functions

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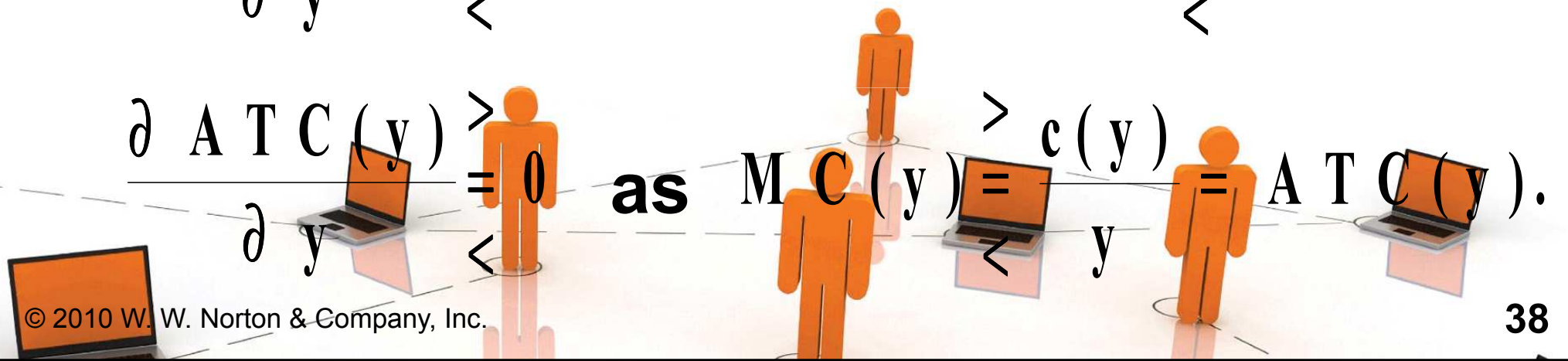
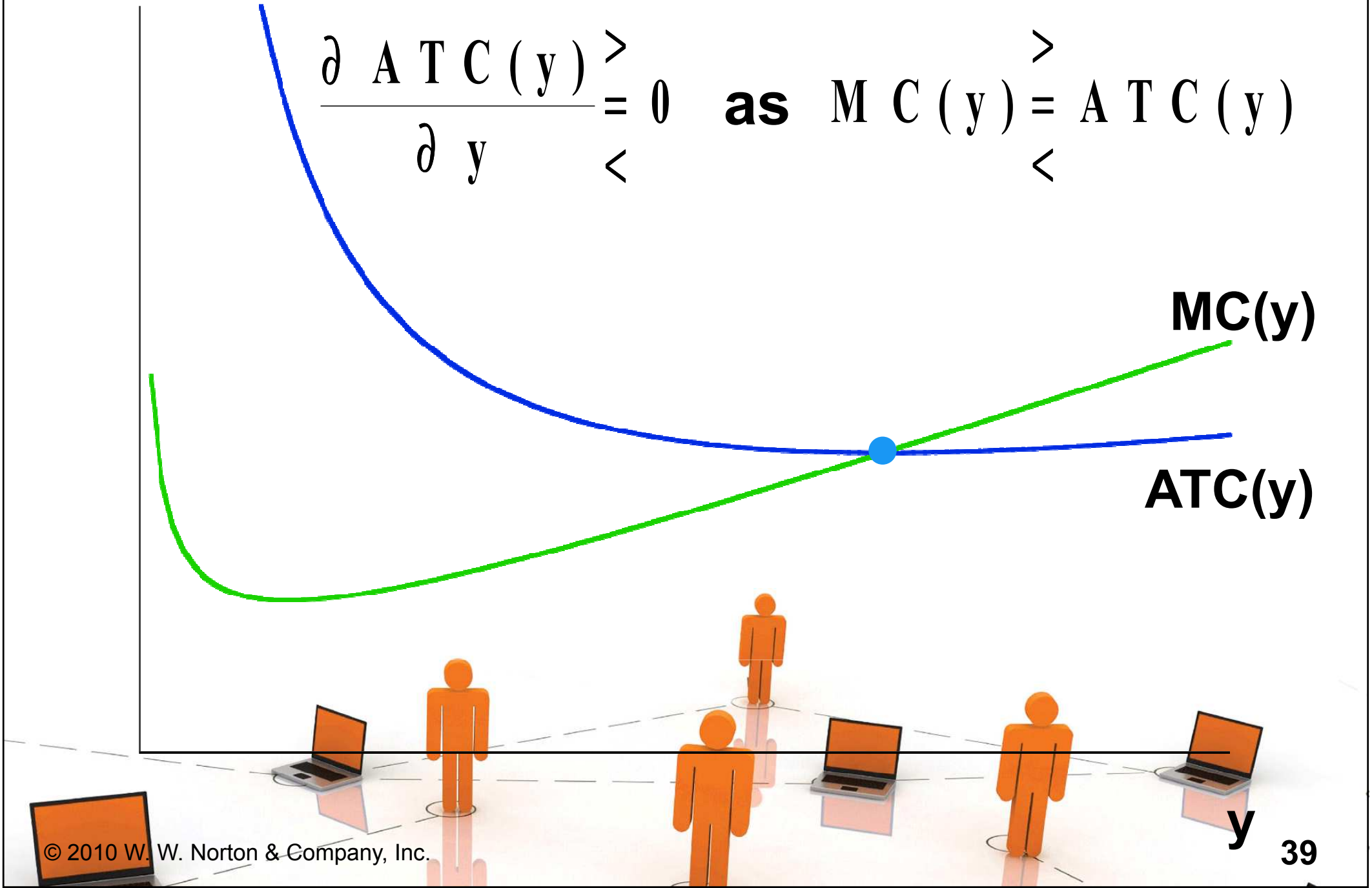


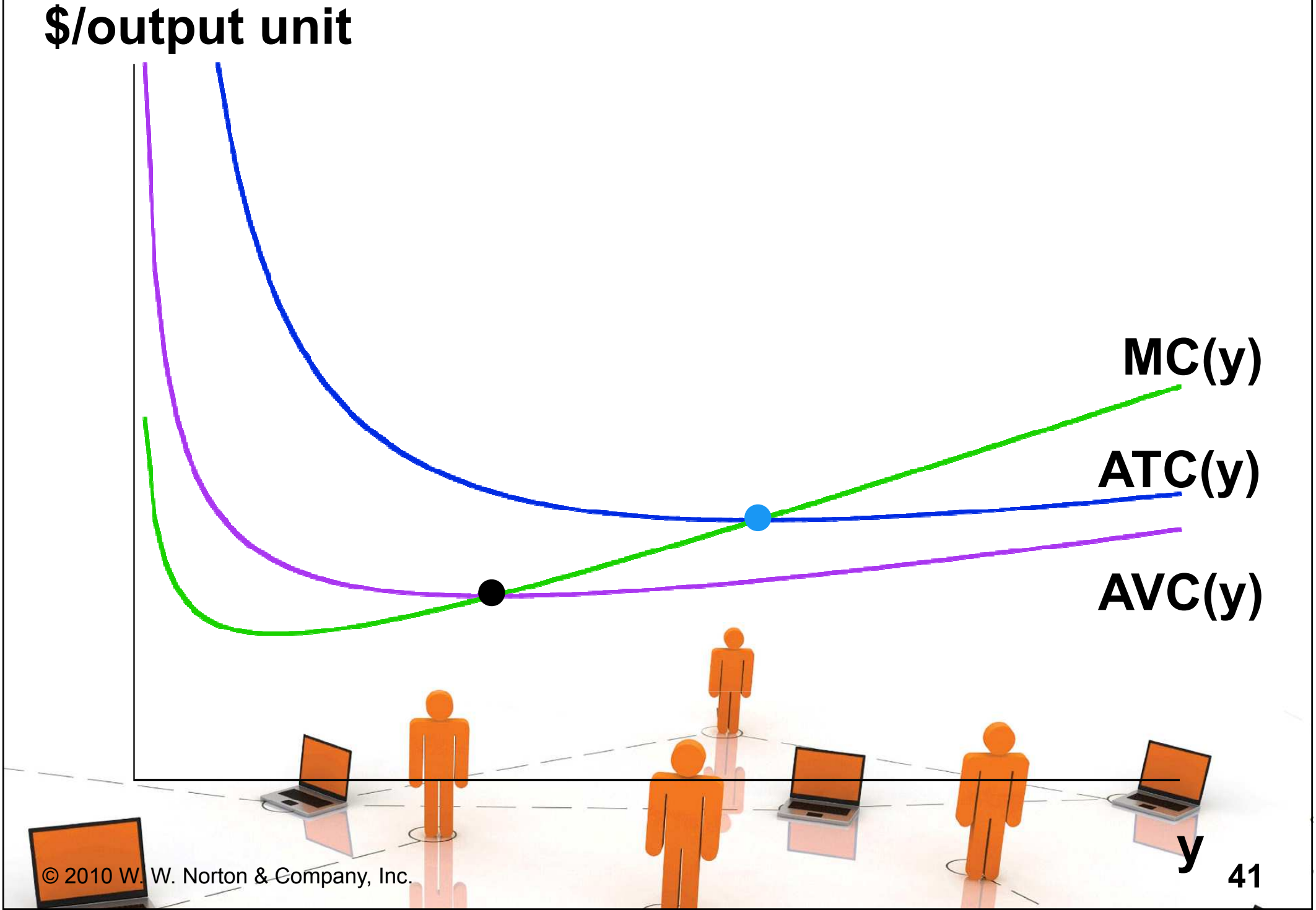
$/output unit

$$\frac{\partial ATC(y)}{\partial y} \begin{matrix} > \\ = \\ < \end{matrix} 0 \quad \text{as} \quad MC(y) \begin{matrix} > \\ = \\ < \end{matrix} ATC(y)$$



Marginal & Average Cost Functions

- **The short-run MC curve intersects the short-run AVC curve from below at the AVC curve's minimum.**
- **And, similarly, the short-run MC curve intersects the short-run ATC curve from below at the ATC curve's minimum.**



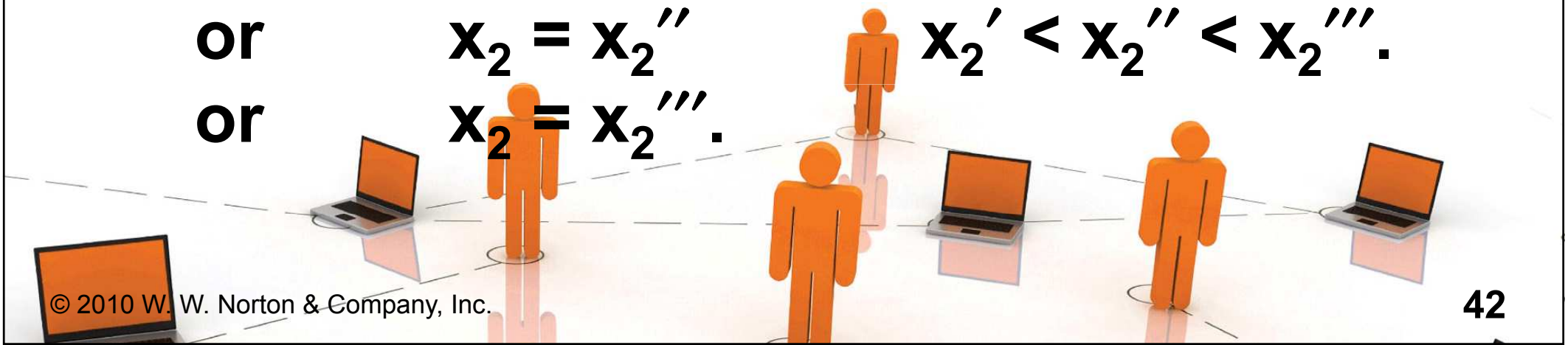
Short-Run & Long-Run Total Cost Curves

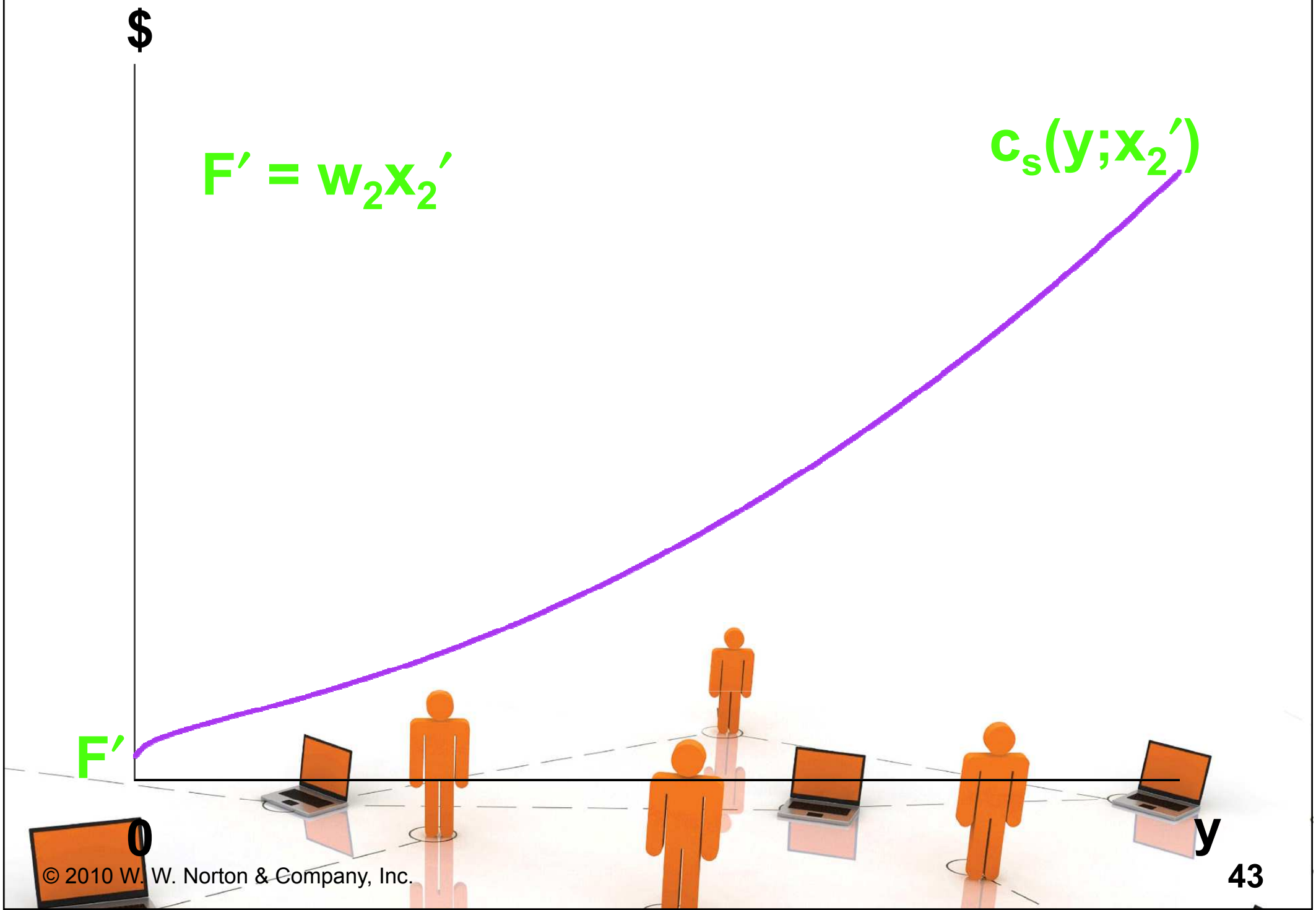
- **A firm has a different short-run total cost curve for each possible short-run circumstance.**
- **Suppose the firm can be in one of just three short-runs;**

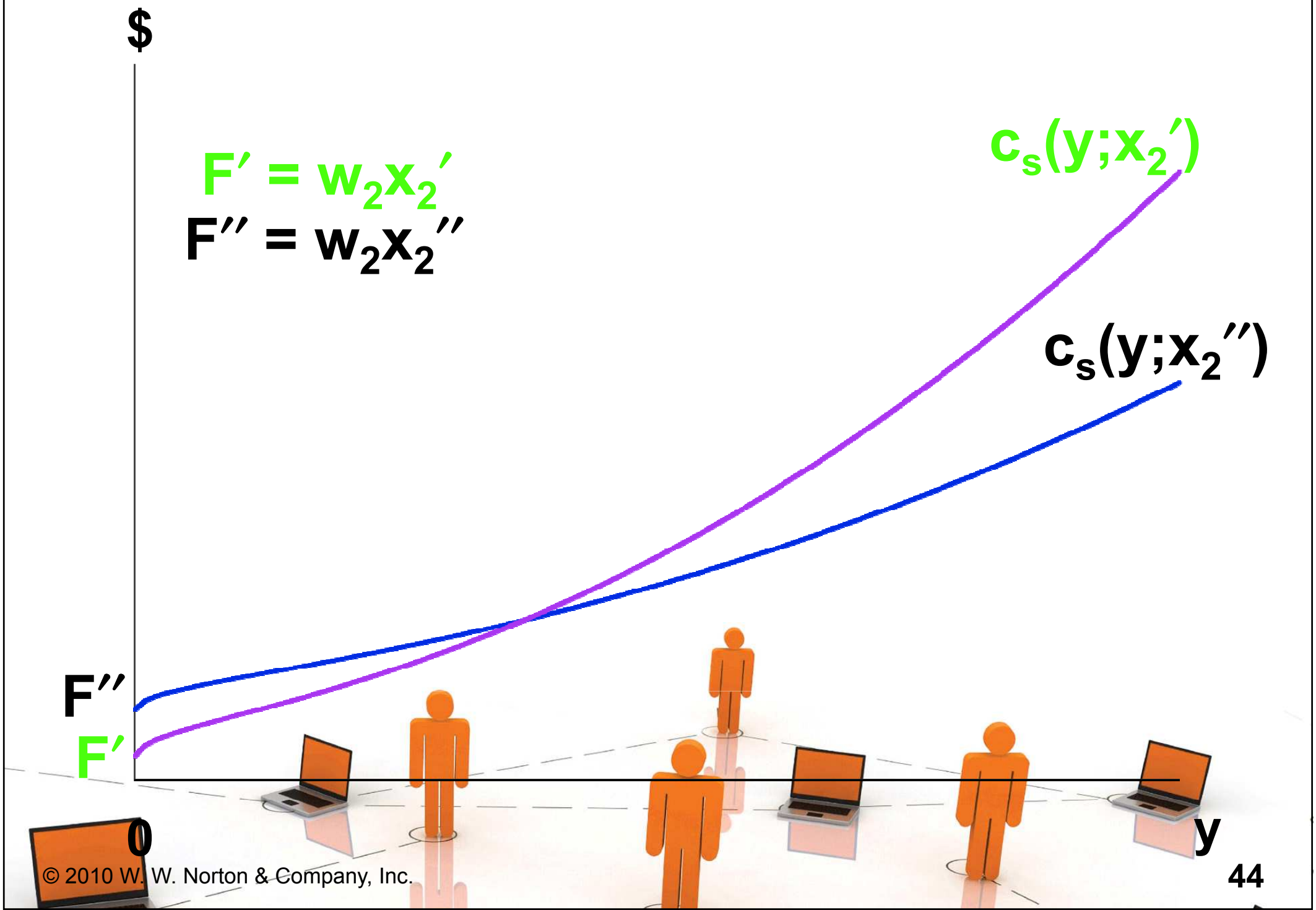
$$x_2 = x_2'$$

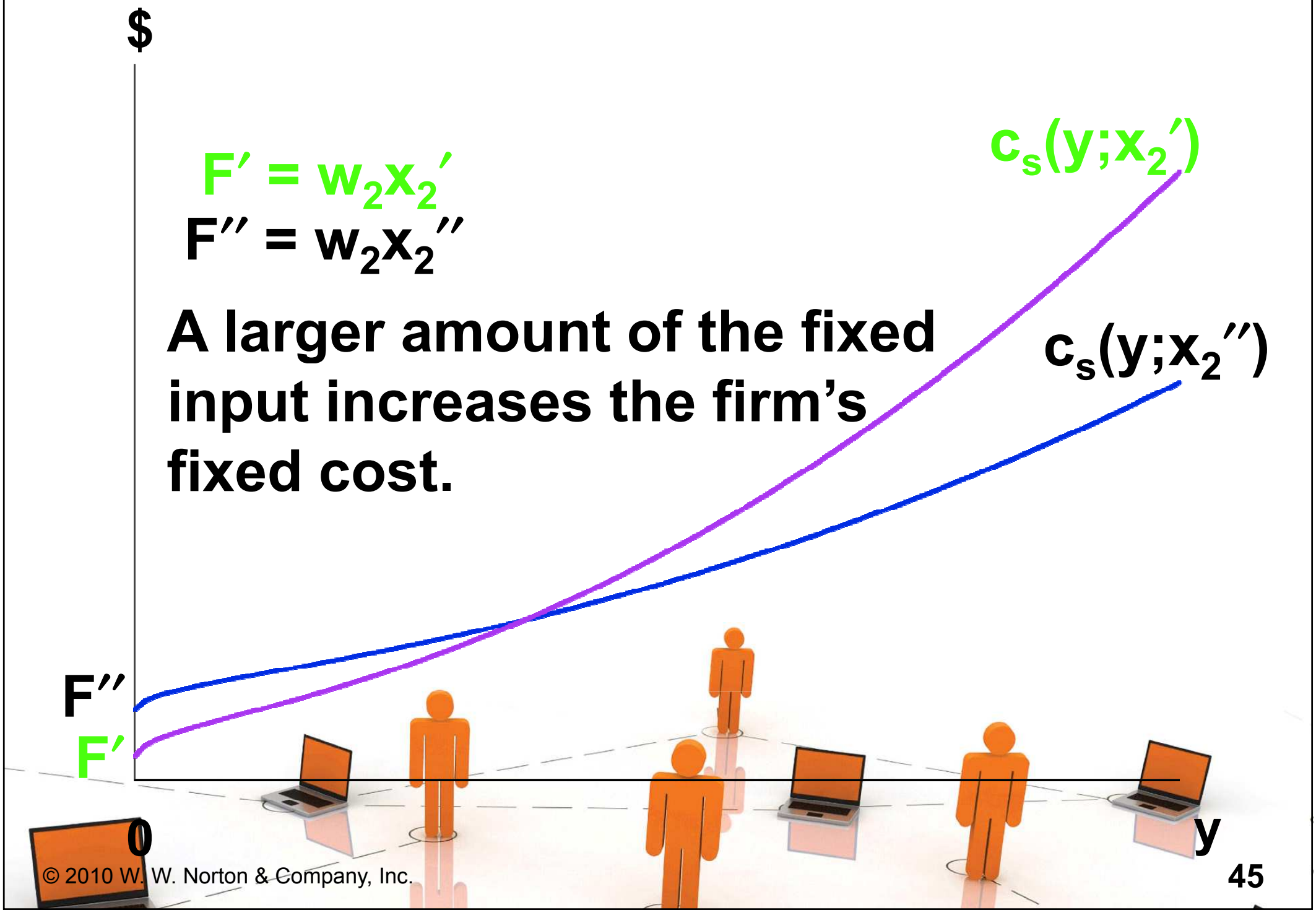
or $x_2 = x_2''$ $\qquad x_2' < x_2'' < x_2'''.$

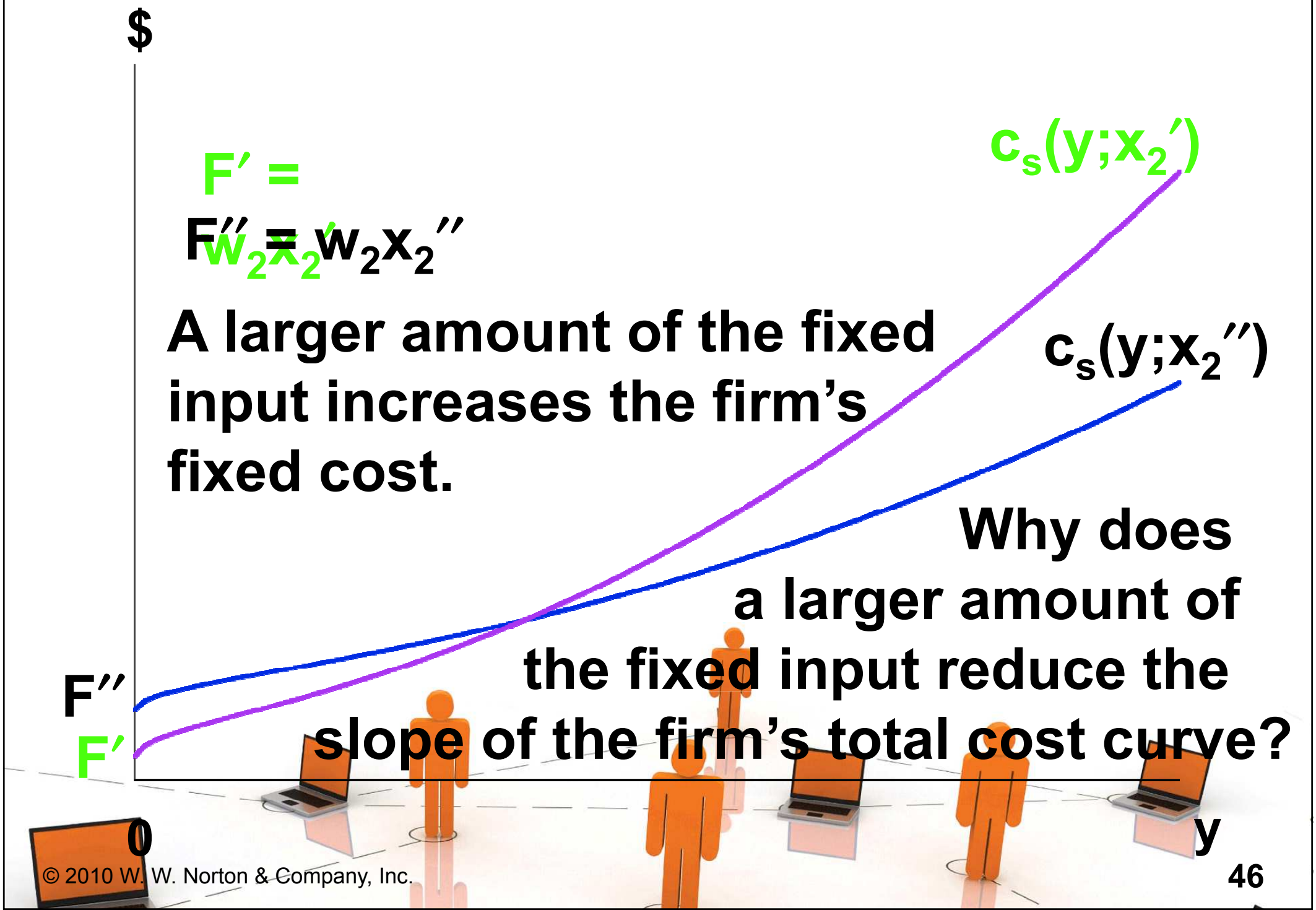
or $x_2 = x_2'''.$





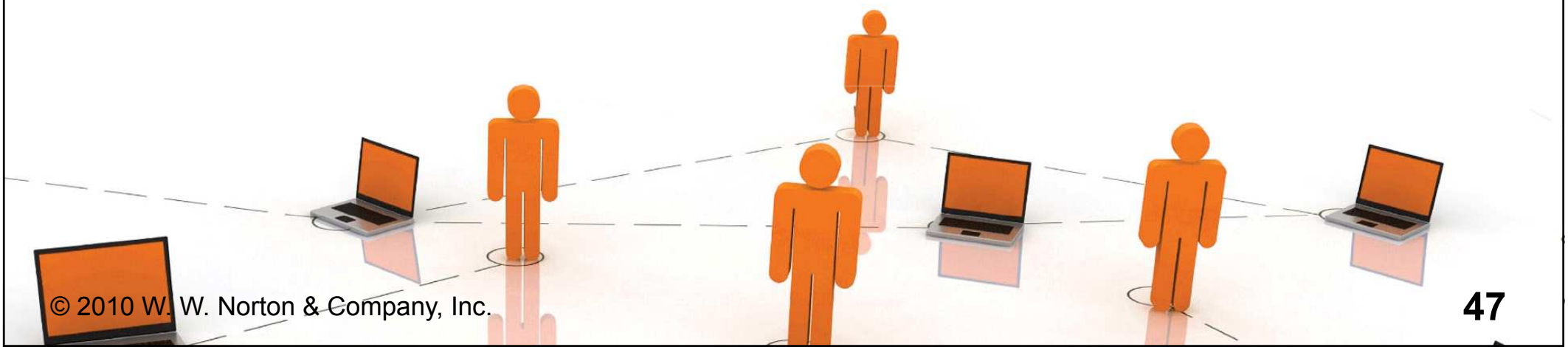






Short-Run & Long-Run Total Cost Curves

MP_1 is the marginal physical productivity of the variable input 1, so one extra unit of input 1 gives MP_1 extra output units. Therefore, the extra amount of input 1 needed for 1 extra output unit is



Short-Run & Long-Run Total Cost Curves

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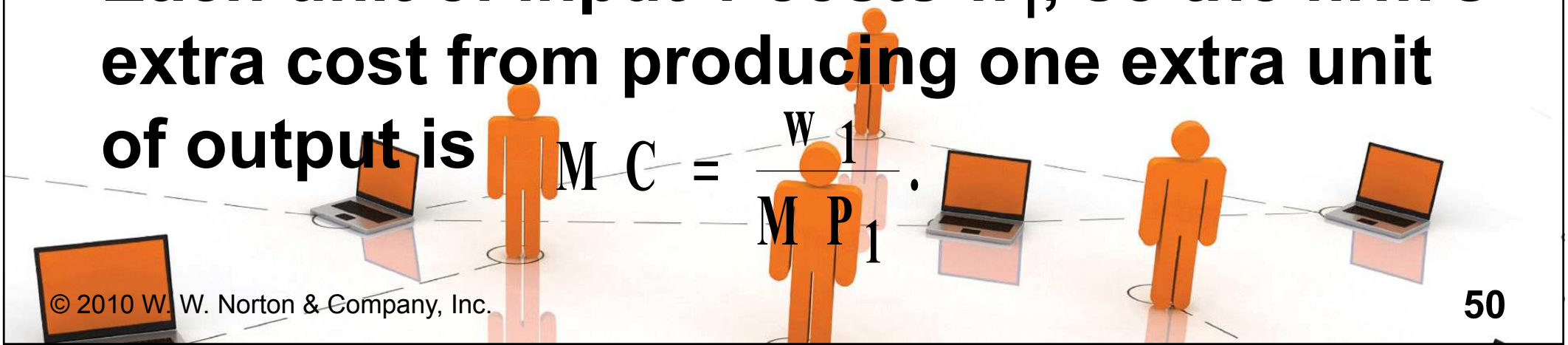
$$MC = \frac{w_1}{MP_1}.$$



Short-Run & Long-Run Total Cost Curves

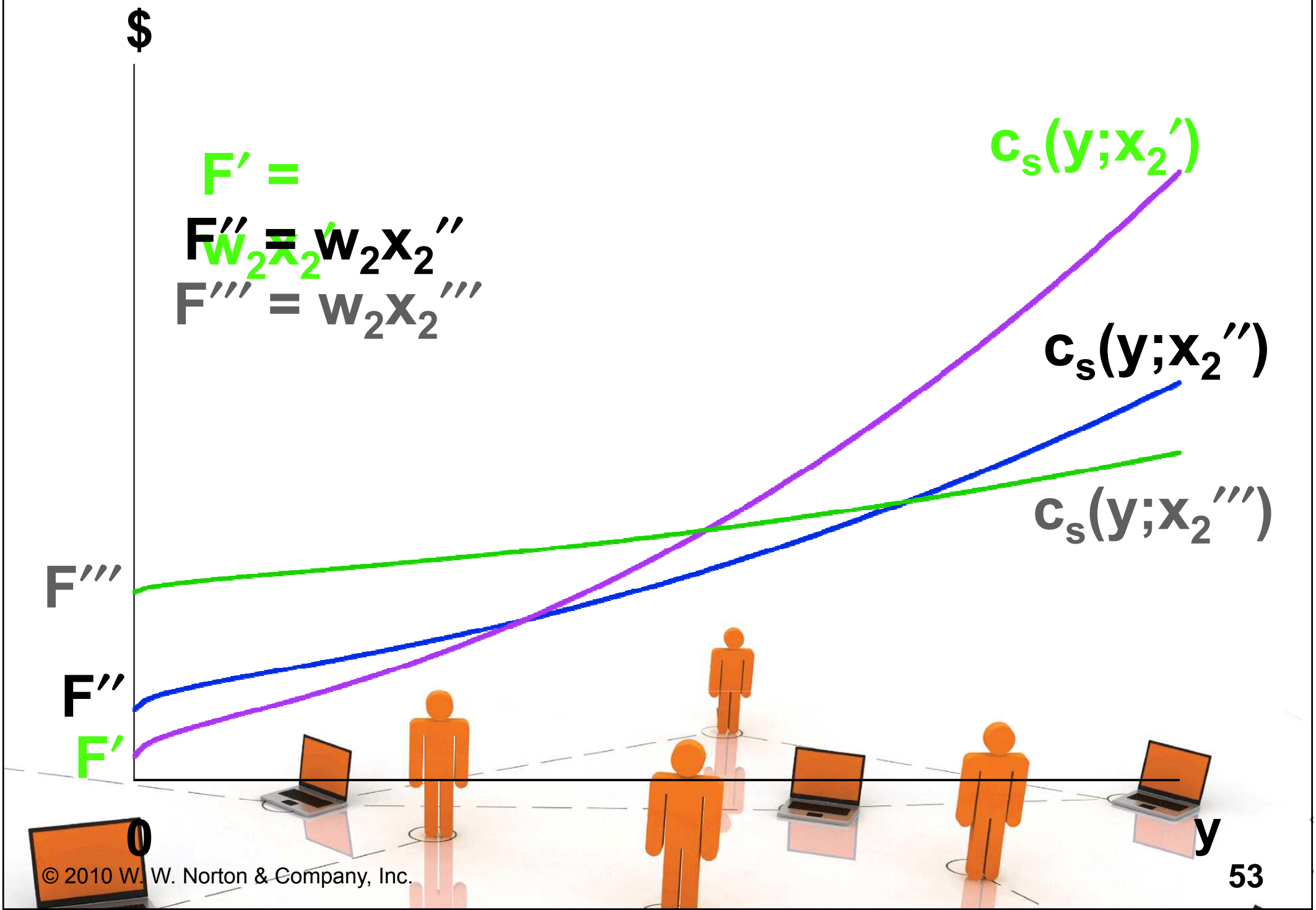
$$MC = \frac{w_1}{MP_1}$$ **is the slope of the firm's total cost curve.**

Short-Run & Long-Run Total Cost Curves

$$MC = \frac{w_1}{MP_1}$$ **is the slope of the firm's total cost curve.**

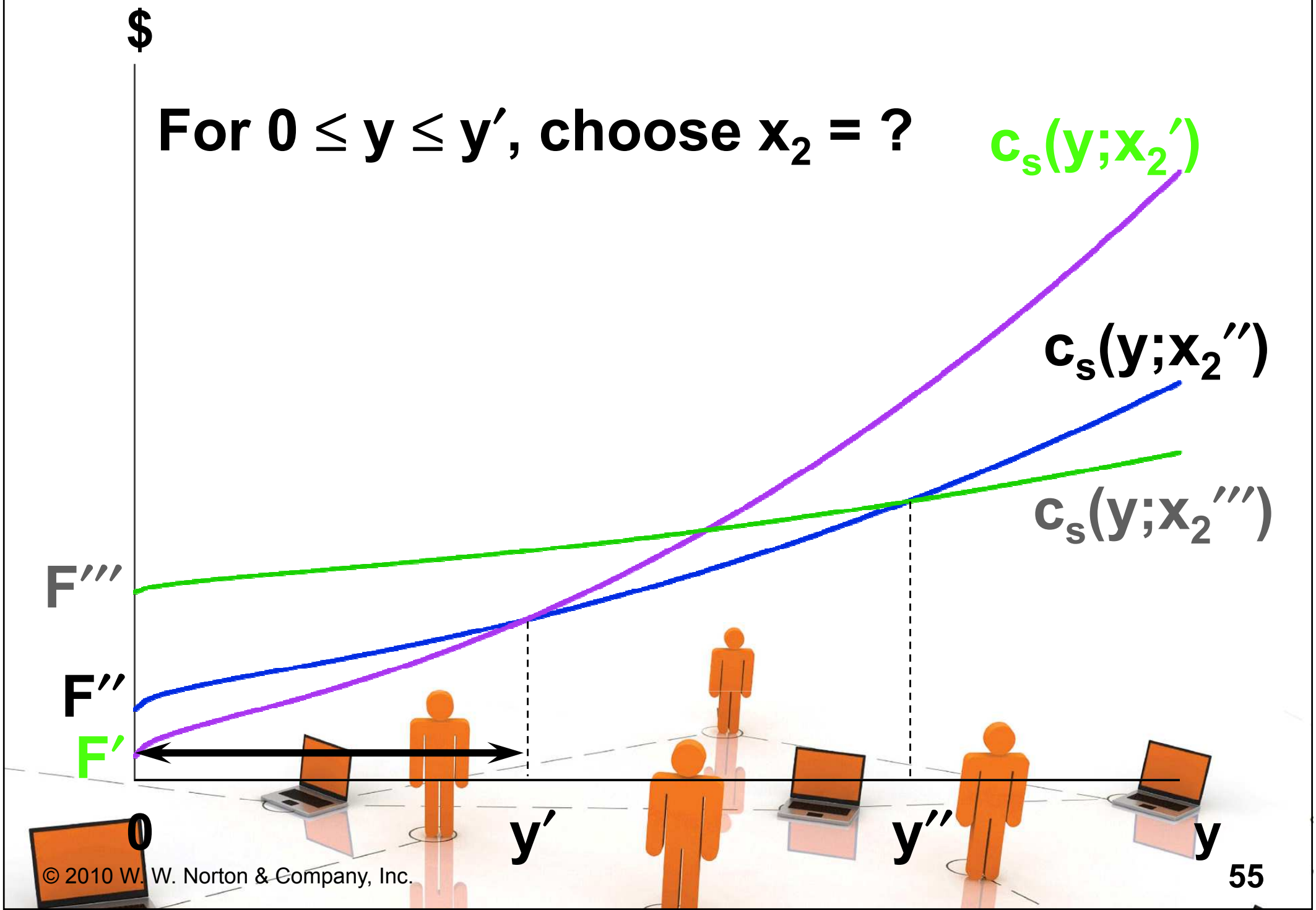
If input 2 is a complement to input 1 then MP_1 is higher for higher x_2.
Hence, MC is lower for higher x_2.

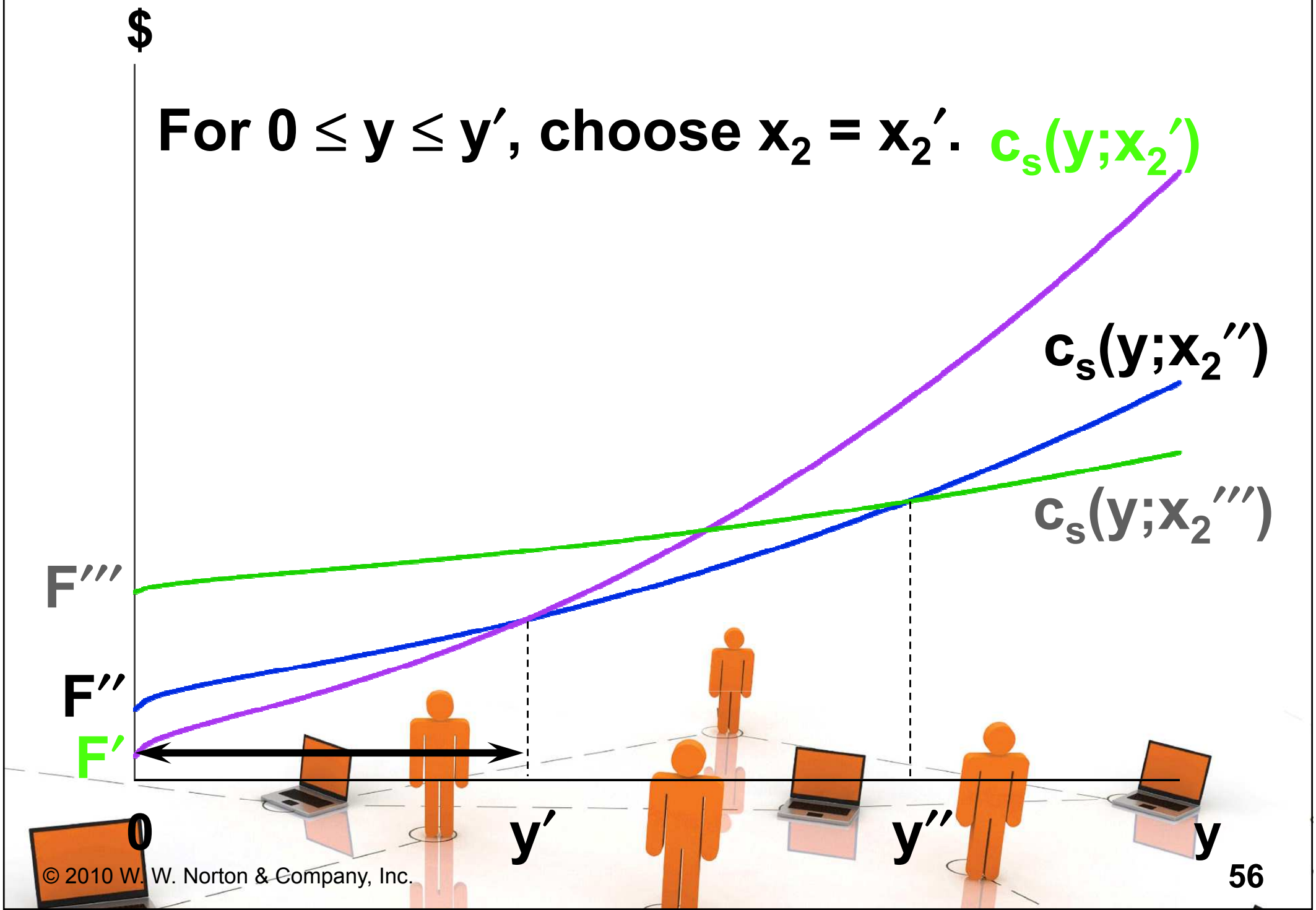
That is, a short-run total cost curve starts higher and has a lower slope if x_2 is larger.

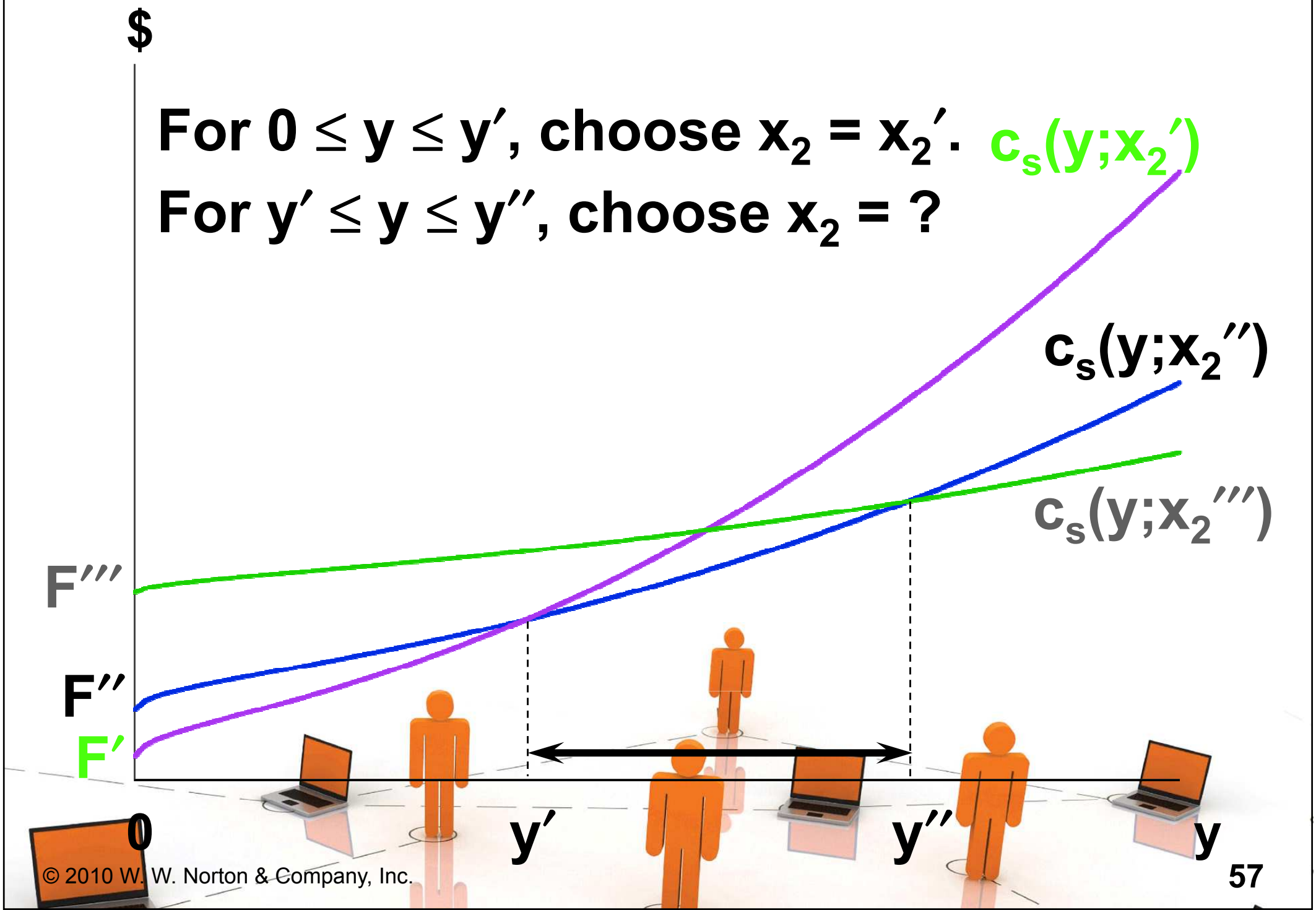


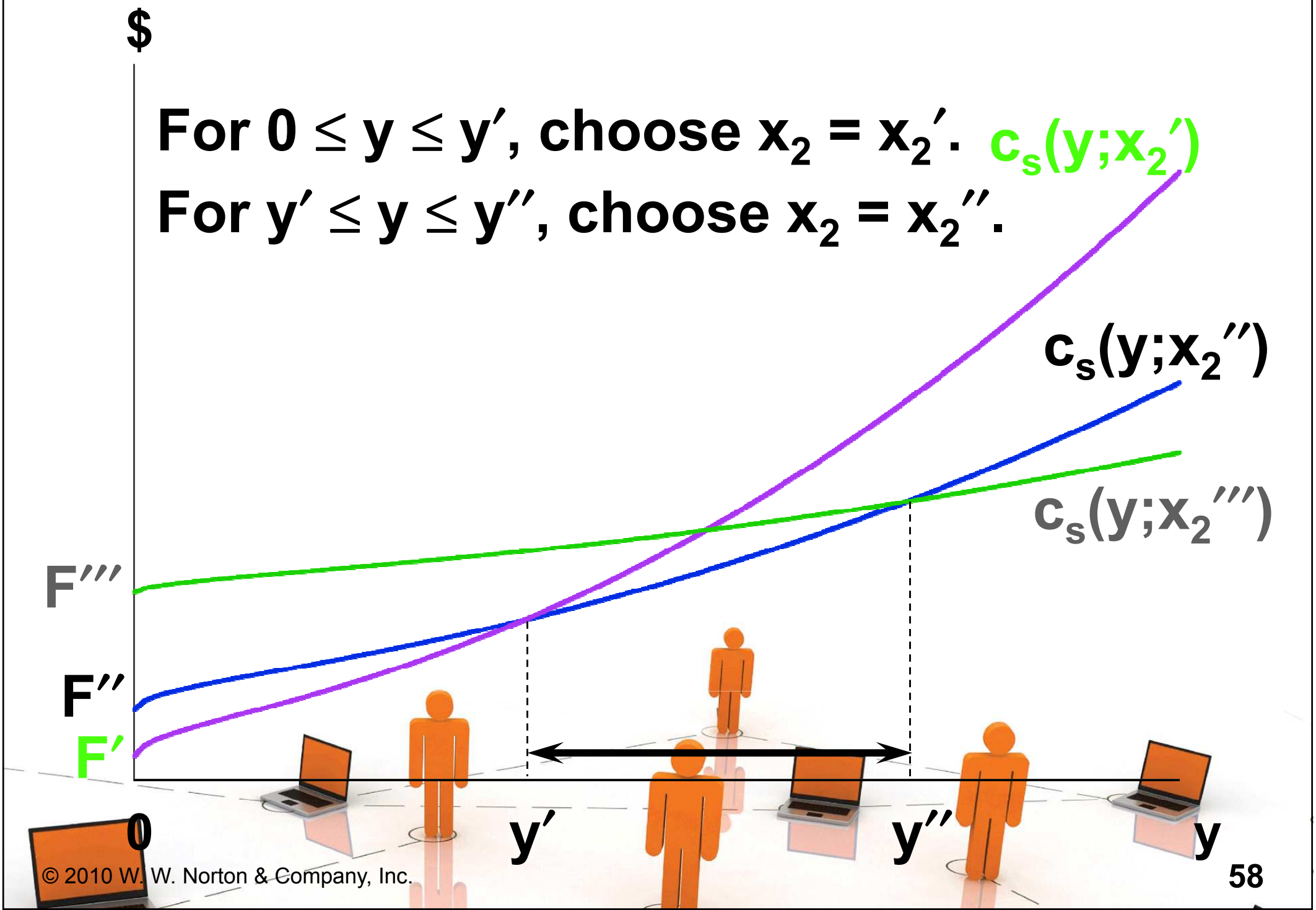
Short-Run & Long-Run Total Cost Curves

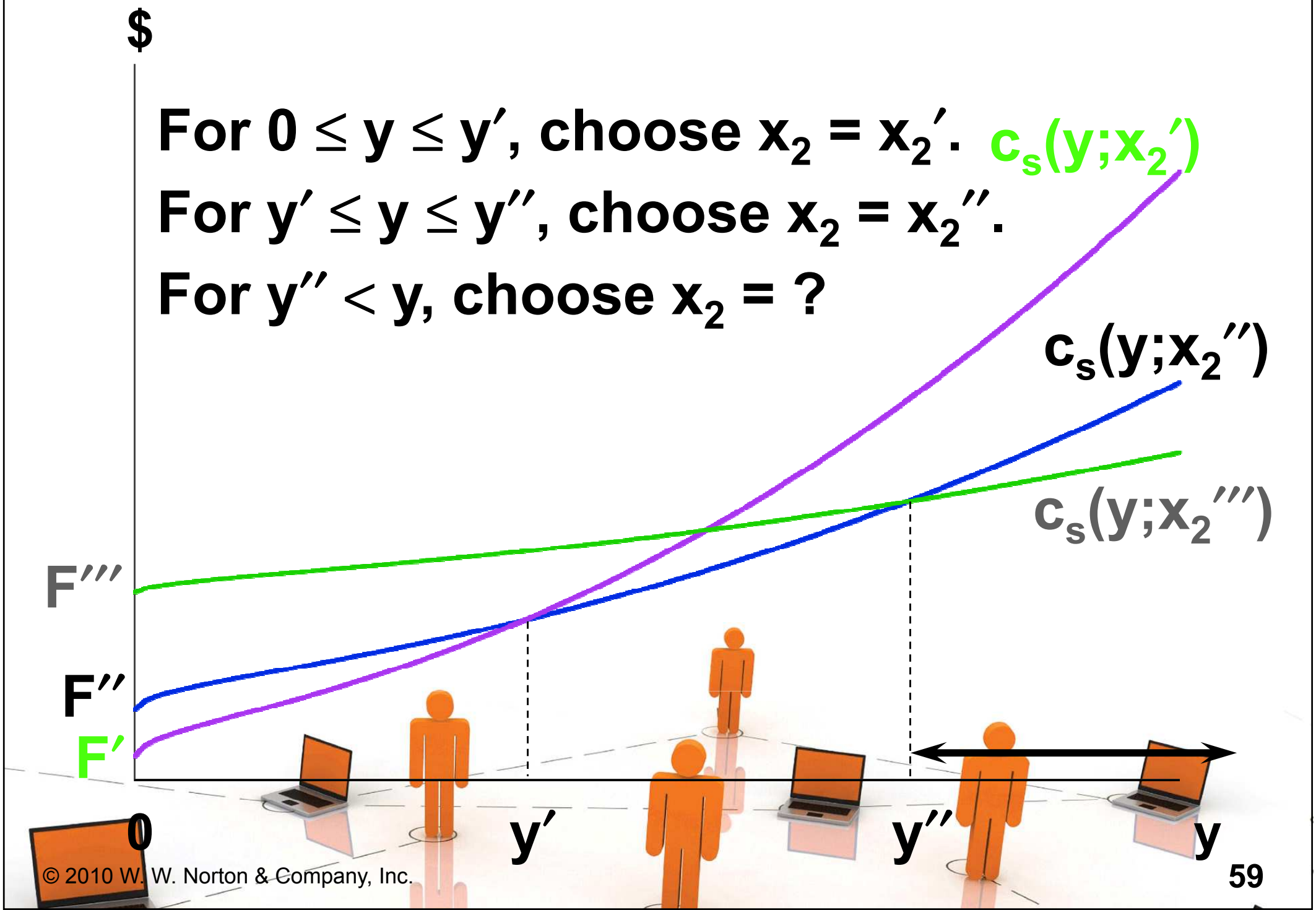
- **The firm has three short-run total cost curves.**
- **In the long-run the firm is free to choose amongst these three since it is free to select x_2 equal to any of x_2', x_2'', or x_2'''.**
- **How does the firm make this choice?**

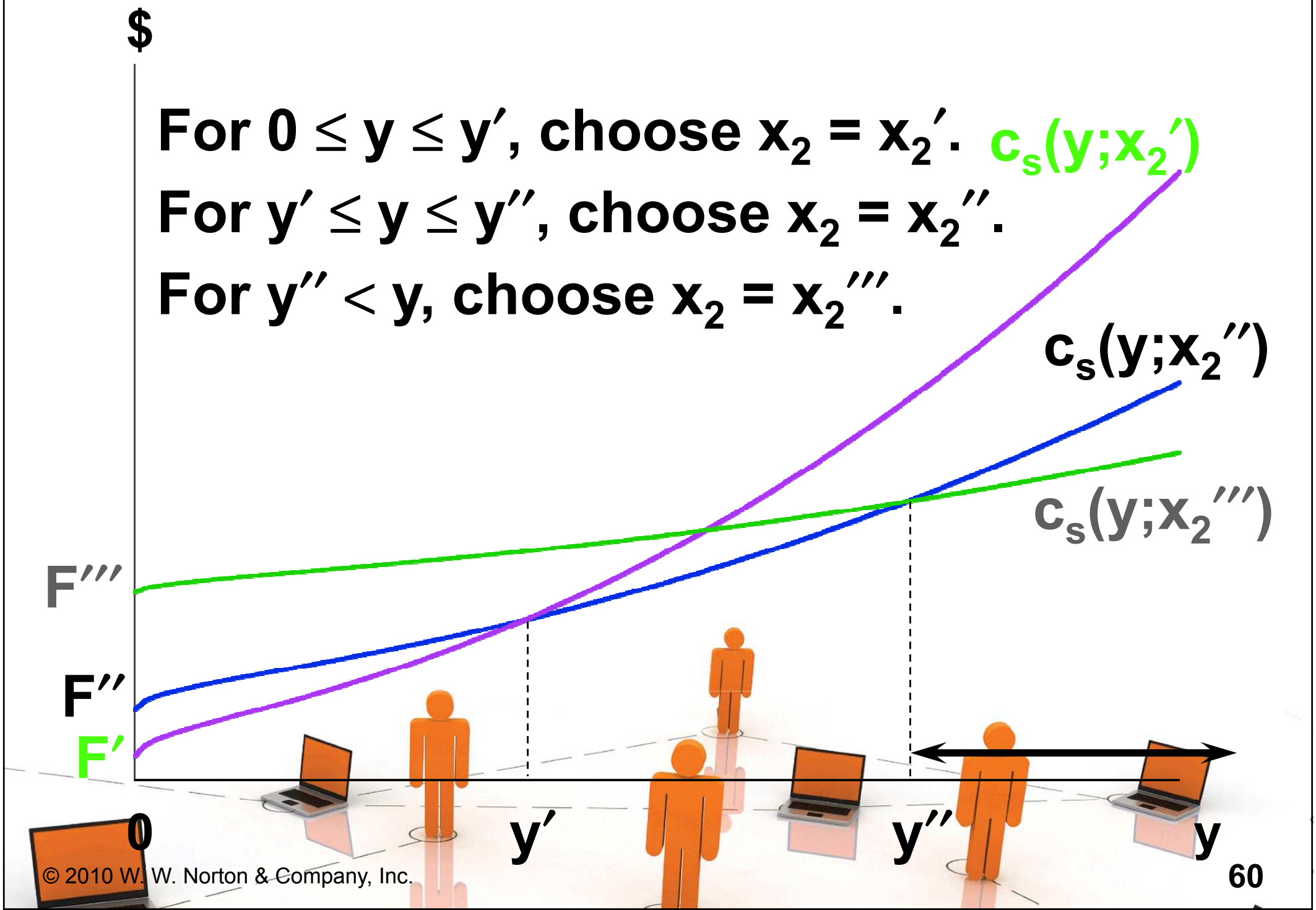


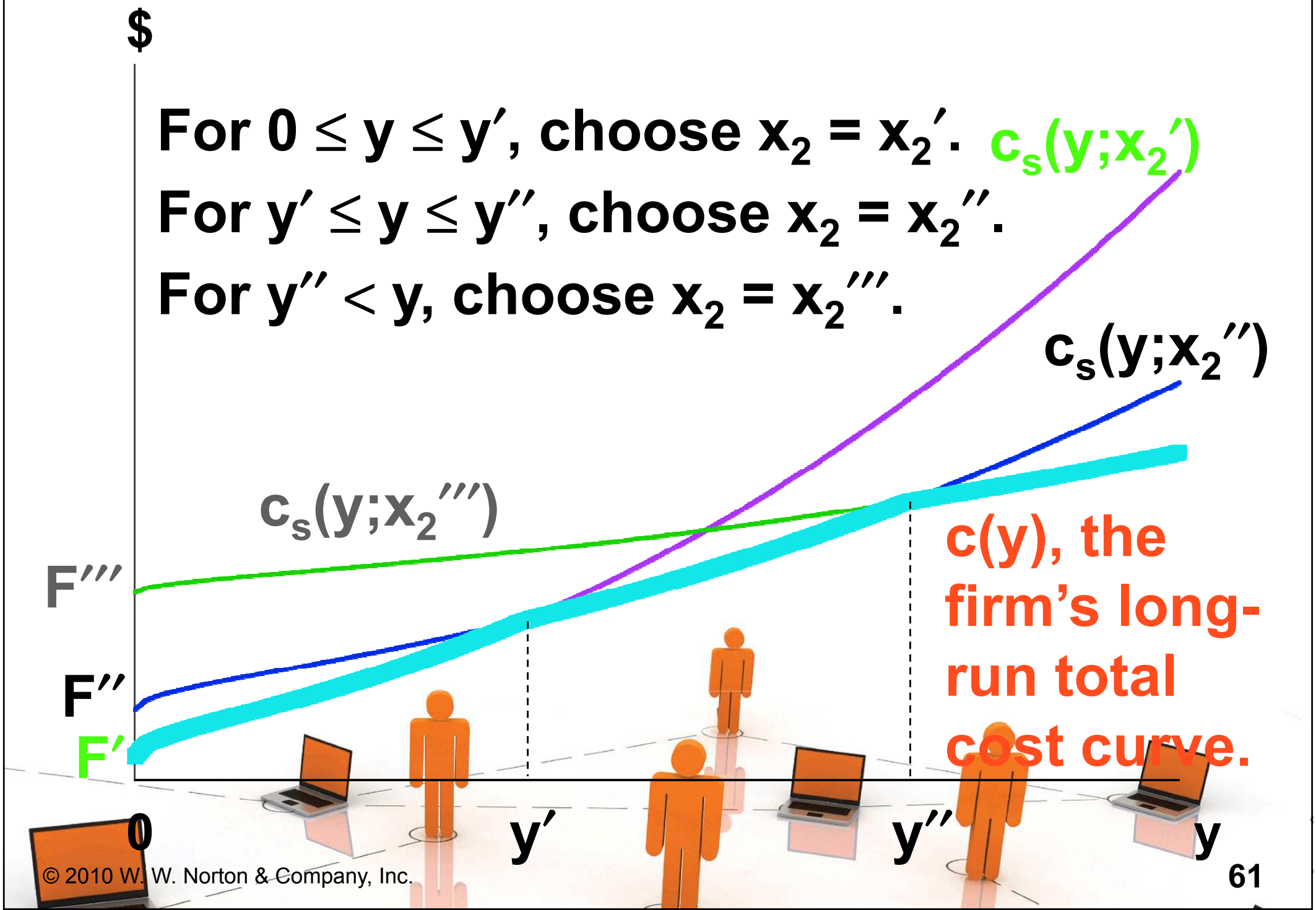










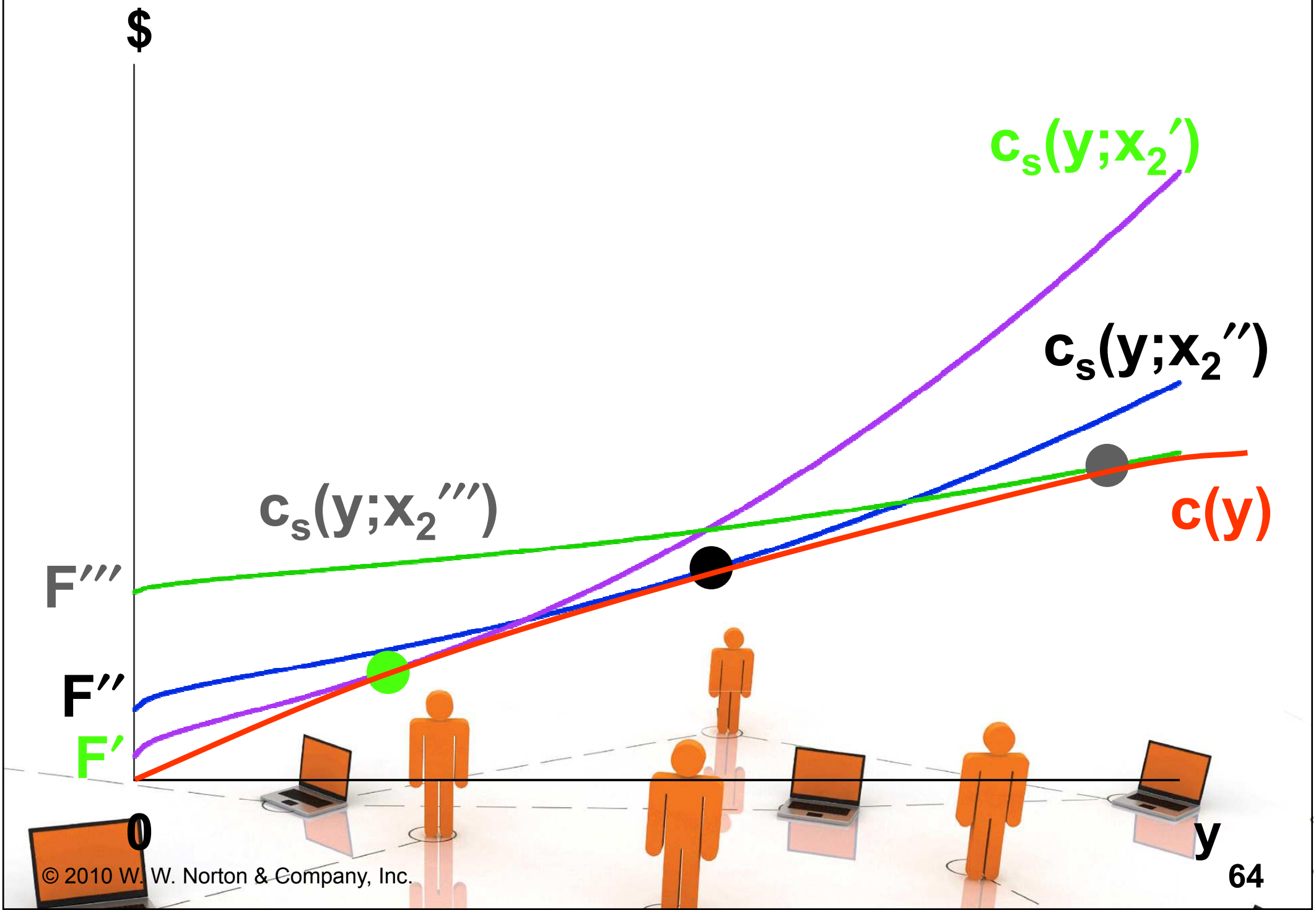


Short-Run & Long-Run Total Cost Curves

- **The firm's long-run total cost curve consists of the lowest parts of the short-run total cost curves. The long-run total cost curve is the lower envelope of the short-run total cost curves.**

Short-Run & Long-Run Total Cost Curves

- **If input 2 is available in continuous amounts then there is an infinity of short-run total cost curves but the long-run total cost curve is still the lower envelope of all of the short-run total cost curves.**



Short-Run & Long-Run Average Total Cost Curves

- **For any output level y, the long-run total cost curve always gives the lowest possible total production cost.**
- **Therefore, the long-run av. total cost curve must always give the lowest possible av. total production cost.**
- **The long-run av. total cost curve must be the lower envelope of all of the firm's short-run av. total cost curves.**

Short-Run & Long-Run Average Total Cost Curves

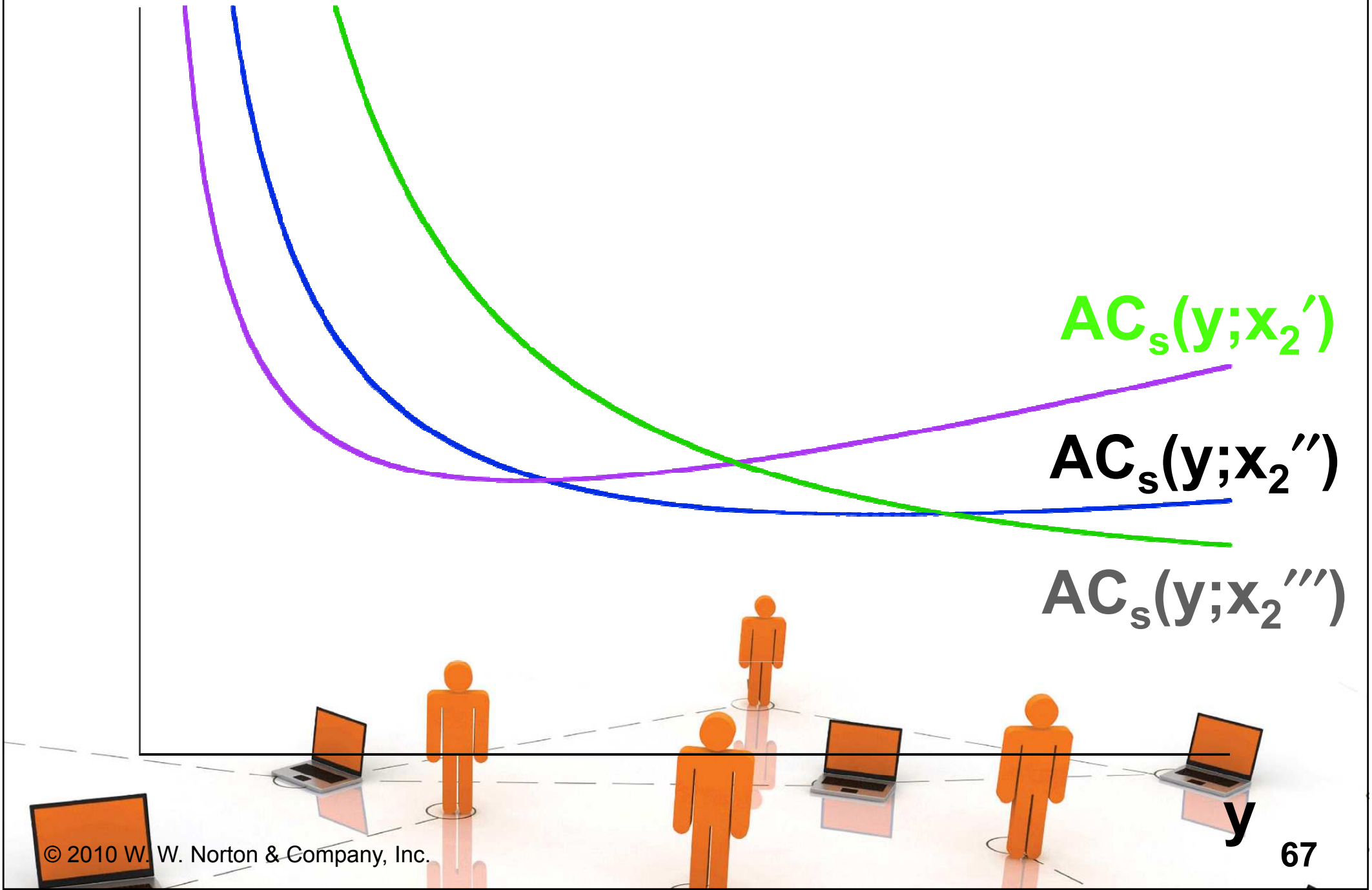
- **E.g. suppose again that the firm can be in one of just three short-runs;**

  $x_2 = x_2'$

  or $x_2 = x_2''$ $\qquad (x_2' < x_2'' < x_2''')$

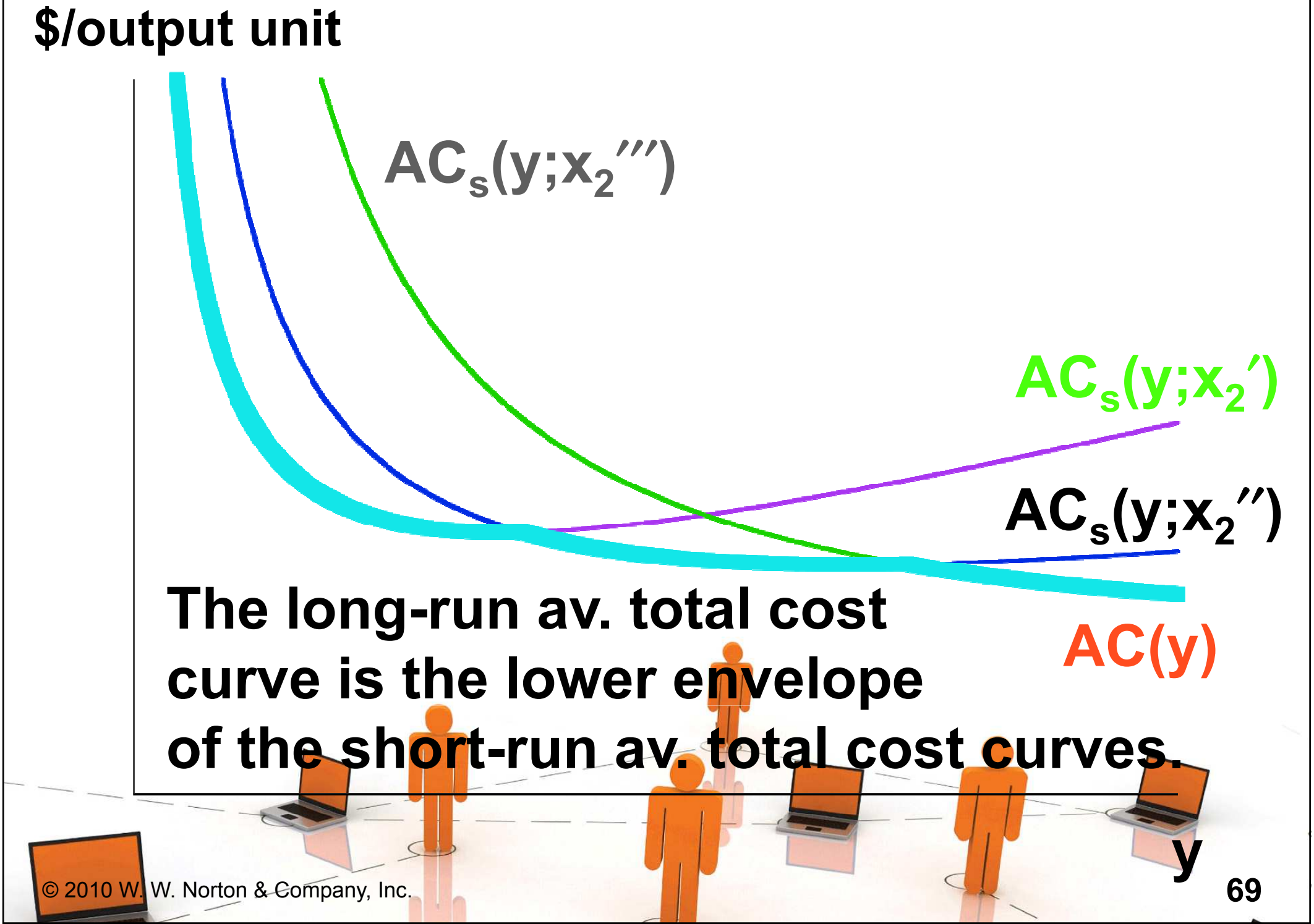
  or $x_2 = x_2'''$

  then the firm's three short-run average total cost curves are ...



Short-Run & Long-Run Average Total Cost Curves

- **The firm's long-run average total cost curve is the lower envelope of the short-run average total cost curves ...**



The long-run av. total cost curve is the lower envelope of the short-run av. total cost curves.

Short-Run & Long-Run Marginal Cost Curves

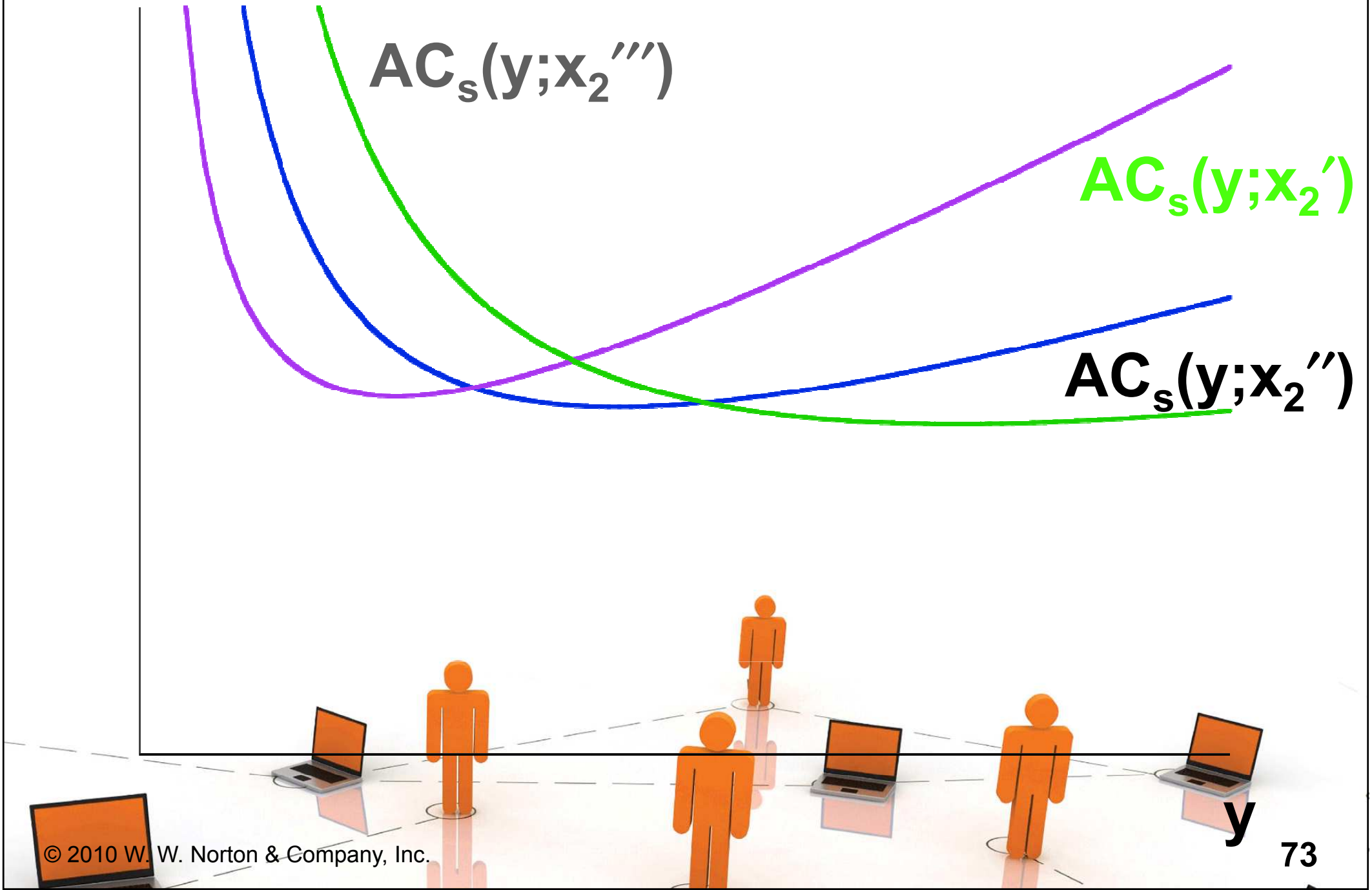
- **Q: Is the long-run marginal cost curve the lower envelope of the firm's short-run marginal cost curves?**

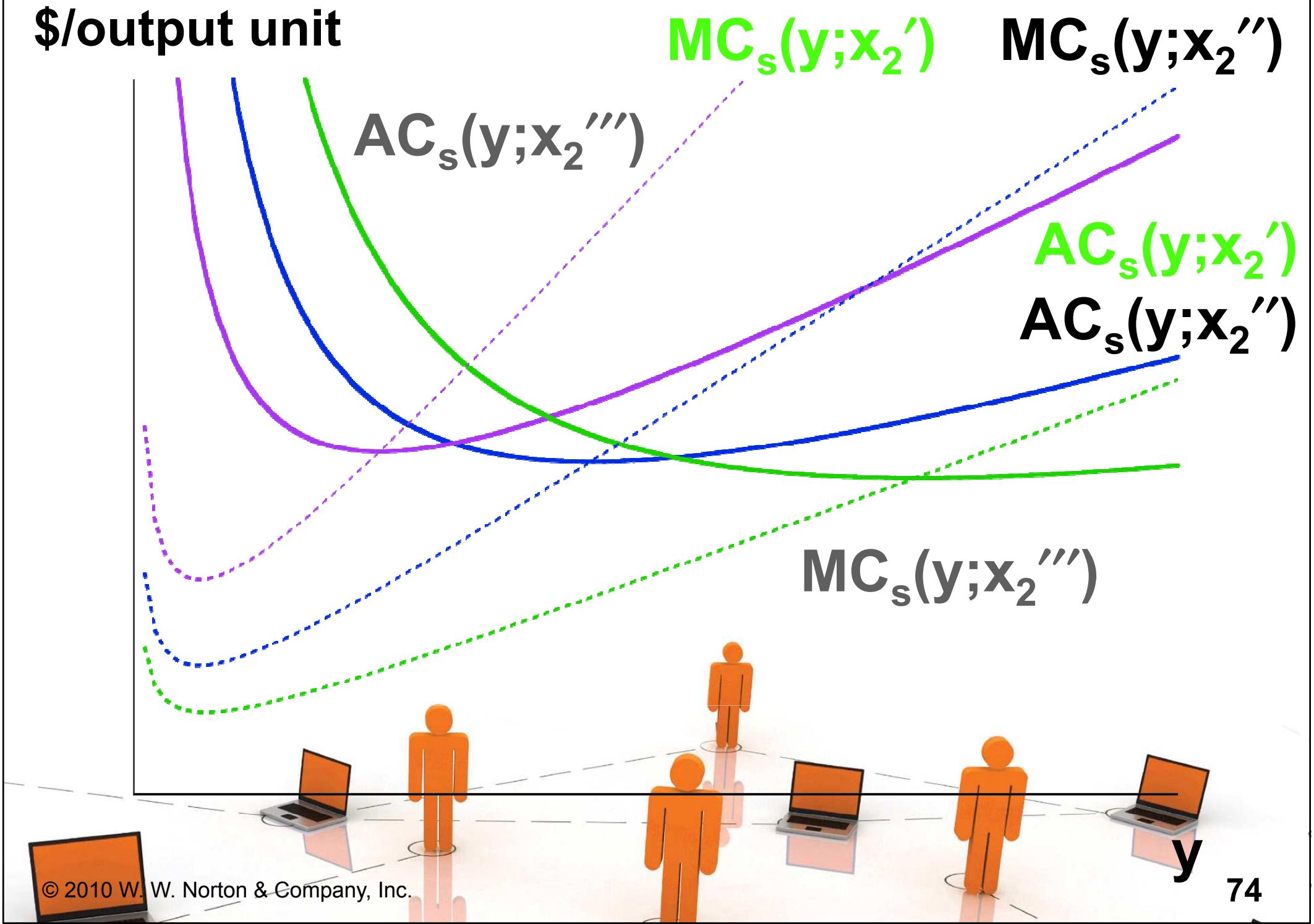
Short-Run & Long-Run Marginal Cost Curves

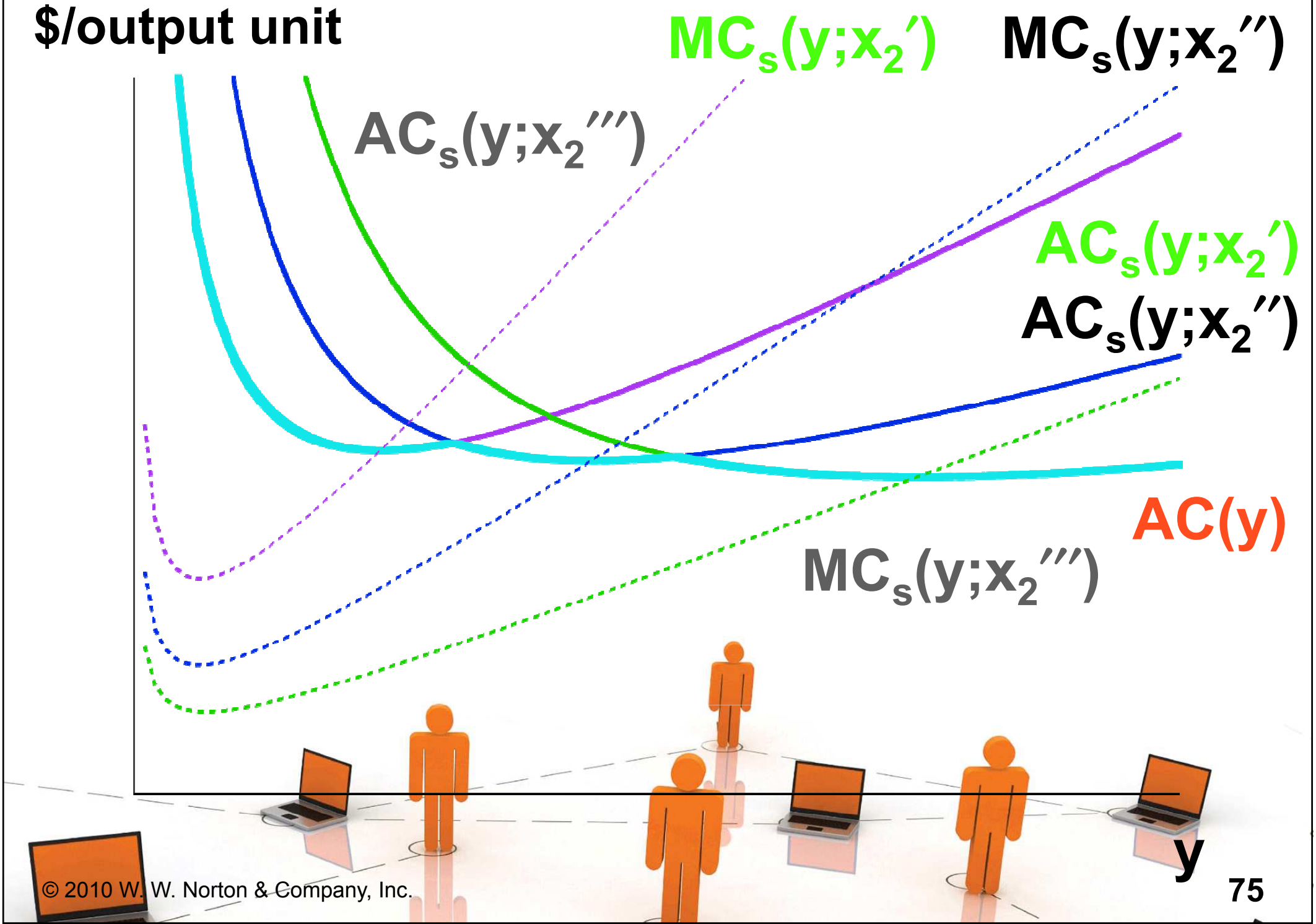
- **Q: Is the long-run marginal cost curve the lower envelope of the firm's short-run marginal cost curves?**
- **A: No.**

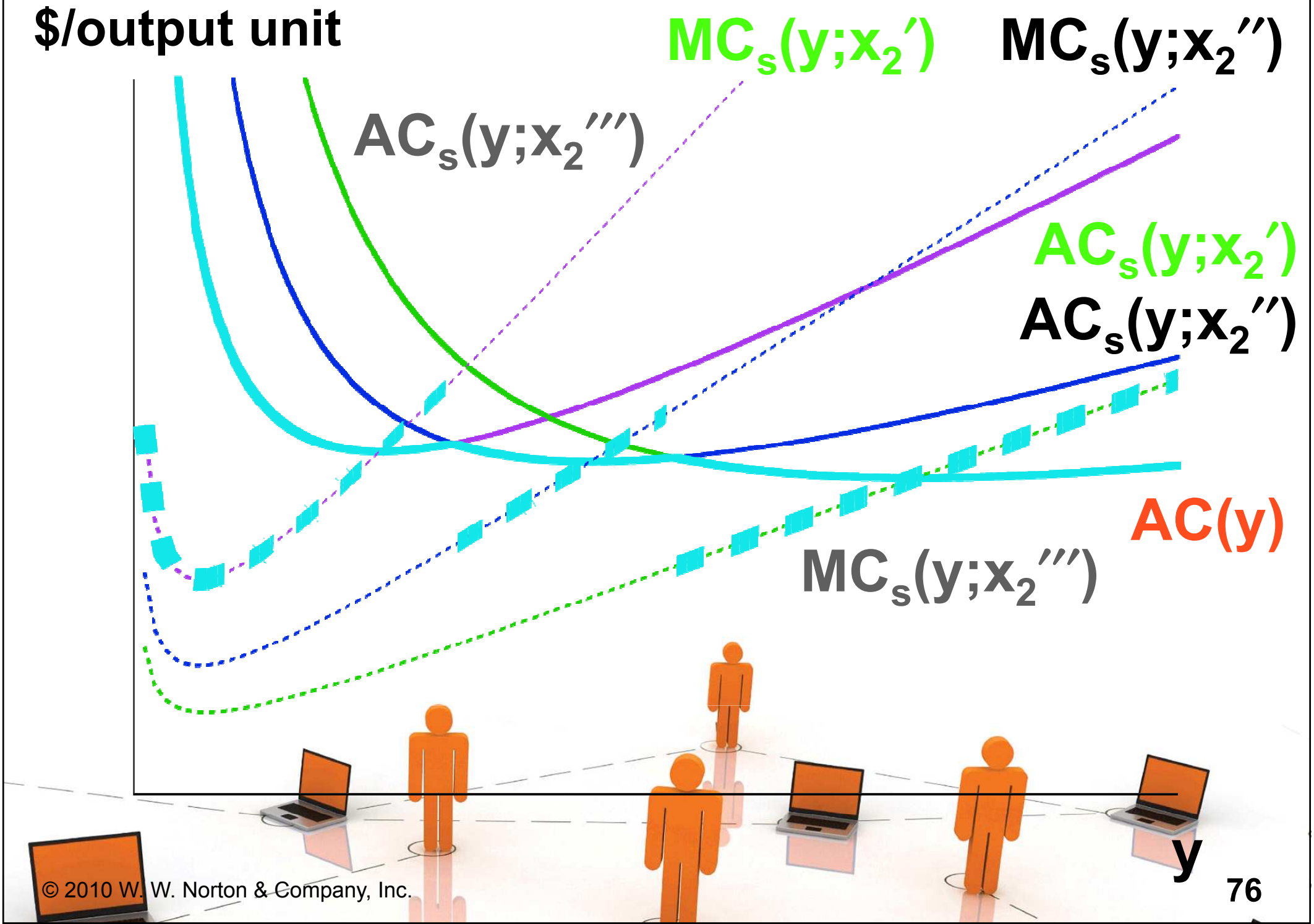
Short-Run & Long-Run Marginal Cost Curves

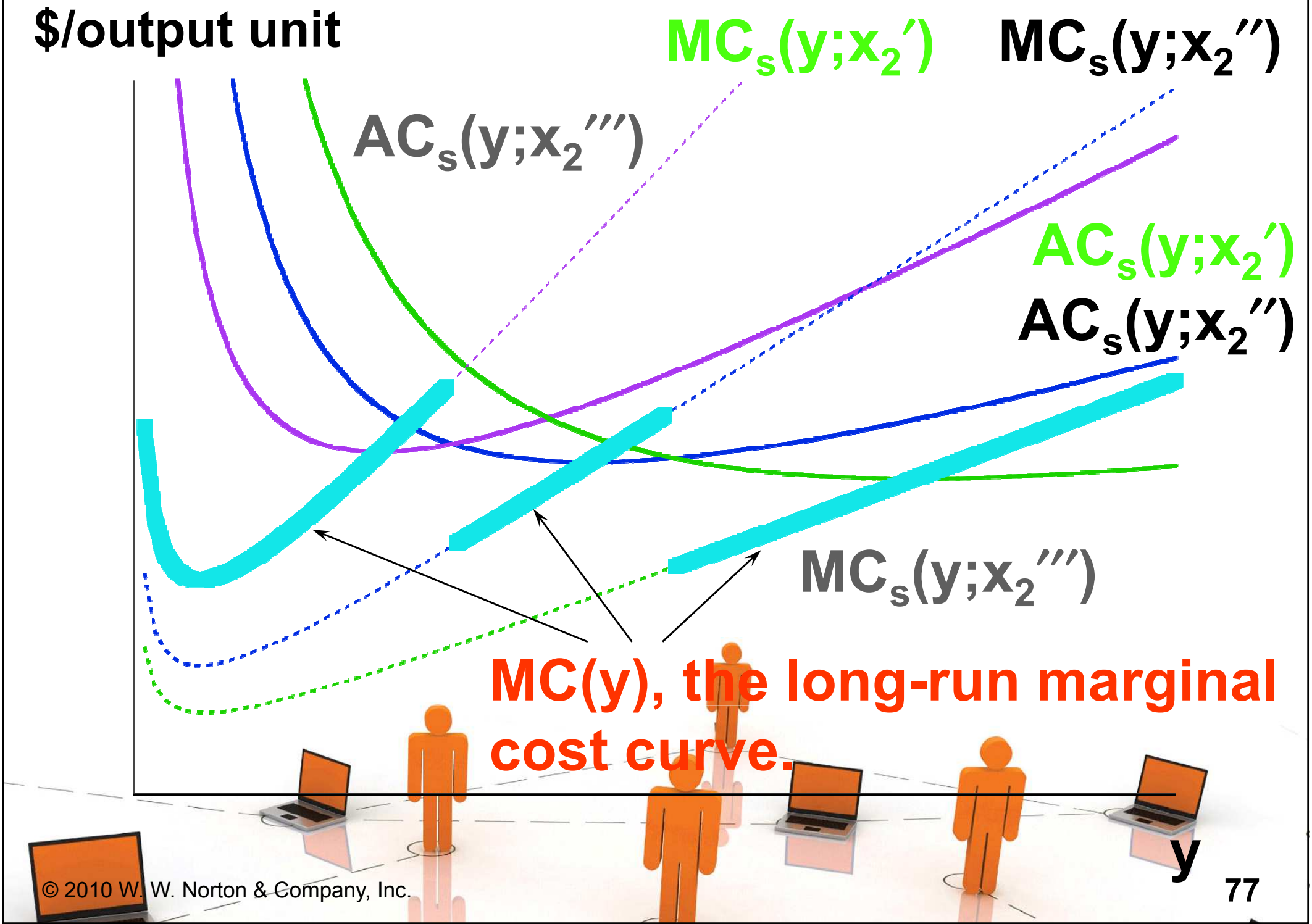
- **The firm's three short-run average total cost curves are ...**





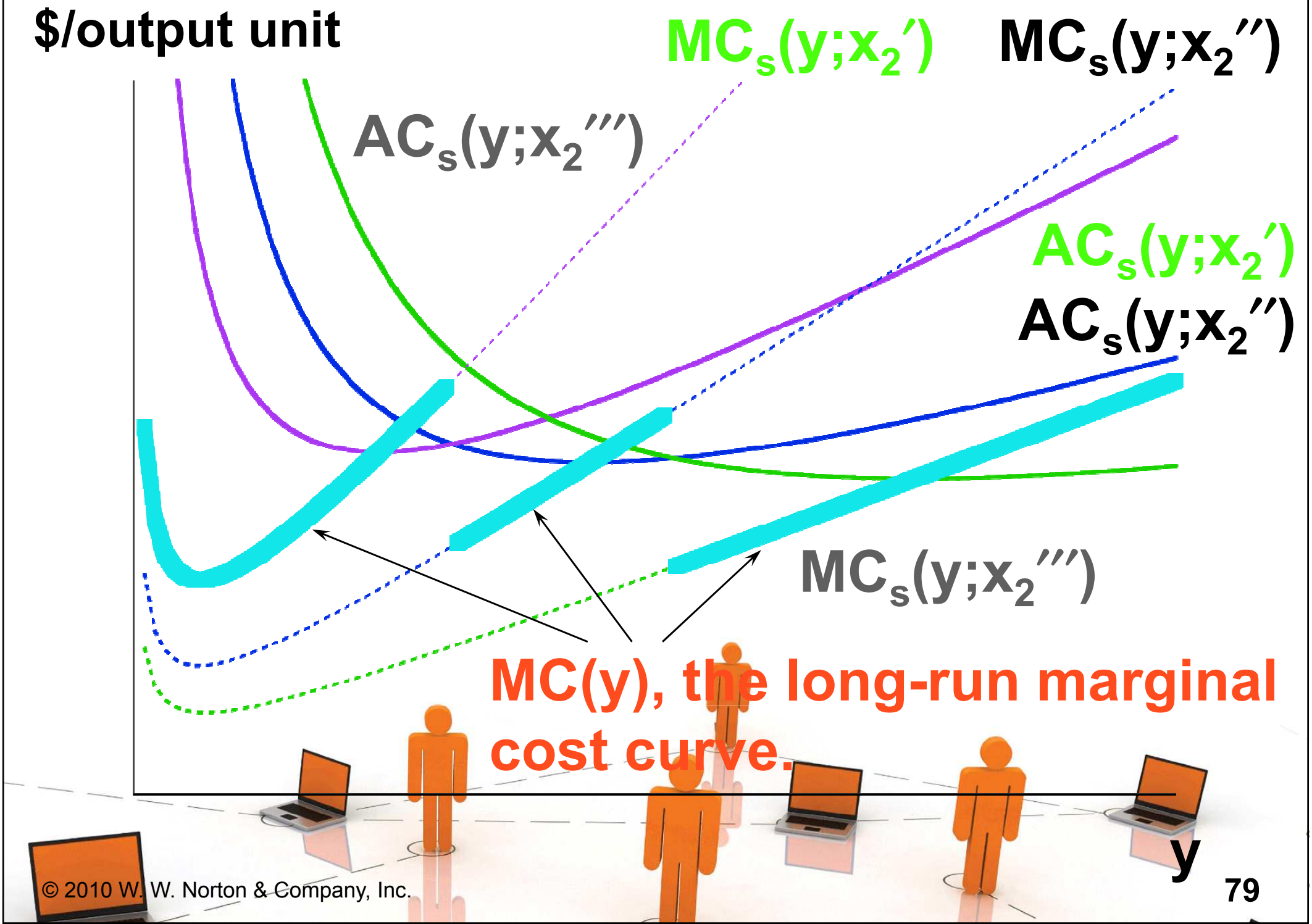






Short-Run & Long-Run Marginal Cost Curves

- **For any output level y > 0, the long-run marginal cost of production is the marginal cost of production for the short-run chosen by the firm.**



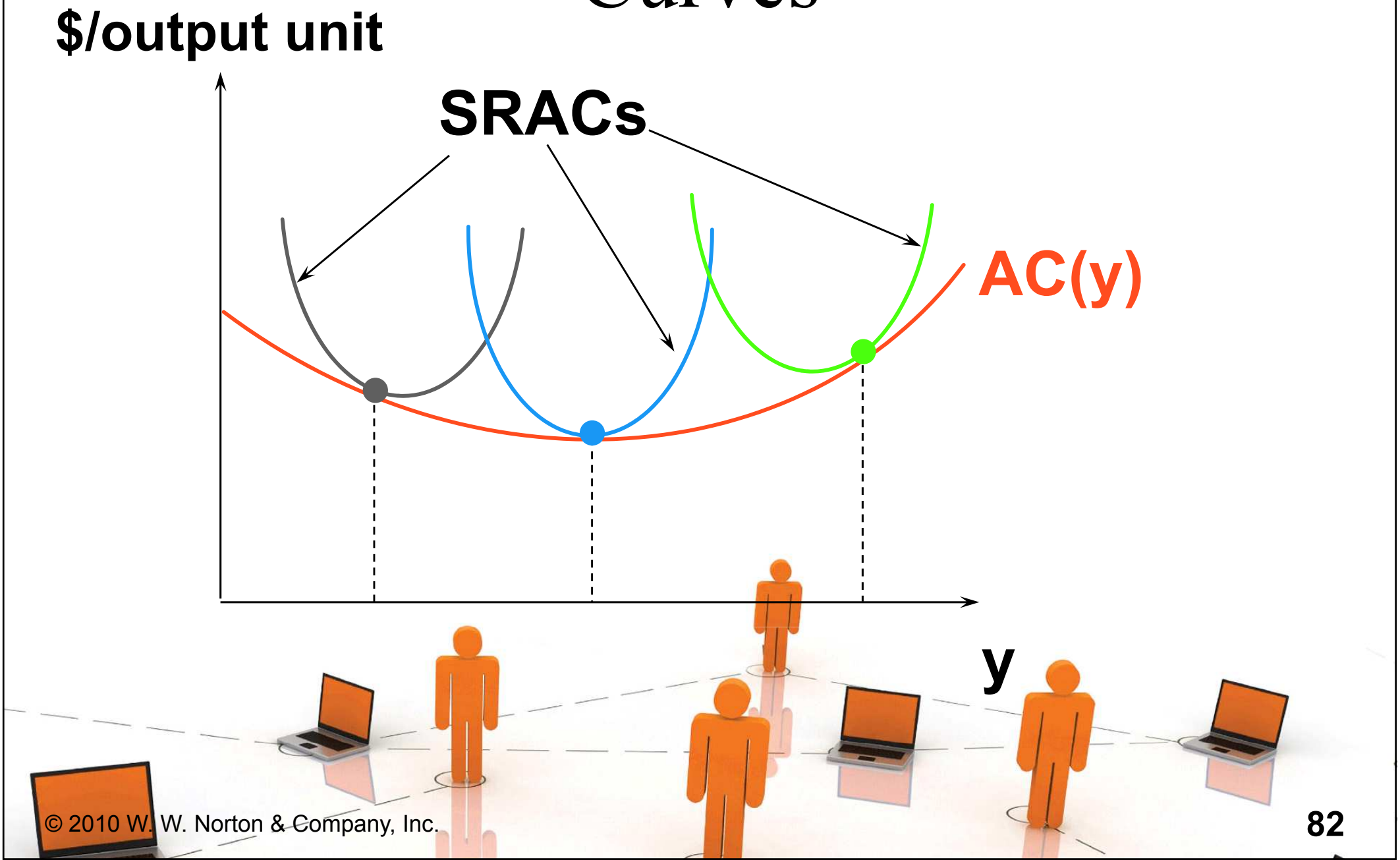
Short-Run & Long-Run Marginal Cost Curves

- **For any output level y > 0, the long-run marginal cost is the marginal cost for the short-run chosen by the firm.**
- **This is always true, no matter how many and which short-run circumstances exist for the firm.**

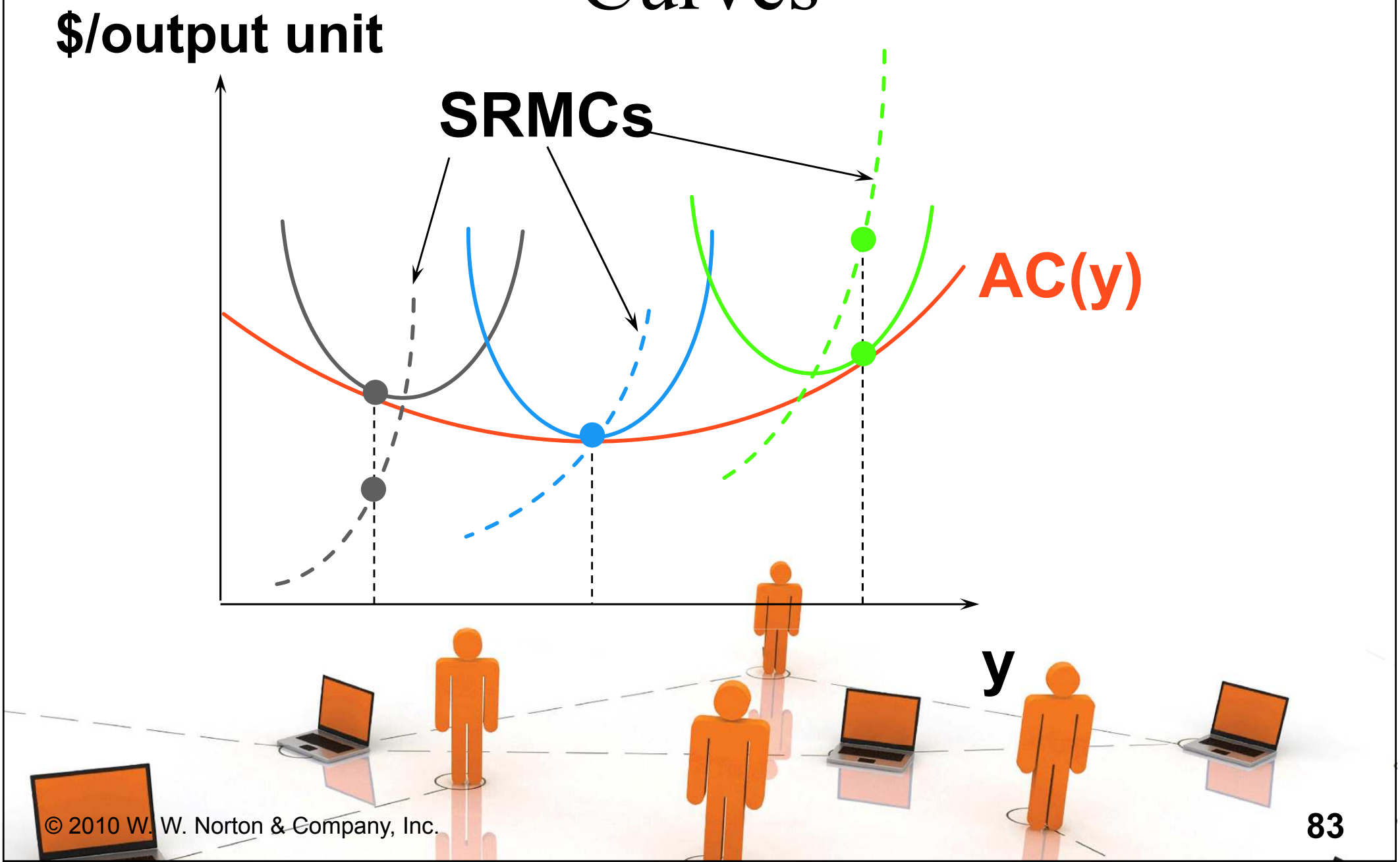
Short-Run & Long-Run Marginal Cost Curves

- **For any output level y > 0, the long-run marginal cost is the marginal cost for the short-run chosen by the firm.**
- **So for the continuous case, where x_2 can be fixed at any value of zero or more, the relationship between the long-run marginal cost and all of the short-run marginal costs is …**

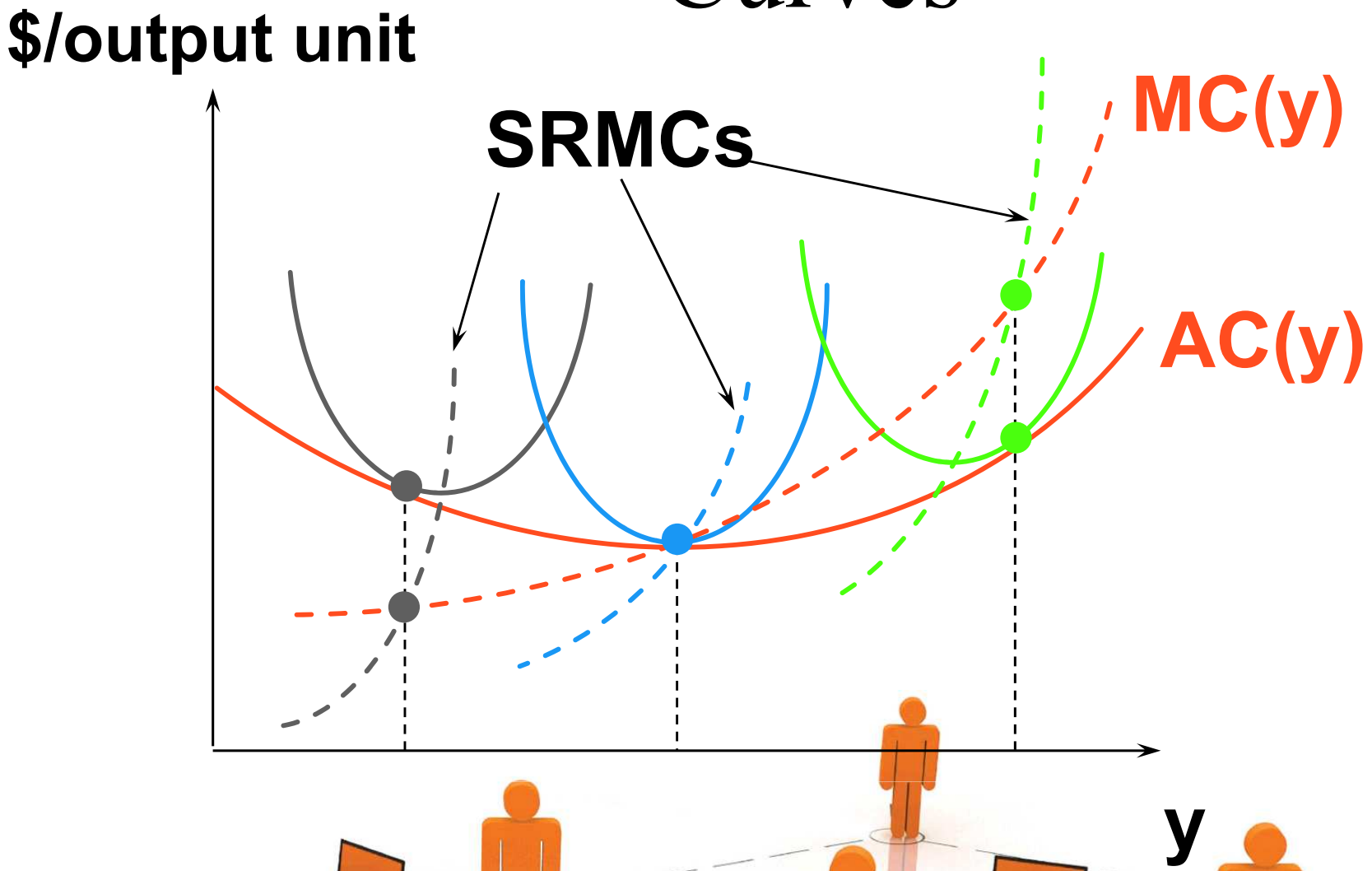
Short-Run & Long-Run Marginal Cost Curves



Short-Run & Long-Run Marginal Cost Curves



Short-Run & Long-Run Marginal Cost Curves



◆For each y > 0, the long-run MC equals the MC for the short-run chosen by the firm.